Integral Error-Based Adaptive Neural Identifier for a Class of Uncertain Nonlinear Systems

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Abstract: This paper presents a robust online neural network (NN) identifier for uncertain nonlinear systems, focusing on online system identification and function approximation. The proposed method employs a cumulative error cost function with a forgetting factor to enable real-time learning of unknown nonlinear dynamics. A rigorous Lyapunov-based stability analysis guarantees Uniform Ultimate Boundedness (UUB) of the identification error. Comparative analysis with state error-based gradient descent methods highlights the improved stability and convergence properties of the proposed approach. The results demonstrate that the method enables effective online identification and function approximation for a wide range of nonlinear systems.

Keywords: Online System Identification, Neural Networks, Adaptive Control, Stability Analysis, Nonlinear Dynamics

1. INTRODUCTION

Online system identification is crucial in control applications where system dynamics are unknown or change over time. Real-time adaptability of system dynamics is essential in fields such as smart materials, human-machine interaction, and fault detection. Conventional offline identification methods require large datasets and lack real-time adaptability, making them unsuitable for dynamically changing environments. Neural networks (NNs) offer powerful capabilities to approximate unknown nonlinearities due to their universal function approximation property. This makes NNs attractive tools for adaptive control and system identification problems where obtaining an accurate model in advance is difficult or impossible. Most real-world systems are nonlinear, and traditional model-based observers rely on prior knowledge of system nonlinearities, which is rarely satisfied in practice. Neural networks provide an adaptive alternative that can learn the dynamics without such prior knowledge.

Existing research on NN-based adaptive control has mainly focused on tracking error convergence (e.g., minimizing e or minimizing cost function $\|e\|^2$). However, this study's major drawback of minimizing instantaneous error is that, although a high learning rate may yield good approximation at a specific moment, it can lead to parameter drift. As a result, the neural network may only approximate the function at that instant, rather than achieving accurate function approximation over the entire time interval.

The main contributions of this study are as follows:

- Development of a robust online neural network identification algorithm for unknown nonlinear dynamics.
- Formal definition and application of a cumulative error cost function with a forgetting factor.
- Rigorous Lyapunov-based stability analysis guaranteeing Uniform Ultimate Boundedness (UUB) of the error.
- Comparative analysis with state error based gradient descent methods, highlighting the advantages of the proposed approach.

This report is organized as follows: introduction, problem definition and neural network identifier structure, proposed online learning algorithm, stability analysis, comparative analysis with related online identification methods, and conclusion and future work.

2. PROBLEM FORMULATION

2.1 Model Dynamics

Consider the nonlinear system

$$\dot{\boldsymbol{x}}(t) = \underbrace{\boldsymbol{f}(\boldsymbol{x},\boldsymbol{u})}_{known} + \underbrace{\boldsymbol{h}(\boldsymbol{x},\boldsymbol{u})}_{unknown} \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^{\ltimes}$ is the state vector and $\boldsymbol{u} \in \mathbb{R}^{\gtrdot}$ is the control input vector, and $\boldsymbol{f}(\cdot)$ represents the known part of the system dynamics, and $\boldsymbol{h}(\cdot)$ denotes the unknown nonlinear dynamics or disturbances.

2.2 Neural Network Identifier

To design a identifier, some assumptions are needed as follows :

Assumption 1: The ideal weights $\mathbf{W}, \mathbf{V}$ are bounded. The disturbance $w(t)$ is bounded by $\|w(t)\| \leq w_M$. The activation function σ and its derivatives are bounded.

Assumption 2 (Open-Loop Stability) The open-loop system (1) is stable, which implies that the state vector $\boldsymbol{x}(t)$ is bounded in L_∞.

By adding and subtracting $\mathbf{A}\boldsymbol{x}$ from . So, the system is described by:

$$\dot{\boldsymbol{x}}(t) = \mathbf{A}\mathbf{x}(\mathbf{t}) + \mathbf{g}(\mathbf{x},\mathbf{u}) + \mathbf{h}(\mathbf{x},\mathbf{u}) \tag{2}$$

where $\mathbf{A} \in \mathbb{R}^{\ltimes\times\ltimes}$ is an arbitrary Hurwitz matrix, $\boldsymbol{g}(\boldsymbol{x},\boldsymbol{u}) = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u}) - \mathbf{A}\boldsymbol{x}$, which is the known nonlinear function.

Then, the identifier model can be selected as:

$$\dot{\hat{\boldsymbol{x}}}(t) = \mathbf{A}\hat{\boldsymbol{x}}(t) + \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}) + \hat{\boldsymbol{h}}(\hat{\bar{\boldsymbol{x}}}) \tag{3}$$

where the NN output is $\hat{\boldsymbol{h}}(\hat{\boldsymbol{x}}, \boldsymbol{u}) = \hat{\mathbf{W}}^T\sigma(\hat{\mathbf{V}}^T\hat{\bar{\boldsymbol{x}}})$. Here, $\hat{\mathbf{W}} \in \mathbb{R}^{\bar{\sim}\times\ltimes}$ and $\hat{\mathbf{V}} \in \mathbb{R}^{\times\bar{\sim}}$ are the estimated weight matrices. The NN input $\hat{\bar{\boldsymbol{x}}}$ is constructed from the estimated states $\hat{\boldsymbol{x}}$ and $\boldsymbol{u}$. Defining the errors $\tilde{\boldsymbol{x}} = \boldsymbol{x} - \hat{\boldsymbol{x}}$, $\tilde{\mathbf{W}} = \mathbf{W} - \hat{\mathbf{W}}$, and $\tilde{\mathbf{V}} = \mathbf{V} - \hat{\mathbf{V}}$, the error dynamics are given by:

$$\dot{\tilde{\boldsymbol{x}}} = \mathbf{A}\tilde{\boldsymbol{x}} + \tilde{\mathbf{W}}^T\sigma(\hat{\mathbf{V}}^T\hat{\bar{\boldsymbol{x}}}) + w(t) \tag{4}$$

where $w(t) = \mathbf{W}^T\left(\sigma(\mathbf{V}^T\bar{\boldsymbol{x}}) - \sigma(\hat{\mathbf{V}}^T\hat{\bar{\boldsymbol{x}}})\right) + \epsilon(x)$ is the lumped disturbance term.

3. METHODOLOGY

3.1 Cumulative Error based Gradient Descent

A common approach to update the weights of a neural network is to minimize the Consider the plant model (2) and the identifier model (3). Given Assumption 2, if the weights of the NLPNN are updated according to

$$\dot{\hat{\mathbf{W}}} = -\eta_1\left(\frac{\partial J}{\partial\hat{\mathbf{W}}}\right) - \rho_1\|\tilde{\boldsymbol{x}}\|\hat{\mathbf{W}}, \tag{5}$$

$$\dot{\hat{\mathbf{V}}} = -\eta_2\left(\frac{\partial J}{\partial\hat{\mathbf{V}}}\right) - \rho_2\|\tilde{\boldsymbol{x}}\|\hat{\mathbf{V}}, \tag{6}$$

where $\eta > 0$ is the learning rate.

$$J = \frac{1}{2}\int_0^t e^{-\lambda(t-\tau)}\tilde{\boldsymbol{x}}(\tau)^T\tilde{\boldsymbol{x}}(\tau)\,\mathrm{d}\tau \tag{7}$$

is the objective function and ρ is a small positive number, then $\tilde{\boldsymbol{x}}$, $\tilde{\mathbf{W}}$, and $\tilde{\mathbf{V}} \in L_\infty$.

Since the cost functional is of an integral form, we first introduce the filtered error signal $\boldsymbol{z}(t)$ to construct the update laws.

$$\boldsymbol{z}(t) = \int_0^t e^{-\lambda(t-\tau)}\tilde{\boldsymbol{x}}(\tau)\,\mathrm{d}\tau \tag{8}$$

$$\dot{\boldsymbol{z}} = -\lambda\boldsymbol{z} + \tilde{\boldsymbol{x}} \tag{9}$$

Let us define

$$\mathrm{net}_{\hat{\mathbf{V}}} = \hat{\mathbf{V}}\hat{\boldsymbol{x}} \tag{10}$$

$$\mathrm{net}_{\hat{\mathbf{W}}} = \hat{\mathbf{W}}\sigma(\hat{\mathbf{V}}\hat{\boldsymbol{x}}). \tag{11}$$

Therefore, by using the chain rule $\frac{\partial J}{\partial\hat{\mathbf{W}}}$ and $\frac{\partial J}{\partial\hat{\mathbf{V}}}$ can be computed according to

$$\frac{\partial J}{\partial\hat{\mathbf{W}}} = \frac{\partial J}{\partial\mathrm{net}_{\hat{\mathbf{W}}}}\cdot\frac{\partial\mathrm{net}_{\hat{\mathbf{W}}}}{\partial\hat{\mathbf{W}}}$$
$$\frac{\partial J}{\partial\hat{\mathbf{V}}} = \frac{\partial J}{\partial\mathrm{net}_{\hat{\mathbf{V}}}}\cdot\frac{\partial\mathrm{net}_{\hat{\mathbf{V}}}}{\partial\hat{\mathbf{V}}},$$

where

$$\frac{\partial J}{\partial\mathrm{net}_{\hat{\mathbf{W}}}} = \frac{\partial J}{\partial\tilde{\boldsymbol{z}}}\frac{\partial\tilde{\boldsymbol{z}}}{\partial\tilde{\boldsymbol{x}}}\frac{\partial\hat{\boldsymbol{x}}}{\partial\mathrm{net}_{\hat{\mathbf{W}}}} = -\boldsymbol{z}^T\frac{\partial\hat{\boldsymbol{x}}}{\partial\mathrm{net}_{\hat{\mathbf{W}}}},$$
$$\frac{\partial J}{\partial\mathrm{net}_{\hat{\mathbf{V}}}} = \frac{\partial J}{\partial\tilde{\boldsymbol{x}}}\frac{\partial\tilde{\boldsymbol{x}}}{\partial\hat{\boldsymbol{x}}}\frac{\partial\hat{\boldsymbol{x}}}{\partial\mathrm{net}_{\hat{\mathbf{V}}}} = -\boldsymbol{z}^T\frac{\partial\hat{\boldsymbol{x}}}{\partial\mathrm{net}_{\hat{\mathbf{V}}}} \tag{12}$$

and

$$\frac{\partial\mathrm{net}_{\hat{\mathbf{W}}}}{\partial\hat{\mathbf{W}}} = \sigma(\hat{\mathbf{V}}\hat{\boldsymbol{x}})$$
$$\frac{\partial\mathrm{net}_{\hat{\mathbf{V}}}}{\partial\hat{\mathbf{V}}} = \hat{\boldsymbol{x}}. \tag{13}$$

We modify the original BP algorithm such that the static approximations of $\frac{\partial\hat{\boldsymbol{x}}}{\partial\mathrm{net}_{\hat{\mathbf{W}}}}$ and $\frac{\partial\hat{\boldsymbol{x}}}{\partial\mathrm{net}_{\hat{\mathbf{V}}}}$ ($\dot{\hat{\boldsymbol{x}}} = 0$) can be used.

$$\frac{\partial\hat{\boldsymbol{x}}}{\partial\mathrm{net}_{\hat{\mathbf{W}}}} \approx -\mathbf{A}^{-\mathbf{1}}$$
$$\frac{\partial\hat{\boldsymbol{x}}}{\partial\mathrm{net}_{\hat{\mathbf{V}}}} \approx -\mathbf{A}^{-\mathbf{1}}\hat{\mathbf{W}}(\mathbf{I} - \boldsymbol{\Lambda}(\hat{\mathbf{V}}\hat{\mathbf{x}})), \tag{14}$$

where

$$\boldsymbol{\Lambda}(\hat{\mathbf{V}}\hat{\mathbf{x}}) = \mathrm{diag}\{\sigma_{\mathbf{i}}^{\mathbf{2}}(\hat{\mathbf{V}}_{\mathbf{i}}\hat{\mathbf{x}})\}, \quad \mathbf{i} = \mathbf{1}, \mathbf{2}, \ldots, \mathbf{m}. \tag{15}$$

$$\dot{\hat{\mathbf{W}}} = -\eta_1\left(\boldsymbol{z}^T\boldsymbol{A}^{-1}\right)^T\sigma(\hat{\mathbf{V}}^T\hat{\bar{\boldsymbol{x}}})^T - \rho_1\|\tilde{\boldsymbol{x}}\|\hat{\mathbf{W}} \tag{16}$$

$$\tag{17}$$

$$\dot{\hat{\mathbf{V}}} = -\eta_2\hat{\bar{\boldsymbol{x}}}\left(\boldsymbol{z}^T\boldsymbol{A}^{-1}\hat{\mathbf{W}}(\mathbf{I} - \boldsymbol{\Lambda}(\hat{\mathbf{V}}\hat{\mathbf{x}}))\right)^T - \rho_2\|\tilde{\boldsymbol{x}}\|\hat{\mathbf{V}} \tag{18}$$

$$\tag{19}$$

where $\eta_W, \eta_V, \rho_1, \rho_2 > 0$ are design parameters.

3.2 Stability Analysis

To analyze the stability of the system described by (4) with the update laws (17)-(19), we will use Lyapunov's direct method. The goal is to show that the errors $\tilde{\boldsymbol{x}}$, $\tilde{\mathbf{W}}$, and $\tilde{\mathbf{V}}$ are Uniformly Ultimately Bounded (UUB).

Theorem 1: For the system given by (4) with the update laws (17)-(19), all signals in the system $(\tilde{\boldsymbol{x}}, \tilde{\mathbf{W}}, \tilde{\mathbf{V}})$ are Uniformly Ultimately Bounded.

Proof: The stability proof is conducted in two steps using a cascaded system approach.

We first prove the boundedness of the state error $\tilde{\boldsymbol{x}}$ and the output layer weight error $\tilde{\mathbf{W}}$. This is possible because the term $\sigma_v = \sigma(\hat{\mathbf{V}}^T\hat{\bar{\boldsymbol{x}}})$ in the error dynamics (4) is always bounded, regardless of the value of $\hat{\mathbf{V}}$, due to the bounded nature of the activation function σ.

Consider the Lyapunov function candidate for the first subsystem:

$$L = \frac{1}{2}\tilde{\boldsymbol{x}}^T\mathbf{P_1}\tilde{\mathbf{x}} + \frac{\mathbf{1}}{\mathbf{2}}\mathrm{tr}(\tilde{\mathbf{W}}^{\mathbf{T}}\rho^{-\mathbf{1}}\tilde{\mathbf{W}})$$
$$+ \frac{1}{2}\int_0^t e^{-\lambda(t-\tau)}\tilde{\boldsymbol{x}}(\tau)^T\mathbf{P_2}\tilde{\mathbf{x}}(\tau)\,\mathrm{d}\tau \tag{20}$$

Its time derivative, after substituting the error dynamics, is:

$$\begin{aligned}\dot{L} = & -\frac{1}{2}\tilde{\boldsymbol{x}}^T(\mathbf{Q_1}-\mathbf{P_2})\tilde{\mathbf{x}}+\tilde{\mathbf{x}}^{\mathbf{T}}\mathbf{P_1}(\tilde{\mathbf{W}}^{\mathbf{T}}\sigma_{\mathbf{v}}+\mathbf{w}) \\ & +\operatorname{tr}(\dot{\tilde{\mathbf{W}}}^{\mathbf{T}}\rho^{-1}\tilde{\mathbf{W}})-\lambda\mathbf{L}_{\text{int}}\end{aligned}$$

where $\sigma_v=\sigma(\hat{\mathbf{V}}^T\hat{\tilde{\boldsymbol{x}}})$, $\mathbf{Q}=\mathbf{Q_1}-\mathbf{P_2}>\mathbf{0}$, and $L_{\text{int}}=\frac{1}{2}\int_0^t e^{-\lambda(t-\tau)}\tilde{\boldsymbol{x}}(\tau)^T\mathbf{P_2}\tilde{\mathbf{x}}(\tau)\,\mathrm{d}\tau$. We substitute the update law (17) using $\dot{\tilde{\mathbf{W}}}=-\dot{\hat{\mathbf{W}}}$.

$$\begin{aligned}\operatorname{tr}(\dot{\tilde{\mathbf{W}}}^{\mathbf{T}}\rho^{-1}\tilde{\mathbf{W}}) &= \operatorname{tr}\left(\left(\eta_W\mathbf{A}^{-\mathbf{T}}\mathbf{z}\sigma_{\mathbf{v}}^{\mathbf{T}}+\rho\|\tilde{\mathbf{x}}\|\hat{\mathbf{W}}\right)^{\mathbf{T}}\rho^{-\mathbf{1}}\tilde{\mathbf{W}}\right) \\ &= \eta_W\operatorname{tr}(\sigma_v\boldsymbol{z}^Tl_1\tilde{\mathbf{W}}) \\ &\quad+\|\tilde{\boldsymbol{x}}\|\operatorname{tr}(\hat{\mathbf{W}}^T\tilde{\mathbf{W}})\end{aligned}$$

where $l_1=\mathbf{A}^{-1}\rho^{-1}$

We expand the leakage term by substituting $\hat{\mathbf{W}}=\mathbf{W}-\tilde{\mathbf{W}}$:

$$\begin{aligned}\|\tilde{\boldsymbol{x}}\|\operatorname{tr}(\hat{\mathbf{W}}^T\tilde{\mathbf{W}}) &= \|\tilde{\boldsymbol{x}}\|\operatorname{tr}((\mathbf{W}-\tilde{\mathbf{W}})^{\mathbf{T}}\tilde{\mathbf{W}}) \\ &= -\|\tilde{\boldsymbol{x}}\|\|\tilde{\mathbf{W}}\|^2+\|\tilde{\boldsymbol{x}}\|\|\mathbf{W}\|\|\tilde{\mathbf{W}}\|\end{aligned}$$

Substituting this back into the $\dot{L}$ expression:

$$\begin{aligned}\dot{L}\leq & -\frac{1}{2}\tilde{\boldsymbol{x}}^T\mathbf{Q}\tilde{\mathbf{x}}-\|\tilde{\mathbf{x}}\|\|\tilde{\mathbf{W}}\|^{\mathbf{2}}-\lambda\mathbf{L}_{\text{int}} \\ & +\tilde{\boldsymbol{x}}^T\mathbf{P_1}(\tilde{\mathbf{W}}^{\mathbf{T}}\sigma_{\mathbf{v}}+\mathbf{w})+\eta_{\mathbf{W}}\operatorname{tr}(\sigma_{\mathbf{v}}\mathbf{z}^{\mathbf{T}}\mathbf{l_1}\tilde{\mathbf{W}}) \\ & +\|\tilde{\boldsymbol{x}}\|\|\mathbf{W}\|\|\tilde{\mathbf{W}}\|\end{aligned}$$

Moreover, we have

$$\begin{aligned}|\tilde{\boldsymbol{x}}^T\mathbf{P_1}\tilde{\mathbf{W}}^{\mathbf{T}}\sigma_{\mathbf{v}}| &\leq \|\tilde{\boldsymbol{x}}\|\|\mathbf{P_1}\|(\|\tilde{\mathbf{W}}\|\sigma_{\mathbf{M}}+\bar{\mathbf{w}}) \\ \|\tilde{\boldsymbol{x}}\|\|\mathbf{W}\|\|\tilde{\mathbf{W}}\| &\leq \|\tilde{\boldsymbol{x}}\|\|\tilde{\mathbf{W}}\|W_M \\ |\eta_W\operatorname{tr}(\sigma_v\boldsymbol{z}^Tl_1\tilde{\mathbf{W}})| &\leq \eta_W\|\sigma_v\|\|\boldsymbol{z}\|\|l_1\|\|\tilde{\mathbf{W}}\| \\ &\leq \eta_W\sigma_M\frac{\sqrt{n}}{\lambda}\|\tilde{\boldsymbol{x}}\|\|l_1\|\|\tilde{\mathbf{W}}\|.\end{aligned}$$

where $\|W\|\leq W_M$, $\|\sigma(\hat{\tilde{\boldsymbol{x}}})\|\leq\sigma_M$, and because $\boldsymbol{z}(t)$ is the state of the first-order filter (8) driven by $\tilde{\boldsymbol{x}}(t)$, its 2-norm satisfies

$$\begin{aligned}\|\boldsymbol{z}(t)\| &= \left\|\int_0^t e^{-\lambda(t-\tau)}\tilde{\boldsymbol{x}}(\tau)\,\mathrm{d}\tau\right\| \\ &\leq \int_0^t e^{-\lambda(t-\tau)}\|\tilde{\boldsymbol{x}}(\tau)\|\,\mathrm{d}\tau \\ &\leq \|\tilde{\boldsymbol{x}}\|_\infty\int_0^t e^{-\lambda(t-\tau)}\,\mathrm{d}\tau \\ &= \frac{1-e^{-\lambda t}}{\lambda}\|\tilde{\boldsymbol{x}}\|_\infty \\ &\leq \frac{\sqrt{n}}{\lambda}\|\tilde{\boldsymbol{x}}\|_\infty \\ &\leq \frac{\sqrt{n}}{\lambda}\|\tilde{\boldsymbol{x}}\| \quad (\text{since } \|\tilde{\boldsymbol{x}}\|_\infty\leq\|\tilde{\boldsymbol{x}}\|).\end{aligned} \tag{21}$$

with n denoting the state dimension. Then, the inequality becomes:

$$\begin{aligned}\dot{L}\leq & -\frac{1}{2}\lambda_{\min}(\mathbf{Q})\|\tilde{\mathbf{x}}\|^{\mathbf{2}}-\|\tilde{\mathbf{x}}\|\|\tilde{\mathbf{W}}\|^{\mathbf{2}}-\lambda\mathbf{L}_{\text{int}} \\ & +\|\tilde{\boldsymbol{x}}\|\|\mathbf{P_1}\|(\|\tilde{\mathbf{W}}\|\sigma_{\mathbf{M}}+\bar{\mathbf{w}}) \\ & +\|\tilde{\boldsymbol{x}}\|W_M\|\tilde{\mathbf{W}}\|+\eta_W\sigma_M\frac{\sqrt{n}}{\lambda}\|\tilde{\boldsymbol{x}}\|\|l_1\|\|\tilde{\mathbf{W}}\|\end{aligned}$$

By completing the squares for the terms involving $\|\hat{W}\|$, we look for conditions on $\|x\|$ which are independent of the neural network weights error and also make the time derivative of the Lyapunov candidate negative.

$$\begin{aligned}\dot{L}\leq & -\|\tilde{\boldsymbol{x}}\|\|\tilde{\mathbf{W}}\|^2+k_b\|\tilde{\boldsymbol{x}}\|\|\tilde{\mathbf{W}}\| \\ & -\frac{1}{2}\lambda_{\min}(\mathbf{Q})\|\tilde{\mathbf{x}}\|^{\mathbf{2}}+\|\tilde{\mathbf{x}}\|\|\mathbf{P_1}\|\mathbf{w_M}-\lambda\mathbf{L}_{\text{int}}\end{aligned}$$

where $k_b=\|\mathbf{P_1}\|\sigma_{\mathbf{M}}+\mathbf{W_M}+\eta_{\mathbf{W}}\sigma_{\mathbf{M}}\frac{\sqrt{\mathbf{n}}}{\lambda}\|\mathbf{A}^{-1}\rho_{\mathbf{1}}^{-1}\|$. The terms involving $\tilde{\mathbf{W}}$ are of the form $-(\|\tilde{\boldsymbol{x}}\|)\|\tilde{\mathbf{W}}\|^2+(k_b\|\tilde{\boldsymbol{x}}\|)\|\tilde{\mathbf{W}}\|$. By completing the square, this is bounded above by $\frac{(k_b\|\tilde{\boldsymbol{x}}\|)^2}{4\|\tilde{\boldsymbol{x}}\|}=\frac{k_b^2}{4}\|\tilde{\boldsymbol{x}}\|$. The final inequality for $\dot{L}$ is:

$$\dot{L}\leq-\frac{1}{2}\lambda_{min}(\mathbf{Q})\|\tilde{\mathbf{x}}\|^{\mathbf{2}}-\lambda\mathbf{L}_{\text{int}}+\left(\|\mathbf{P_1}\|\bar{\mathbf{w}}+\frac{\mathbf{k_b^2}}{\mathbf{4}}\right)\|\tilde{\mathbf{x}}\| \tag{22}$$

To find a sufficient condition that guarantees $\dot{L}\leq 0$ and subsequently derive the ultimate bound, we can analyze a simpler upper bound. Since the term $-\lambda L_{int}$ is always non-positive, it can be omitted from the right-hand side while the inequality still holds. The analysis thus proceeds with the remaining terms :

$$\|\tilde{\boldsymbol{x}}\|\geq\frac{2\left(\|\mathbf{P_1}\|\bar{\mathbf{w}}+\mathbf{k_b^2}\right)}{\lambda_{min}(\mathbf{Q})}=b \tag{23}$$

Furthermore, the above condition on $\|\tilde{\boldsymbol{x}}\|$ guarantees the negative semi-definiteness of $\dot{L}$ and therefore, ultimate boundedness of $\tilde{\boldsymbol{x}}$. In fact, $\dot{L}$ is negative definite outside the ball with radius b.

■

parameter tuning of the learning rates η_1,η_2 and the forgetting factors ρ_1,ρ_2 is crucial for ensuring convergence and stability. The values of these parameters should be chosen based on the specific characteristics of the system being identified, such as the dynamics and noise levels.

4. FIGURES, TABLES, AND EQUATIONS

The dynamics of an n-degree-of-freedom robot manipulator can be described by the following nonlinear state-space model:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+\boldsymbol{G}(\boldsymbol{q})+\boldsymbol{\tau_d}=\boldsymbol{\tau} \tag{24}$$

where $\boldsymbol{q} \in \mathbb{R}^{K}$ is the joint position vector, $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$ are the joint velocity and acceleration vectors, $\boldsymbol{M}(\boldsymbol{q}) \in \mathbb{R}^{K\times K}$ is the positive definite inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{K\times K}$ is the Coriolis/centrifugal matrix, $\boldsymbol{G}(\boldsymbol{q}) \in \mathbb{R}^{K}$ is the gravity vector, and $\boldsymbol{\tau} \in \mathbb{R}^{K}$ is the control input. To generalize the system for identification of an unknown nonlinear function, we consider a Hamiltonian structure where the dynamics are partitioned into known and unknown components.

$$\begin{bmatrix} \dot{\boldsymbol{q}} \\ \dot{\boldsymbol{p}} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial p} \\ \frac{\partial H}{\partial q} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \boldsymbol{\tau}}_{\text{known}} - \underbrace{\begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \boldsymbol{\tau_d}}_{\text{unknown}} \tag{25}$$

where $\boldsymbol{p} = \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}}$ is the generalized momentum, $\boldsymbol{x} = [\boldsymbol{q}^T \ \boldsymbol{p}^T]$ is the state vector composed of joint positions and momenta, and $H(\boldsymbol{q}, \boldsymbol{p})$ is the Hamiltonian (total energy) of the system. The first term on the right-hand side represents the intrinsic system dynamics derived from the Hamiltonian structure, while the second and third terms correspond to the control input and dissipative torque, respectively.

4.1 Figures and tables

All figures and tables should be placed after their first mention in the text. Large figures and tables may span across both columns. Scanned images (e.g., line art, photos) can be used if the output resolution is at least 600 dpi.

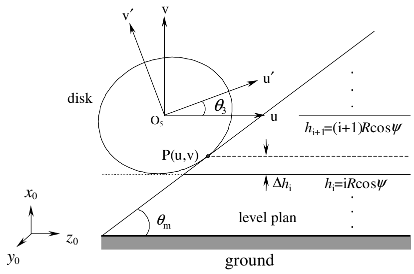

Fig. 1. The caption should be placed after the figure.

Figure captions should be below the figures; table captions should be above the tables. They should be referred to in the text as, for example, Fig. 1, or Figs. 1∼3.

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Table 1. Your table caption here

4.2 Equations

Equation numbers should be Arabic numerals enclosed in parentheses on the right-hand margin. They should be cited in the text as, for example, Eq. (1), or Eqs. (1)∼(3). Equations are located in the middle and equation numbers are located at the end. Punctuate equations with commas or periods when they are part of a sentence. For example,

$$\dot{x} = Ax + Bu, \tag{26}$$

$$y = Cx + Du. \tag{27}$$

4.3 References

References should appear in a separate bibliography at the end of the paper, with items referred to by numerals in square brackets [1, 4-5]. Times New Roman 10pt is used for references.

5. PAGE NUMBERS

Do not put a page number in the manuscript PDF.

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