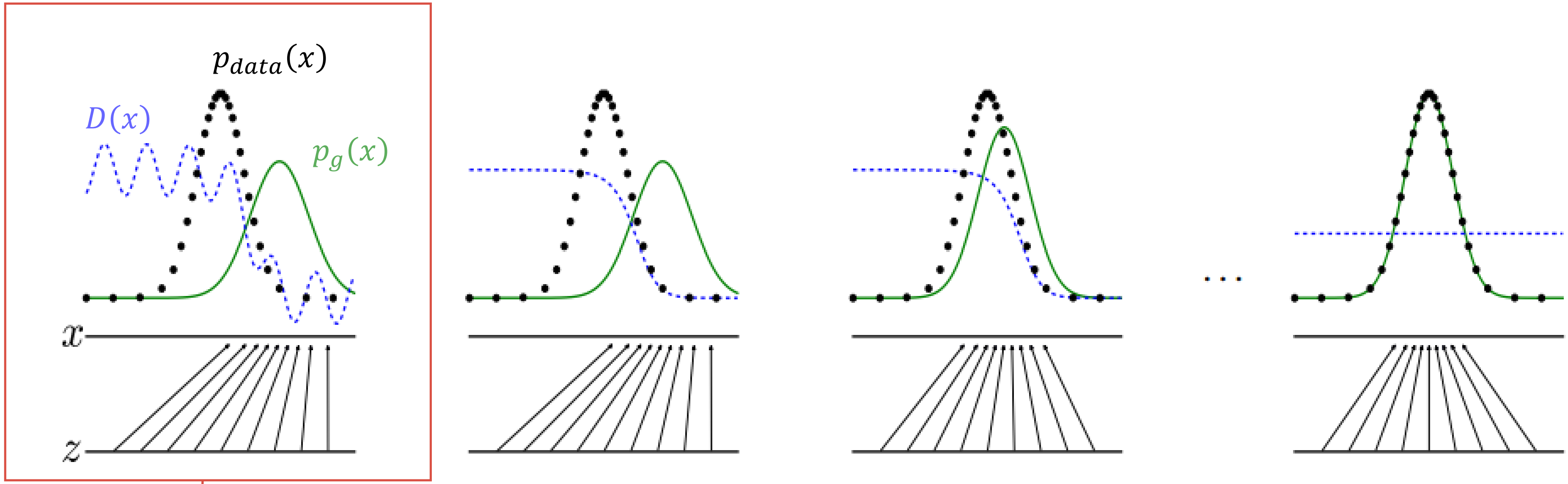
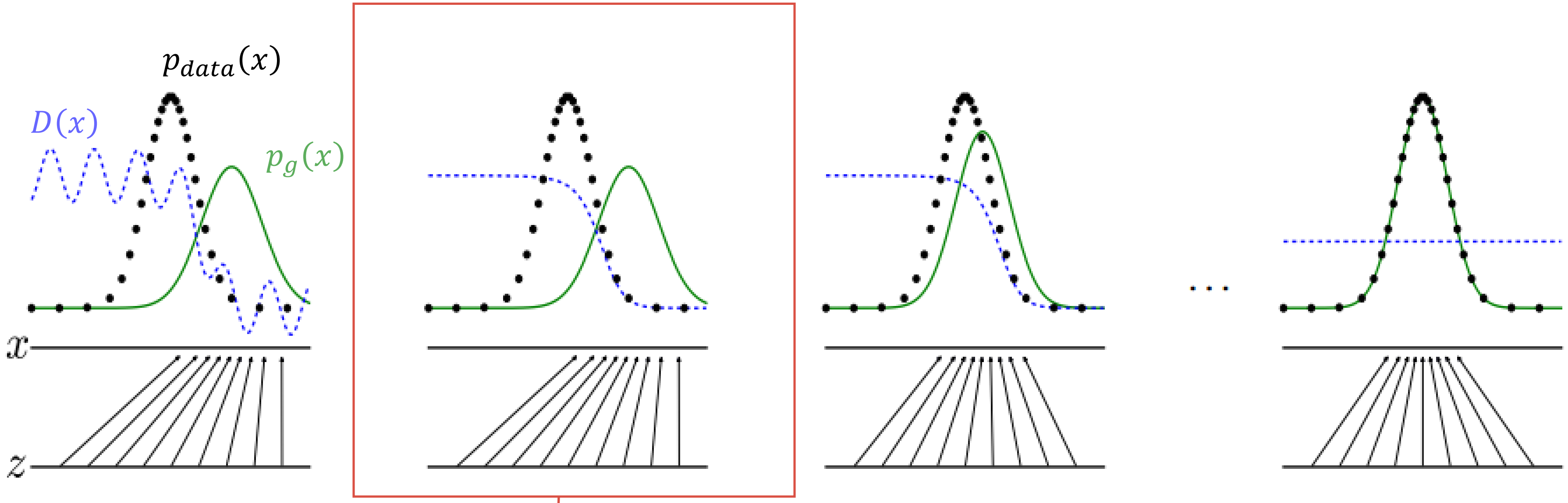
Lecture
Generative Adversarial Nets
(Theoretical Part)

Generative Adversarial Nets



학습을 아예 하지 않은 초기 상태

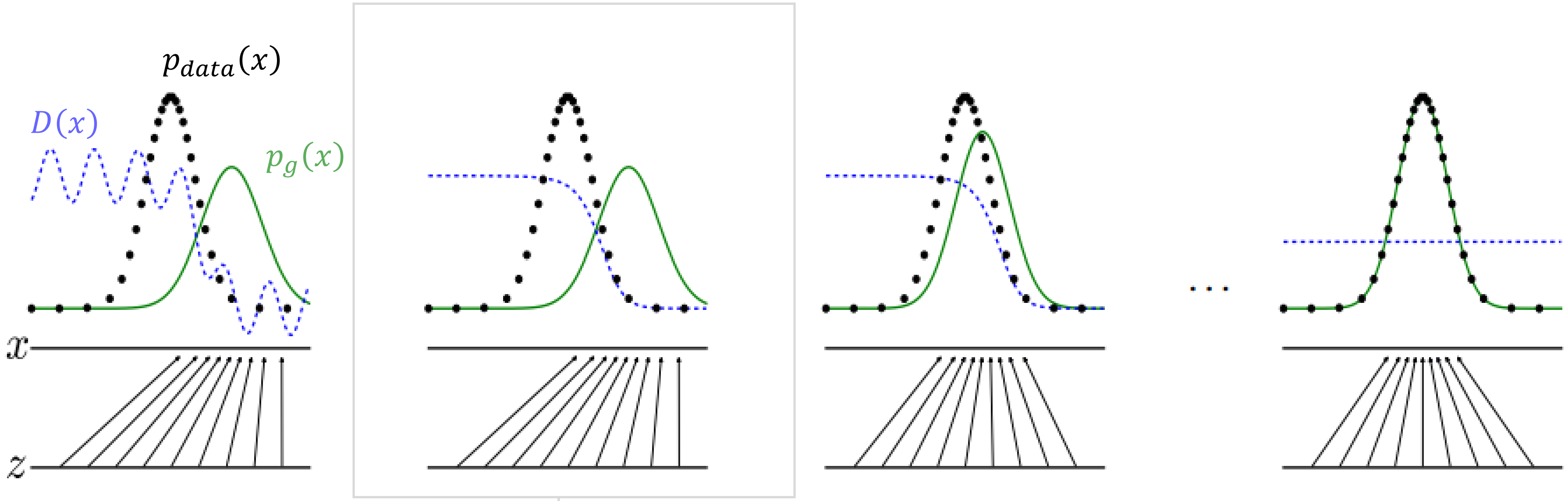
Generative Adversarial Nets



D를 학습시켰을 때

$$D(x) = \frac{p_{data}(x)}{p_{data}(x) + p_g(x)}$$

Generative Adversarial Nets



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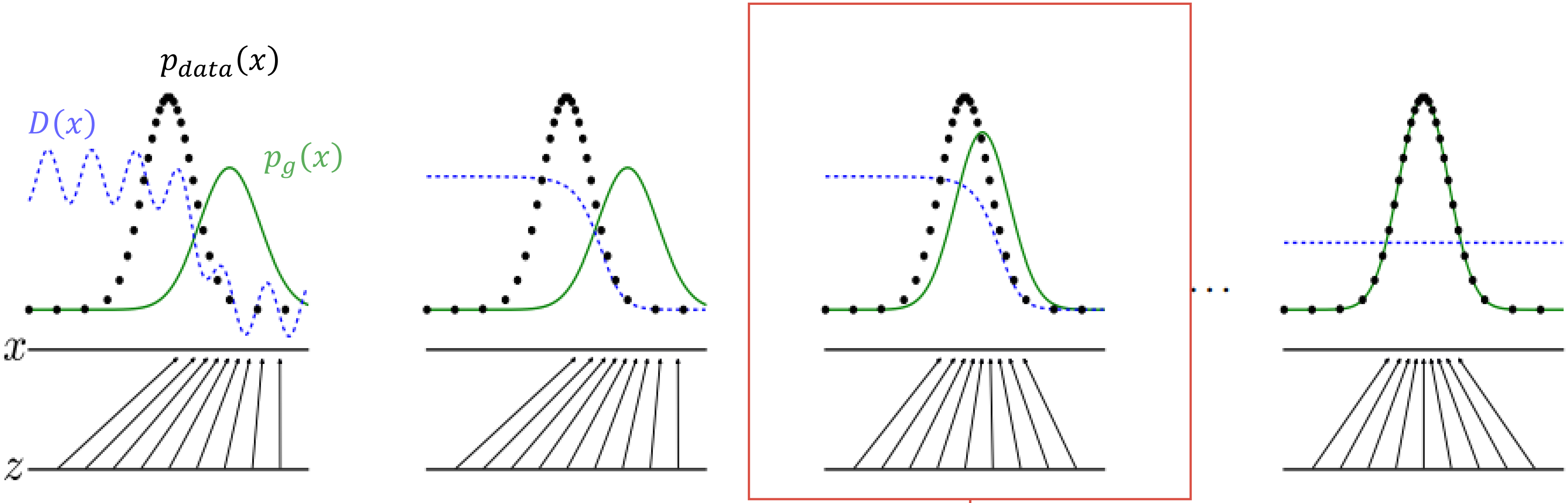
왜 이렇게 될까?
(뒤에서 증명)

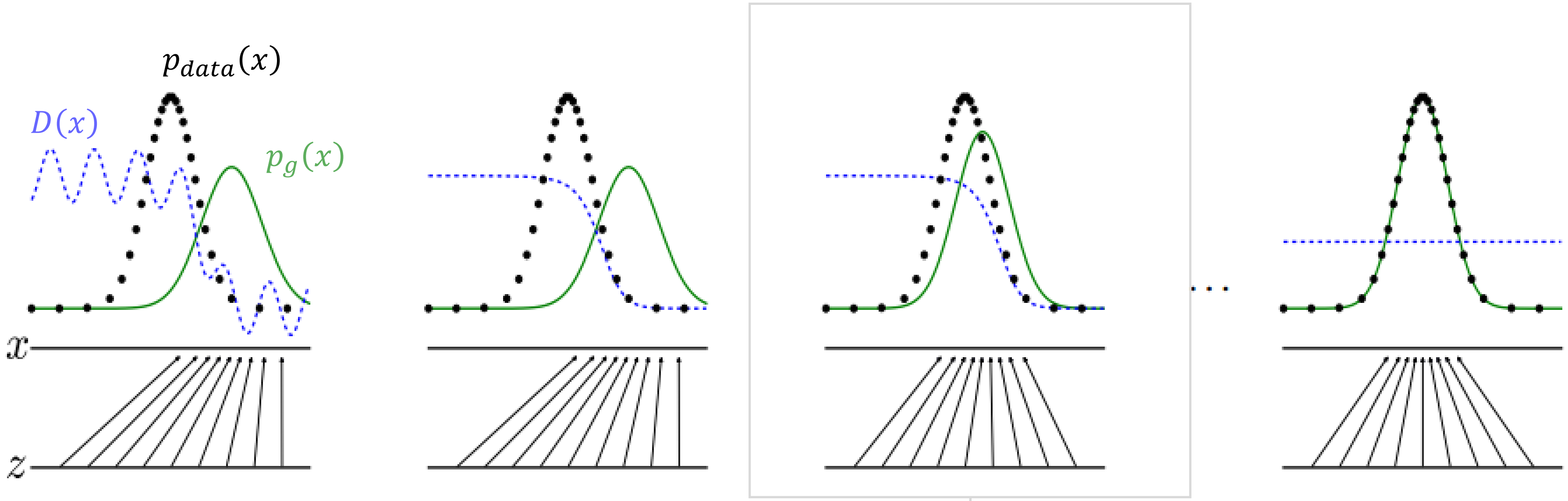
Generative Adversarial Nets



G를 학습시켰을 때

p_{data}와 p_g간의 차이가 줄어든다

Generative Adversarial Nets



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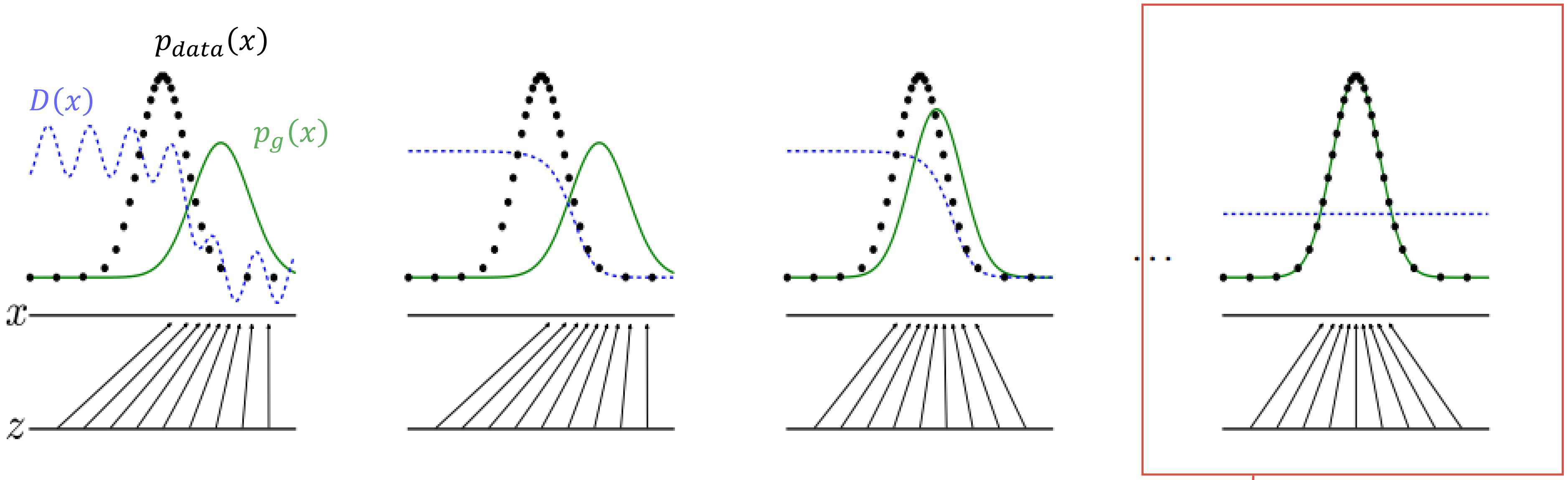


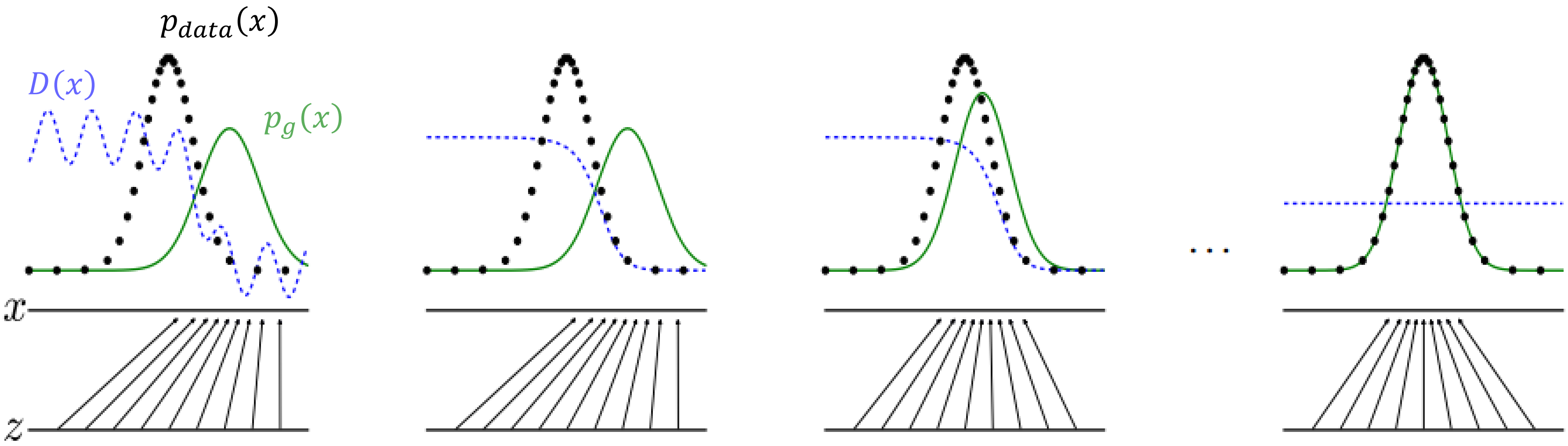
Global optimum에 도달했을 때

$$p_{data}(x) = p_g(x)$$

뒤에서 증명

$$D(x) = \frac{p_{data}(x)}{p_{data}(x) + p_g(x)}$$
$$= \frac{1}{2}$$

Generative Adversarial Nets



$$D^*(x) = \frac{p_{data}(x)}{p_{data}(x) + p_g(x)}$$

Theoretical Results

$$\min_G \max_D V(D,G) = E_{x \sim p_{data}(x)}[\log D(x)] + E_{z \sim p_z(z)}[\log(1 - D(G(z))]$$

Theoretical Results

$$\cancel{\min_G} \max_D V(D, \cancel{G}) = E_{x \sim p_{data}(x)}[\log D(x)] + E_{z \sim p_z(z)}[\log(1 - D(G(z))]$$

$$\downarrow \; \color{red}{G\text{를 고정}}$$

$$D^*(x) = \arg\max_D V(D) = E_{x \sim p_{data}(x)}[\log D(x)] + E_{z \sim p_z(z)}[\log(1 - D(G(z))]$$

Theoretical Results

$$\min_G \max_D V(D,G) = E_{x\sim p_{data}(x)}[\log D(x)] + E_{z\sim p_z(z)}[\log(1 - D(G(z))]$$

$\downarrow$ G를 고정

$$D^*(x) = arg\max_D V(D) = E_{x\sim p_{data}(x)}[\log D(x)] + E_{z\sim p_z(z)}[\log(1 - D(G(z))]$$

$\downarrow$ z 를 $sampling$ 하는 대신 x를 G에서 $sampling$

$$= E_{x\sim p_{data}}[\log D(x)] + E_{x\sim p_g}[\log(1 - D(x))]$$

Theoretical Results

$$\min_G \max_D V(D,G) = E_{x\sim p_{data}(x)}[\log D(x)] + E_{z\sim p_z(z)}[\log(1 - D(G(z))]$$

$\downarrow$ G를 고정

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$$= E_{x\sim p_{data}}[\log D(x)] + E_{x\sim p_g}[\log(1 - D(x))]$$

$$= \int_x p_{data}(x) \log D(x)\, dx + \int_x p_g(x) \log\big(1 - D(x)\big)\, dx$$

Theoretical Results

$$\min_G \max_D V(D,G) = E_{x \sim p_{data}(x)}[\log D(x)] + E_{z \sim p_z(z)}[\log(1 - D(G(z))]$$

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Expectation의 정의

$$E_{x \sim p(x)}[f(x)] = \int_x p(x) f(x) dx \quad \text{if } x \text{ is continuous r.v}$$

$$= \sum_x p(x) f(x) \quad \text{if } x \text{ is discrete r.v}$$

Theoretical Results

$$\min_G \max_D V(D,G) = E_{x\sim p_{data}(x)}[\log D(x)] + E_{z\sim p_z(z)}[\log(1 - D(G(z))]$$

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$$= \int_x p_{data}(x)\log D(x)\,dx + \int_x p_g(x)\log\big(1 - D(x)\big)\,dx$$

$$= \int_x \underline{p_{data}(x)\log D(x) + p_g(x)\log\big(1 - D(x)\big)\,dx}$$

→ 적분 안의 값이 최대가 되는 점을 찾자
($D(x)$로 미분해서 0이 되는 지점, 극대값)

Theoretical Results

$$p_{data}(x)\log D(x) + p_g(x)\log\big(1 - D(x)\big)$$

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$\downarrow$ $a = p_{data}(x),\ y = D(x),\ b = p_g(x)$ 로 치환

$$a\log y + b\log(1 - y)$$

Theoretical Results

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$$\frac{d}{dx}\log x = \frac{1}{x}$$

$$\frac{d}{dx}\log f(x) = \frac{f'(x)}{f(x)}$$

$$a\log y + b\log(1 - y)$$

$\downarrow$ y에 대하여 미분

$$\frac{a}{y} + \frac{-b}{1-y}$$

Theoretical Results

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$$\frac{a-(a+b)y}{y(1-y)} = 0$$

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$$\frac{a-(a+b)y}{y(1-y)} = 0 \longrightarrow y = \frac{a}{a+b}$$ 에서 극대이자 최대값을 가짐 ()

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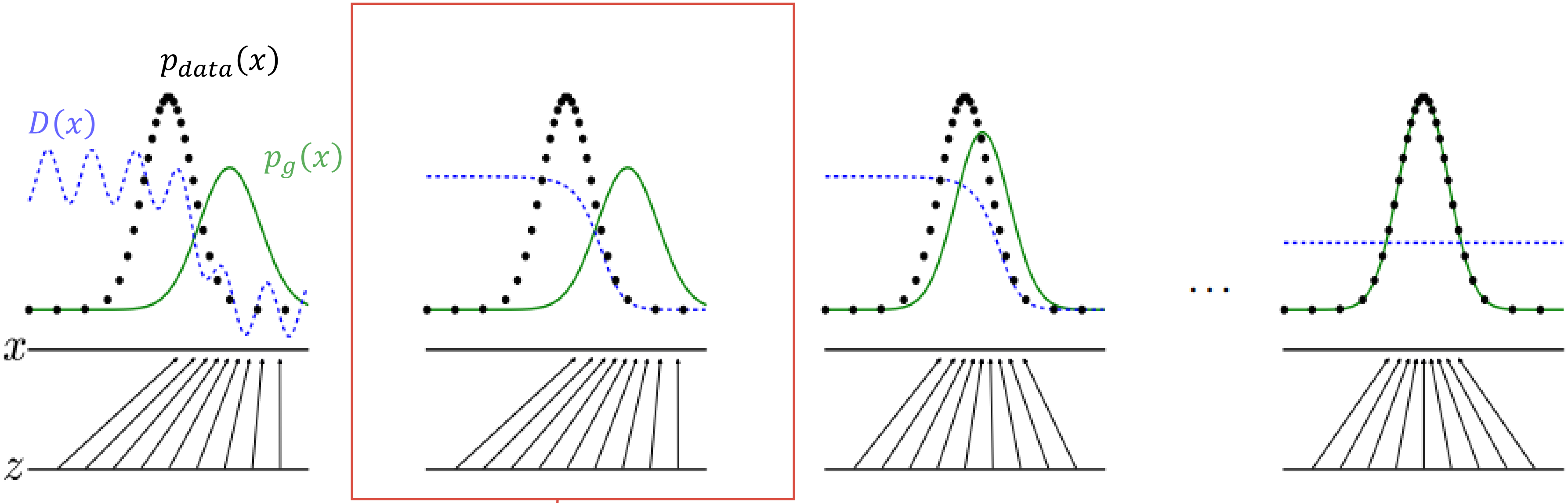

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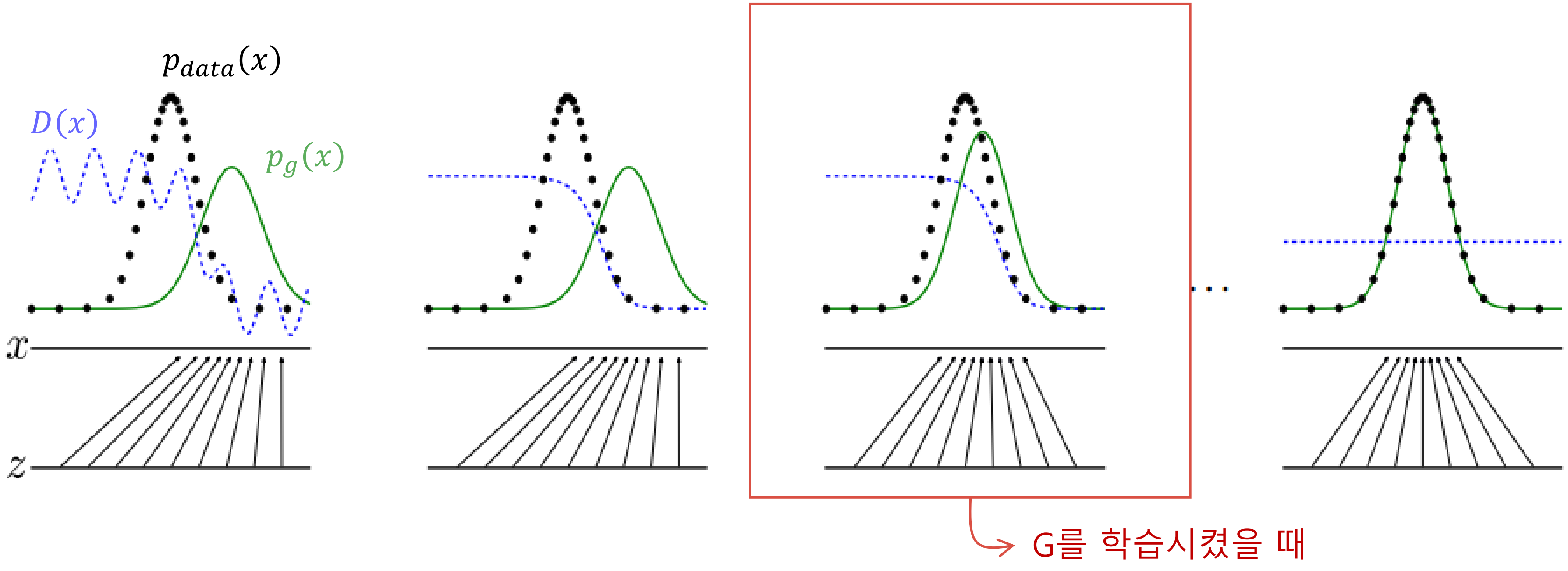
증명 끝

Generative Adversarial Nets



p_{data}와 p_g간의 차이가 줄어든다

Theoretical Results

이런 $minimax\ algorithm$을 통해 정말 $p_{data}(x)$ 에 가까운 $p_g(x)$를 얻을 수 있을까?

$$\min_G \max_D V(D, G) = E_{x\sim p_{data}(x)}[\log D(x)] + E_{z\sim p_z(z)}[\log(1 - D(G(z))]$$

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$$\min_{G,D} JSD(p_{data} || p_g)$$

$Jenson\ Shannon\ Divergence$

Theoretical Results

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$$\downarrow$$

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$$JSD(P||Q) = \frac{1}{2}KL(P||M) + \frac{1}{2}KL(Q||M)$$

$$where\ M = \frac{1}{2}(P+Q)$$

Theoretical Results

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$KL\ Divergence$

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$$KL(P||Q) = \sum_{x_i} P(x_i) \log \frac{P(x_i)}{Q(x_i)}$$

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$$KL(P||Q) = \sum_{x_i} P(x_i)\log\frac{P(x_i)}{Q(x_i)}$$

$$= \sum_{x_i} -P(x_i)\log Q(x_i) - (-P(x_i)\log P(x_i))$$

$Cross\ Entropy$ $Entropy$

Theoretical Results

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$$KL(P||Q) = \sum_{x_i} P(x_i) \log \frac{P(x_i)}{Q(x_i)}$$

더 필요한 bit 수

P를 Q로 추정하는데 치뤄야하는 penalty

$$= \sum_{x_i} \boxed{-P(x_i)\log Q(x_i) - (-P(x_i)\log P(x_i))}$$

$Cross\ Entropy$ $\quad$ $Entropy$

Theoretical Results

$$C(G) = \max_{D} V(D, G)$$

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$$C(G) = \max_D V(D,G)$$
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$$= \int p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{p_g(x)}{p_{data}(x) + p_g(x)} dx$$

Theoretical Results

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$$= -log4 + log4 + \int p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{p_g(x)}{p_{data}(x) + p_g(x)} dx$$

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$$= -log4 + \int p_{data}(x) \log \frac{2 \cdot p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{2 \cdot p_g(x)}{p_{data}(x) + p_g(x)} dx$$

Theoretical Results

$$C(G) = \max_D V(D, G)$$

$$= E_{x \sim p_{data}}[\log D^*(x)] + E_{x \sim p_g}[\log(1 - D^*(x))]$$

$$\int p_{data}(x)dx = 1$$

$$= -log4 + log4 + \int p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{p_g(x)}{p_{data}(x) + p_g(x)} dx$$

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$$\int p_{data}(x)dx = 1 \qquad log2 = \int p_{data}(x) \cdot log2\, dx$$

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$$\int p_{data}(x)dx = 1 \qquad log2 = \int p_{data}(x) \cdot log2\, dx$$

$$log2 + \int p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)} dx = \int p_{data}(x) \log \frac{2 \cdot p_{data}(x)}{p_{data}(x) + p_g(x)} dx$$

$$= -log4 + log4 + \int p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{p_g(x)}{p_{data}(x) + p_g(x)} dx$$

$$= -log4 + \int p_{data}(x) \log \frac{2 \cdot p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{2 \cdot p_g(x)}{p_{data}(x) + p_g(x)} dx$$

Theoretical Results

$$C(G) = \max_D V(D, G)$$

$$= E_{x\sim p_{data}}[\log D^*(x)] + E_{x\sim p_g}[\log(1 - D^*(x))]$$

$$= E_{x\sim p_{data}}\left[\log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)}\right] + E_{x\sim p_g}\left[\log \frac{p_g(x)}{p_{data}(x) + p_g(x)}\right]$$

$$= \int p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{p_g(x)}{p_{data}(x) + p_g(x)} dx$$

$$= -log4 + log4 + \int p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{p_g(x)}{p_{data}(x) + p_g(x)} dx$$

$$= -log4 + \int p_{data}(x) \log \frac{2 \cdot p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{2 \cdot p_g(x)}{p_{data}(x) + p_g(x)} dx$$

Theoretical Results

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$$= \int p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{p_g(x)}{p_{data}(x) + p_g(x)} dx$$

$$= -log4 + log4 + \int p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{p_g(x)}{p_{data}(x) + p_g(x)} dx$$

$$= -log4 + \int p_{data}(x) \log \frac{2 \cdot p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{2 \cdot p_g(x)}{p_{data}(x) + p_g(x)} dx$$

$$= -log4 + KL(p_{data} || \frac{p_{data} + p_g}{2}) + KL(p_g || \frac{p_{data} + p_g}{2})$$

Theoretical Results

$$C(G) = \max_D V(D, G)$$

$$= E_{x\sim p_{data}}[\log D^*(x)] + E_{x\sim p_g}[\log(1 - D^*(x))]$$

$$= E_{x\sim p_{data}}\left[\log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)}\right] + E_{x\sim p_g}\left[\log \frac{p_g(x)}{p_{data}(x) + p_g(x)}\right]$$

$$KL(P||Q) = \int P(x) \log \frac{P(x)}{Q(x)} dx$$

$$= -log4 + \int p_{data}(x) \log \frac{2 \cdot p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{2 \cdot p_g(x)}{p_{data}(x) + p_g(x)} dx$$

$$= -log4 + KL(p_{data}|| \frac{p_{data} + p_g}{2}) + KL(p_g|| \frac{p_{data} + p_g}{2})$$

Theoretical Results

$$C(G) = \max_D V(D,G)$$

$$= E_{x\sim p_{data}}[\log D^*(x)] + E_{x\sim p_g}[\log(1 - D^*(x))]$$

$$= E_{x\sim p_{data}}\left[\log \frac{p_{data}(x)}{p_{data}(x)+p_g(x)}\right] + E_{x\sim p_g}\left[\log \frac{p_g(x)}{p_{data}(x)+p_g(x)}\right]$$

$$KL(P||Q) = \int P(x)\log\frac{P(x)}{Q(x)}dx$$

$$\boxed{\int p_{data}(x)\log\frac{p_{data}(x)}{\frac{p_{data}(x)+p_g(x)}{2}}dx} = KL(p_{data}||\frac{p_{data}+p_g}{2})$$

$$= -log4 + \boxed{\int p_{data}(x)\log\frac{2\cdot p_{data}(x)}{p_{data}(x)+p_g(x)}dx} + \int p_g(x)\log\frac{2\cdot p_g(x)}{p_{data}(x)+p_g(x)}dx$$

$$= -log4 + KL(p_{data}||\frac{p_{data}+p_g}{2}) + KL(p_g||\frac{p_{data}+p_g}{2})$$

Theoretical Results

$$C(G) = \max_D V(D, G)$$

$$= E_{x\sim p_{data}}[\log D^*(x)] + E_{x\sim p_g}[\log(1 - D^*(x))]$$

$$= E_{x\sim p_{data}}\left[\log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)}\right] + E_{x\sim p_g}\left[\log \frac{p_g(x)}{p_{data}(x) + p_g(x)}\right]$$

$$= \int p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{p_g(x)}{p_{data}(x) + p_g(x)} dx$$

$$= -log4 + log4 + \int p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{p_g(x)}{p_{data}(x) + p_g(x)} dx$$

$$= -log4 + \int p_{data}(x) \log \frac{2 \cdot p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{2 \cdot p_g(x)}{p_{data}(x) + p_g(x)} dx$$

$$= -log4 + KL(p_{data}|| \frac{p_{data} + p_g}{2}) + KL(p_g|| \frac{p_{data} + p_g}{2})$$

$$= -log4 + 2 \cdot JSD(p_{data}||p_g)$$

Theoretical Results

$$C(G) = \max_D V(D, G)$$

$$= E_{x\sim p_{data}}[\log D^*(x)] + E_{x\sim p_g}[\log(1 - D^*(x))]$$

$$= E_{x\sim p_{data}}\left[\log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)}\right] + E_{x\sim p_g}\left[\log \frac{p_g(x)}{p_{data}(x) + p_g(x)}\right]$$

$$= \int p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{p_g(x)}{p_{data}(x) + p_g(x)} dx$$

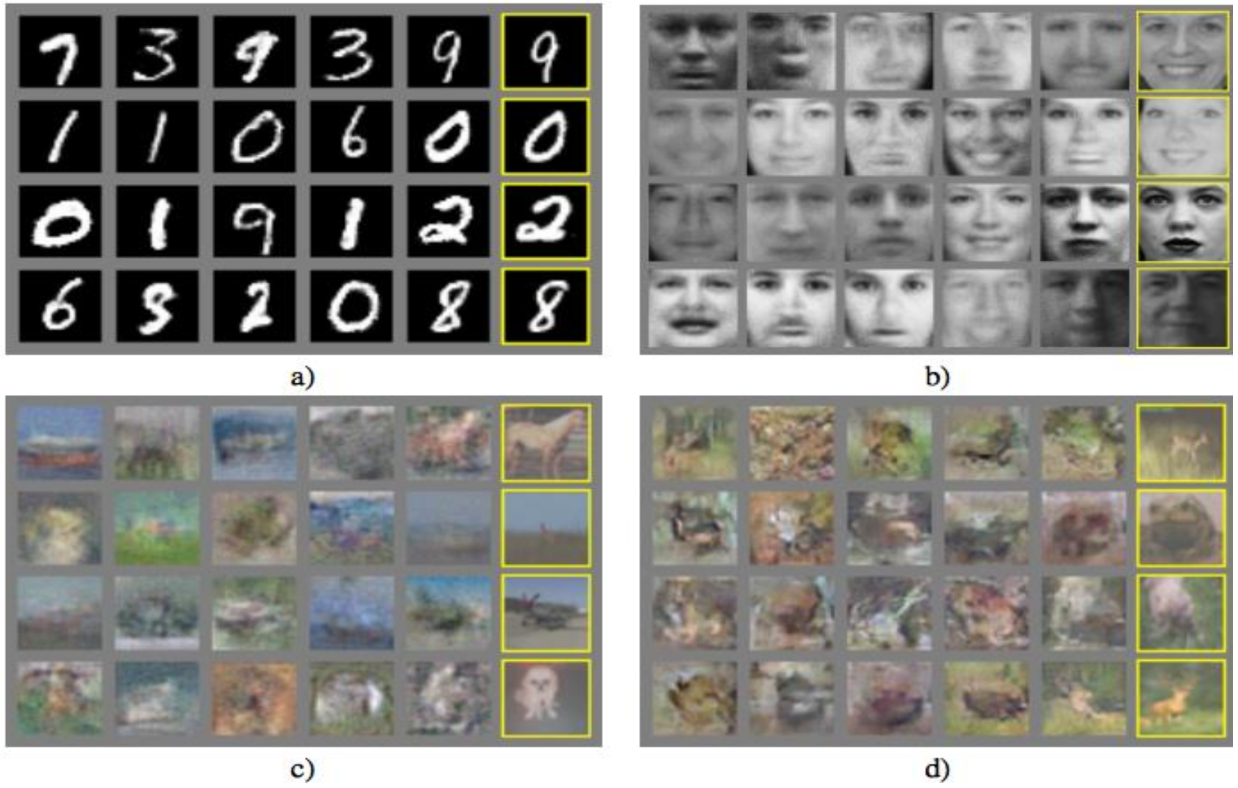
$$= -log4 + log4 + \int p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{p_g(x)}{p_{data}(x) + p_g(x)} dx$$

$$= -log4 + \int p_{data}(x) \log \frac{2 \cdot p_{data}(x)}{p_{data}(x) + p_g(x)} dx + \int p_g(x) \log \frac{2 \cdot p_g(x)}{p_{data}(x) + p_g(x)} dx$$

$$= -log4 + KL(p_{data}||\frac{p_{data} + p_g}{2}) + KL(p_g||\frac{p_{data} + p_g}{2})$$

$$= -log4 + 2 \cdot JSD(p_{data}||p_g)$$

Experiments



a) MNIST
b) TFD
c) CIFAR-10 (fully connected model)
d) CIFAR-10 (convolutional discriminator and “deconvolutional” generator)