

Introduction on Gravitational Waves and Detectors

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General Relativity: Einstein Equation

Einstein Equation

$$R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu},$$

- $g_{\mu\nu}$: metric tensor,
- $R_{\mu\nu}$: Ricci tensor,
- $R = g^{\mu\nu}R_{\mu\nu}$: Ricci scalar,
- $T_{\mu\nu}$: stress-energy tensor.

Measure of interval in spacetime

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu.$$

General Relativity: Linearized Theory

Perturbation of metric tensor

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1.$$

Linearized Einstein Equation

$$\square \bar{h}_{\mu\nu} + \eta_{\mu\nu} \partial^\rho \partial^\sigma \bar{h}_{\rho\sigma} - \partial^\rho \partial_\nu \bar{h}_{\mu\rho} - \partial^\sigma \partial_\mu \bar{h}_{\nu\rho} = -\frac{16\pi G}{c^4} T_{\mu\nu}.$$

- $h = \eta^{\mu\nu} h_{\mu\nu}$,
- $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$.

Harmonic Gauge and Transverse-Traceless Gauge

Harmonic Gauge

$$\partial^\nu \bar{h}_{\mu\nu} = 0.$$

Linearized Einstein Equation

$$\square \bar{h}_{\mu\nu} = 0,$$

$T_{\mu\nu} = 0$ if we observe the GW far away from the source.

Transverse-Traceless Gauge

$$h^{0\mu}, \quad h_i^i = 0, \quad \partial^j h_{ij} = 0,$$

which can only be chosen away from the source.

Solution of the Gravitational Wave

We can choose the propagation direction along \hat{z} , with the wave vector:

$$k^\mu = (\omega/c, \mathbf{k}),$$

and the solution of GW:

$$h_{\mu\nu}^{\text{TT}}(t, z) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cos [\omega(t - z/c)].$$

The we have the perturbed metric tensor:

$$\begin{aligned} ds^2 = & -c^2 dt^2 + dz^2 + \{1 + h_+ \cos[\omega(t - z/c)]\} dx^2 \\ & - \{1 + h_+ \cos[\omega(t - z/c)]\} dy^2 + 2h_\times \cos[\omega(t - z/c)] dx dy. \end{aligned}$$

Gravitational Wave Interacting with Test Masses

$$0 = -c^2 dt^2 + (1 + h_+) dx^2, \quad \rightarrow cdt \simeq \left(1 + \frac{h_+}{2}\right) |dx|.$$

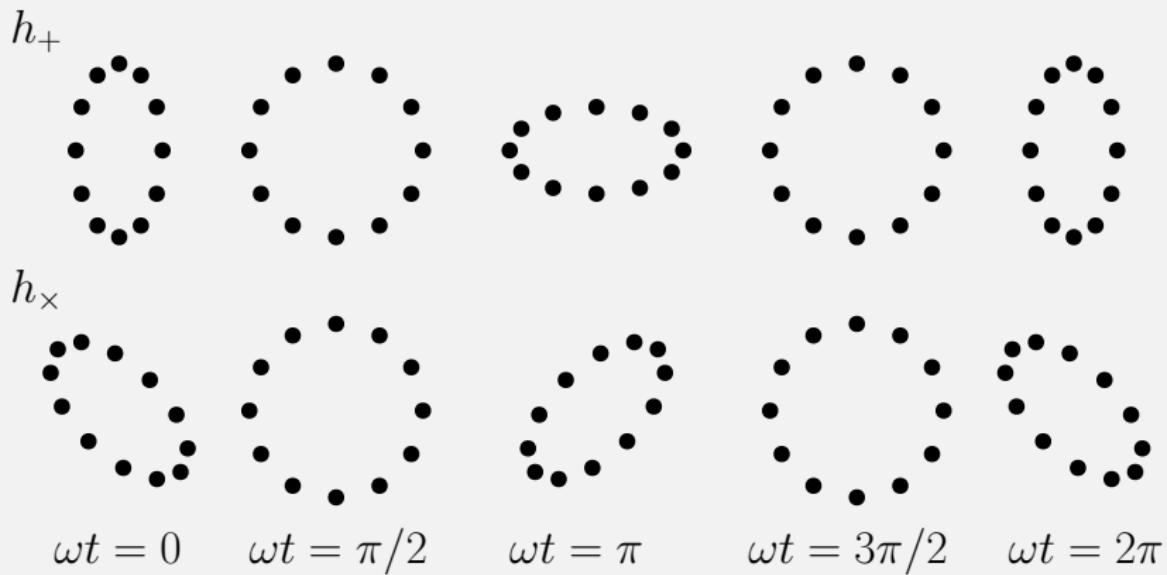
$$\Delta T_x = \frac{h_+ L_x}{2c}, \quad \Delta L_x = \frac{h_+ L_x}{2}.$$

$$0 = -c^2 dt^2 + (1 - h_+) dy^2, \quad \rightarrow cdt \simeq \left(1 - \frac{h_+}{2}\right) |dy|.$$

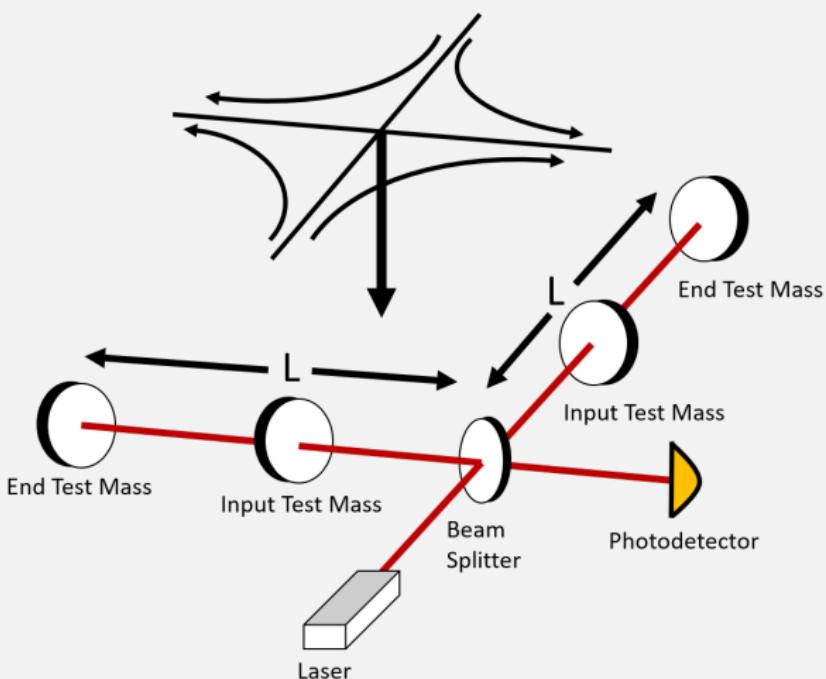
$$\Delta T_y = -\frac{h_+ L_y}{2c}, \quad \Delta L_x = -\frac{h_+ L_y}{2}.$$

$$\Delta L = \Delta L_x - \Delta L_y = h_+ \cdot \frac{L_x + L_y}{2} \equiv h_+ L.$$

Gravitational Wave Interacting with Test Masses

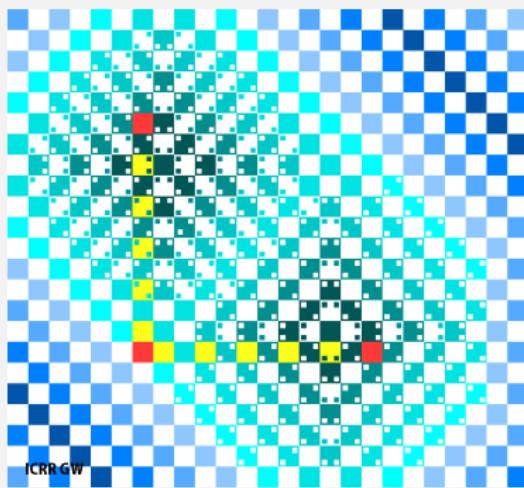
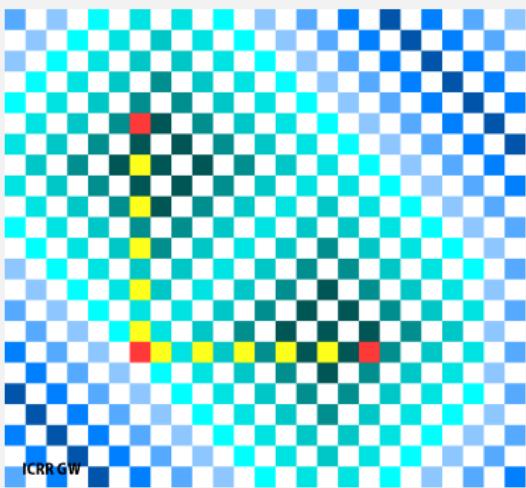


Gravitational Wave Detector



Gravitational Wave Detector

Most Precise Ruler Ever Constructed



Gravitational Wave Signal

The signal detected by a GW detector:

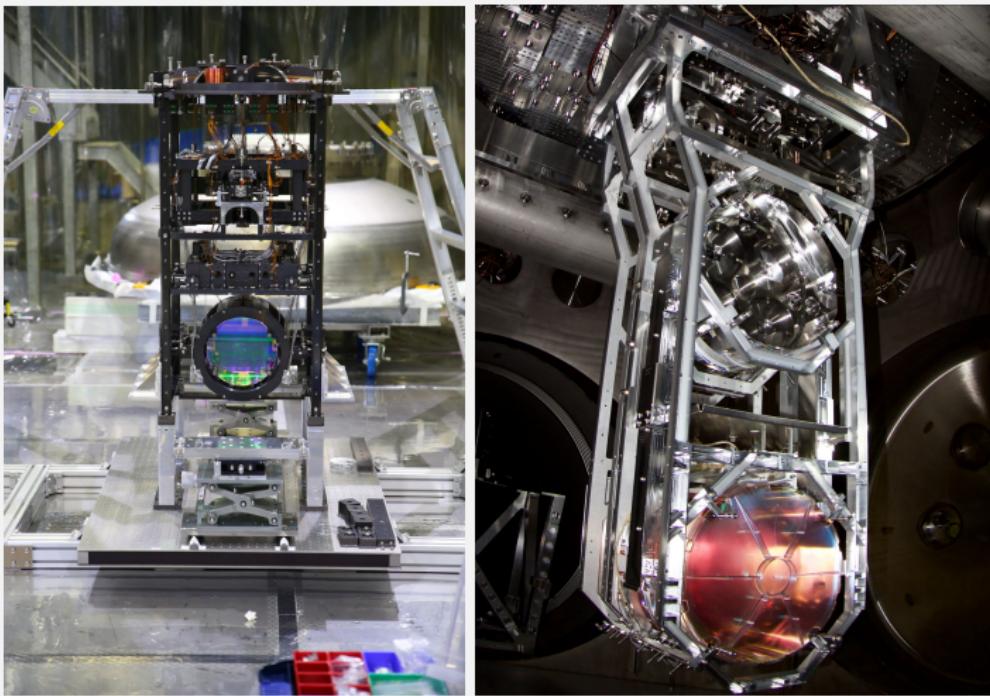
$$h(t) = D^{ij} h_{ij}(t),$$

where D^{ij} (detector tensor) depends on the detector geometry.

$$s(t) = h(t) + n(t),$$

where $|h(t)| \sim 10^{-22}$.

Test Mass Mirror



KAGRA



Virgo



LIGO

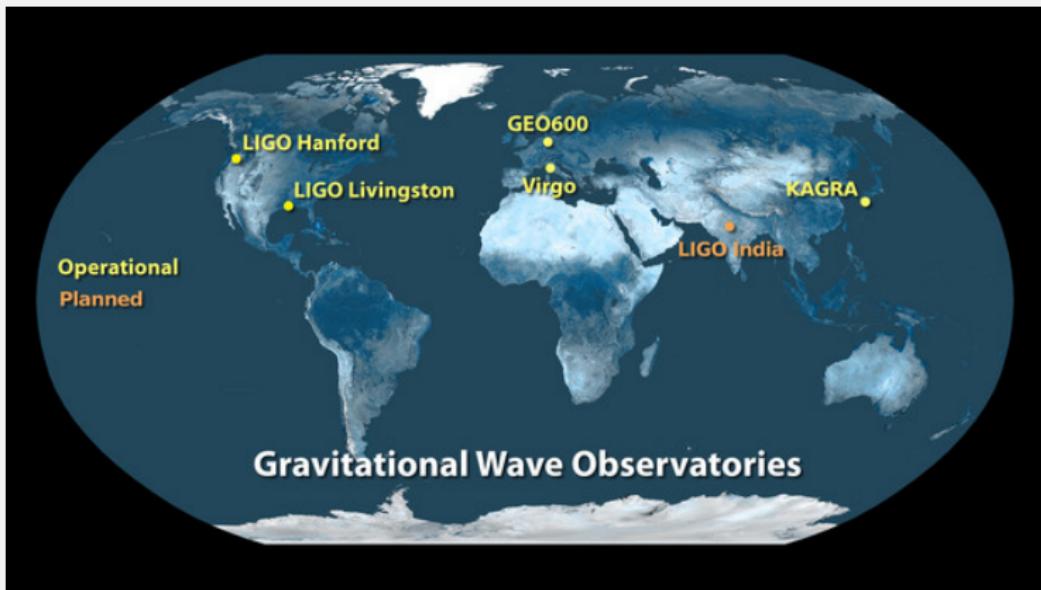


(a) Hanford

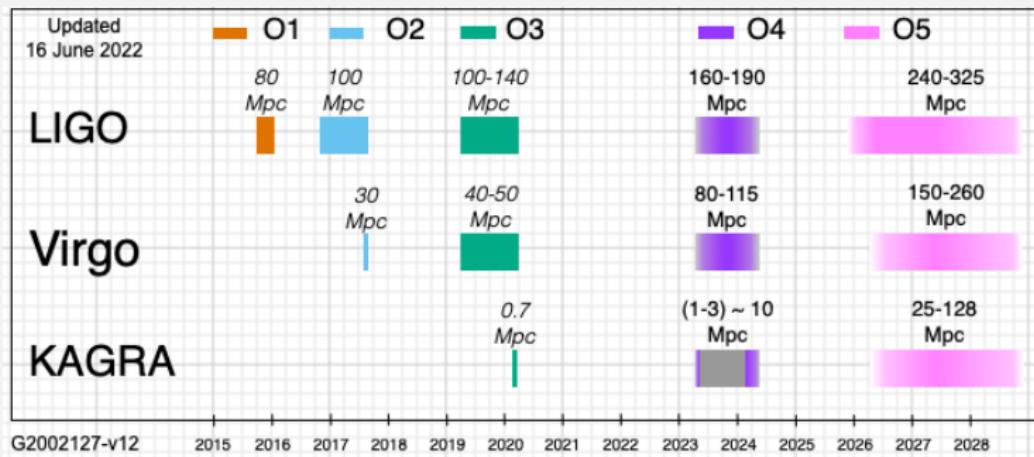


(b) Livingston

Gravitational Wave Detector Network



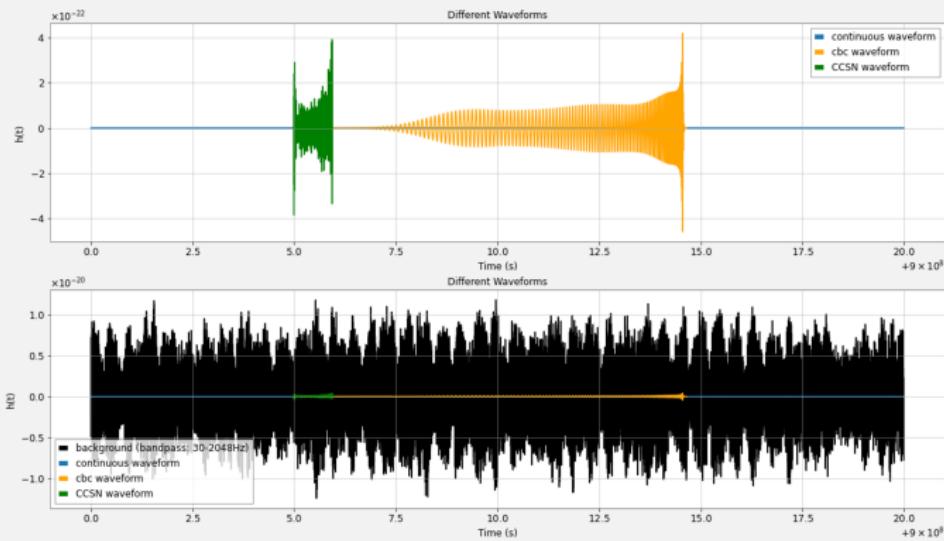
Observation Timeline



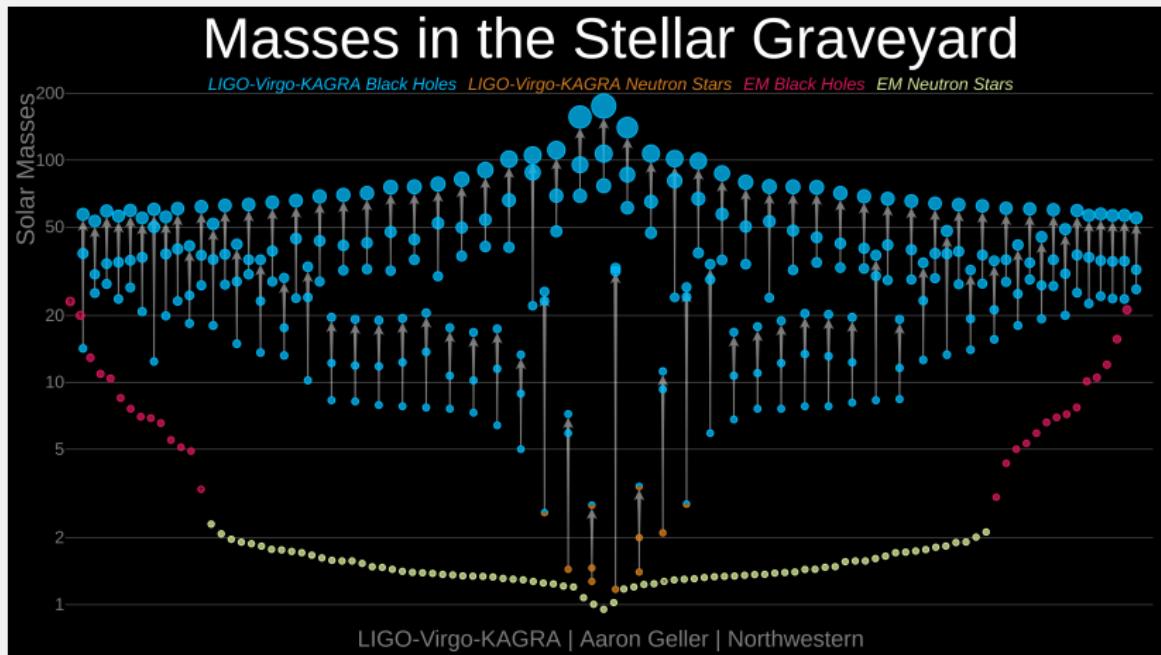
Sources of Gravitational Waves

- Compact Binary Coalescence:
 - Black Hole-Black Hole Merger,
 - Neutron Star-Neutron Star Merger,
 - Black Hole-Neutron Star Merger.
- Burst Sources:
 - Supernovae,
 - Gamma-ray Bursts.
- Continuous Wave Sources:
 - Spinning Neutron Stars.
- Stochastic Background:
 - Cosmological background,
 - Astrophysical background.

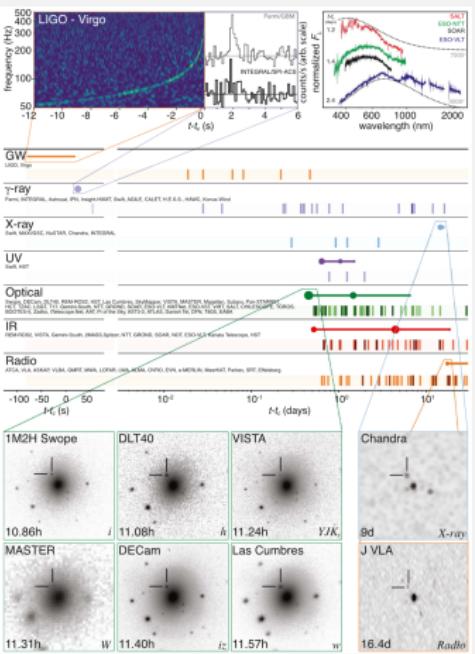
Gravitational Wave Data



Detections from O1 to O3



Multi-Messenger Astronomy



from [1710.05833](#)

Summary

- Multi-Messenger Astronomy
 - Gravitational Waves from Binary Neutron Star Merger
 - Gravitational Waves from Core-Collapse Supernovae
- Low-latency Pipelines
 - Detection
 - Sky Localization
 - Low-latency Parameter Estimation using C00 data
- Machine Learning Pipelines
 - Noise Subtraction (NonSENS, DeepClean)
 - GW Detection (MLy)
 - Sky Localization

Thank you