Some setting

- Data

```
xn = {-27.02, 3.57, 8.191, 9.898, 9.603, 9.945, 10.056}
```

```
{-27.02, 3.57, 8.191, 9.898, 9.603, 9.945, 10.056}
```

- Sample mean

```
Mean[xn]
```

```
3.46329
```

- Normalizing constant

$$\int_{-\infty}^{\infty} \mathbf{Exp}\left[\frac{-\mathbf{N}\,(\mu-\mathbf{x})^2}{2\,\sigma^2}\right] \mathrm{d}\mu$$

$$\mathtt{ConditionalExpression}\left[\frac{\sqrt{2\,\pi}}{\sqrt{\frac{\mathtt{N}}{\sigma^2}}}, \mathtt{Re}\left[\frac{\mathtt{N}}{\sigma^2}\right] \geq 0\right]$$

Posterior distribution function w.o. normalizing constant

$$\int_{0}^{\infty} \sigma^{c}\, \mathbf{Exp}\left[\frac{-(\mathbf{x}-\mu)^2}{2\,\sigma^2} - \frac{\sigma}{\mathbf{s}}\right] \mathrm{d}\sigma$$

$$\text{ConditionalExpression}\left[11.976796597153522` \text{ HypergeometricPFQ}\left[\{\},\ \{-0.050000000000000044`,\ 0.44999999999999996`\},\ -\frac{1}{800}(x-\mu)^2\right] - \frac{1}{\left(\frac{1}{(x-\mu)^2}\right)^{0.55`}}\right.$$
$$1.2220697972757049` \text{ HypergeometricPFQ}\left[\{\},\ \{1.55`,\ 0.5`\},\ -\frac{1}{800}(x-\mu)^2\right] -$$
$$\frac{1}{\left(\frac{1}{(x-\mu)^2}\right)^{1.05`}} 0.47443725266903375` \text{ HypergeometricPFQ}\left[\{\},\right.$$
$$\left.\left.\{2.05`,\ 1.4999999999999998`\},\ -\frac{1}{800}(x-\mu)^2\right],\ \text{Re}\left[(x-\mu)^2\right] > 0\right] /. x \to 10$$

$$\text{ConditionalExpression}\left[11.9768\ \text{HypergeometricPFQ}\left[\{\},\ \{-0.05,\ 0.45\},\ -\frac{1}{800}(10-\mu)^2\right] -\right.$$
$$\frac{1}{\left(\frac{1}{(10-\mu)^2}\right)^{0.55}} 1.22207\ \text{HypergeometricPFQ}\left[\{\},\ \{1.55,\ 0.5\},\ -\frac{1}{800}(10-\mu)^2\right] - \frac{1}{\left(\frac{1}{(10-\mu)^2}\right)^{1.05}}$$
$$\left.0.474437\ \text{HypergeometricPFQ}\left[\{\},\ \{2.05,\ 1.5\},\ -\frac{1}{800}(10-\mu)^2\right],\ \text{Re}\left[(10-\mu)^2\right] > 0\right]$$

$$\text{Assuming}\left[\text{Re}\left[(x-\mu)^2\right] > 0,\ \int_0^\infty \sigma^{0.1}\ \text{Exp}\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]\text{Exp}\left[-\frac{\sigma}{10}\right] d\sigma\right]$$

Define function for posterior distribution of one data

```
Clear[plot]
```

$$\text{plot}[x\_] := 11.976796597153522` \text{ HypergeometricPFQ}\left[\{\},\ \{-0.050000000000000044`,\ 0.44999999999999996`\},\ -\frac{1}{800}(x-\mu)^2\right] - \frac{1}{\left(\frac{1}{(x-\mu)^2}\right)^{0.55`}}$$
$$1.2220697972757049` \text{ HypergeometricPFQ}\left[\{\},\ \{1.55`,\ 0.5`\},\ -\frac{1}{800}(x-\mu)^2\right] -$$
$$\frac{1}{\left(\frac{1}{(x-\mu)^2}\right)^{1.05`}} 0.47443725266903375` \text{ HypergeometricPFQ}\left[\{\},\ \{2.05`,\ 1.4999999999999998`\},\ -\frac{1}{800}(x-\mu)^2\right]$$

$$\text{HypergeometricPFQ}\left[\{\},\ \{-0.050000000000000044`,\ 0.44999999999999996`\},\ -\frac{1}{800}(0.1)^2\right]$$

$$1.00056$$

$$\int \sigma^{0.1}\ \text{Exp}\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]\text{Exp}\left[\frac{\sigma}{10}\right] d\sigma$$

$$\int e^{-\frac{(x-\mu)^2}{2\sigma^2}+\frac{\sigma}{10}} \sigma^{0.1}\, d\sigma$$

$$\int \sigma^{0.1}\, \text{Exp}[\sigma]\, d\sigma$$

$$-\frac{1.\,\sigma^{1.1}\,\text{Gamma}[1.1, -1.\,\sigma]}{(-1.\,\sigma)^{1.1}}$$

Plot

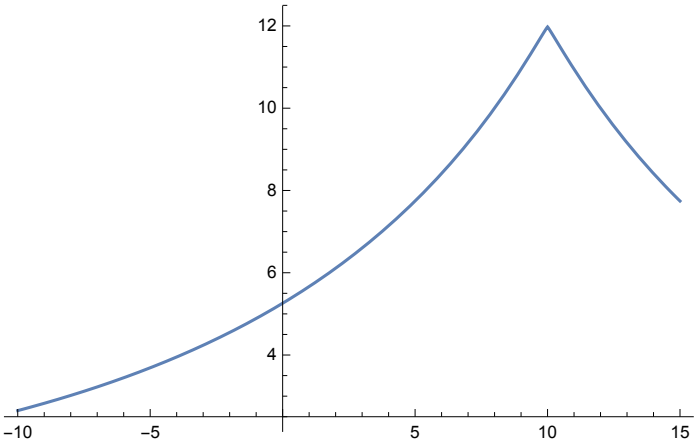
Parameters

```
Clear[x, μ]
```

```
s = 10; c = 0.1; x = 12.1; μ = 10;
```

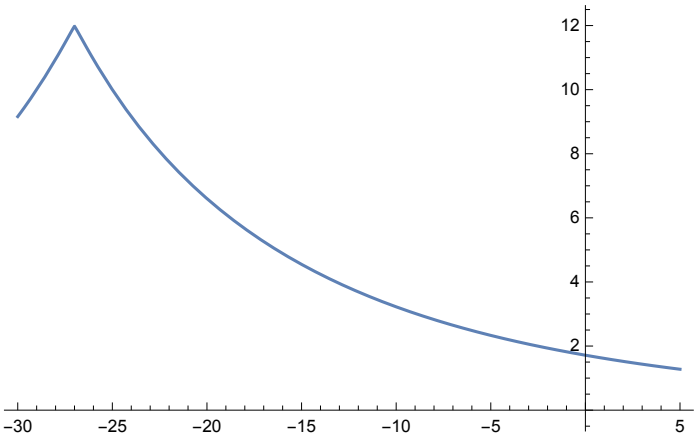
Posterior P(μ|x) for one datum from Xn = -10

```
Plot[plot[10], {μ, -10, 15}]
```



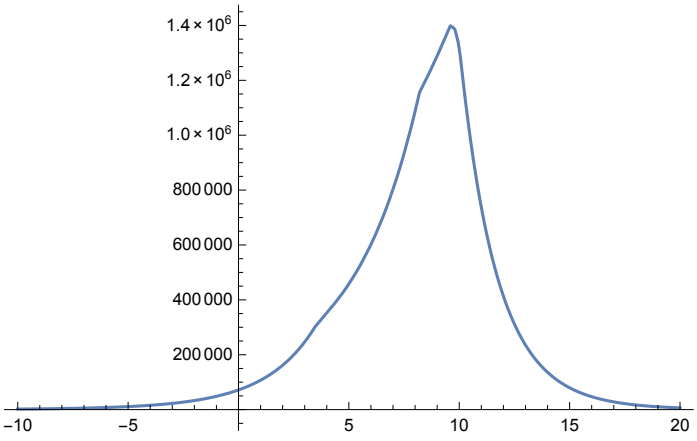
Posterior P(μ|x) for one datum from Xn = -27

```
Plot[plot[-27], {μ, -30, 5}]
```

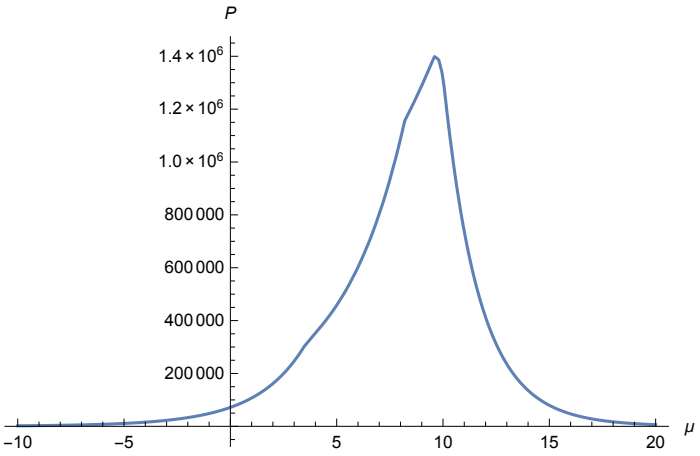


Posterior P(μ|x) for all 7 data

```
Plot[plot[-27] plot[3.6] plot[8.2]
  plot[9.8] plot[9.6] plot[9.95] plot[10.05], {μ, -10, 20}]
```

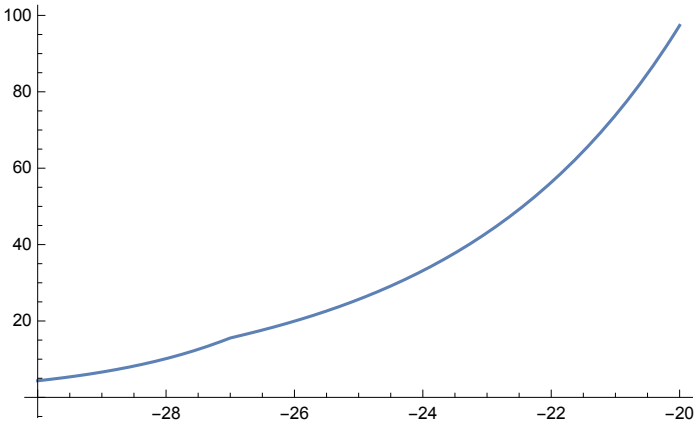


```
Show[%67, AxesLabel → {HoldForm[μ], HoldForm[P]},
 PlotLabel → None, LabelStyle → {GrayLevel[0]}]
```



- check a small bump around μ=-27

```
Plot[plot[-27] plot[3.6] plot[8.2]
  plot[9.8] plot[9.6] plot[9.95] plot[10.05], {μ, -30, -20}]
```



```
Show[%64, AxesLabel → {HoldForm[μ], HoldForm[P]},
 PlotLabel → None, LabelStyle → {GrayLevel[0]}]
```