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# Model Predictive Torque Control of Synchronous Machines Without a Current or Stator Flux Reference Generator

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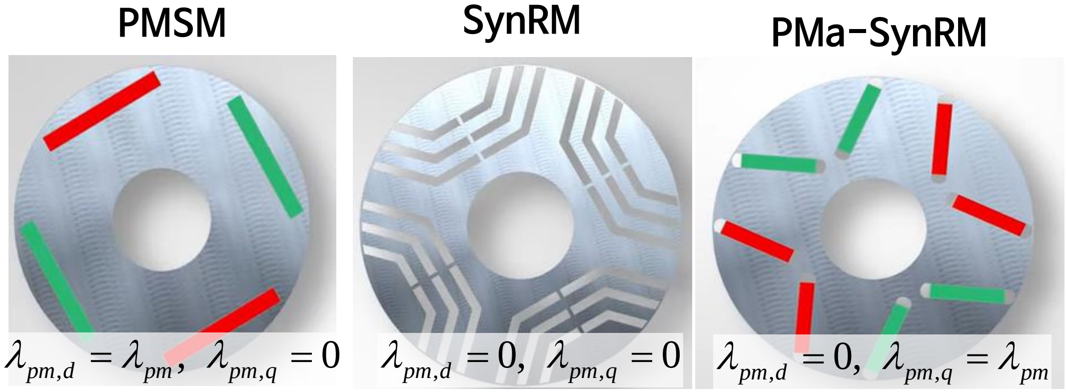
# 1. Introduction

# Synchronous Machines (SMs)

- **Modeling**

$$\lambda_d = L_d i_d + \lambda_{pm,d}$$
$$\lambda_q = L_q i_q + \lambda_{pm,q}$$



$\lambda_{pm,d} = \lambda_{pm},\ \lambda_{pm,q} = 0$ $\lambda_{pm,d} = 0,\ \lambda_{pm,q} = 0$ $\lambda_{pm,d} = 0,\ \lambda_{pm,q} = \lambda_{pm}$

**Flux-based model**

$$\frac{d\lambda_d}{dt} = -R_s i_d + w_r \lambda_q + v_d,$$
$$\frac{d\lambda_q}{dt} = -R_s i_q - w_r \lambda_d + v_q,$$
$$T_e = 1.5P(\lambda_d i_q - \lambda_q i_d).$$

**Current-based model**

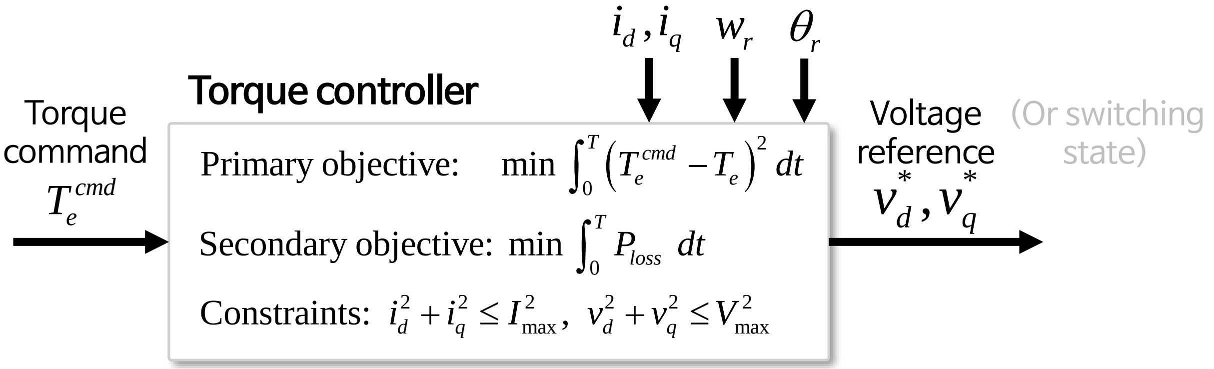
$$\frac{d(L_d i_d)}{dt} = -R_s i_d + w_r L_q i_q + v_d,$$
$$\frac{d(L_q i_q)}{dt} = -R_s i_q - w_r (L_d i_d + \lambda_{pm}) + v_q,$$
$$T_e = 1.5P(\lambda_{pm} + (L_d - L_q) i_d) i_q.$$

---

- $L_{d(q)}$ : $d(q)$-axis inductance, $\lambda_{pm}$ : $d(q)$-axis magnetic flux linkage, $R_s$ : stator resistance, $P$ : number of pole pairs, $J$ : rotational inertia, $B$ : damping coefficient, $T_l$ : load torque.

# Synchronous Machines (SMs)

- **Torque control**



Torque command $T_e^{cmd}$

**Torque controller**

$i_d, i_q$ $\quad w_r$ $\quad \theta_r$

Primary objective: $\min \int_0^T \left(T_e^{cmd} - T_e\right)^2 dt$

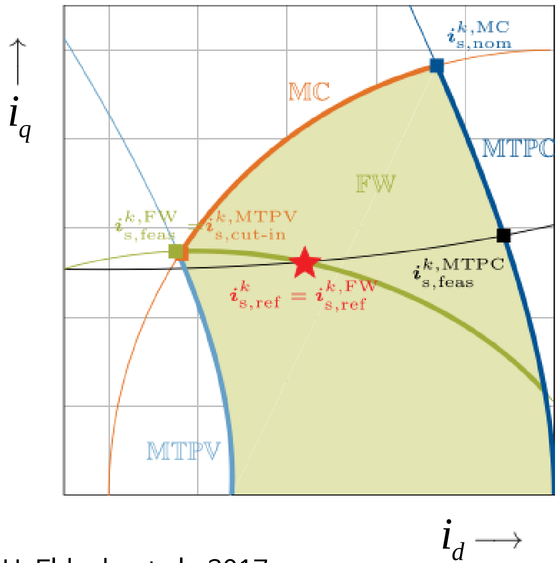
Secondary objective: $\min \int_0^T P_{loss}\ dt$

Constraints: $i_d^2 + i_q^2 \le I_{\max}^2,\ v_d^2 + v_q^2 \le V_{\max}^2$

Voltage reference $v_d^*, v_q^*$ (Or switching state)

**Note** DOF in the torque equation: $T_e = 1.5P(\lambda_{pm} + (L_d - L_q)\boxed{i_d})i_q$

## ➢ Need to optimize operating points [1]



## ➢ How to determine the operating point?

1) Solve the optimization analytically [1] / numerically [2]

**Case 1.** $T_e^* \le T_e^{\max}$

$\min\ i_d^2 + i_q^2$

subject to $T_e^* = (k_1 + k_2 i_d) i_q,$

$v_d^2 + v_q^2 \le V_{\max}^2.$

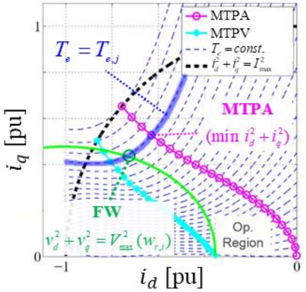
**Case 2.** $T_e^* > T_e^{\max}$

$\max\ \operatorname{sgn}(T_e) T_e$

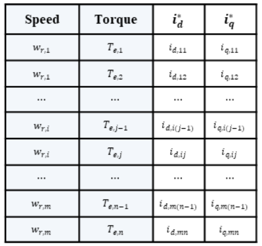
subject to $v_d^2 + v_q^2 \le V_{\max}^2,$

$i_d^2 + i_q^2 \le I_{\max}^2.$

2) Identify experimentally [3]



Stored in LUTs

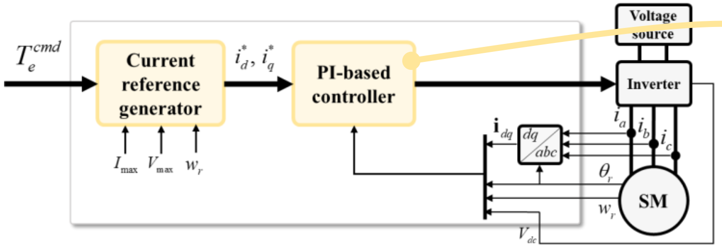


| **Speed** | **Torque** | $i_d^*$ | $i_q^*$ |
|---|---|---|---|
| $w_{r,1}$ | $T_{e,1}$ | $i_{d,11}$ | $i_{q,11}$ |
| $w_{r,1}$ | $T_{e,2}$ | $i_{d,12}$ | $i_{q,12}$ |
| … | … | … | … |
| $w_{r,i}$ | $T_{e,j-1}$ | $i_{d,i(j-1)}$ | $i_{q,i(j-1)}$ |
| $w_{r,i}$ | $T_{e,j}$ | $i_{d,ij}$ | $i_{q,ij}$ |
| … | … | … | … |
| $w_{r,m}$ | $T_{e,n-1}$ | $i_{d,m(n-1)}$ | $i_{q,m(n-1)}$ |
| $w_{r,m}$ | $T_{e,n}$ | $i_{d,mn}$ | $i_{q,mn}$ |

[1] H. Eldeeb, et al., 2017.
[2] K. Choi, et al., 2020.
[3] B. Gallert, et al., 2017

# Existing schemes for Torque control of SMs

## Reference generator + PI-based controller [4]



**PI control**

$$v^*_{d,fb} = K_{pd}e_d + K_{id}\int e_d$$
$$v^*_{q,fb} = K_{pq}e_q + K_{iq}\int e_q$$

+

**Anti-windup**

$$e_d \leftarrow e_d - K_{ad}(v^*_d - v_d)$$
$$e_q \leftarrow e_q - K_{aq}(v^*_q - v_q)$$

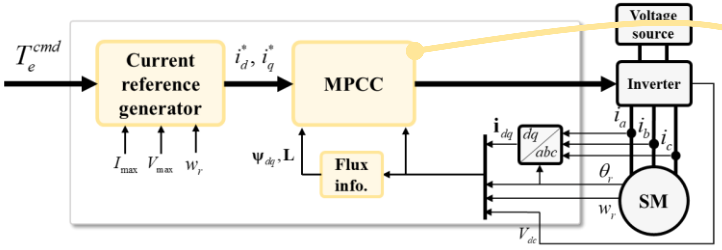
+

**FF comp.**

$$v^*_{d,ff} = -w_r\hat{L}_q i_q$$
$$v^*_{q,ff} = w_r(\hat{L}_d i_d + \hat{\lambda}_{pm})$$

## Reference generator + MPCC [5]

MPCC: Model Predictive Current Control

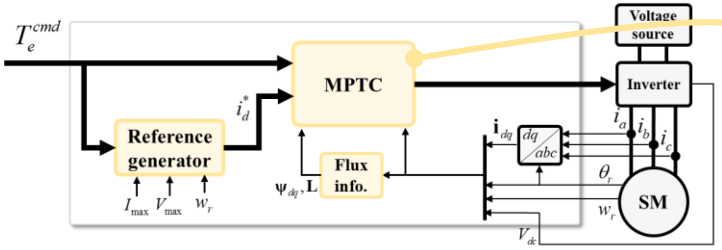


$$\min_{u_k,u_{k+1},\cdots,u_{k+N-1}} \sum_{j=1}^{N_p} \left(i_d^* - i_{d,k+j}\right)^2 + \left(i_q^* - i_{q,k+j}\right)^2 + \alpha J_{p,k+j}$$

$$\text{s.t. } i^2_{d,k+j} + i^2_{q,k+j} \le I^2_{\max},\ v^2_{d,k+j-1} + v^2_{q,k+j-1} \le V^2_{\max}$$

## Reference generator + MPTC [6]

MPTC: Model Predictive Torque Control



$$\min_{u_k,u_{k+1},\cdots,u_{k+N-1}} \sum_{j=1}^{N_p} \left(T_e^{cmd} - T_{e,k+j}\right)^2 + \left(i_d^* - i_{d,k+j}\right)^2 + \alpha J_{p,k+j}$$

$$\text{s.t. } i^2_{d,k+j} + i^2_{q,k+j} \le I^2_{\max},\ v^2_{d,k+j-1} + v^2_{q,k+j-1} \le V^2_{\max}$$

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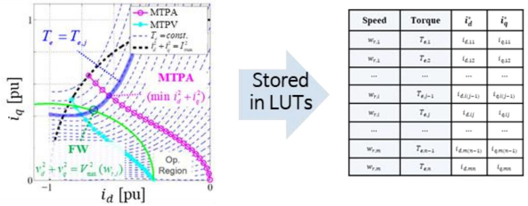
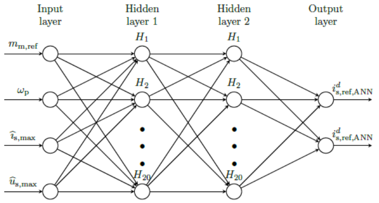
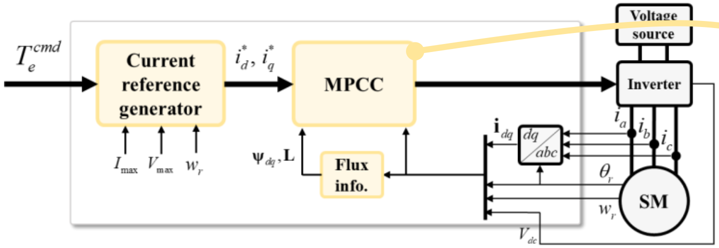
[4] S.-K. Sul, Control of electric machine drive systems. *John Wiley & Sons*, 2011, vol. 88.
[5] J. Rodriguez, et al., “Predictive current control of a voltage source inverter,” *IEEE TIE*, vol. 54, no. 1, pp. 495 - 503, 2007.
[6] T. Englert and K. Graichen, “Nonlinear model predictive torque control of PMSMs for high performance applications,” *CEP Eng. Prac.*, vol. 81, pp. 43 - 54, 2018.

# Existing schemes for Torque control of SMs

- **Most schemes rely on a reference generator.**
  - However, implementing a reference generator requires a lot of resources.

| Experimental [3] | Analytical [1] /numerical [2] | ANN-based [7] |
|---|---|---|
|  <br>➢ Experiment time (e.g., 3 months) | **Case 1**. $T_e^* \le T_e^{max}$<br>min $i_d^2 + i_q^2$<br>subject to $T_e^* = (k_1 + k_2 i_d) i_q$, $v_d^2 + v_q^2 \le V_{max}^2$.<br>**Case 2**. $T_e^* > T_e^{max}$<br>max $\mathrm{sgn}(T_e) T_e$<br>subject to $v_d^2 + v_q^2 \le V_{max}^2$, $i_d^2 + i_q^2 \le I_{max}^2$.<br>➢ Computation time [7] (e.g., ~400 us) |  <br>➢ Training [7] |

- In addition, using a reference generator restricts MPC from fully utilizing the DOF.
  - E.g.,



**MTPA point** vs **Inverter loss**

$$\min_{u_k, u_{k+1}, \cdots, u_{k+N-1}} \sum_{j=1}^{N_p} \left( i_d^* - i_{d,k+j} \right)^2 + \left( i_q^* - i_{q,k+j} \right)^2 + \alpha J_{p,k+j}$$

$$\text{s.t. } i_{d,k+j}^2 + i_{q,k+j}^2 \le I_{max}^2,\ v_{d,k+j-1}^2 + v_{q,k+j-1}^2 \le V_{max}^2$$

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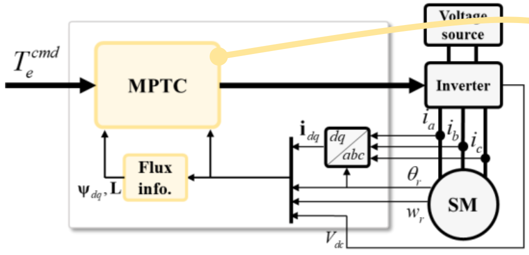
[7] M. A. Buettner, N. Monzen, and C. M. Hackl, "Artificial neural network based optimal feedforward torque control of interior permanent magnet synchronous machines: A feasibility study and comparison with the state-of-the-art," *Energies,* vol. 15, no. 5, p. 1838, 2022.

# Existing schemes for Torque control of SMs

- **One of the few attempts not to use a reference generator.**

## MPTC [8]

MPTC: Model Predictive Torque Control

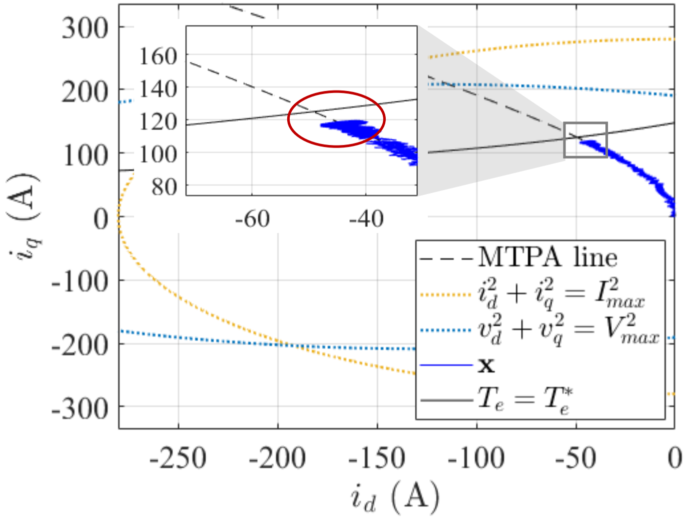


$$\min_{u_k,u_{k+1},\cdots,u_{k+N-1}} \sum_{j=1}^{N_p} \left(T_e^{cmd} - T_{e,k+j}\right)^2 + \alpha J_{p,k+j}$$

$$\text{s.t. } i_{d,k+j}^2 + i_{q,k+j}^2 \le I_{\max}^2,\ v_{d,k+j-1}^2 + v_{q,k+j-1}^2 \le V_{\max}^2$$

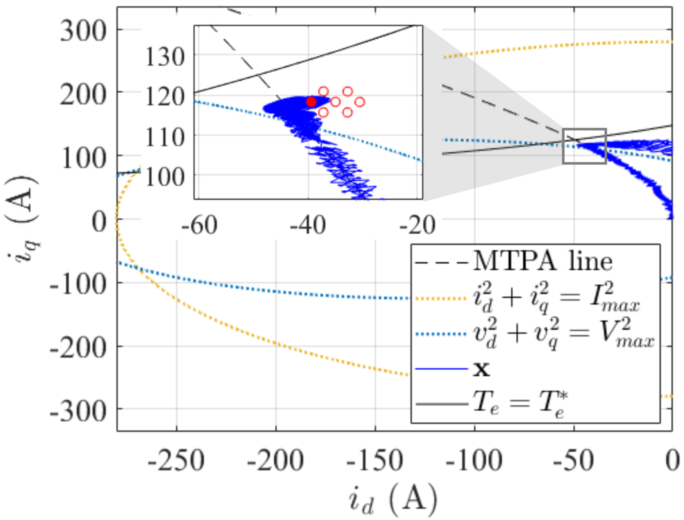
- $J_{p,k+j} = 1.5R_s(i_{d,k+j}^2 + i_{q,k+j}^2)$

**Limitation 1. Trade-off b/w the objectives**



**Limitation 2. Instability in the FW region**

FW: Flux-weakening



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[8] L. Samaranayake and S. Longo, “Degradation control for electric vehicle machines using nonlinear model predictive control,” *IEEE TCST,* vol. 26, no. 1, pp. 89 - 101, 2017.

# Research Objectives

- **Develop a MPTC scheme for SMs that**
  - Does not rely on a reference generator
  - But guarantees optimal operation under all operating regions
  - Can be implemented based on both the FCS and CCS
    - FCS: finite control set, CCS: continuous control set
  - Can be implemented with various performance indices ($J_p$)

**Primary objectives**

**Secondary objectives**

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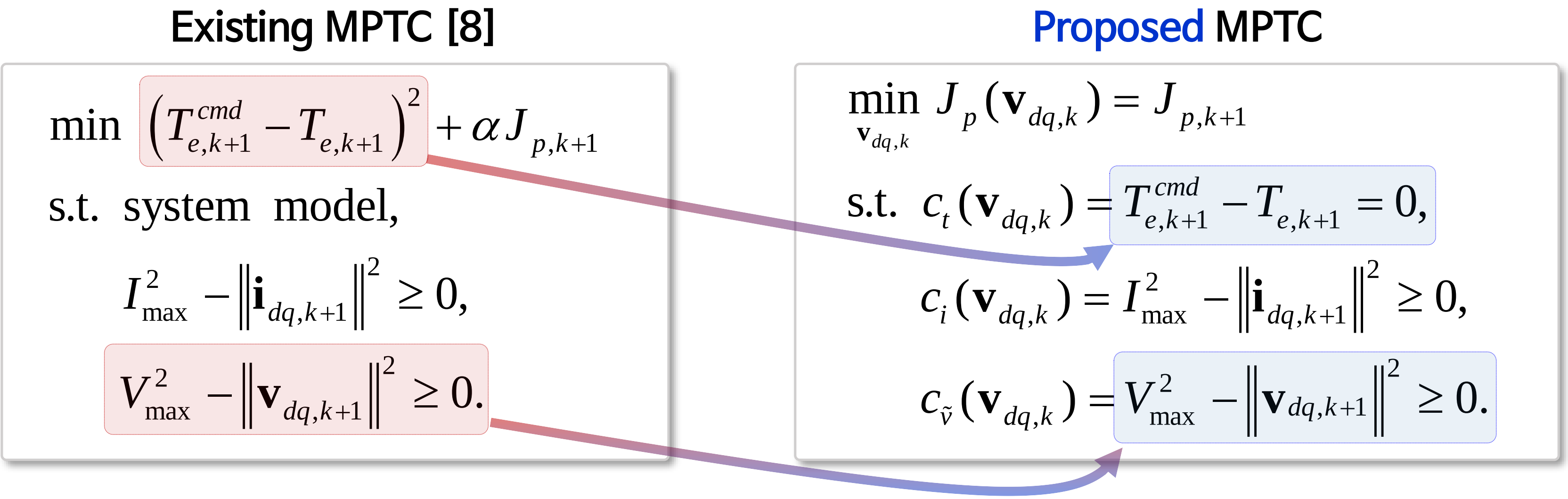
[5] J. Rodriguez, et al., “Predictive current control of a voltage source inverter,” *IEEE TIE*, vol. 54, no. 1, pp. 495 - 503, 2007.
[6] T. Englert and K. Graichen, “Nonlinear model predictive torque control of PMSMs for high performance applications,” *Control Eng. Prac.,* vol. 81, pp. 43 - 54, 2018.
[7] L. Samaranayake and S. Longo, “Degradation control for electric vehicle machines using nonlinear model predictive control,” *IEEE TCST*, vol. 26, no. 1, pp. 89 - 101, 2017.

# 2. Proposed MPTC

# Problem Formulation

- **Overcome two limitations of the existing MPTC [8] by modifying it.**
  - **Limitation 1.** Trade-off in the objective function
    - Resolved by moving the torque error term to the equality constraint.
  - **Limitation 2.** Instability in the FW region
    - Resolved by modifying the voltage constraint.



**Existing MPTC [8]**

$$\min \ \left(T_{e,k+1}^{cmd} - T_{e,k+1}\right)^2 + \alpha J_{p,k+1}$$

$$\text{s.t. system model,}$$

$$I_{\max}^2 - \left\|\mathbf{i}_{dq,k+1}\right\|^2 \geq 0,$$

$$V_{\max}^2 - \left\|\mathbf{v}_{dq,k+1}\right\|^2 \geq 0.$$

**Proposed MPTC**

$$\min_{\mathbf{v}_{dq,k}} \ J_p(\mathbf{v}_{dq,k}) = J_{p,k+1}$$

$$\text{s.t.} \ c_t(\mathbf{v}_{dq,k}) = T_{e,k+1}^{cmd} - T_{e,k+1} = 0,$$

$$c_i(\mathbf{v}_{dq,k}) = I_{\max}^2 - \left\|\mathbf{i}_{dq,k+1}\right\|^2 \geq 0,$$

$$c_{\tilde{v}}(\mathbf{v}_{dq,k}) = V_{\max}^2 - \left\|\mathbf{v}_{dq,k+1}\right\|^2 \geq 0.$$

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[8] L. Samaranayake and S. Longo, “Degradation control for electric vehicle machines using nonlinear model predictive control,” *IEEE TCST,* vol. 26, no. 1, pp. 89 - 101, 2017.

# Problem Formulation

- **Overcome two limitations of the existing MPTC [8] by modifying it.**
  - **Limitation 1**. Trade-off in the objective function
    - Resolved by moving the torque error term to the equality constraint.
    - *Is it possible to consider the tracking error as an equality constraint?*

      **Yes**, with a solver that allows tolerance during the optimization process.

**Existing MPTC [8]**

$$\begin{aligned} \min \ & \left(T_{e,k+1}^{cmd} - T_{e,k+1}\right)^2 + \alpha J_{p,k+1} \\ \text{s.t.} \ & \text{system model}, \\ & I_{\max}^2 - \left\|\mathbf{i}_{dq,k+1}\right\|^2 \geq 0, \\ & V_{\max}^2 - \left\|\mathbf{v}_{dq,k}\right\|^2 \geq 0. \end{aligned}$$

**Proposed MPTC**

$$\begin{aligned} \min_{\mathbf{v}_{dq,k}} \ & J_p(\mathbf{v}_{dq,k}) = J_{p,k+1} \\ \text{s.t.} \ & c_t(\mathbf{v}_{dq,k}) = T_{e,k+1}^{cmd} - T_{e,k+1} = 0, \\ & c_i(\mathbf{v}_{dq,k}) = I_{\max}^2 - \left\|\mathbf{i}_{dq,k+1}\right\|^2 \geq 0, \\ & c_{\tilde{v}}(\mathbf{v}_{dq,k}) = V_{\max}^2 - \left\|\mathbf{v}_{dq,k+1}\right\|^2 \geq 0. \end{aligned}$$

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[8] L. Samaranayake and S. Longo, "Degradation control for electric vehicle machines using nonlinear model predictive control," *IEEE TCST,* vol. 26, no. 1, pp. 89 - 101, 2017.

# Problem Formulation

- **Overcome two limitations of the existing MPTC [8] by modifying it.**
  - **Limitation 2.** Instability in the FW region
    - Resolved by modifying the voltage constraint.

**System model**

$$d\lambda_d / dt = -R_s i_d + w_r \lambda_q + v_d,$$
$$d\lambda_q / dt = -R_s i_q - w_r \lambda_d + v_q.$$

**System model in steady-states**

$$0 = -R_s i_d + w_r \lambda_q + \tilde{v}_d,$$
$$0 = -R_s i_q - w_r \lambda_d + \tilde{v}_q.$$

**Existing MPTC [8]**

$$\min \ \left(T_{e,k+1}^{cmd} - T_{e,k+1}\right)^2 + \alpha J_{p,k+1}$$
$$\text{s.t. system model,}$$
$$I_{\max}^2 - \left\|\mathbf{i}_{dq,k+1}\right\|^2 \geq 0,$$
$$V_{\max}^2 - \left\|\mathbf{v}_{dq,k}\right\|^2 \geq 0.$$

**Proposed MPTC**

$$\min_{\mathbf{v}_{dq,k}} J_p(\mathbf{v}_{dq,k}) = J_{p,k+1}$$
$$\text{s.t. } c_t(\mathbf{v}_{dq,k}) = T_{e,k+1}^{cmd} - T_{e,k+1} = 0,$$
$$c_i(\mathbf{v}_{dq,k}) = I_{\max}^2 - \left\|\mathbf{i}_{dq,k+1}\right\|^2 \geq 0,$$
$$c_{\tilde{v}}(\mathbf{v}_{dq,k}) = V_{\max}^2 - \left\|\mathbf{v}_{dq,k+1}\right\|^2 \geq 0.$$

[8] L. Samaranayake and S. Longo, “Degradation control for electric vehicle machines using nonlinear model predictive control,” *IEEE TCST,* vol. 26, no. 1, pp. 89 - 101, 2017.

# Problem Formulation

- **Overcome two limitations of the existing MPTC [8] by modifying it.**
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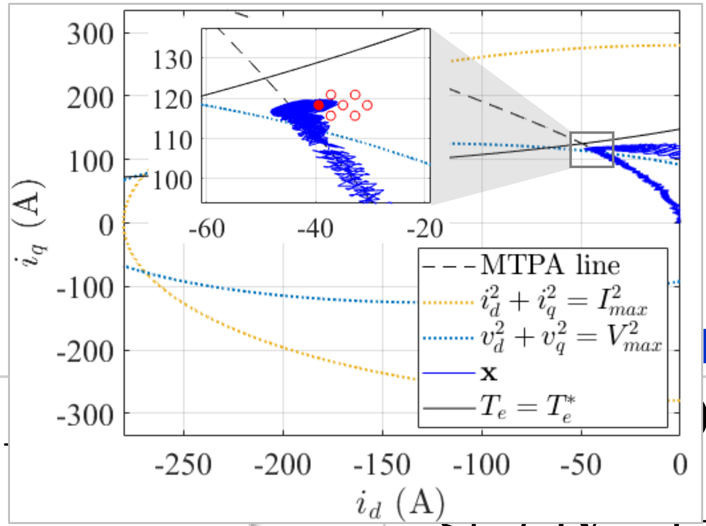

**Existing MPTC**

$$\min \left(T_{e,k+1}^{cmd} - T_{e,k+1}\right)^2$$

s.t. system model,

$$I_{\max}^2 - \left\|\mathbf{i}_{dq,k+1}\right\|^2 \geq 0,$$

$$V_{\max}^2 - \left\|\mathbf{v}_{dq,k}\right\|^2 \geq 0.$$

**posed MPTC**

$$) = J_{p,k+1}$$

$$\text{s.t.} \quad c_t(\mathbf{v}_{dq,k}) = T_{e,k+1}^{cmd} - T_{e,k+1} = 0,$$

$$c_i(\mathbf{v}_{dq,k}) = I_{\max}^2 - \left\|\mathbf{i}_{dq,k+1}\right\|^2 \geq 0,$$

$$c_{\tilde{v}}(\mathbf{v}_{dq,k}) = V_{\max}^2 - \left\|\mathbf{v}_{dq,k+1}\right\|^2 \geq 0.$$

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[8] L. Samaranayake and S. Longo, “Degradation control for electric vehicle machines using nonlinear model predictive control,” *IEEE TCST,* vol. 26, no. 1, pp. 89 - 101, 2017.

# Solver

- **The augmented Lagrangian method is adopted to solve the proposed MPTC.**
  - **Concept**

**Nonlinear Programming (NLP)**

$$\begin{aligned} &\min_{x} \; J_p(x) \\ &\text{s.t.} \quad c_i(x) = 0,\; i \in \mathrm{E}, \\ &\qquad\;\; c_i(x) \geq 0,\; i \in \mathrm{I}. \end{aligned}$$

**Minimize the augmented Lagrangian function instead**

$$\begin{aligned} &\min_{x} \; L_A(x, \lambda_k; \mu_k) \\ &\quad = J_p(x) - \sum_{i \in \mathrm{E}} \lambda_i c_i(x) + \frac{1}{2\mu} \sum_{i \in \mathrm{E}} \lambda_i c_i^2(x) + \sum_{i \in \mathrm{I}} \psi(c_i(x), \lambda_i; \mu) \end{aligned}$$

**Multiplier update:**

$$\lambda_{i,k+1} = \lambda_{i,k} - c_i(x_k) / \mu_k, \quad i \in \mathrm{E},$$

$$\lambda_{i,k+1} = \max(\lambda_{i,k} - c_i(x_k) / \mu_k, 0), \quad i \in \mathrm{I}.$$

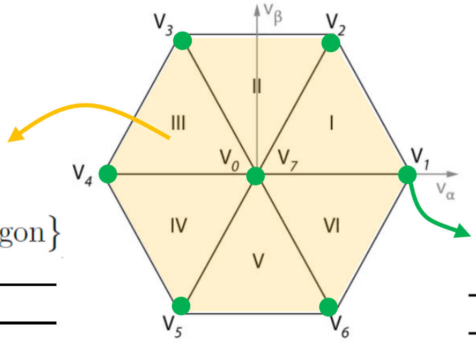
  - **Properties**
    - 1. Can handle NLP → Allow $J_p$ to be any function (e.g., copper loss or inverter loss)
    - 2. Allow tolerances to constraints
      → Allow the tracking error to be the equality constraint
      → Allow smooth transitions b/w operating regions (i.e., MTPA, FW, MC, MTPV)

---

- $\psi(c_i(x), \lambda_i; \mu) = \begin{cases} -\lambda_i c_i(x) + c_i^2(x) / (2\mu), & \text{if } c_i(x) - \mu\lambda_i \leq 0, \\ -\mu\lambda_i^2 / 2, & \text{otherwise.} \end{cases}$

# Solver

- **Implementation**



## Proposed MPTC based on CCS

$$\mathbf{V}_c = \left\{ \mathbf{v}_{dq,k} \in \mathbb{R}^2 \middle| \, \mathbf{v}_{dq,k} \text{ is within the limit hexagon} \right\}$$

**Algorithm 1:** CCS-based GMPTC solver

```
Determine the expression of $J_p$;
Choose positive parameters $\mu_t, \mu_{\tilde{v}}, \mu_i$
Set $\lambda_{t,1} \leftarrow 0,\ \lambda_{i,1} \leftarrow 0,\ \lambda_{\tilde{v},1} \leftarrow 0,\ \mathbf{v}^*_{dq,0} \leftarrow 0$;
for $k = 1, 2, 3, \dots$ do
    Input: $T^{cmd}_{e,k+1}$
    Set $\mathbf{v}^*_{dq,k} \leftarrow \mathbf{v}^*_{dq,k-1}$;
    repeat
        $\mathbf{v}^*_{dq,k} \leftarrow \mathbf{v}^*_{dq,k} - \nabla^2 L_{A1}^{-1}\left(\mathbf{v}^*_{dq,k}, \boldsymbol{\lambda}_k, \boldsymbol{\mu}\right) \nabla L_{A1}\left(\mathbf{v}^*_{dq,k}, \boldsymbol{\lambda}_k, \boldsymbol{\mu}\right)$;
    until $\left\| \nabla L_{A1}\left(\mathbf{v}^*_{dq,k}, \boldsymbol{\lambda}_k, \boldsymbol{\mu}\right) \right\| \leq \tau$;
    $\lambda_{t,k+1} = \lambda_{t,k} - c_t(\mathbf{v}^*_{dq,k})/\mu_t$;
    $\lambda_{i,k+1} = \max\left(\lambda_{i,k} - c_i(\mathbf{v}^*_{dq,k})/\mu_i, 0\right)$;
    $\lambda_{\tilde{v},k+1} = \max\left(\lambda_{\tilde{v},k} - c_{\tilde{v}}(\mathbf{v}^*_{dq,k})/\mu_{\tilde{v}}, 0\right)$;
    Limit $\mathbf{v}^*_{dq,k}$ within the CCS;
    Output: $\mathbf{v}^*_{dq,k}$
```

Tuning parameters

**Any combinations of**
- Copper loss
- Iron loss
- Inverter loss
- Temperature
- …

## Proposed MPTC based on FCS

$$\mathbf{V}_f = \left\{ \mathbf{v}^{(0)}_{dq,k}, \mathbf{v}^{(1)}_{dq,k}, \mathbf{v}^{(2)}_{dq,k}, \mathbf{v}^{(3)}_{dq,k}, \mathbf{v}^{(4)}_{dq,k}, \mathbf{v}^{(5)}_{dq,k}, \mathbf{v}^{(6)}_{dq,k}, \mathbf{v}^{(7)}_{dq,k} \right\}$$

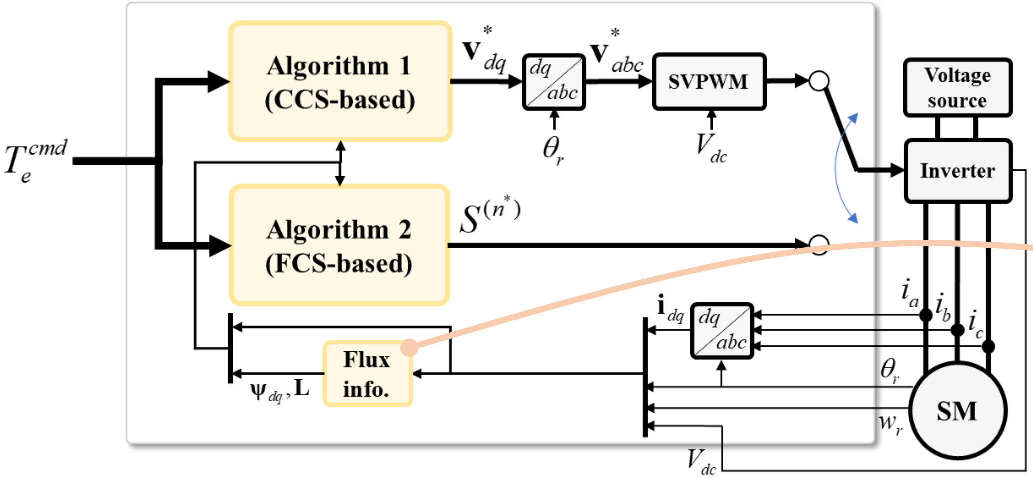
**Algorithm 2:** FCS-based GMPTC solver

```
Determine the expression of $J_p$;
Choose positive parameters $\mu_t, \mu_{\tilde{v}}, \mu_i$
Set $\lambda_{t,1} \leftarrow 0,\ \lambda_{i,1} \leftarrow 0,\ \lambda_{\tilde{v},1} \leftarrow 0,\ \mathbf{v}^*_{dq,0} \leftarrow 0$;
for $k = 1, 2, 3, \dots$ do
    Input: $T^{cmd}_{e,k+1}$
    Compute $L_{A1}\left(\mathbf{v}^{(n)}_{dq,k}, \boldsymbol{\lambda}_k, \boldsymbol{\mu}\right)$ for $n = 0, \cdots, 7$;
    $n^* = \arg\min_n L_{A1}(\mathbf{v}^{(n)}_{dq,k}, \boldsymbol{\lambda}_k, \boldsymbol{\mu})$;
    $\lambda_{t,k+1} = \lambda_{t,k} - c_t(\mathbf{v}^{(n^*)}_{dq,k})/\mu_t$;
    $\lambda_{i,k+1} = \max\left(\lambda_{i,k} - c_i(\mathbf{v}^{(n^*)}_{dq,k})/\mu_i, 0\right)$;
    $\lambda_{\tilde{v},k+1} = \max\left(\lambda_{\tilde{v},k} - c_{\tilde{v}}(\mathbf{v}^{(n^*)}_{dq,k})/\mu_{\tilde{v}}, 0\right)$;
    Set $\mathbf{v}^*_{dq,k} \leftarrow \mathbf{v}^{(n^*)}_{dq,k}$;
    Output: $S^{(n^*)}$
```

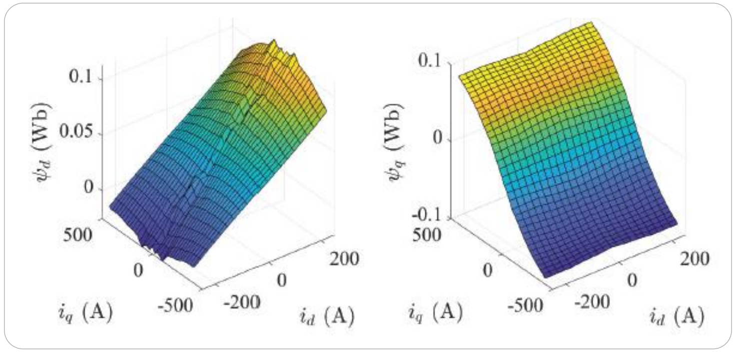
FCS: finite control set
CCS: continuous control set

# Schematic Diagrams

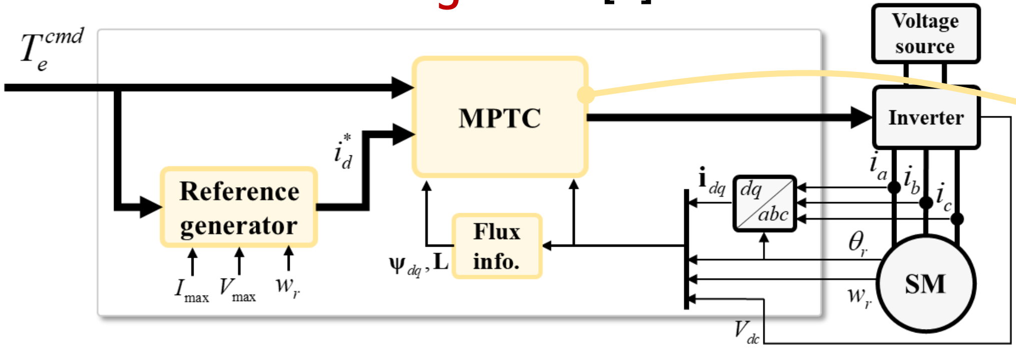
## Proposed MPTC



## Required information



## Existing MPTC [6]



$$\min_{u_k,u_{k+1},\cdots,u_{k+N-1}} \sum_{j=1}^{N_p} \left(T_e^{cmd} - T_{e,k+j}\right)^2 + \left(i_d^* - i_{d,k+j}\right)^2 + \alpha J_{p,k+j}$$

s.t. $i_{d,k+j}^2 + i_{q,k+j}^2 \le I_{\max}^2,\ v_{d,k+j-1}^2 + v_{q,k+j-1}^2 \le V_{\max}^2$

---

[6] T. Englert and K. Graichen, "Nonlinear model predictive torque control of PMSMs for high performance applications," *CEP Eng. Prac.*, vol. 81, pp. 43 - 54, 2018.

# 3. Validation

# Simulation Setup

- **Use MATLAB/SIMULINK example and modify the control part.**

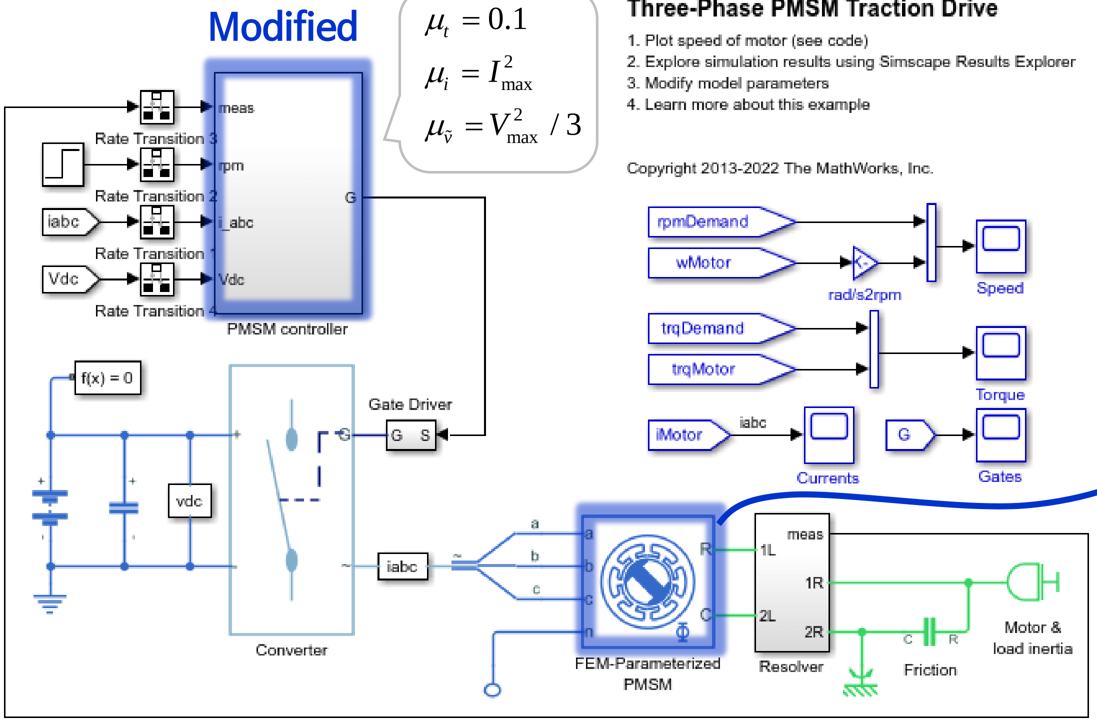
**Tuning parameters**

$$\mu_t = 0.1$$

$$\mu_i = I_{\max}^2$$

$$\mu_{\tilde{v}} = V_{\max}^2 / 3$$

**Modified**



**Three-Phase PMSM Traction Drive**

1. Plot speed of motor (see code)
2. Explore simulation results using Simscape Results Explorer
3. Modify model parameters
4. Learn more about this example

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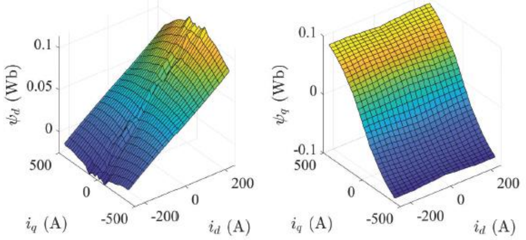
## 50 kW IPMSM model

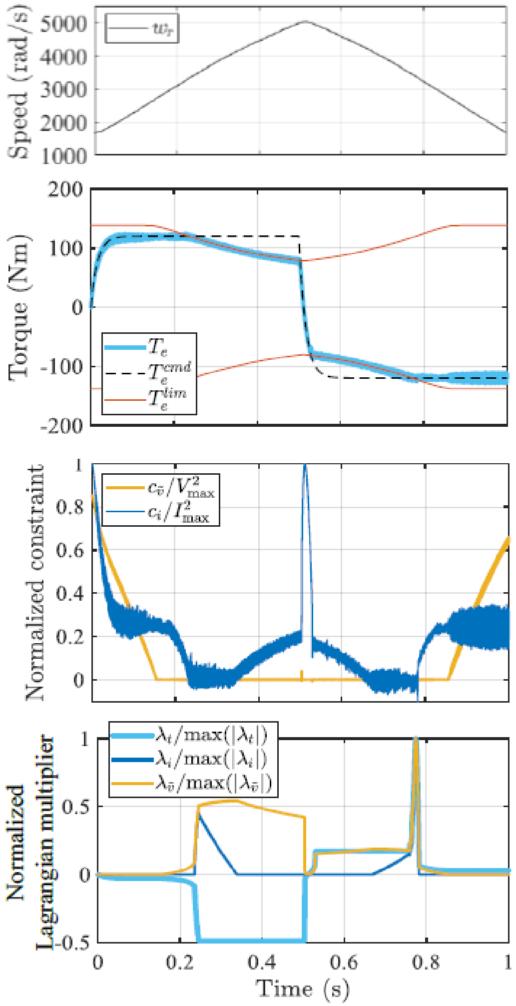
| | |
|---|---|
| Base speed | 3200 RPM |
| Maximum torque | 150 Nm |
| Rotor inertia | 0.1234 kg·m$^2$ |
| $I_{max}$ | 250 A |
| $V_{dc}$ | 325 V |
| $R_s$ | 10 mΩ |
| $P$ | 8 |
| $T_s$ | 20 $\mu$s |



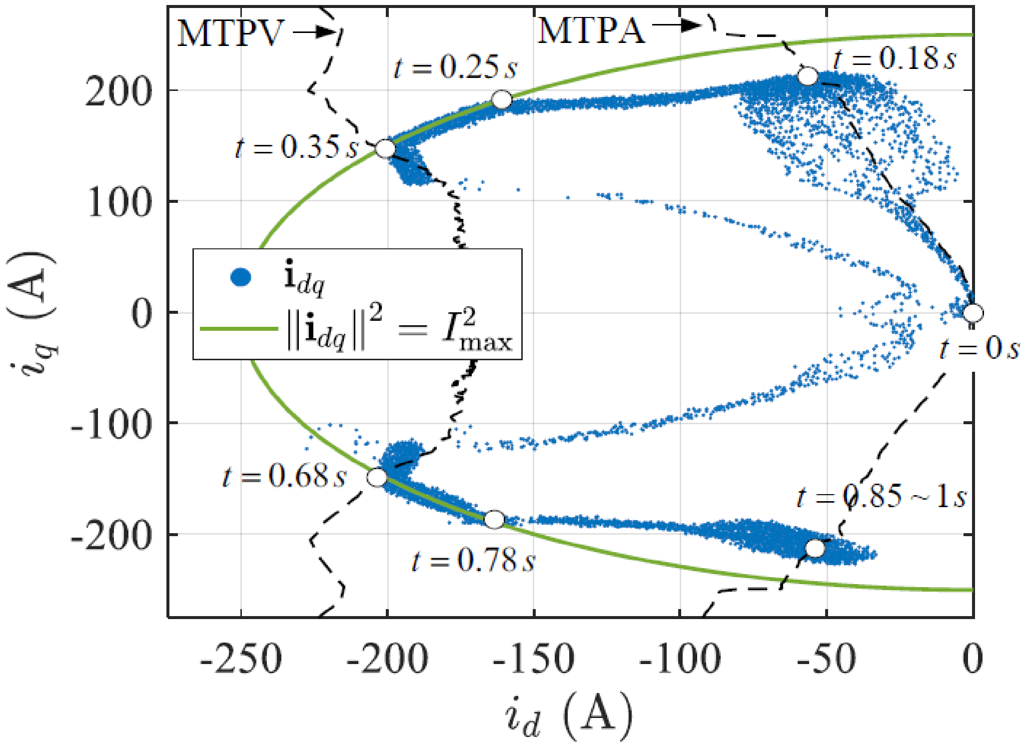
# Simulation Results

$J_p = P_{cu}$

- **Simulation 1. Validation of the Proposed MPTC**
  - Based on the CCS $(T_s = 50\mu s)$



Current locus

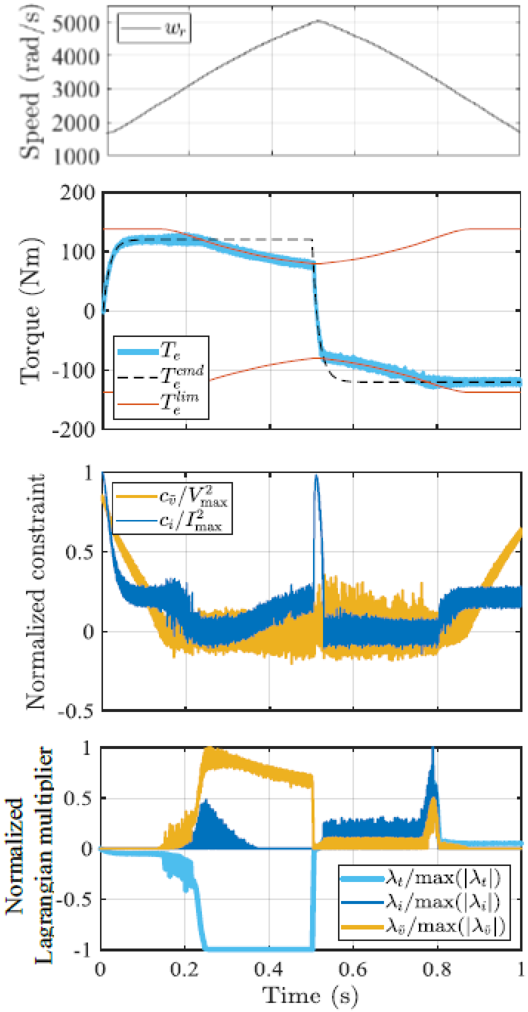


➢ Guarantee optimal operations in all regions.

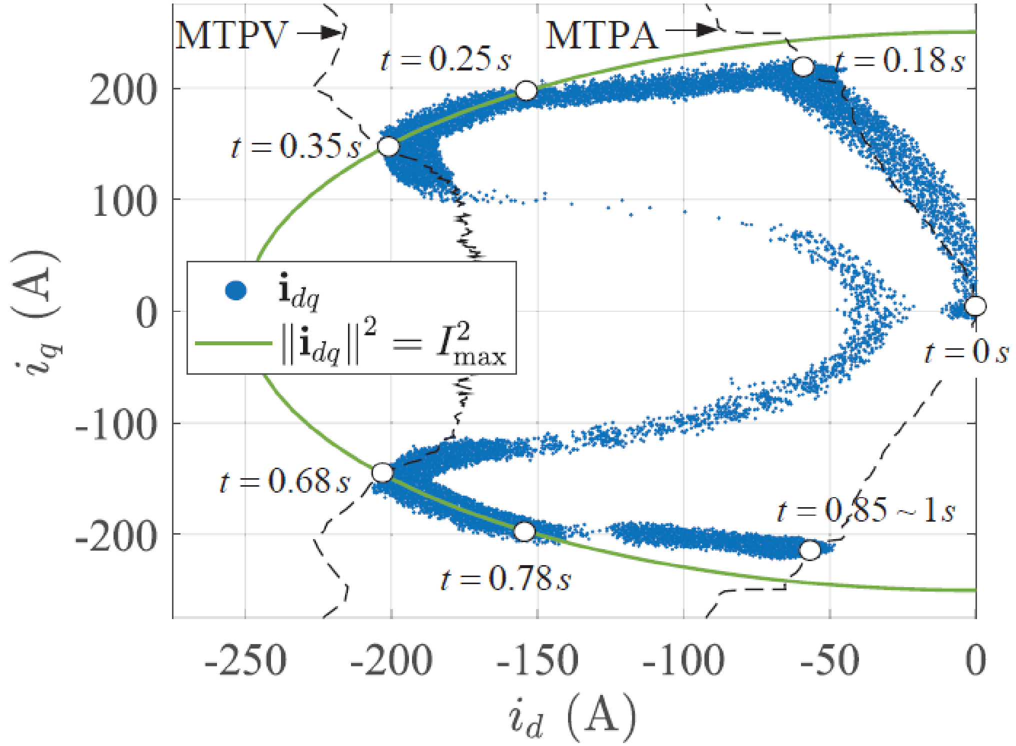
# Simulation Results

- **Simulation 1. Validation of the Proposed MPTC**
  - Based on the FCS ($T_s = 20\mu s$)

$$J_p = P_{cu}$$



**Current locus**



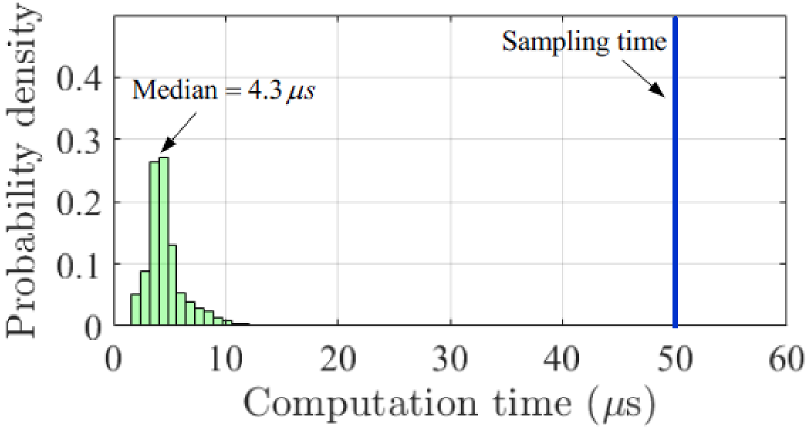
➢ Guarantee optimal operations in all regions.

# Simulation Results

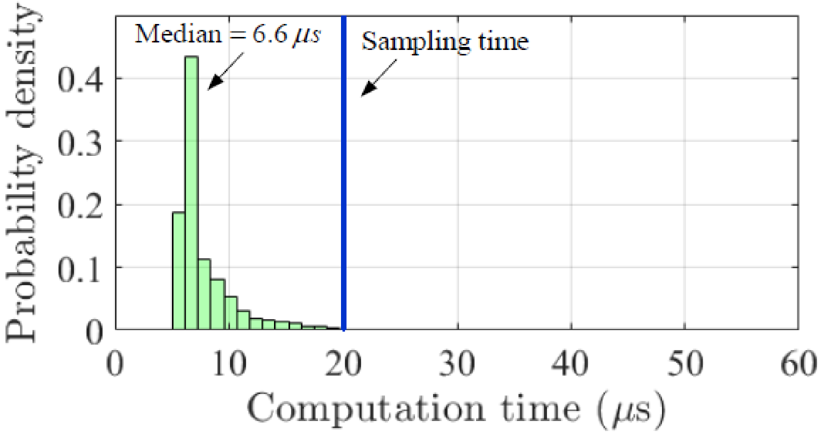
- **Simulation 1. Validation of the Proposed MPTC**
  - Computation time

$$J_p = P_{cu}$$

CCS



FCS

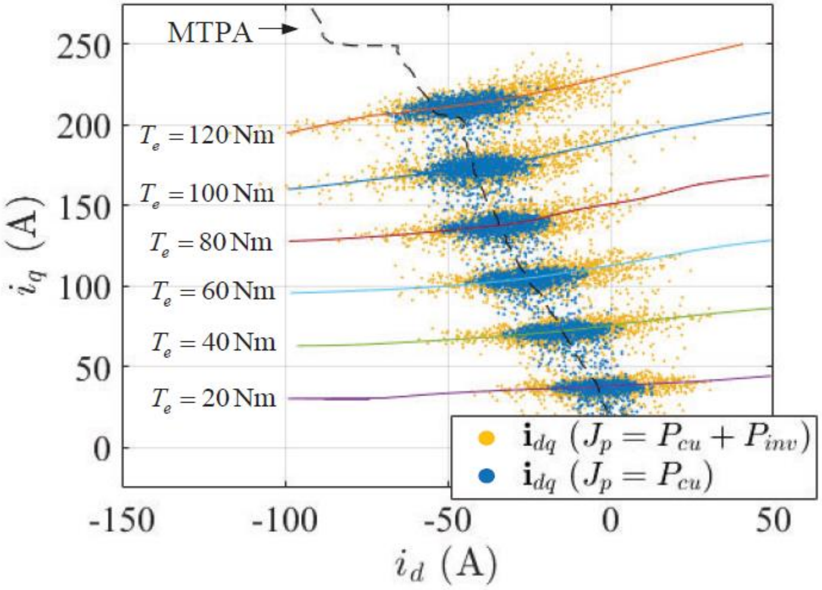


# Simulation Results

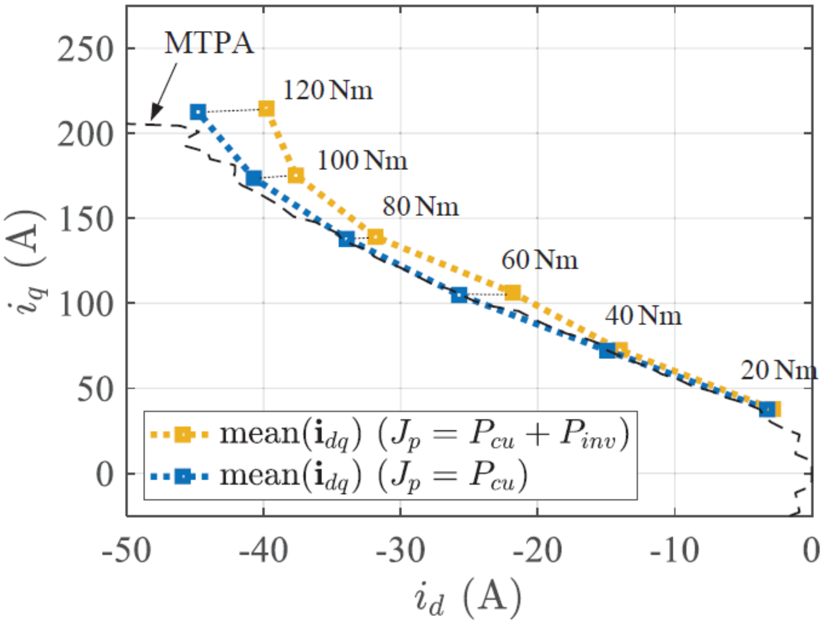
$J_p = P_{cu}$ vs. $J_p = P_{cu} + P_{inv}$

- **Simulation 2. Effects of Using Different Performance Indices**
  - Based on the FCS ($T_s = 20\mu s$)

Current locus



Mean current locus



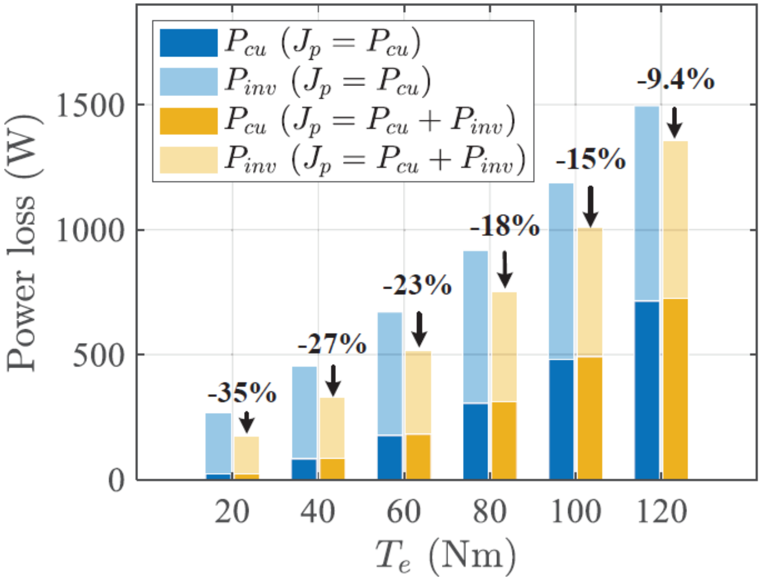
- Different performance indices result in different current loci.

# Simulation Results

$J_p = P_{cu}$ vs. $J_p = P_{cu} + P_{inv}$

- **Simulation 2. Effects of Using Different Performance Indices**
  - Based on the FCS ($T_s = 20\mu s$)

**Power loss**



– Using a different performance index can improve performance significantly.

# 4. Conclusion & Further Work

# Conclusion

- **A novel MPTC scheme was presented that**
  - Does not rely on a reference generator,
  - Guarantees optimal operation under all operating regions.

- **Two key ideas were**
  - Moving the torque error term to the equality constraint,
  - Redefining the voltage constraint.

- **The proposed MPTC is a general approach in that**
  - It can be implemented without a reference generator,
  - It can be implemented  based on both the FCS and CCS,
  - Various performance indices can be used.

# Further Work – Toward Intelligent SM controller

- **How to control SMs without prior information and offline tuning?**

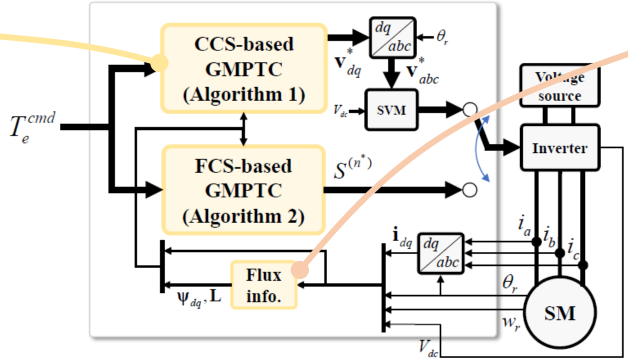
**Proposed MPTC + @**

**Tuning parameters:**

$\mu_t, \mu_i, \mu_{\tilde{v}}$

**Future work 2.**

Applying AI-based auto gain tuning



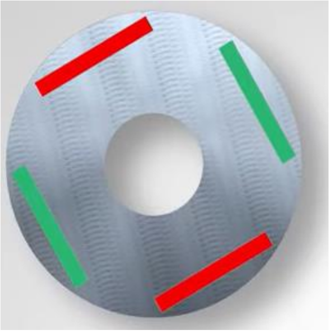
**Required information:**

$\lambda_d, \lambda_q$

**Future work 1.**

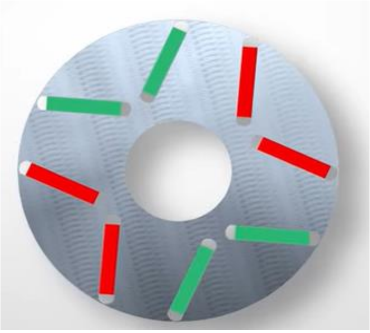
Applying a stator flux linkage estimator

**PMSM**



**SynRM**

**PMa-SynRM**



# Appendix

# When the torque command ($T_e^{cmd}$) is not achievable

- **Need to solve a different MPTC problem**

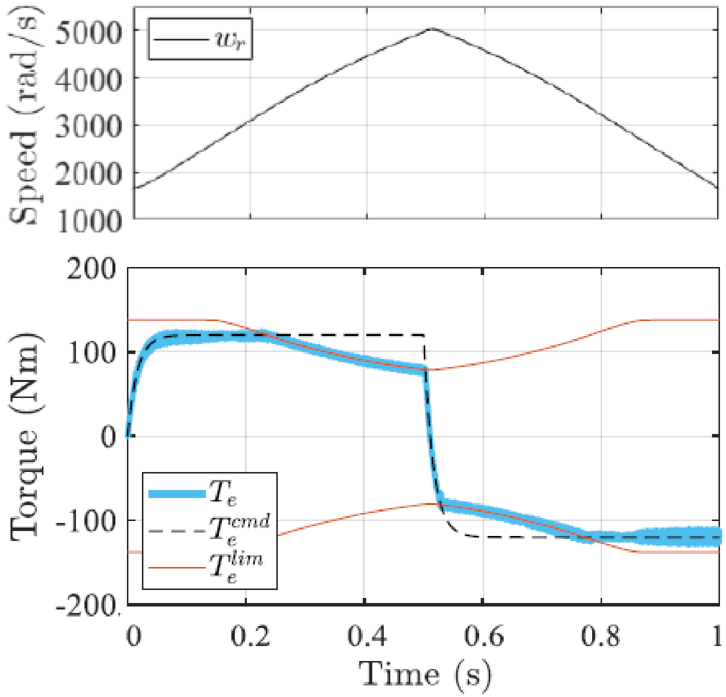
### When $T_e^{cmd}$ is achievable

$$\begin{aligned}
\min_{\mathbf{v}_{dq,k}} \quad & J_p(\mathbf{v}_{dq,k}) = J_{p,k+1} && \text{(10a)}\\
\text{s.t.} \quad & c_t(\mathbf{v}_{dq,k}) = T_{e,k+1}^{cmd} - T_{e,k+1} = 0, && \text{(10b)}\\
& c_i(\mathbf{v}_{dq,k}) = I_{\max}^2 - \|\mathbf{i}_{dq,k+1}\|^2 \geq 0, && \text{(10c)}\\
& c_{\tilde{v}}(\mathbf{v}_{dq,k}) = V_{\max}^2 - \|\tilde{\mathbf{v}}_{dq,k+1}\|^2 \geq 0. && \text{(10d)}
\end{aligned}$$

### When $T_e^{cmd}$ is not achievable

$$\begin{aligned}
\min_{\mathbf{v}_{dq,k}} \quad & \operatorname{sgn}(T_{e,k+1}^{cmd})\, c_t(\mathbf{v}_{dq,k}) && \text{(11a)}\\
\text{s.t.} \quad & c_i(\mathbf{v}_{dq,k}) = I_{\max}^2 - \|\mathbf{i}_{dq,k+1}\|^2 \geq 0, && \text{(11b)}\\
& c_{\tilde{v}}(\mathbf{v}_{dq,k}) = V_{\max}^2 - \|\tilde{\mathbf{v}}_{dq,k+1}\|^2 \geq 0. && \text{(11c)}
\end{aligned}$$

### Simulation result



# Modified voltage constraint

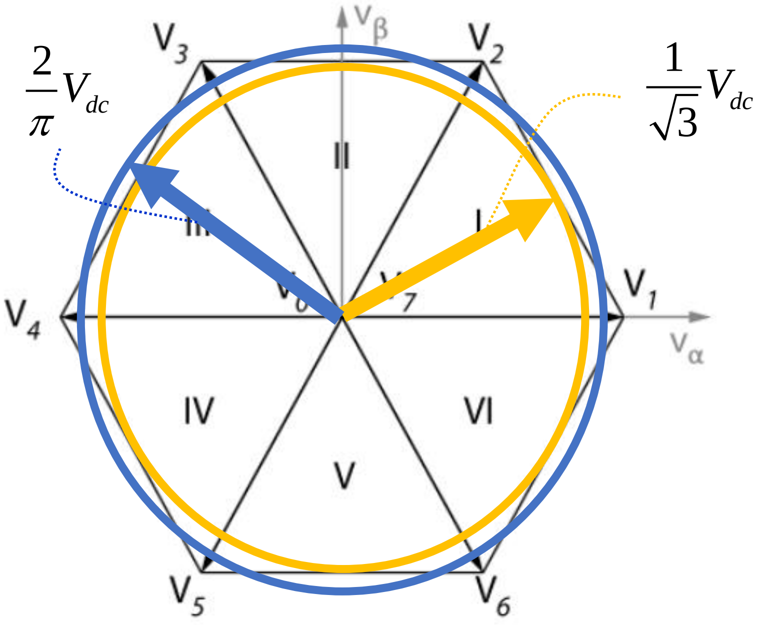
**Proposed MPTC**

$$\min_{\mathbf{v}_{dq,k}} \; J_p(\mathbf{v}_{dq,k}) = J_{p,k+1}$$

$$\text{s.t.} \quad c_t(\mathbf{v}_{dq,k}) = T^{cmd}_{e,k+1} - T_{e,k+1} = 0,$$

$$c_i(\mathbf{v}_{dq,k}) = I^2_{\max} - \left\|\mathbf{i}_{dq,k+1}\right\|^2 \geq 0,$$

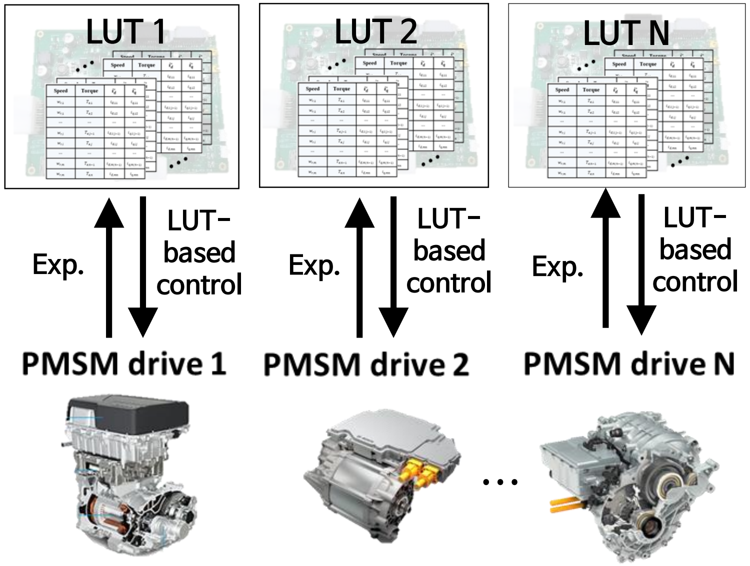
$$c_{\tilde{v}}(\mathbf{v}_{dq,k}) = \boxed{V^2_{\max} - \left\|\mathbf{v}_{dq,k+1}\right\|^2 \geq 0.}$$



# Intelligent SM controller

## AS-IS

### Experiment-based controllers



LUT 1

LUT 2

LUT N

Exp.

LUT-based control

PMSM drive 1

PMSM drive 2

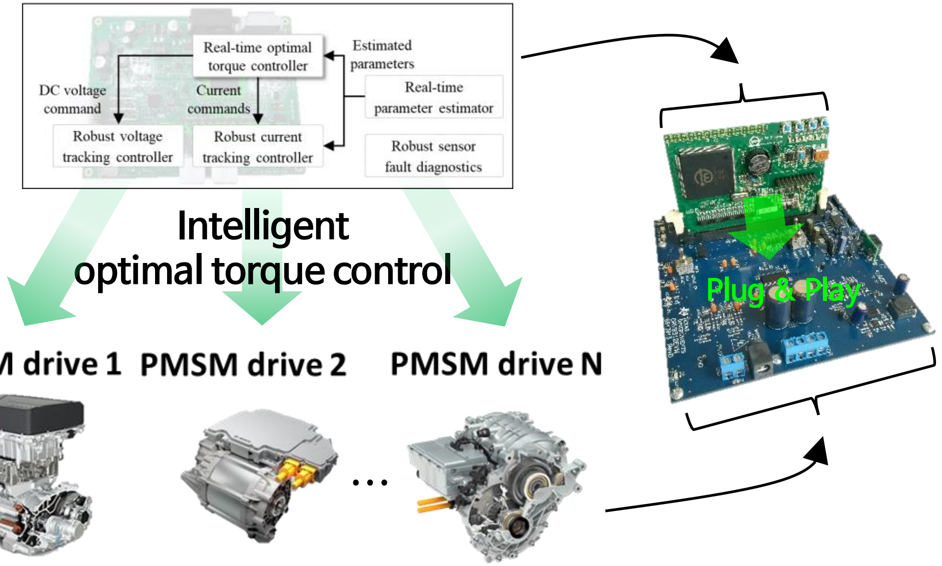
PMSM drive N

…

- ➢ Require extensive experiments (costs)
- ➢ Different controllers for different SMs

## TO-BE

### Intelligent SM controller



Intelligent optimal torque control

Plug & Play

PMSM drive 1

PMSM drive 2

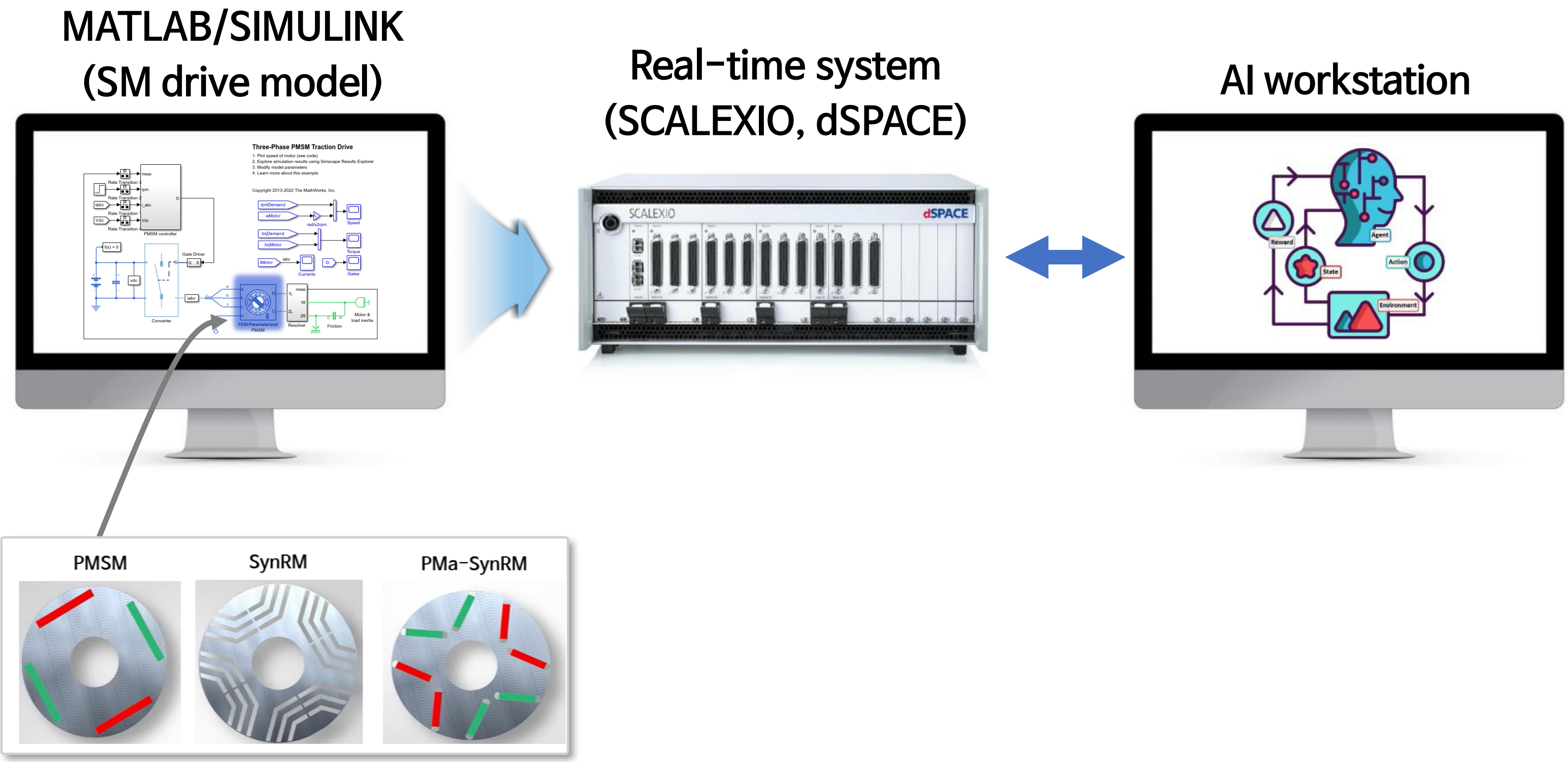
PMSM drive N

…

- ➢ Control any SM drives w/o prior experimental data
- ➢ Significant reduction in development costs

**SM**: Synchronous Machine
**LUT**: Look-Up Table

# Experimental setup for AI-based automatic gain tuning



MATLAB/SIMULINK
(SM drive model)

Real-time system
(SCALEXIO, dSPACE)

AI workstation

PMSM

SynRM

PMa-SynRM

Three-Phase PMSM Traction Drive

1. Plot speed of motor (see code)
2. Explore simulation results using Simscape Results Explorer
3. Modify model parameters
4. Learn more about this example

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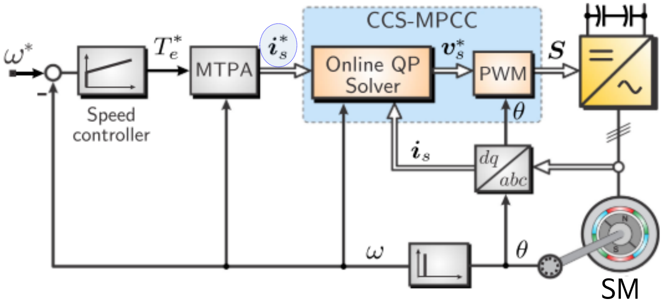
# MPC application to SM torque control

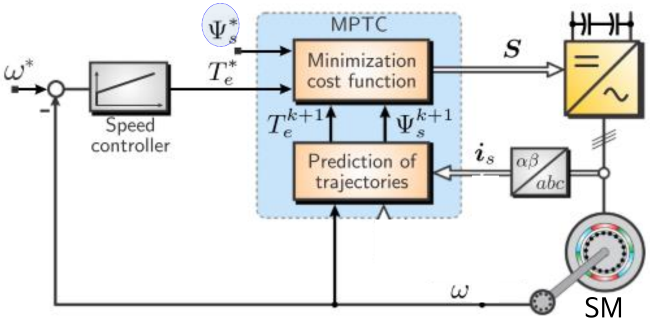
- **Two control schemes [11]**
  - 1) Model predictive current control (MPCC) w/ current reference generator



**Primary objective**

$$\min_{u_k, u_{k+1}, \cdots, u_{k+N-1}} \sum_{j=0}^{N_p-1} \left(i_d^* - i_{d,k+j+1}\right)^2 + \left(i_q^* - i_{q,k+j+1}\right)^2$$

  - 2) Model predictive torque control (MPTC) w/ flux reference generator



**Primary objective**

$$\min_{u_k, u_{k+1}, \cdots, u_{k+N-1}} \sum_{j=0}^{N_p-1} \left(T_e^* - T_{e,k+j+1}\right)^2 + \rho\left(\left\|\lambda_{dq}^*\right\| - \left\|\lambda_{dq,k+j+1}\right\|\right)^2$$

---

**SM**: Synchronous Machine

[11] J. Rodriguez *et al.*, "Latest advances of model predictive control in electrical drives—Part II: Applications and benchmarking with classical control methods," *IEEE TPE*, 2021.

# Optimal current reference generation

- Problem statement

**Case 1.** $T_e^* \le T_e^{\max}$

$\min \; i_d^2 + i_q^2$

subject to $T_e^* = (k_1 + k_2 i_d) i_q,$

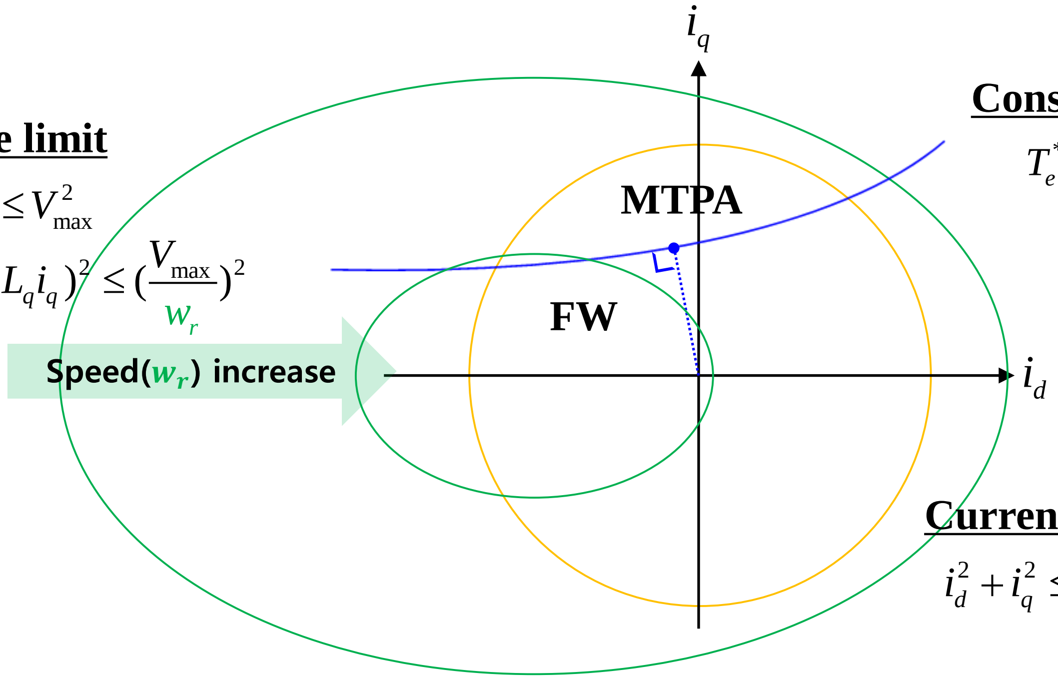
$v_d^2 + v_q^2 \le V_{\max}^2.$

**Case 2.** $T_e^* > T_e^{\max}$

$\max \; \pm_t T_e$

subject to $v_d^2 + v_q^2 \le V_{\max}^2,$

$i_d^2 + i_q^2 \le I_{\max}^2.$



**Voltage limit**

$$v_d^2 + v_q^2 \le V_{\max}^2$$

$$\Rightarrow (L_d i_d + \lambda_{pm})^2 + (L_q i_q)^2 \le (\frac{V_{\max}}{w_r})^2$$

**Constant torque line**

$$T_e^* = (k_1 + k_2 i_d) i_q$$

**Current limit**

$$i_d^2 + i_q^2 \le I_{\max}^2$$

# Optimal current reference generation

- Problem statement

**Case 1.** $T_e^* \le T_e^{\max}$

$\min \; i_d^2 + i_q^2$

subject to $T_e^* = (k_1 + k_2 i_d) i_q,$

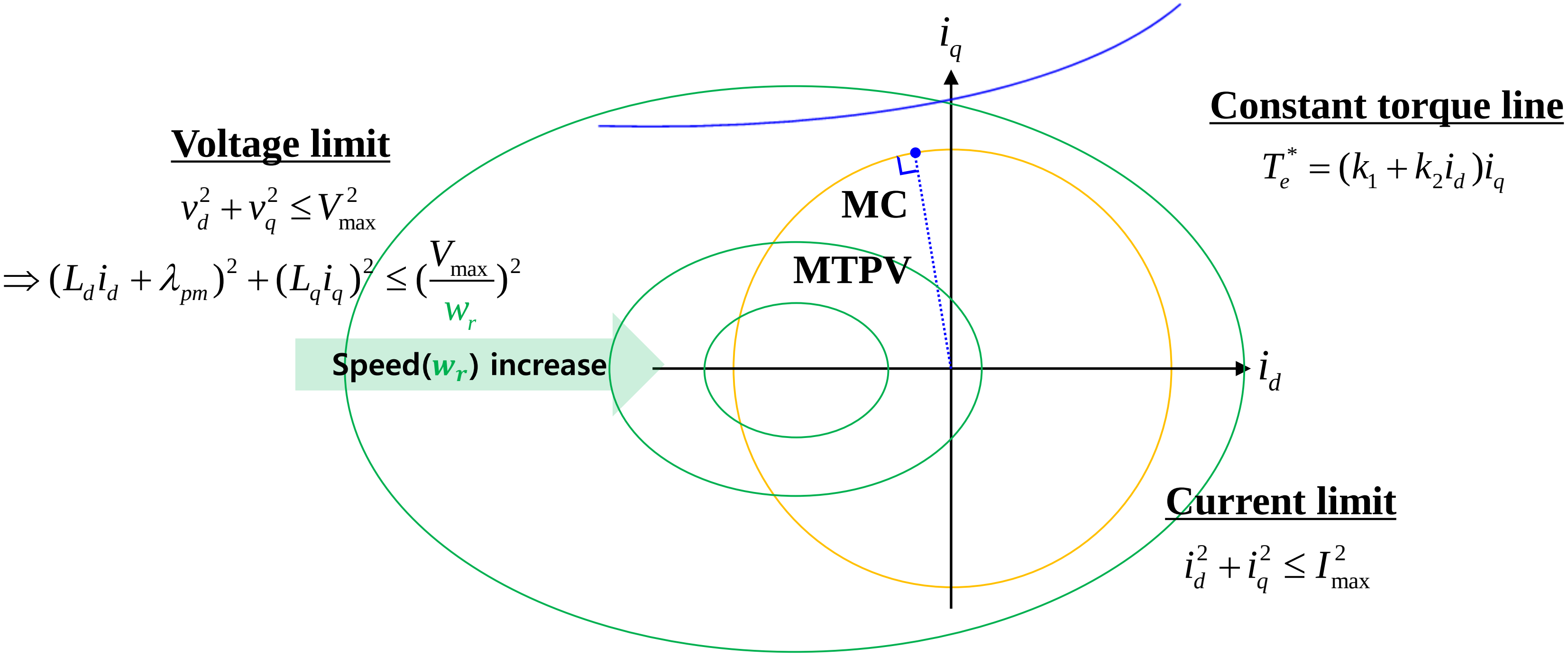
$v_d^2 + v_q^2 \le V_{\max}^2.$

**Case 2.** $T_e^* > T_e^{\max}$

$\max \; \pm_t T_e$

subject to $v_d^2 + v_q^2 \le V_{\max}^2,$

$i_d^2 + i_q^2 \le I_{\max}^2.$



**Voltage limit**

$$v_d^2 + v_q^2 \le V_{\max}^2$$

$$\Rightarrow (L_d i_d + \lambda_{pm})^2 + (L_q i_q)^2 \le \left(\frac{V_{\max}}{w_r}\right)^2$$

**Constant torque line**

$$T_e^* = (k_1 + k_2 i_d) i_q$$

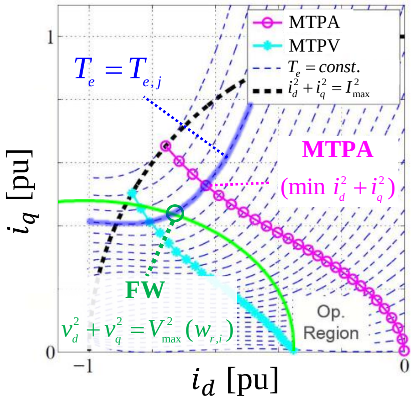
**Current limit**

$$i_d^2 + i_q^2 \le I_{\max}^2$$

- $\pm_t = \mathrm{sgn}(T_e^*)$, **MC** : Maximum Current, **MTPV** : Maximum Torque Per Voltage

# Optimal current reference generation

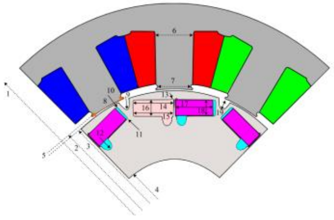
- **Experimental solution [12]**
  - Experimentally find solutions and store them in a **LUT**



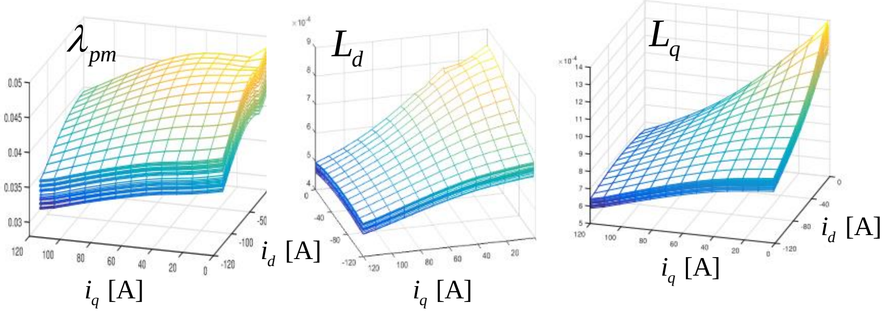
stored
in a LUT

| Speed | Torque | $i_d^*$ | $i_q^*$ |
|---|---|---|---|
| $w_{r,1}$ | $T_{e,1}$ | $i_{d,11}$ | $i_{q,11}$ |
| $w_{r,1}$ | $T_{e,2}$ | $i_{d,12}$ | $i_{q,12}$ |
| … | … | … | … |
| $w_{r,i}$ | $T_{e,j-1}$ | $i_{d,i(j-1)}$ | $i_{q,i(j-1)}$ |
| $w_{r,i}$ | $T_{e,j}$ | $i_{d,ij}$ | $i_{q,ij}$ |
| … | … | … | … |
| $w_{r,m}$ | $T_{e,n-1}$ | $i_{d,m(n-1)}$ | $i_{q,m(n-1)}$ |
| $w_{r,m}$ | $T_{e,n}$ | $i_{d,mn}$ | $i_{q,mn}$ |

- Require extensive experiments
- A specific solution just for one product,
  which cannot handle parameter deviations resulting from tolerances


design variables

manufacturing
tolerances [13]



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[1] B. Gallert, et al., "Maximum efficiency control strategy of PM traction machine drives in GM hybrid and electric vehicles," in IEEE ECCE, 2017.
[13] H. Khreis, et al., "Sensitivity analysis on electrical parameters for permanent magnet synchronous machine manufacturing tolerances in EV and HEV," in ITEC, 2016.

# Synchronous Machines in Electric Vehicles

- **Example of EVs on the market from 2010 to 2020 and their machines [14]**

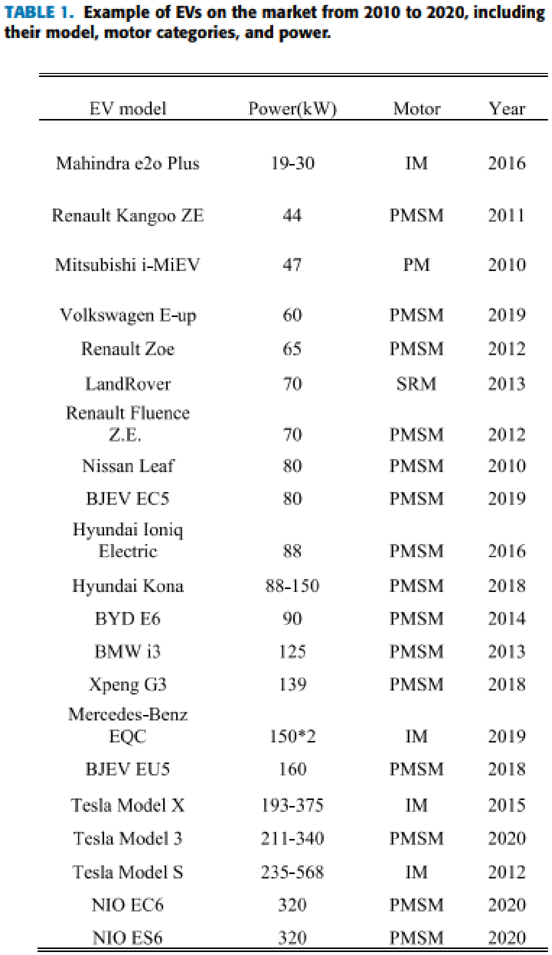

**TABLE 1. Example of EVs on the market from 2010 to 2020, including their model, motor categories, and power.**

| EV model | Power(kW) | Motor | Year |
|---|---|---|---|
| Mahindra e2o Plus | 19-30 | IM | 2016 |
| Renault Kangoo ZE | 44 | PMSM | 2011 |
| Mitsubishi i-MiEV | 47 | PM | 2010 |
| Volkswagen E-up | 60 | PMSM | 2019 |
| Renault Zoe | 65 | PMSM | 2012 |
| LandRover | 70 | SRM | 2013 |
| Renault Fluence Z.E. | 70 | PMSM | 2012 |
| Nissan Leaf | 80 | PMSM | 2010 |
| BJEV EC5 | 80 | PMSM | 2019 |
| Hyundai Ioniq Electric | 88 | PMSM | 2016 |
| Hyundai Kona | 88-150 | PMSM | 2018 |
| BYD E6 | 90 | PMSM | 2014 |
| BMW i3 | 125 | PMSM | 2013 |
| Xpeng G3 | 139 | PMSM | 2018 |
| Mercedes-Benz EQC | 150*2 | IM | 2019 |
| BJEV EU5 | 160 | PMSM | 2018 |
| Tesla Model X | 193-375 | IM | 2015 |
| Tesla Model 3 | 211-340 | PMSM | 2020 |
| Tesla Model S | 235-568 | IM | 2012 |
| NIO EC6 | 320 | PMSM | 2020 |
| NIO ES6 | 320 | PMSM | 2020 |

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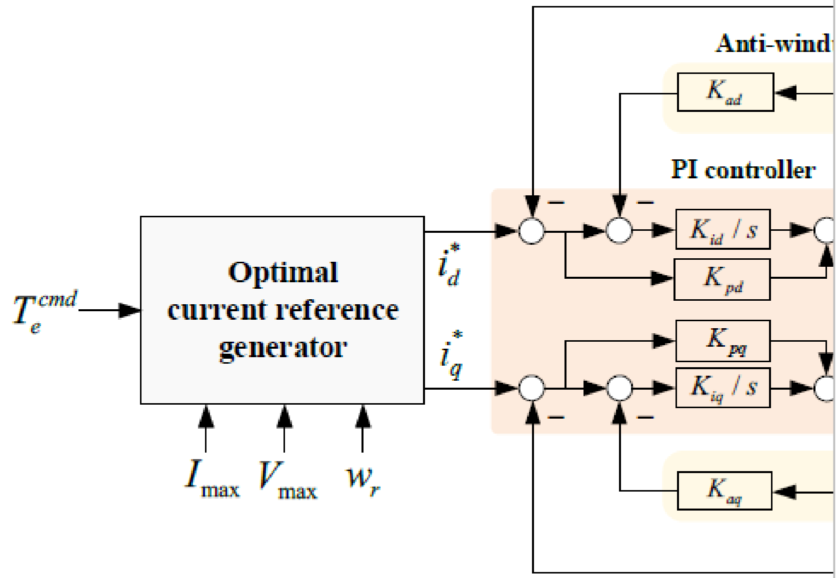
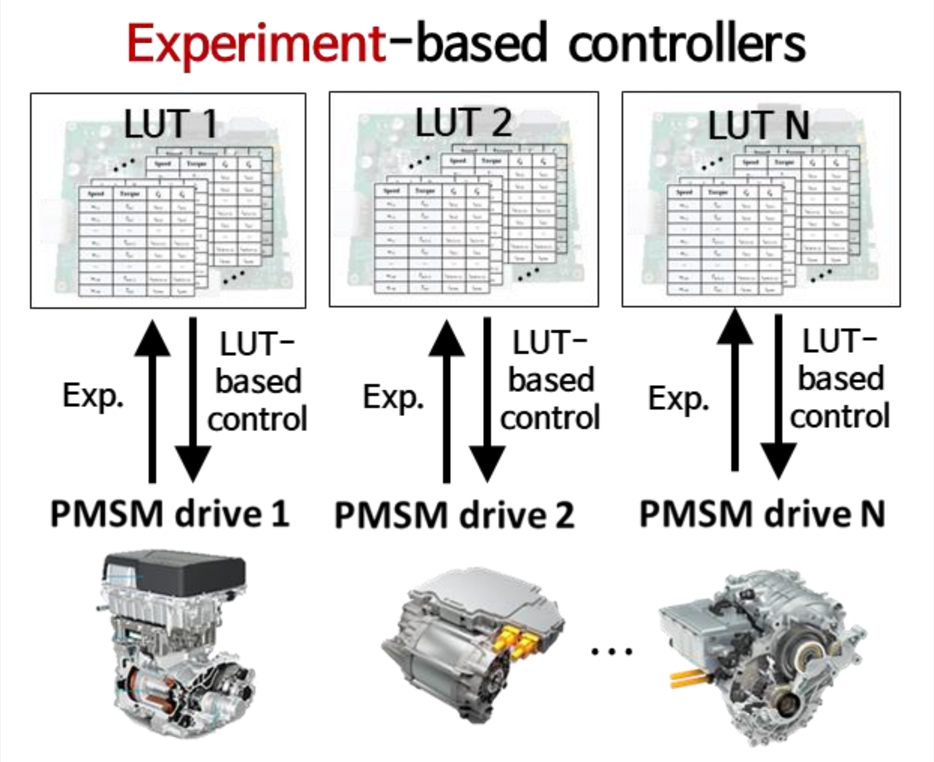
[14] Wang, Z., Ching, T. W., Huang, S., Wang, H., & Xu, T. (2020). Challenges faced by electric vehicle motors and their solutions. *IEEE Access*, *9*, 5228-5249.

# Existing schemes for Torque control of SMs

- **1. Conventional scheme [4]**

  : Current reference generator + PI-





- Involve a number of components
- Require a lot of time (e.g., 3 months) to finalize this scheme

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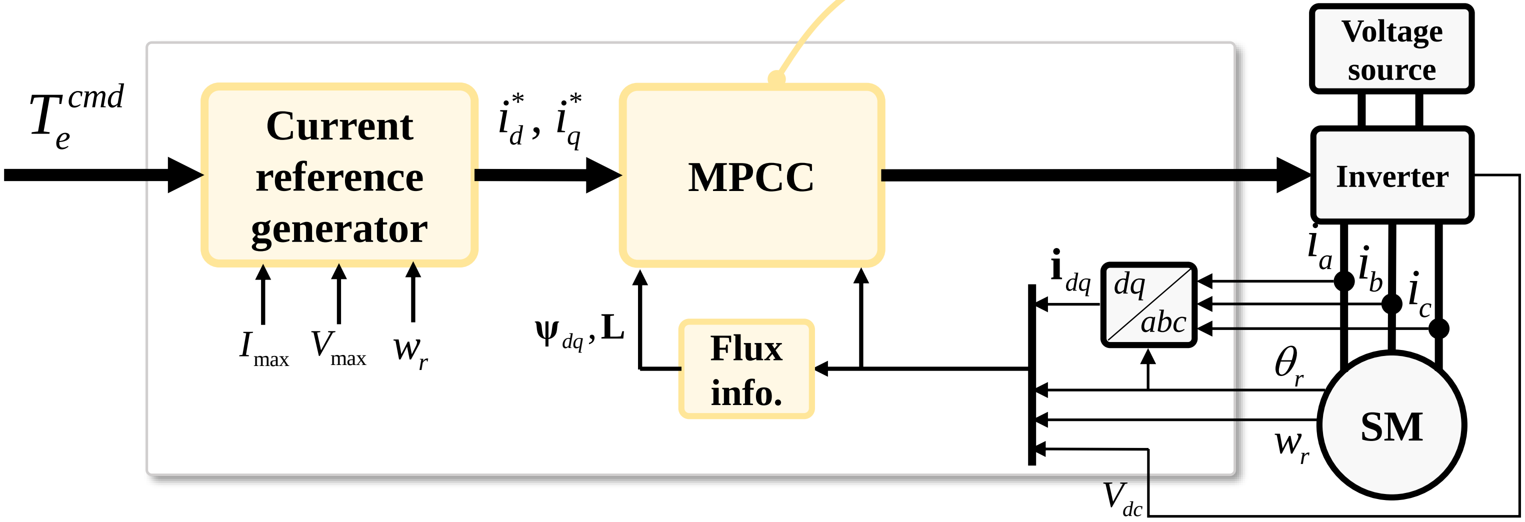
[4] S.-K. Sul, Control of electric machine drive systems. *John Wiley & Sons*, 2011, vol. 88.

# Existing schemes for Torque control of SMs

- **2. Model Predictive Current Control (MPCC) [5]**

$$\min_{u_k,u_{k+1},\cdots,u_{k+N-1}} \sum_{j=1}^{N_p} \overbrace{\left(i_d^* - i_{d,k+j}\right)^2 + \left(i_q^* - i_{q,k+j}\right)^2}^{\text{MTPA point}} + \alpha \overbrace{J_{p,k+j}}^{\text{Inverter loss}}$$

$$\text{s.t. } i_{d,k+j}^2 + i_{q,k+j}^2 \le I_{\max}^2,\ v_{d,k+j-1}^2 + v_{q,k+j-1}^2 \le V_{\max}^2$$



- Guarantee improved current tracking performance *But torque?*
- Still rely on a current reference generator
  - **Problem 1.** Need to solve **another optimization problem** other than MPC
  - **Problem 2.** Two optimization problems **handle the DOF separately**

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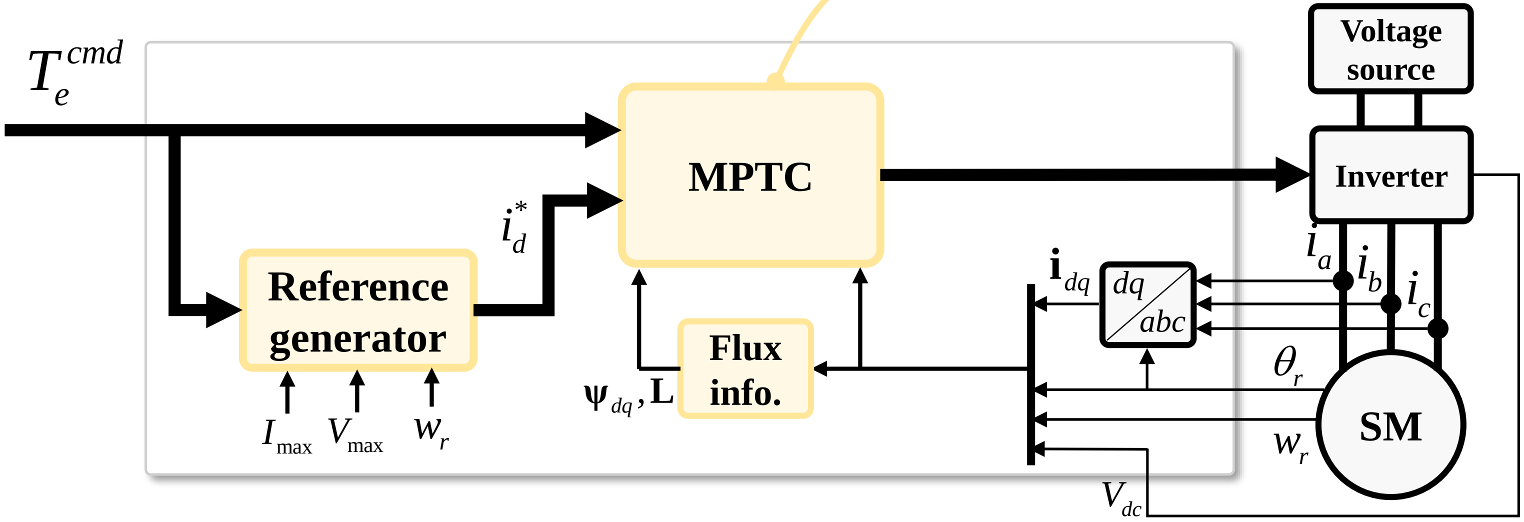
[5] J. Rodriguez, et al., "Predictive current control of a voltage source inverter," *IEEE TIE*, vol. 54, no. 1, pp. 495 - 503, 2007.

# Existing schemes for Torque control of SMs

- 3. Model Predictive Torque Control (MPTC)
  1) w/ reference generator [6]

$$\min_{u_k,u_{k+1},\cdots,u_{k+N-1}} \sum_{j=1}^{N_p} \left(T_e^{cmd} - T_{e,k+j}\right)^2 + \left(i_d^* - i_{d,k+j}\right)^2 + \alpha J_{p,k+j}$$

$$\text{s.t. } i_{d,k+j}^2 + i_{q,k+j}^2 \le I_{\max}^2,\quad v_{d,k+j-1}^2 + v_{q,k+j-1}^2 \le V_{\max}^2$$



- Guarantee improved torque tracking performance
- Still rely on a current reference generator

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[6] T. Englert and K. Graichen, “Nonlinear model predictive torque control of PMSMs for high performance applications,” *CEP Eng. Prac.*, vol. 81, pp. 43 - 54, 2018.

# Existing schemes for Torque control of SMs

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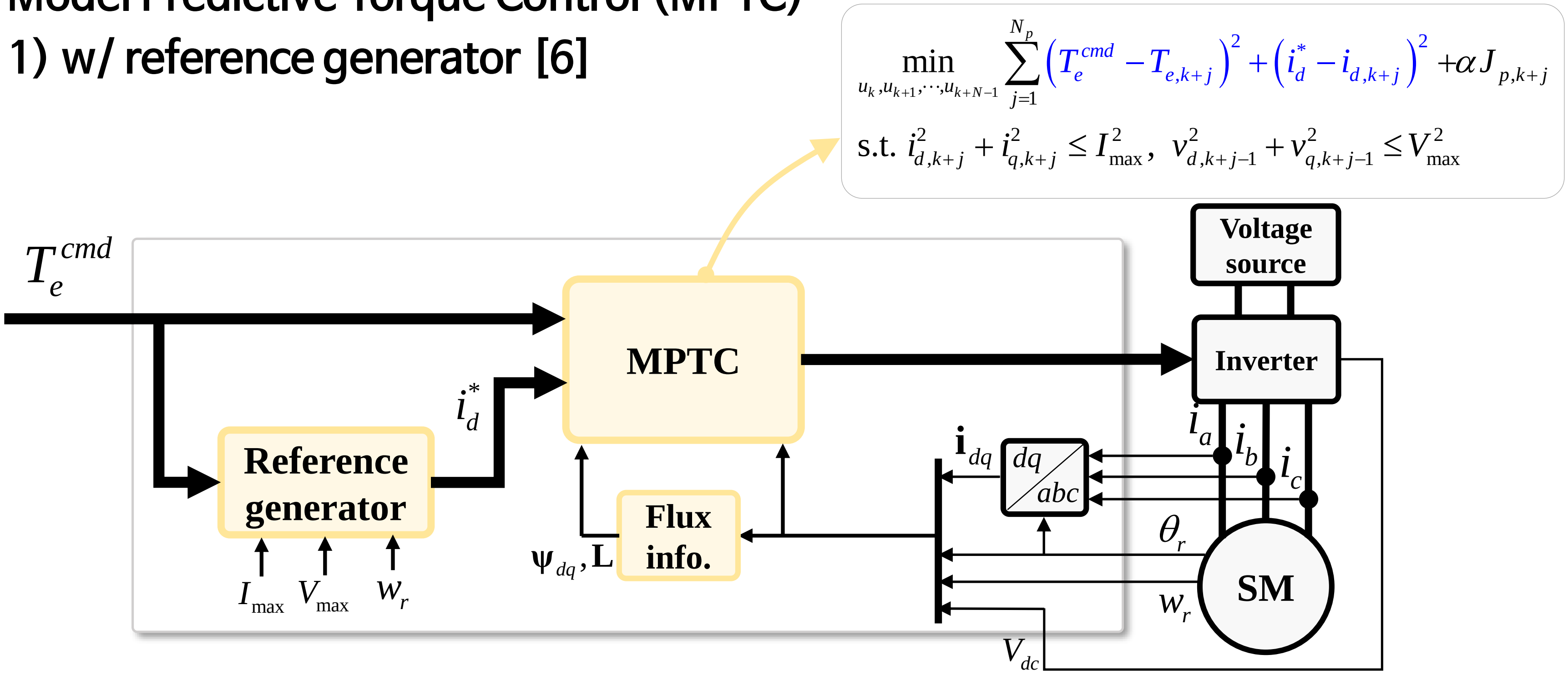

$$\min_{u_k, u_{k+1}, \cdots, u_{k+N-1}} \sum_{j=1}^{N_p} \left(T_e^{cmd} - T_{e,k+j}\right)^2 + \left(i_d^* - i_{d,k+j}\right)^2 + \alpha J_{p,k+j}$$

$$\text{s.t. } i_{d,k+j}^2 + i_{q,k+j}^2 \le I_{\max}^2, \quad v_{d,k+j-1}^2 + v_{q,k+j-1}^2 \le V_{\max}^2$$

- Guarantee improved torque tracking performance
- Still rely on a current reference generator
  - **Problem 1.** Need to solve **another optimization problem** other than MPC
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# Simulation Results

$$J_p = P_{cu} \quad \text{vs.} \quad J_p = P_{cu} + P_{inv}$$

## Simulation 2. Effects of Using Different Performance Indices

- Discussion

**Proposed MPTC**

$$\min_{\mathbf{v}_{dq,k}} J_p(\mathbf{v}_{dq,k}) = P_{cu,k+1} + P_{inv,k+1}$$

$$\text{s.t.} \;\; c_t(\mathbf{v}_{dq,k}) = T_{e,k+1}^{cmd} - T_{e,k+1} = 0,$$

$$c_i(\mathbf{v}_{dq,k}) = I_{\max}^2 - \left\| \mathbf{i}_{dq,k+1} \right\|^2 \geq 0,$$

$$c_{\tilde{v}}(\mathbf{v}_{dq,k}) = V_{\max}^2 - \left\| \mathbf{v}_{dq,k+1} \right\|^2 \geq 0.$$

**vs.**

**Existing MPTC [5]**

$$\min_{\mathbf{v}_{dq,k}} J_p(\mathbf{v}_{dq,k}) = \left\| \mathbf{i}_{dq,k+1} - \mathbf{i}_{dq}^{MTPA} \right\|^2 + \alpha P_{inv,k+1}$$

$$\text{s.t.} \;\; c_i(\mathbf{v}_{dq,k}) = I_{\max}^2 - \left\| \mathbf{i}_{dq,k+1} \right\|^2 \geq 0,$$

$$c_{\tilde{v}}(\mathbf{v}_{dq,k}) = V_{\max}^2 - \left\| v_{dq,k} \right\|^2 \geq 0.$$

- ➢ The MPTC fully utilizes the DOF.
- ➢ The MPTC partially utilizes the DOF.

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[5] J. Rodriguez, et al., “Predictive current control of a voltage source inverter,” *IEEE TIE*, vol. 54, no. 1, pp. 495 - 503, 2007.

# Research Objectives

## Summary of Literature review

| | | |
|---|---|---|
| **Concept** | **Reference generator** + **MPCC [5] or MPTC [6]** | **MPTC [7]** |
| **Property** | ➢ Good tracking performance<br>➢ Two optimization problems | ➢ Steady-state error, Instability<br>➢ One optimization problem |

## Research objectives

- Develop a MPTC scheme for SMs that
  - Does not rely on a reference generator
  - But guarantees optimal operation under all operating regions

**Primary objectives**

  - Can be implemented based on both the FCS and CCS
  - Can be implemented with various performance indices ($J_p$)

**Secondary objectives**

- FCS: finite control set
- CCS: continuous control set

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[6] T. Englert and K. Graichen, “Nonlinear model predictive torque control of PMSMs for high performance applications,” *Control Eng. Prac.,* vol. 81, pp. 43 - 54, 2018.
[7] L. Samaranayake and S. Longo, “Degradation control for electric vehicle machines using nonlinear model predictive control,” *IEEE TCST*, vol. 26, no. 1, pp. 89 - 101, 2017.