Financial Engineering2

PROJECT01

-Valuation of callable monthly S&P500 Index-linked range accrual security-

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1. Terms of the callable range accrual securities with fixed percentage buffered

Issuer		Wells Fargo & Company
Market measure		S&P500 Index
Pricing date		January 28 2019
Issue date		January 31 2019
Published price		953.22
Face value		$1000
Initial Index value		2643.85
Barrier value		2115.08 (= 0.8 * Initial value)
Maximum Coupon Rate		6.15% per annually 0.5125% per monthly
Accrual days		$observation\ \ days\ \ on\ \ S_t \geq B$
Observation period end dates		Monthly on the 26th day of each month
Contingent coupon payment		FV*0.5125% * $\frac{Number\ \ of\ \ accrual\ \ days\ \ during\ \ observation\ \ period}{Number\ \ of\ \ observation\ \ days\ \ during\ \ observation\ \ period}$
Optional redemption dates		Monthly, from January 2020 to December 2023
Optional redemption payoff	$\boldsymbol{S_T > B}$	Min(1000, holding Notes value) + coupon
	$\boldsymbol{S_T < B}$	Min(1000, holding Notes value)
Payoff at maturity	$\boldsymbol{S_T > B}$	1000
	$\boldsymbol{S_T < B}$	$1000 * (1 - \frac{B-S_T}{S_0})$

2. Parameter for pricing the security

We will estimate this product through binomial model. In addition to the information given above, the required parameters are interest rate, dividend rate and volatility. To estimate the parameters we use linear interpolation and extrapolation.

1) Interest rate

In the case of interest rate, there is no interest rate exactly matched from pricing date to maturity. After receiving the interest rate rata from Bloomberg, we calculated data for the period through interpolation and use it as constant. The interpolation formula is below.

$$r^* = \frac{(t^* - t_1) * (r_2 - r_1)}{t_2 - t_1} + r_1$$

2) dividend

In the case of dividends, we obtain it from the dividend curve of Bloomberg and use as constant.

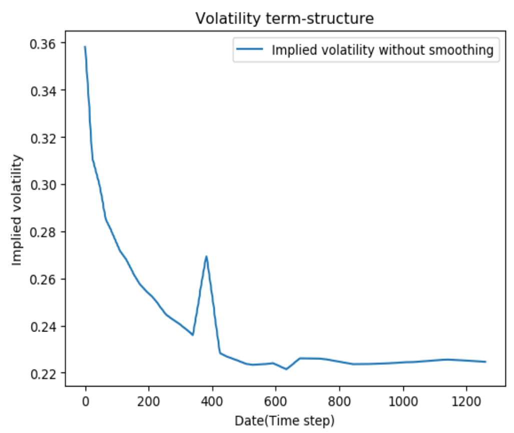
It is used to calculate the probability that stock will increase.

3) Volatility

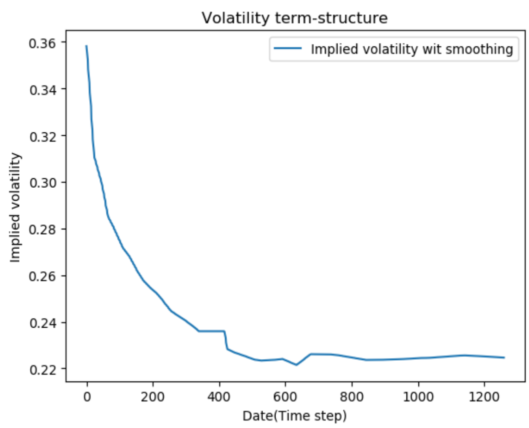
We could not obtain the volatility corresponding barrier value from Bloomberg. Therefore, after obtaining the volatility data in Bloomberg, we extrapolated it to moneyness for getting the volatility corresponding to barrier. Extrapolation formula is below and interpolation method is the same with interest rate interpolation formula.

$$\sigma^* = \frac{(M^* - M_1) * (\sigma_2 - \sigma_1)}{M_2 - M_1} + \sigma_1, \qquad \text{M: Moneyness}$$

In addition, since volatility is an important parameter in pricing, we created volatility structure by interpolating it to maturity. The resulting figure is follow. As you can see in the figure, we observed spikes at certain intervals. Since there are some spike value in raw data. We smoothed it to get rid of it.



Before smoothing



After smoothing

By doing like this, we can obtain more sophisticated structure of volatility, which create more accurately the stock path. The importance of volatility will be examined through the analysis of the scenario.

4) Model and Time step selection

We selected the 'CRR' model among lot of binomial tree models. In case of time step, we selected 1259 because the number of business days until maturity except holiday is 1259. Although we should consider weekend, this is excluded for convenience of calculation. In the scenario analysis, we will look how prices change with the change in models and time steps.

The table below summarizes the parameters and please refer to the attached code for more details.

Interest rate	3.05%
Dividend rate	2%
Volatility	Volatility Term-structure
Binomial model	CRR
Number of nodes	1259

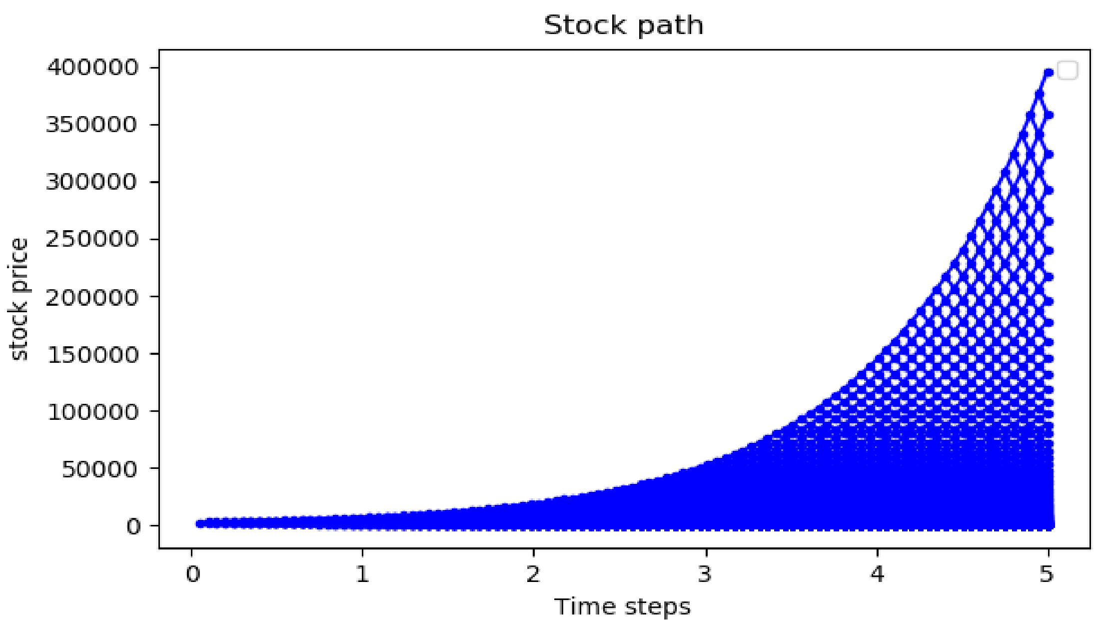
3. Pricing for the security

1) Stock path tree

We will briefly explain the process of pricing and if you want to know more detail, please see the attached code. Since we have estimated the parameters, we now have to create a stock path tree. We should consider volatility structure and dividend yield to make stock path tree, so the process for creating the 'CRR' model is as follows.

$$u_j = e^{\sigma_j * \sqrt{\Delta t}} \ , d_j = \frac{1}{u_j}, p_j = \frac{e^{(r-q)*\Delta t} - d_j}{u_j - d_j} \quad S_{j,i} = S_0 * u^j * d^{i-j}$$

The below figure is final tree.



2) The security tree

We divided time nodes because it has different payoff for periods. To evaluate each security nodes we have to consider coupon payment and its amount depends on accrued days. The core of the payoff structure is as follows. The original coupon payment is paid by calculating the accrued days for that period. However, we use other way to create security tree. If the stock value of corresponding node is larger than barrier, then coupons are paid in discounted form. Therefore, we do not have to count the accrued days because it is already reflected at tree. The below table is payoff structure according to time node and condition.

Time node	Condition		Value of the security($V_{j,i}$)
Maturity	$S_{j,i} \geq B$		1000
	$S_{j,i} < B$		$1000 * (1 - \frac{B - S_{j,i}}{S_0})$
Between Last coupon date and maturity			$e^{-r*\Delta t} * [p_j * V_{j+1,i+1} + (1 - p_j) * V_{j,i+1}]$
From Jan 2020 to Feb 2023	Early redemption node	$S_{j,i} \geq B$	Min(1000* $e^{(-r*(last\ coupon\ payment\ node-i)*\Delta t}$, $e^{-r*\Delta t} * [p_j * V_{j+1,i+1} + (1 - p_j) * V_{j,i+1}])$ $+ Contigent\ coupon\ payment * e^{(-r*(last\ coupon\ payment\ node-i)*\Delta t}$
	Early redemption node	$S_{j,i} < B$	Min(1000* $e^{(-r*(last\ coupon\ payment\ node-i)*\Delta t}$, $e^{-r*\Delta t} * [p_j * V_{j+1,i+1} + (1 - p_j) * V_{j,i+1}])$
	No Early redemption node	$S_{j,i} \geq B$	$e^{-r*\Delta t} * [p_j * V_{j+1,i+1} + (1 - p_j) * V_{j,i+1}] + Contigent\ coupon\ payment * e^{(-r*(last\ coupon\ payment\ node-i)*\Delta t}$
	No Early redemption node	$S_{j,i} < B$	$e^{-r*\Delta t} * [p_j * V_{j+1,i+1} + (1 - p_j) * V_{j,i+1}]$
From Feb 2019 to Dec 2019	$S_{j,i} \geq B$		$e^{-r*\Delta t} * [p_j * V_{j+1,i+1} + (1 - p_j) * V_{j,i+1}] + Contigent\ coupon\ payment * e^{(-r*(last\ coupon\ payment\ node-i)*\Delta t}$
	$S_{j,i} < B$		$e^{-r*\Delta t} * [p_j * V_{j+1,i+1} + (1 - p_j) * V_{j,i+1}]$

3) Price of security

As a result of drawing the security tree according to the above Payoff, **the final price of securities was $958.02.** This is very close to the price suggested by Wells Fargo (estimated price is $953.22), which the difference is about $5. The reason for this difference is that we do not consider every weekend and it means that the discounts for every node were slightly less. So when we consider weekend, the final price will decrease and be closer to the published price.

4. Scenario analysis

Although we estimated the price close to the published price, we will analyze various scenarios to identify parameters that affect price.

1) Volatility Scenario analysis

To analyze how the volatility change affects the price, other variables are fixed to the existing values. As a result of the scenario analysis, it can be seen that the volatility has a big influence on the price. The lower the volatility, the greater the error. Also 22.4% is the interpolated volatility for the maturity and the price calculated through this value had some errors. Finally, it can be seen that the price calculated using the volatility term-structure is closest to the published price.

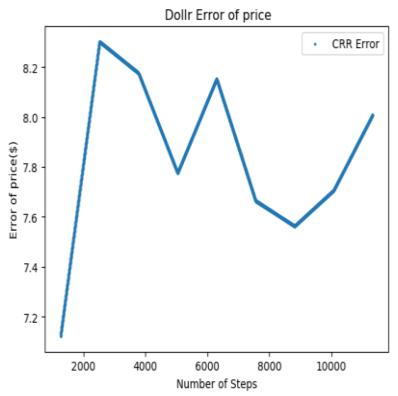
Volatility	19%	20%	22%	22.4%	Term Structure
Price	984.50	977.58	965.34	960.34	958.02

2) Changes in model and time step

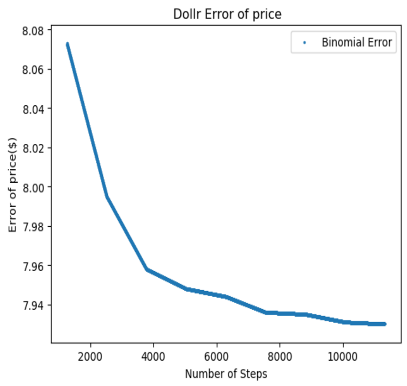
We have analyzed the scenario to see how the price changes according to models and the number of nodes. Prior to the analysis, we assumed that the volatility is constant due to the speed and convenience of calculation. The table below shows the results of the scenario analysis. When the number of node is 1259 and model is CRR, the value is nearest to the published price.

Node / **Model**	**1,259**	**2,518**	**3,777**	**5,036**	**6,295**	**7,554**	**8,813**	**10,072**	**11,331**
CRR	960.341	961.522	961.396	960.996	961.373	960.884	960.783	960.926	961.229
Binomial	961.293	961.215	961.178	961.168	961.164	961.156	961.155	961.151	961.150
Rendleman&Bartter	961.313	961.193	961.176	961.168	961.164	961.159	961.153	961.152	961.146

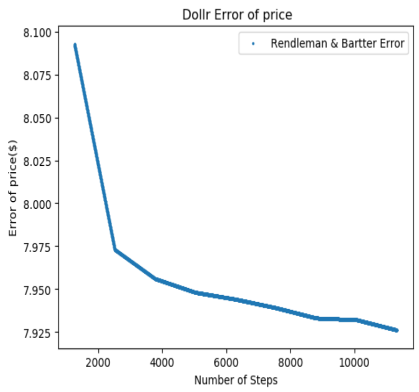
The 1% confidence interval based on the published price of 953.22 is range in [943.687, 962.752]. In the figure below, the price of all three models, regardless of the number of nodes, falls within the confidence interval. Error of Binomial and Rendleman&Bartter model seems to converge.



CRR Error



Binomial Error



Rendleman&Bartter Error

However, error of CRR model does not converge and has an odd-even effect. The mean error of each model is seen as below and we observe taht the average error of CRR model is the smallest.

	CRR	Binomial	Rendleman&Bartter
Mean of error	7.863	7.956	7.954