SKY LOCALIZATION BY BAYESTAR

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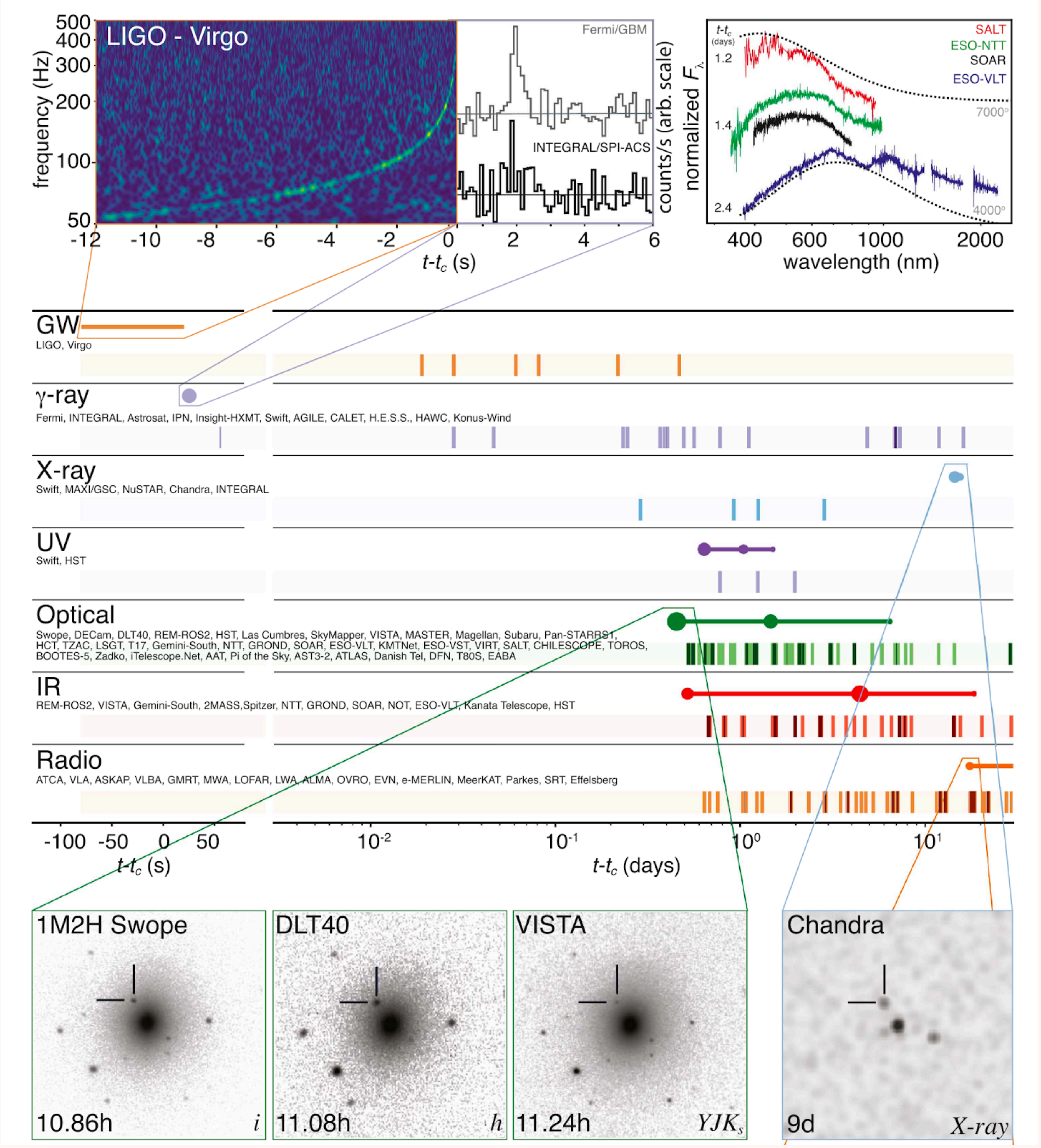
CONTENTS

WHY IS SKY LOCALIZATION IMPORTANT

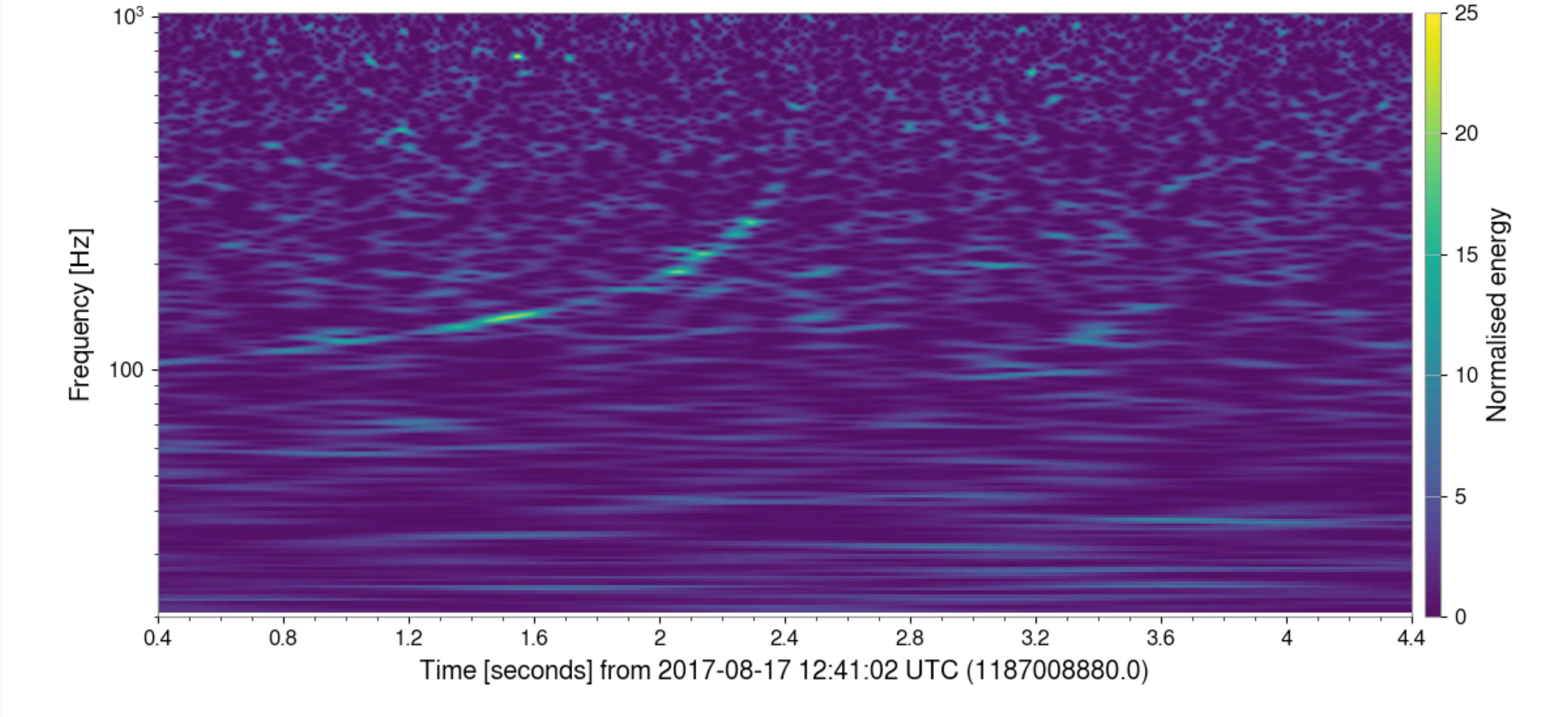
- The early alert from the GW detector can provide EM counterpart information to the optic telescope.
- The follow-up gamma-ray burst was detected 2 seconds after the merger.
- Fast parameter estimation that provides the sky location for the optic telescope to point at is required for multi-messenger astrophysics(MMA) study



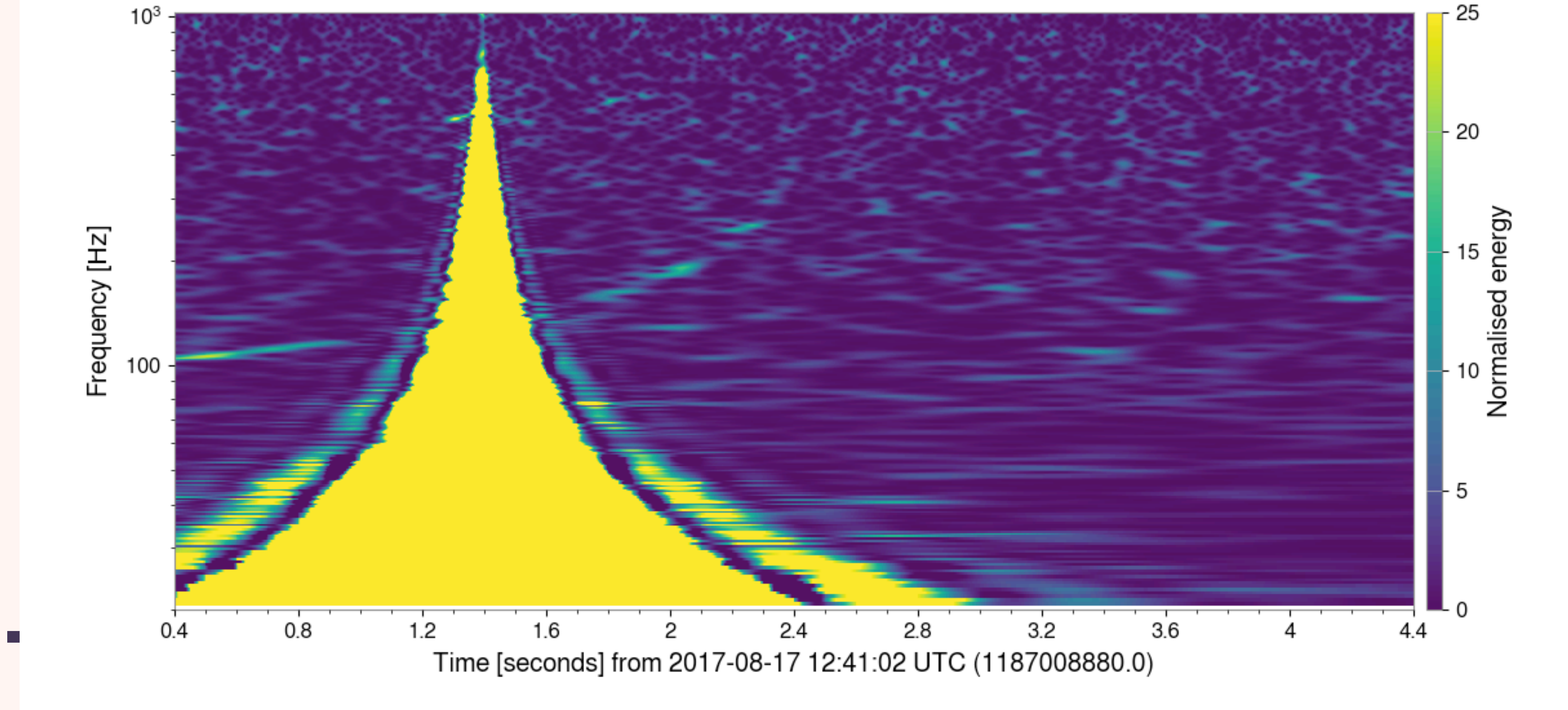
https://arxiv.org/abs/1710.05833

DETECTION OF GW170817

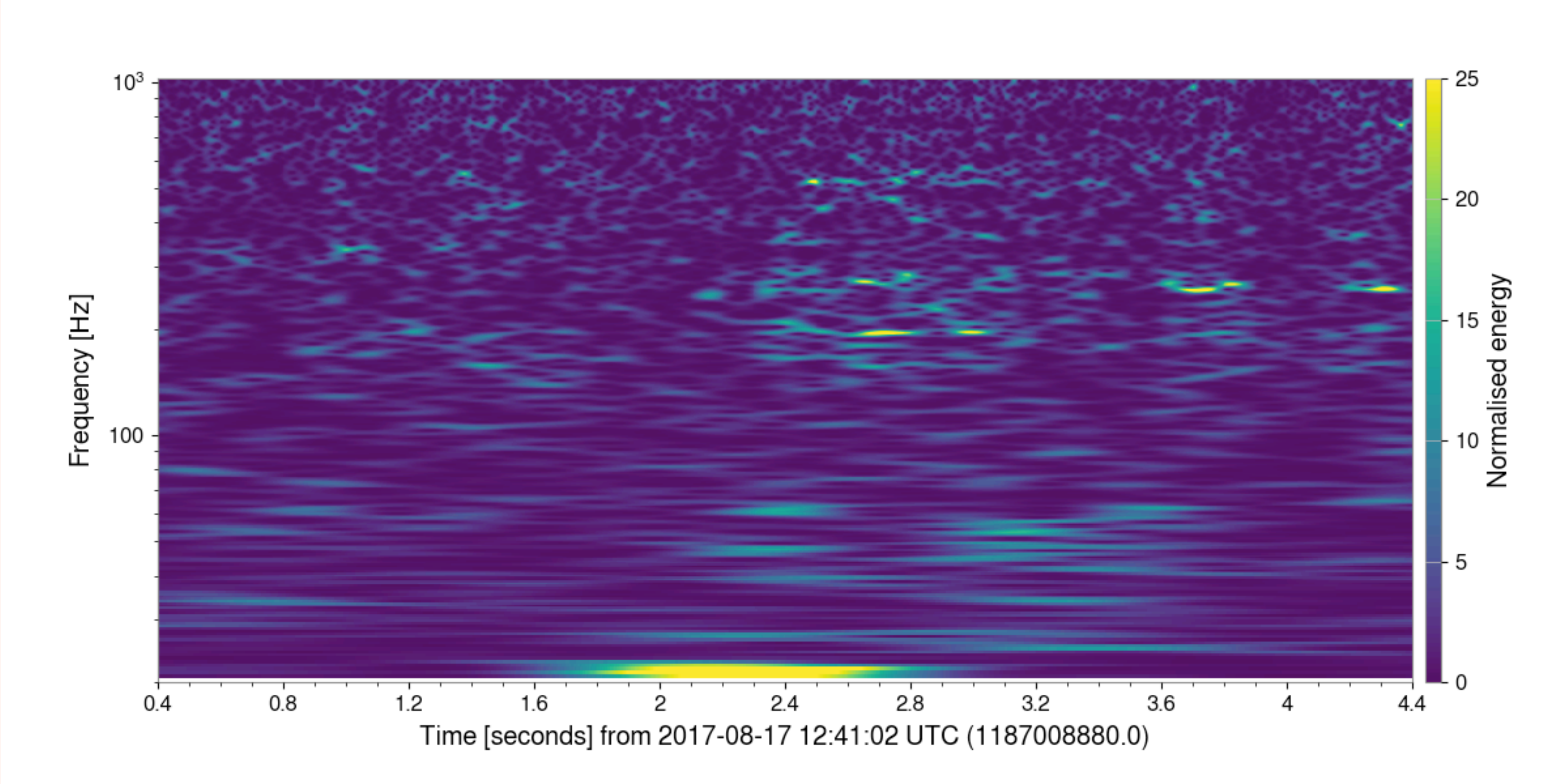
Hanford



Livinston



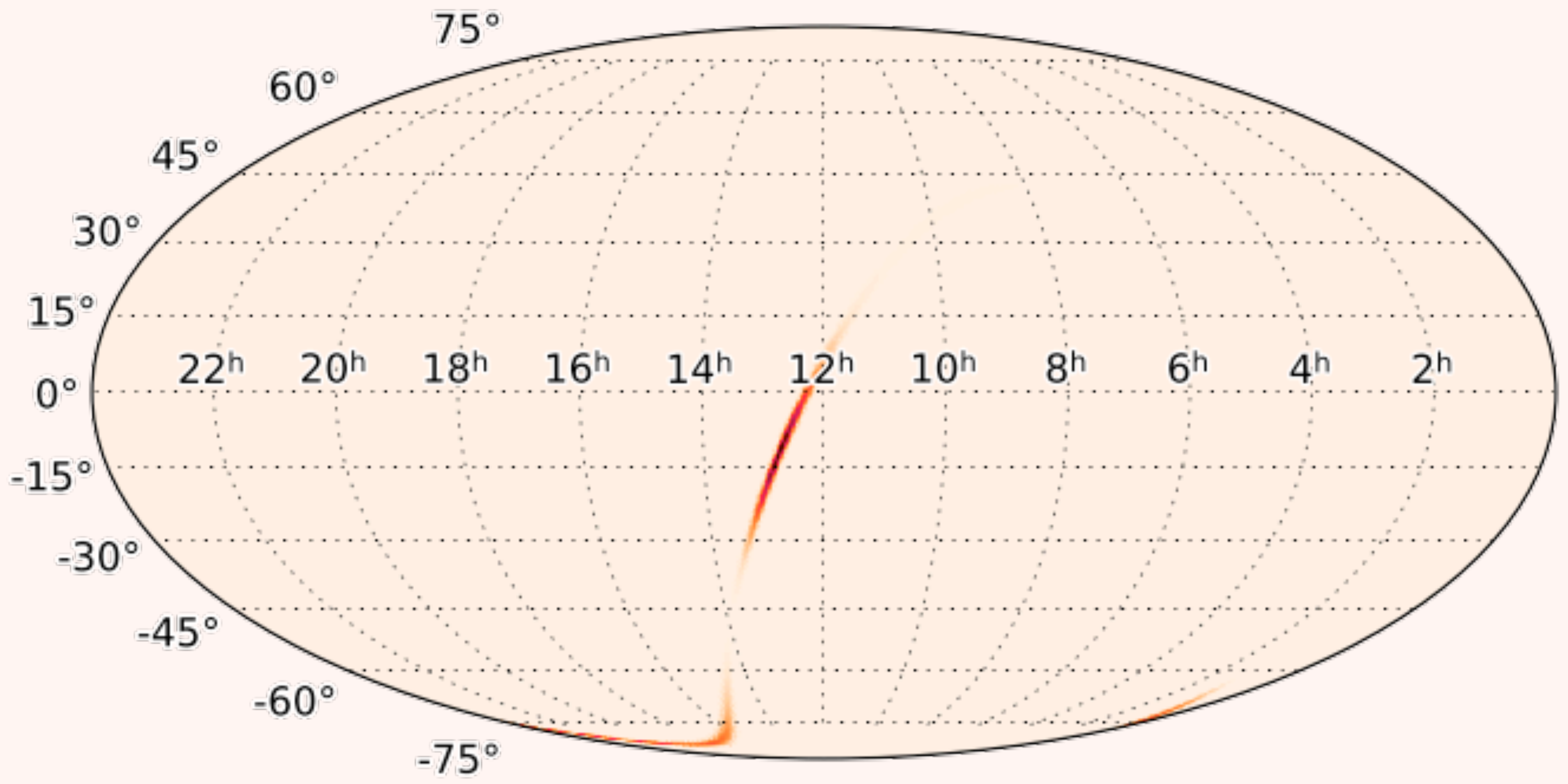
Virgo



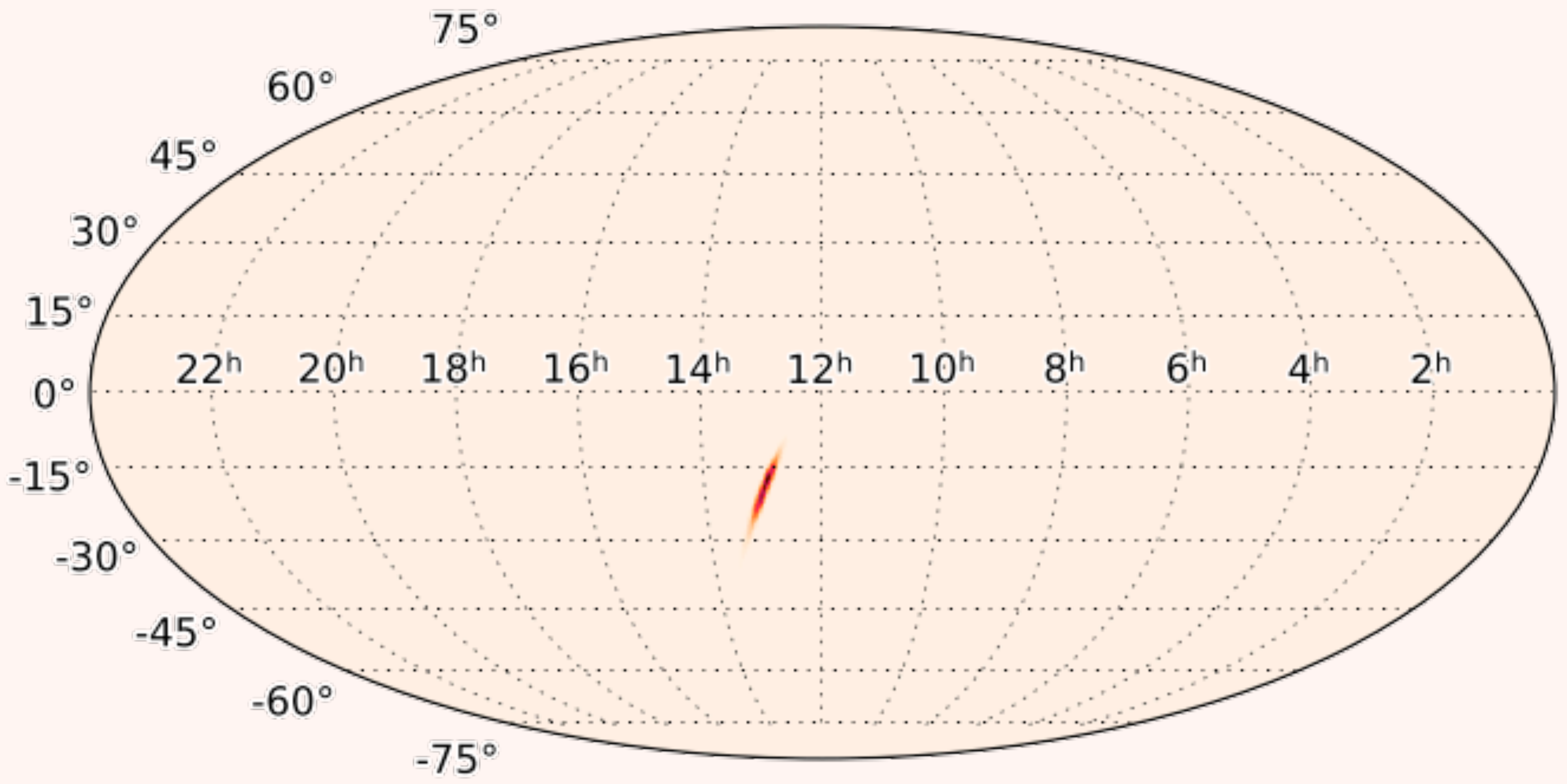
https://gwosc.org/eventapi/html/GWTC-1-confident/GW170817/v3/

SKY LOCATION WITH & WITHOUT VIRGO

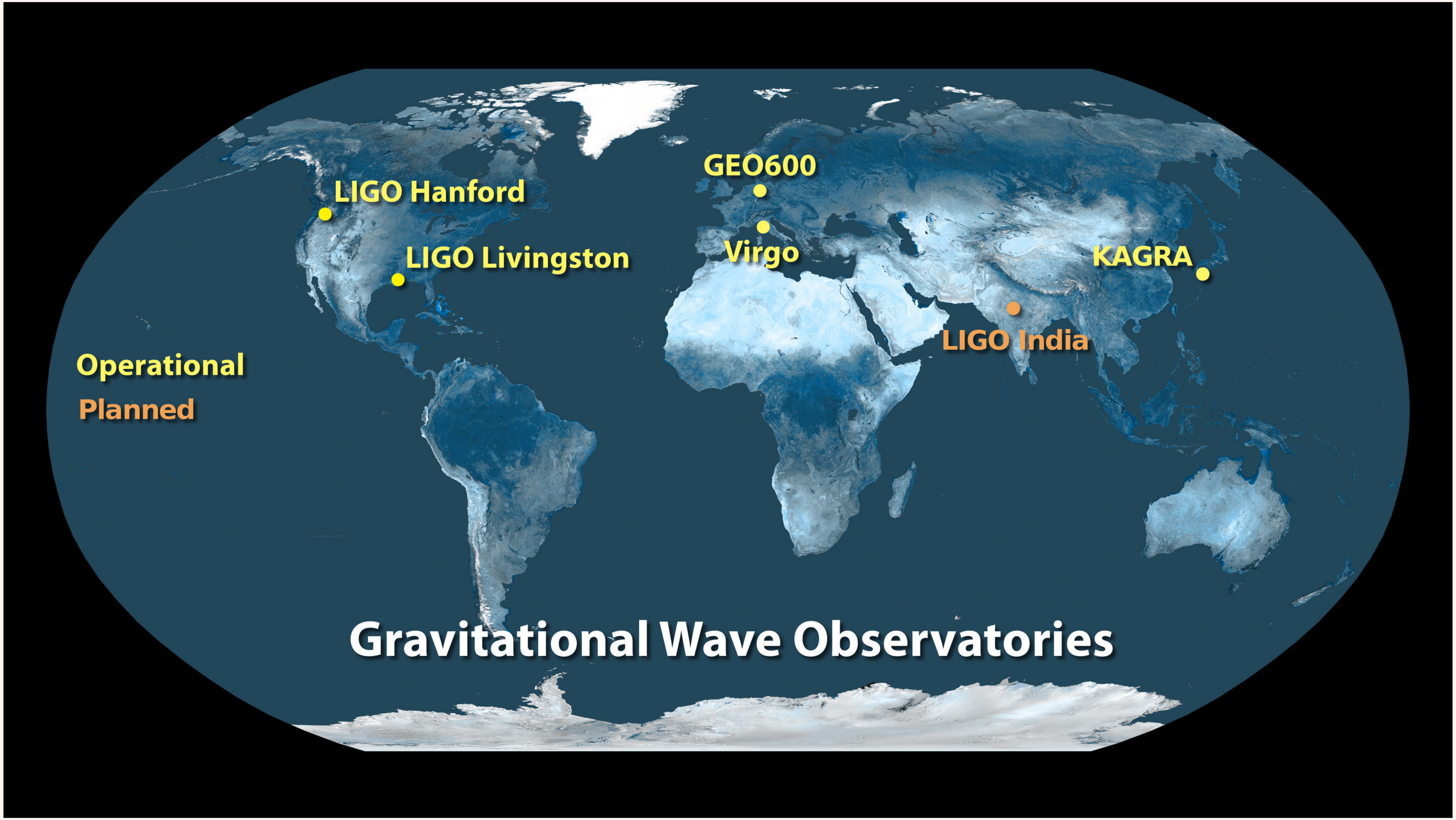
Skymap without Virgo data



Skymap with Virgo data

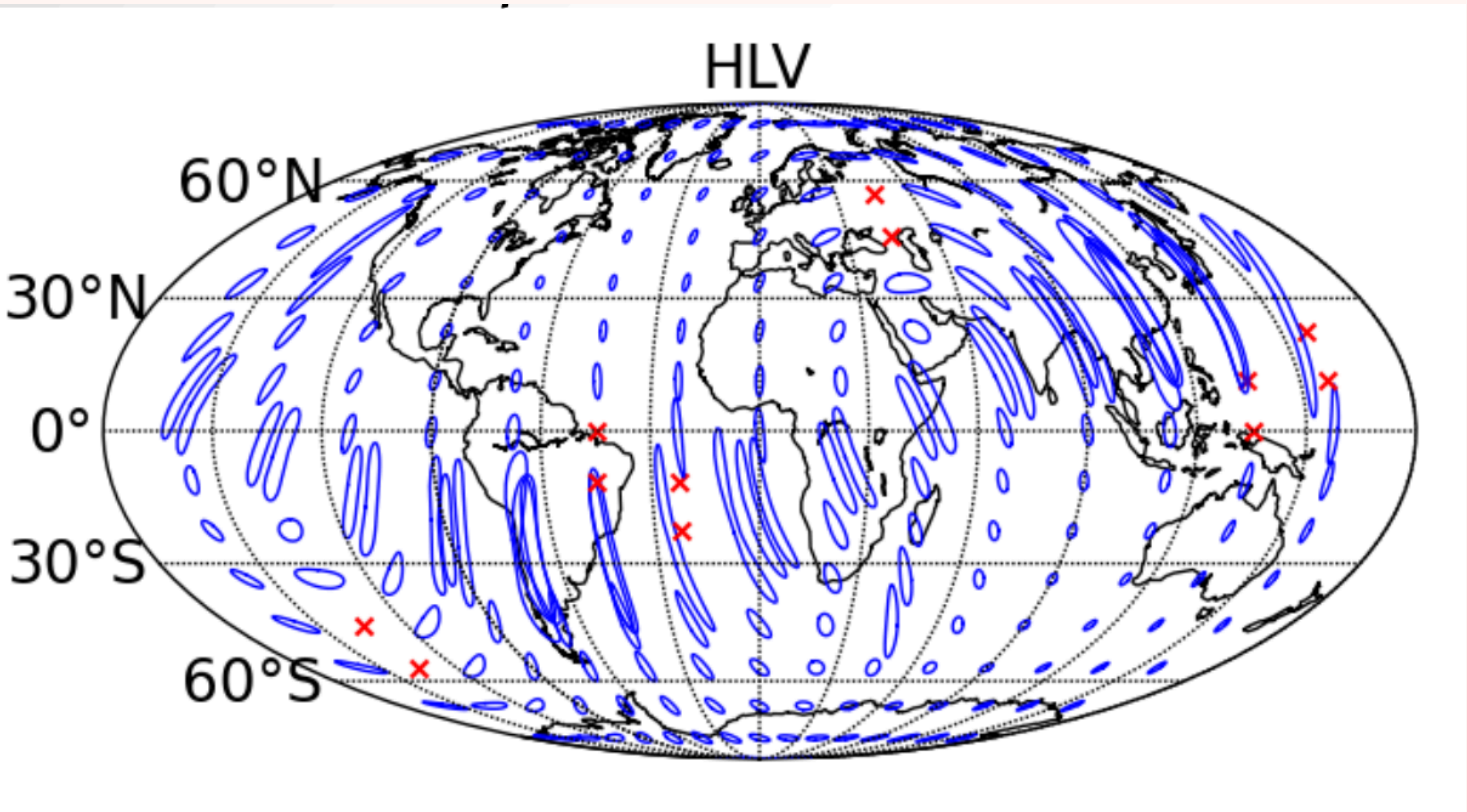


https://dcc.ligo.org/LIGO-G1701985/public

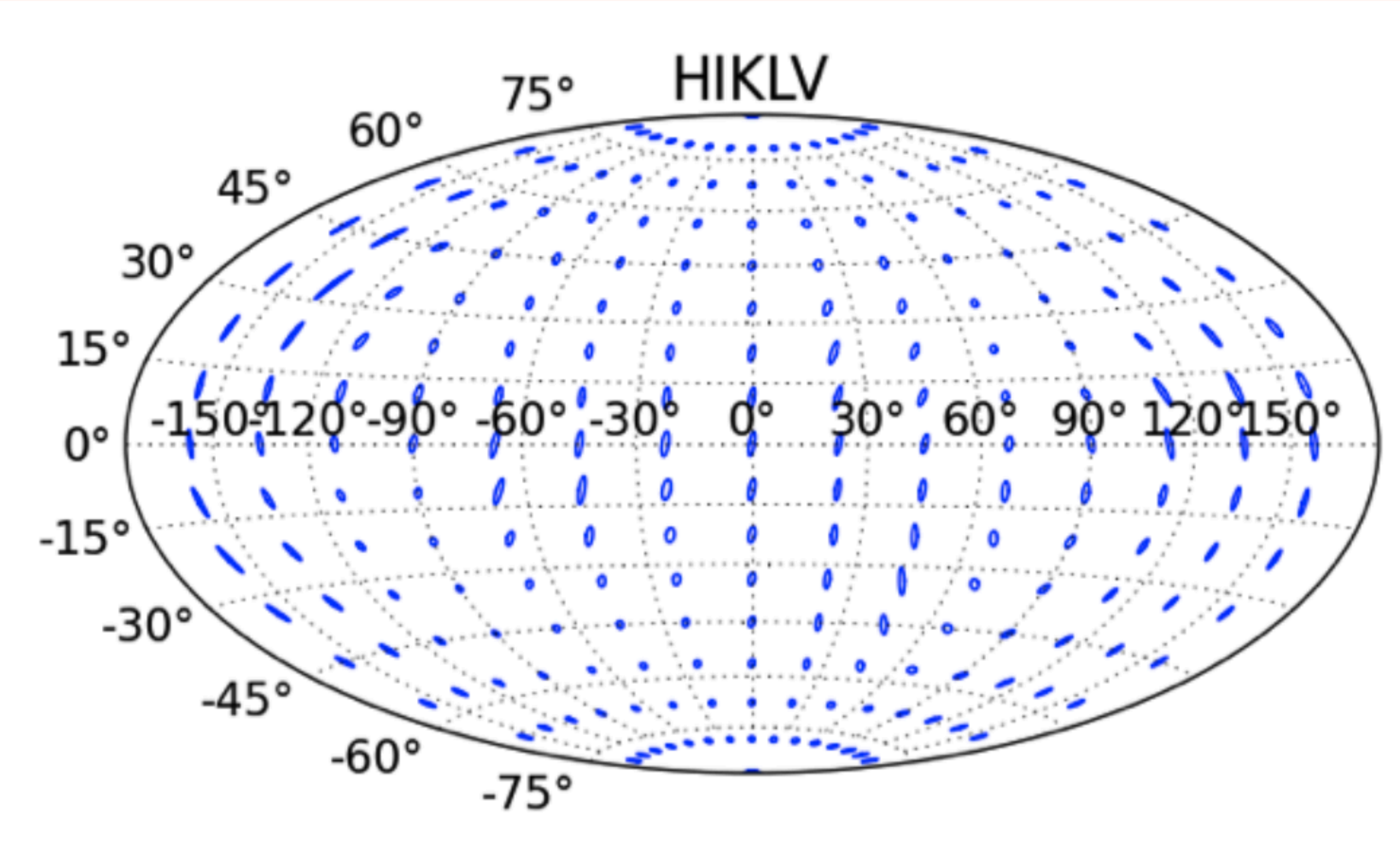

GEO600
LIGO Hanford
Virgo
LIGO Livingston
KAGRA
LIGO India
Operational
Planned
Gravitational Wave Observatories

RESOLUTIONS OF SKYMAP

LIGO-Virgo joint observation



LIGO-Virgo-KAGRA joint observation



https://indico.in2p3.fr/event/16310/contributions/60642/attachments/48045/60588/FuturGW_SVOM2018_Leroy.pdf

TO ARCHIVE MMA

- Loud enough gravitational wave with EM follow up
- Coincident detection across multiple detectors
- Fast and accurate sky localization parameter estimation

BAYESTAR

https://arxiv.org/abs/1508.03634

BAYES' THEOREM

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

- $Posterior : P(A \mid B)$
- $Likelihood : P(B \mid A)$
- $Prior : P(A)$
- $Evidence : P(B)$
- $A = \boldsymbol{\theta}; B = \mathbf{Y}$

- **The ultimate goal of parameter estimation is to find a set of parameters that maximize the likelihood $\mathscr{L}(\mathbf{Y};\boldsymbol{\theta})$ given the prior distribution**

$$\mathscr{L}(\mathbf{Y};\boldsymbol{\theta}) = \prod_i p\left(Y_i | \boldsymbol{\theta}\right), \quad i : detector$$

LIKELIHOOD FUNCTION

Time series strain data of the ith detector can express as $y_i = x_i(t;\boldsymbol{\theta}) + n_i(t)$

And its Fourier transform can express as $Y_i(\omega) = \int_{-\infty}^{\infty} y(t)e^{-i\omega t}dt = X_i(\omega;\theta) + N_i(\omega)$

The likelihood can express as:

$$\mathscr{L}(\mathbf{Y};\boldsymbol{\theta}) = \prod_i p\left(Y_i|\boldsymbol{\theta}\right), \quad i : detector\,;$$

$$p\left(Y_i|\boldsymbol{\theta}\right) \propto exp[-\frac{1}{2}\sum_i \int_0^{\infty} \frac{|Y_i(\omega) - X_i(\omega;\boldsymbol{\theta})|^2}{S_i(\omega)} d\omega]$$

With $X(\omega;\boldsymbol{\theta})$ as the projected hypothesis waveform

WAVEFORM PARAMETERS θ

$Extrinsic\ parameters : \boldsymbol{\theta_{ex}}$

α : right ascension

δ : declination

r : distance

t_a : arrival time at geocenter

ι : inclination angle

ψ : polarization angle

ϕ_c : coalescence phase

$Intrinisic\ parameters : \boldsymbol{\theta_{in}}$

m_1 : first component's mass

m_2 : second component's mass

$\boldsymbol{S_1}$: first component's spin

$\boldsymbol{S_2}$: second component's spin

$$H(\omega; \boldsymbol{\theta_{in}})$$

GW WAVEFORMS IN DETECTOR

$$X_i(\omega;\boldsymbol{\theta}) = e^{-i\omega(t_a - \boldsymbol{d}_i\cdot\boldsymbol{n})}\frac{r_{1,i}}{r}e^{2i\phi_c}[\frac{1}{2}(1+cos^2\imath)\mathscr{R}\{\zeta\} - i(cos(\imath))\mathscr{I}\{\zeta\}]H(\omega;\boldsymbol{\theta_{in}})$$

$$\zeta = e^{-2i\psi}(F_{+,i}(\alpha,\delta,t_a) + iF_{\times,i}(\alpha,\delta,t_a))$$

$$With\ the\ antenana\ patten\ as\ F_{+,i}, F_{-,i}$$

GW WAVEFORMS IN DETECTOR

$$X_i(\omega;\boldsymbol{\theta}) = e^{-i\omega(t_a-\boldsymbol{d}_i\cdot\boldsymbol{n})}\frac{r_{1,i}}{r}e^{2i\phi_c}[\frac{1}{2}(1+cos^2\iota)\mathscr{R}\{\zeta\} - i(cos(\iota))\mathscr{I}\{\zeta\}]H(\omega;\boldsymbol{\theta_{in}})$$

$$r_{1,i} = \frac{1}{\sigma_i}$$

$$\sigma_i^2 = \int_0^\infty \frac{|H(\omega;\boldsymbol{\theta_{in}})|^2}{S_i(\omega)}d\omega$$

ρ_i : *amplitude*

γ_i : *phase*

τ_i : *time delay*

$$X_i(\omega;\boldsymbol{\theta_i},\boldsymbol{\theta_{in}}) = X_i(\omega;\rho_i,\gamma_i,\tau_i,\boldsymbol{\theta_{in}}) = \frac{\rho_i}{\sigma_i}e^{i(\gamma_i-\omega\tau_i)}H(\omega;\boldsymbol{\theta_{in}})$$

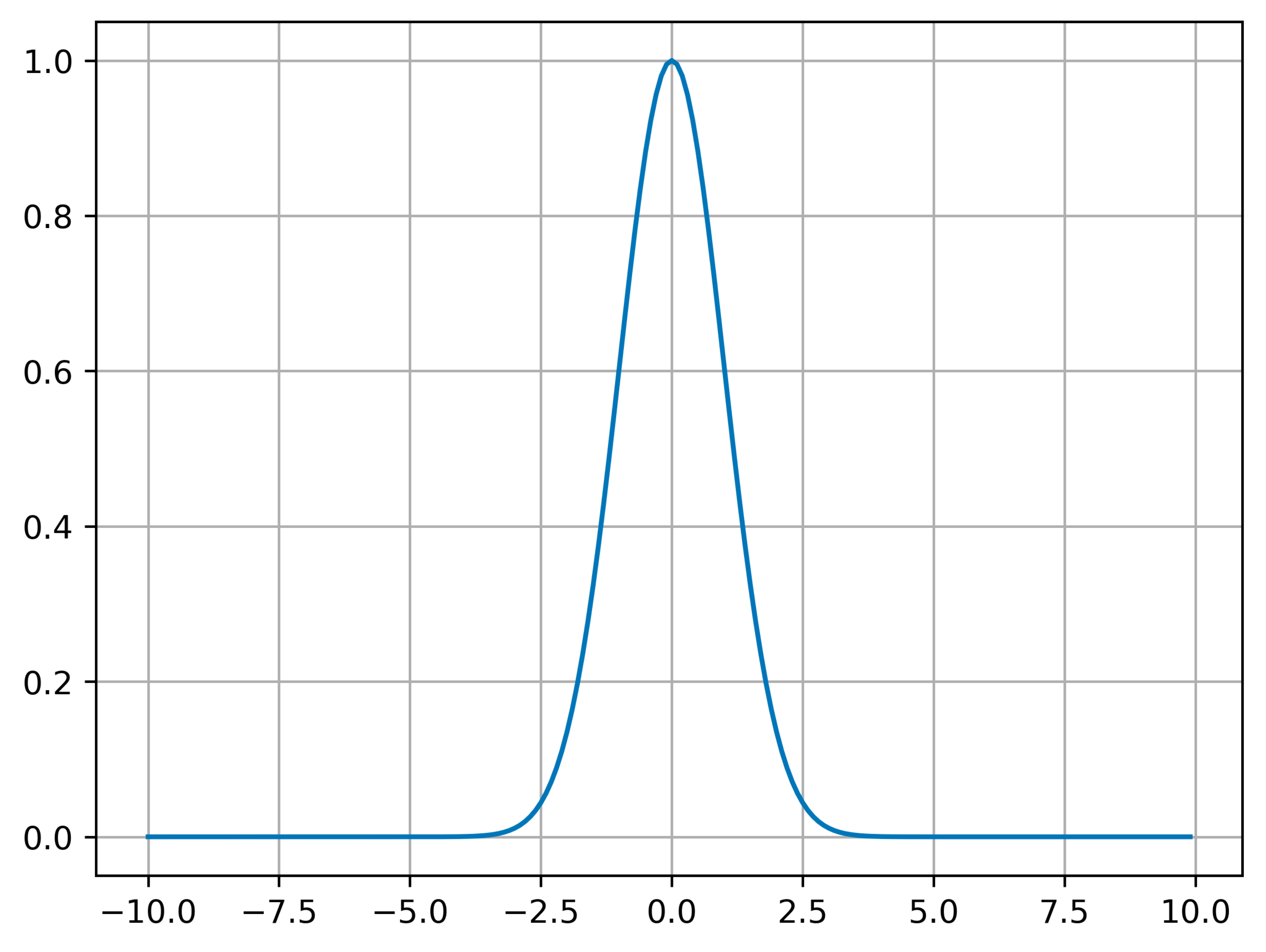
AUTOCORRELATION LIKELIHOOD

Likelihood

$$p\left(Y_i|\boldsymbol{\theta}\right) \propto exp[-\frac{1}{2}\sum_i \int_0^\infty \frac{|Y_i(\omega) - X_i(\omega;\boldsymbol{\theta})|^2}{S_i(\omega)} d\omega]$$

Autocorrelation likelihood

$$p(\hat{\boldsymbol{\theta}}_i|\boldsymbol{\theta}) := p(Y_i(\omega) = X_i(\omega;\hat{\boldsymbol{\theta}}|\boldsymbol{\theta}))$$



AUTOCORRELATION LIKELIHOOD

$$p\left(Y_i|\boldsymbol{\theta}\right) \propto exp[-\frac{1}{2}\sum_i \int_0^\infty \frac{|Y_i(\omega) - X_i(\omega;\boldsymbol{\theta})|^2}{S_i(\omega)} d\omega]$$

New likelihood to estimate:

$$p(\hat{\boldsymbol{\theta}}_{\boldsymbol{i}}|\boldsymbol{\theta}) := p(Y_i(\omega) = X_i(\omega;\hat{\boldsymbol{\theta}}|\boldsymbol{\theta}))$$

$$\propto exp[-\frac{1}{2}\int_0^\infty |\frac{\hat{\rho}_i}{\sigma_i(\hat{\boldsymbol{\theta}}_{\boldsymbol{in}})} e^{i(\hat{\gamma}_i-\omega\hat{\tau}_i)}\frac{H(\omega;\hat{\boldsymbol{\theta}}_{in})}{S_i(\omega)} - \frac{\rho_i}{\sigma_i(\boldsymbol{\theta}_{\boldsymbol{in}})} e^{i(\gamma_i-\omega\tau_i)}\frac{H(\omega;\boldsymbol{\theta}_{\boldsymbol{in}})}{S_i(\omega)}|^2 d\omega]$$

AUTOCORRELATION LIKELIHOOD

We assume that $\boldsymbol{\theta}_{in} = \hat{\boldsymbol{\theta}}_{in}$

$$p(\hat{\rho}_i, \hat{\gamma}_i, \hat{\tau}_i \,|\, \rho_i, \gamma_i, \tau_i) \propto exp[-\frac{1}{2}\hat{\rho}_i{}^2 - \frac{1}{2}\rho_i^2 + \hat{\rho}_i\rho_i\mathscr{R}\{e^{i\tilde{\gamma}_i}a_i^*(\tilde{\tau}_i)\}]$$

$\tilde{\gamma}_i = \hat{\gamma}_i - \gamma_i, \quad \tilde{\tau}_i = \hat{\tau}_i - \tau_i$ are the errors produced when making the assumption

$a_i(t; \boldsymbol{\theta}_{in}) := \dfrac{1}{\sigma_i^2(\boldsymbol{\theta}_{in})} \displaystyle\int_0^\infty \dfrac{|H(\omega; \hat{\boldsymbol{\theta}}_{in})|^2}{S_i(\omega)} e^{i\omega t} d\omega$ is the autocorrection function

MARGINAL POSTERIOR

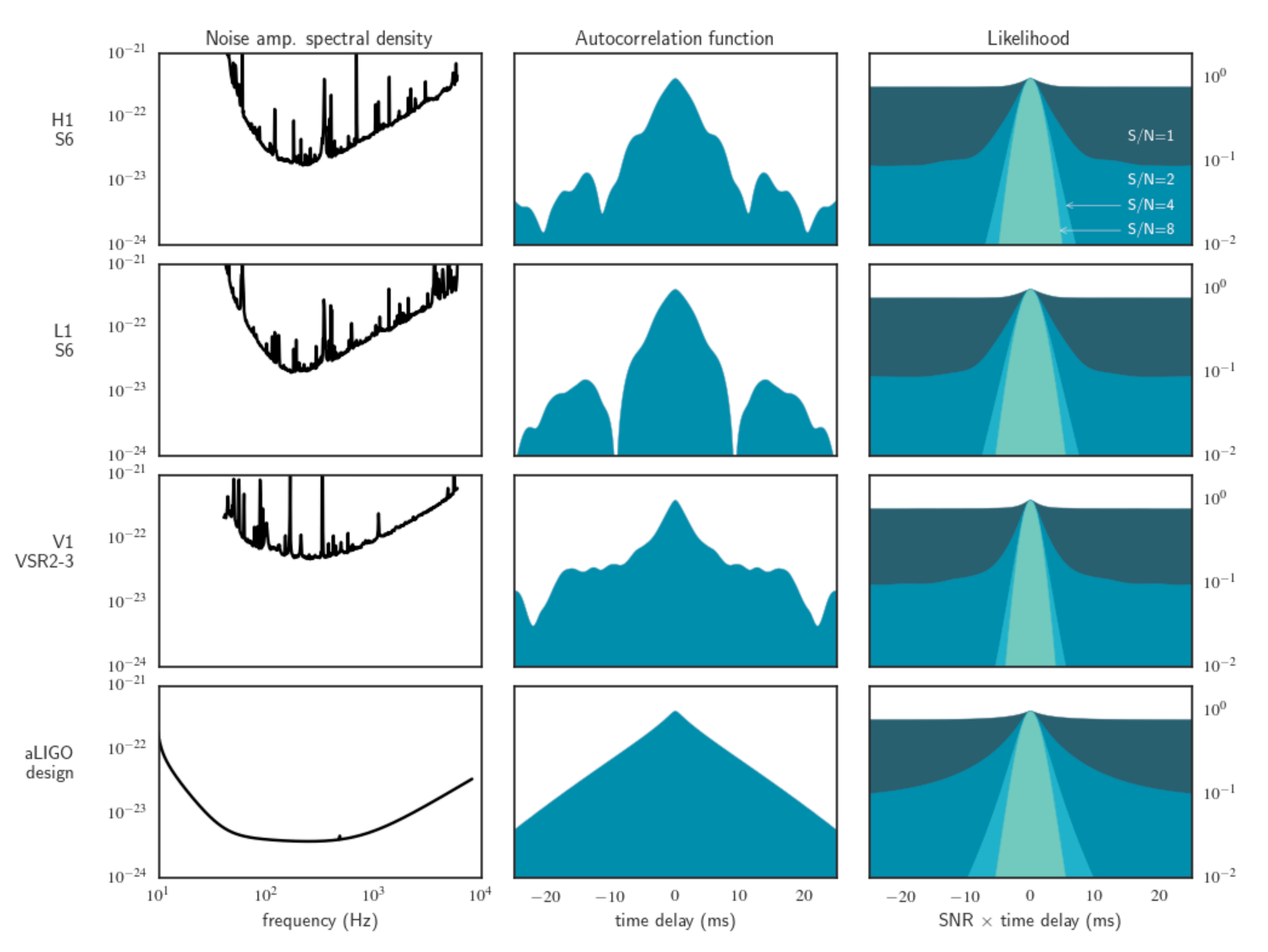
$$p(\hat{\rho}_i, \hat{\gamma}_i, \hat{\tau}_i \,|\, \rho_i, \gamma_i, \tau_i) \propto exp[-\cancel{\frac{1}{2}\sum_i \hat{\rho}_i{}^2} - \frac{1}{2}\sum_i \rho_i^2 + \sum_i \hat{\rho}_i \rho_i \mathscr{R}\{e^{i\tilde{\gamma}_i} a_i^*(\tilde{\tau}_i)\}]$$

Posterior for skymap

$$f(\alpha, \delta) \propto \int_0^{\pi} \int_{-1}^{1} \int_{-T}^{T} \int_{r_{min}}^{r_{max}} \int_0^{1\pi} exp[-\frac{1}{2}\sum_i \rho_i^2 + \sum_i \hat{\rho}_i \rho_i \mathscr{R}\{e^{i\tilde{\gamma}_i} a^*(\tilde{\tau}_i)\}] r^2 \; d\phi_c dr dt_a dcos_\iota d\psi$$

AUTOCORRELATION FUNCTION

$$a_i(t;\boldsymbol{\theta}_{in}) := \frac{1}{\sigma_i^2(\boldsymbol{\theta}_{in})} \int_0^\infty \frac{|H(\omega;\hat{\boldsymbol{\theta}}_{in})|^2}{S_i(\omega)} e^{i\omega t} d\omega$$

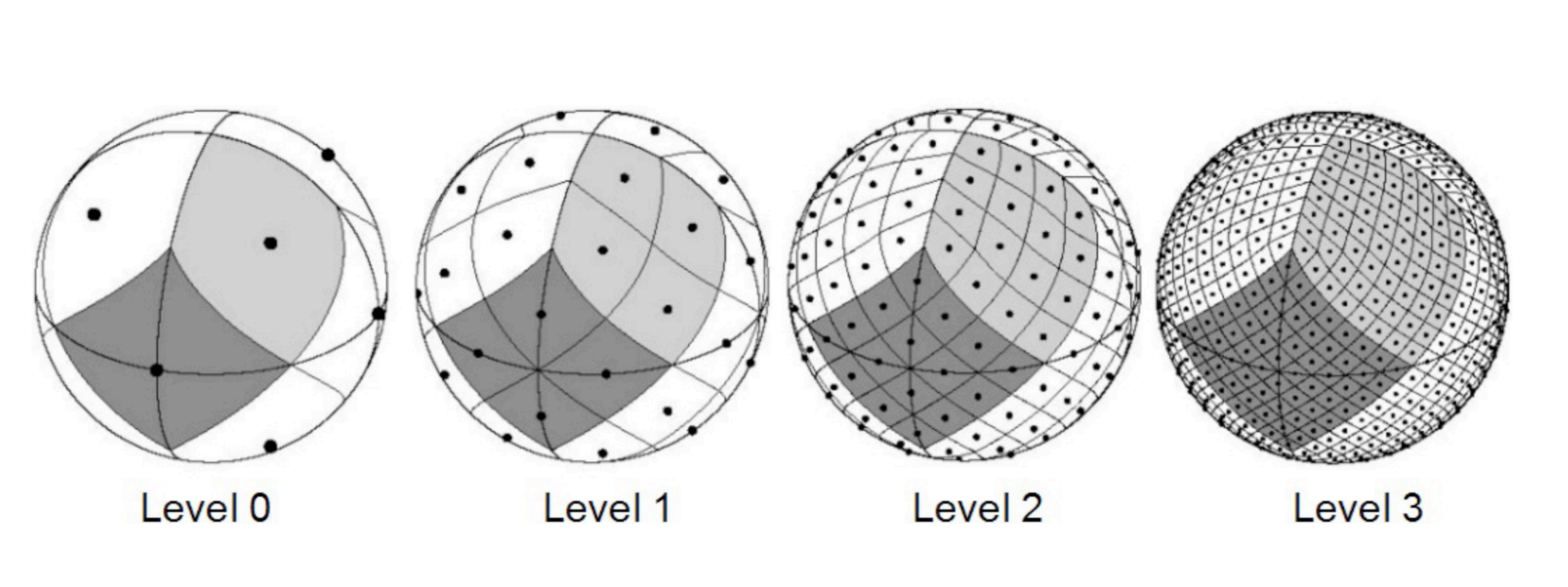


https://arxiv.org/abs/1508.03634

SETTING THE PRIORS

- $\boldsymbol{\theta}_{in} = \hat{\boldsymbol{\theta}}_{\boldsymbol{in}}$
- $Distance\ : r : 0 \sim \frac{1}{4} \max_i r_{1,i}$
- $Inclination\ angle : \iota :\ uniform\ across\ all\ parameter\ space$
- $Polarization\ angle : \psi :\ uniform\ across\ all\ parameter\ space$
- $Coalescence\ phase : \phi_c :\ uniform\ across\ all\ parameter\ space$
- $Arrival\ time\ at\ geocenter : t_a : -20ms \sim 20ms$

HEALPIX



https://www.ivoa.net/documents/MOC/20190215/WD-MOC-1.1-20190215.pdf

ADAPTIVE HEALPIX SAMPLING

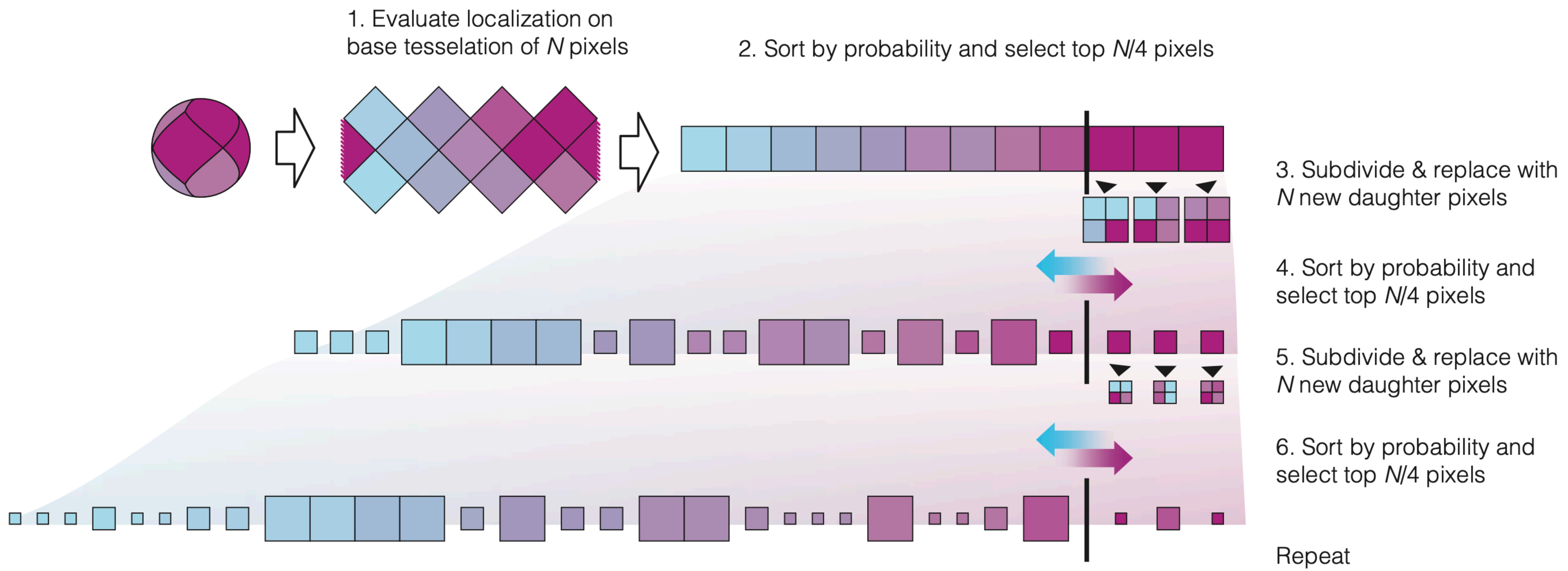


FIG. 8. Illustration of the BAYESTAR adaptive HEALPix sampling scheme.

https://arxiv.org/abs/1508.03634

RESOLUTION

- $N_{pix,0} = 3072$
- Evaluate the posterior probability density of a total of $8N_{pix,0}$ pixel
- The resolution per pixel is about $10^{-3} deg$

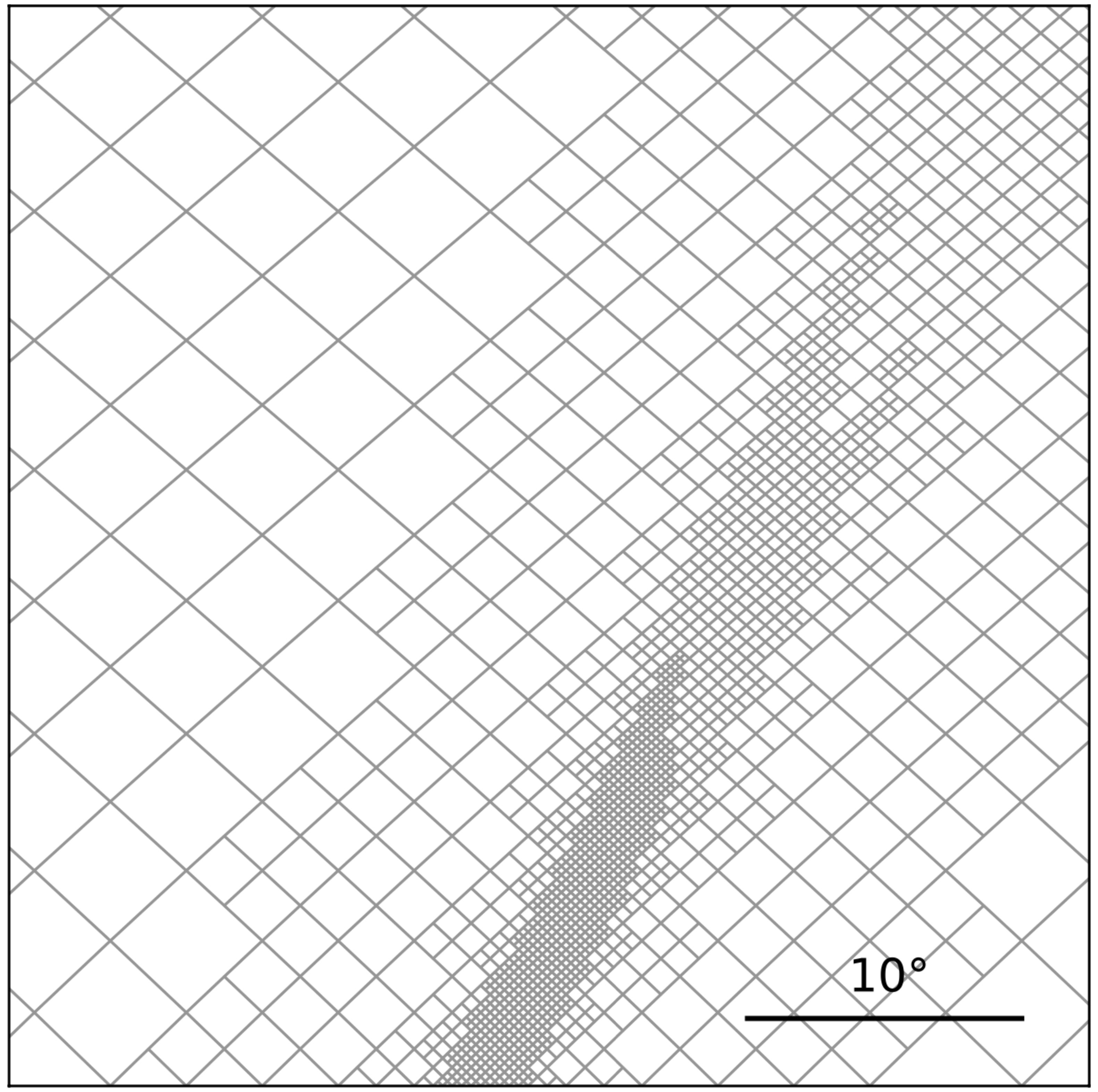


FIG. 9. An example multiresolution HEALPix mesh arising from the BAYESTAR sampling scheme (plotted in a cylindrical projection). This is event 18951 from Ref. [27].

https://arxiv.org/abs/1508.03634

HANDS ON

Sky localization for GW170817 using estimated data from

https://dcc.ligo.org/LIGO-G1701985/public