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Lyapunov-based Nonlinear Programming (LBNLP)

Myeongseok Ryu [*], Kyunghwan Choi [*], Christoph Hackl [†]

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Abstract

This research aims to

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1 Introduction

1.1 Background

1.2 Research Objectives

The main objectives of this research are as follows:

- Mathematical stability analysis of the controller and estimator with deep neural networks using the contraction theory.
- Development of the controller and estimator with deep neural networks using the contraction theory.

2 Notations and Preliminaries

The following notations are used throughout this document:

- $:=$ denotes *defined as*.
- $(\cdot)^\top$ denotes the transpose of a matrix or a vector.
- $\boldsymbol{x} := [x_i]_{i\in\{1,\cdots,n\}} \in \mathbb{R}^n$ denotes the state vector.
- $\boldsymbol{A} := [a_{ij}]_{i,j\in\{1,\cdots,n\}} \in \mathbb{R}^{n\times n}$ denotes a matrix.
- $\lambda_i(\boldsymbol{A}),\ i \in \{\max, \min\}$ denotes the maximum and minimum singular value of $\boldsymbol{A}$, respectively.
- $\boldsymbol{I}_n$ denotes the identity matrix of size n and $\boldsymbol{0}_{n\times m}$ denotes the zero matrix of size $n \times m$.
- sym denotes the symmetric part of a matrix, i.e., $\mathrm{sym}(\boldsymbol{A}) := \frac{1}{2}(\boldsymbol{A} + \boldsymbol{A}^\top)$ (see, [1]).

[*]Myeongseok Ryu and Kyunghwan Choi are with the Department of Mechanical and Robotics Engineering, Gwangju Institute of Science and Technology, 61005 Gwangju, Republic of Korea dding_98@gm.gist.ac.kr, khchoi@gist.ac.kr
[†]Christoph Hackl is , 000, 000, Germany example@mail.com

We introduce the following lemmas.

Lemma 2.1 (Comparison Lemma)**.** *Suppose that a continuously differentiable function* $f : \mathbb{R}^n \to \mathbb{R}$ *satisfies the following inequality:*

$$\tfrac{\mathrm{d}}{\mathrm{d}t} f(t) \le -af(t) + b, \quad \forall t \in \mathbb{R}_{\ge 0},$$

where $a, b > 0$*. Then, the following inequality holds:*

$$f(t) \le -af(0)e^{-at} + \tfrac{b}{a}(1 - e^{-at}), \quad \forall t \in \mathbb{R}_{\ge 0}$$

and remains in a compact set $f(t) \in \{\|f(t)\| \mid \|f(0)\| \le \frac{b}{a}\}$*.*

Proof. This is a simple special case of the comparison lemma [2, pp. 102-103]. See [2, pp. 659-660]. □

3 Conclusion

References

[1] H. Tsukamoto, S.-J. Chung, and J.-J. E. Slotine, "Neural stochastic contraction metrics for learning-based control and estimation," *IEEE Control Systems Letters*, vol. 5, no. 5, pp. 1825–1830, 2021.

[2] H. K. Khalil, *Nonlinear systems; 3rd ed.* Upper Saddle River, NJ: Prentice-Hall, 2002. The book can be consulted by contacting: PH-AID: Wallet, Lionel.