Some setting

■ Data and plots

We have 10 data points

```
n = 10;
```

Give the data

In[1]:=
```
x = Table[i + RandomReal[], {i, 0, 10}]
```

Out[1]=
```
{0.301554, 1.51353, 2.82512, 3.8655, 4.46734,
 5.26735, 6.80425, 7.35982, 8.69651, 9.10802, 10.2167}
```

In[58]:=
```
Mean[x]
```

Out[58]=
```
5.49324
```

In[2]:=
```
t = Table[ (x[[i]] * 3 + 2) + RandomReal[{-2, 2}], {i, 1, 10}]
```

Out[2]=
```
{1.25094, 6.05802, 10.954, 13.8093,
 15.242, 18.6502, 21.3727, 23.5615, 28.9253, 29.8955}
```

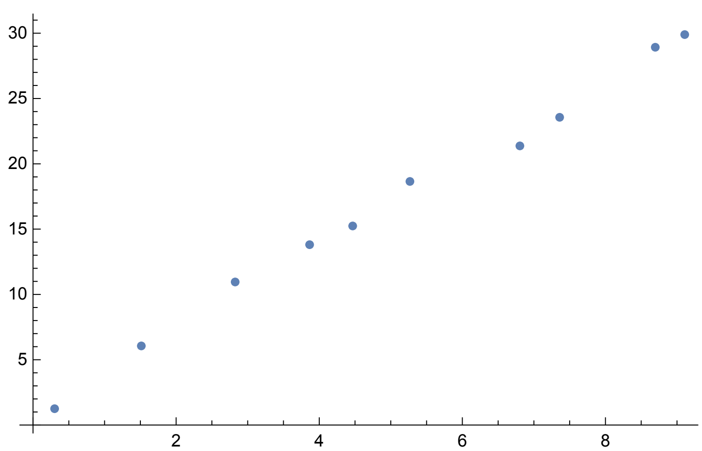
In[3]:=
```
l = Table[{x[[i]], t[[i]]}, {i, 1, 10}]
```

Out[3]=
```
{{0.301554, 1.25094}, {1.51353, 6.05802}, {2.82512, 10.954},
 {3.8655, 13.8093}, {4.46734, 15.242}, {5.26735, 18.6502}, {6.80425, 21.3727},
 {7.35982, 23.5615}, {8.69651, 28.9253}, {9.10802, 29.8955}}
```

Plot of the data

In[4]:=
```
ListPlot[l]
```

Out[4]=



Variance

In[5]:= $\sigma = \sqrt{\text{Variance[t]}}$

Out[5]= 9.3748

The coefficient 3 is chosen by assumption in the next equation. It gives Log $P(\omega_i \mid t, x, \sigma)$

In[63]:= `term = 1 / (2 σ^2) (Sum[(t[[i]] - w0 - w1 x[[i]])^2, {i, 1, 10}] + 3 (w0^2 + w1^2))`

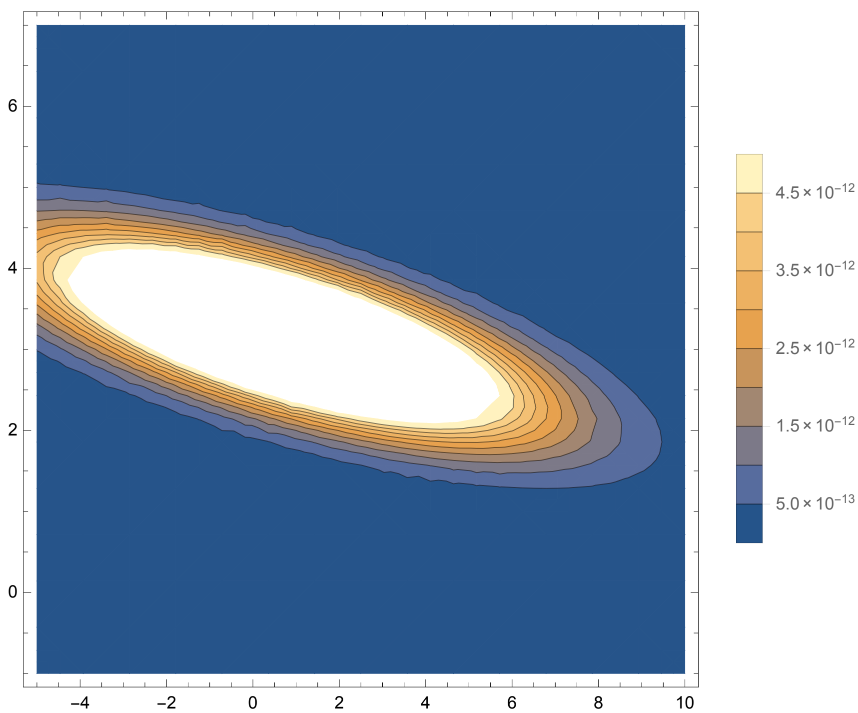
Out[63]= $0.00568913 \left((29.8955 - w0 - 9.10802\,w1)^2 + (28.9253 - w0 - 8.69651\,w1)^2 + (23.5615 - w0 - 7.35982\,w1)^2 + (21.3727 - w0 - 6.80425\,w1)^2 + (18.6502 - w0 - 5.26735\,w1)^2 + (15.242 - w0 - 4.46734\,w1)^2 + (13.8093 - w0 - 3.8655\,w1)^2 + (10.954 - w0 - 2.82512\,w1)^2 + (6.05802 - w0 - 1.51353\,w1)^2 + (1.25094 - w0 - 0.301554\,w1)^2 + 3\left(w0^2 + w1^2\right)\right)$

■ Posterior

Contour plot of $P(\omega_i \mid t, x, \sigma)$, posterior

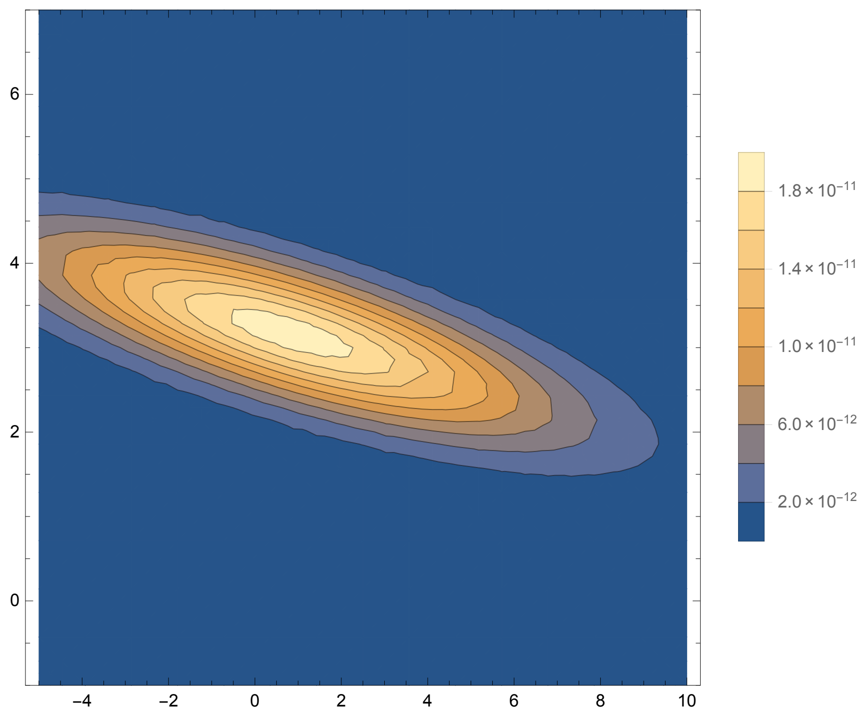
In[61]:= `ContourPlot[`$\left(\frac{1}{2\pi\sigma}\right)^{\frac{n}{2}+1}$` Exp[-term], {w0, -5, 10}, {w1, -1, 7}, PlotLegends → Automatic]`

Out[61]=



In[64]:=
```
ContourPlot[(1/(2 π σ))^(n/2+1) Exp[-term], {w0, -5, 10},
  {w1, -1, 7}, PlotLegends → Automatic, PlotRange → {2 * 10^-11, 0}]
```

Out[64]=



Has a maximum at

In[33]:=
```
Solve[{D[term, w0] == 0, D[term, w1] == 0}, {w0, w1}]
```

Out[33]=
```
{{w0 → 0.812961, w1 → 3.16977}}
```

Review for likelihood, Log P(t |ω,..)

In[34]:=
```
original = 1/(2 σ^2) (Sum[(t[[i]] - w0 - w1 x[[i]])^2, {i, 1, 10}])
```

Out[34]=
```
0.00568913
 ((29.8955 - w0 - 9.10802 w1)^2 + (28.9253 - w0 - 8.69651 w1)^2 + (23.5615 - w0 - 7.35982 w1)^2 +
   (21.3727 - w0 - 6.80425 w1)^2 + (18.6502 - w0 - 5.26735 w1)^2 + (15.242 - w0 - 4.46734 w1)^2 +
   (13.8093 - w0 - 3.8655 w1)^2 + (10.954 - w0 - 2.82512 w1)^2 +
   (6.05802 - w0 - 1.51353 w1)^2 + (1.25094 - w0 - 0.301554 w1)^2)
```

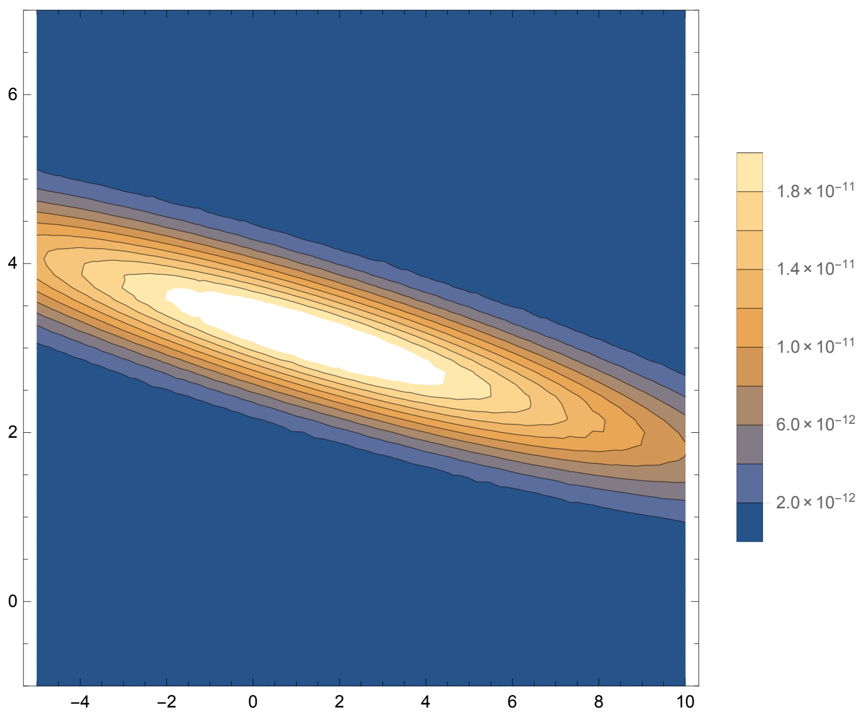
In[35]:=
```
Solve[{D[original, w0] == 0, D[term, w1] == 0}, {w0, w1}]
```

Out[35]=
```
{{w0 → 1.79794, w1 → 3.02217}}
```

Contour plot of P(t |ω,..), likelihood

In[46]:=
```
ContourPlot[(1/(2 π σ))^(n/2+1) Exp[-original], {w0, -5, 10},
 {w1, -1, 7}, PlotLegends → Automatic, PlotRange → {2 * 10^-11, 0}]
```

Out[46]=



Laplace approximation of posterior

Laplace approximation the constant factor.

In[48]:=
```
G0 = (1/(2 π σ))^(n/2+1) Exp[-term] /. {w0 → 0.8129608926369966`, w1 → 3.16977096459527`}
```

Out[48]= 1.93281×10^{-11}

```
P = (1/(2 π σ))^(n/2+1) Exp[-term]
```

3.75562×10^{-11}

$e^{-0.00661034\left((31.1715-w0-9.56193\,w1)^2+(27.8925-w0-8.40024\,w1)^2+(24.7902-w0-7.1589\,w1)^2+(21.1721-w0-6.37539\,w1)^2+(19.7\ldots\right.}$

Exponential part

In[37]:= `D[D[-term, w0], w0]`

Out[37]= -0.147917

In[38]:= `D[D[-term, w0], w1]`

Out[38]= -0.571291

In[39]:= `D[D[-term, w1], w0]`

Out[39]= -0.571291

In[40]:= `D[D[-term, w1], w1]`

Out[40]= -3.81237

In[47]:= $A = \begin{pmatrix} \texttt{D[D[-term, w0], w0]} & \texttt{D[D[-term, w0], w1]} \\ \texttt{D[D[-term, w1], w0]} & \texttt{D[D[-term, w1], w1]} \end{pmatrix}$

Out[47]= {{-0.147917, -0.571291}, {-0.571291, -3.81237}}

```
{{w0 → 0.8129608926369966`, w1 → 3.16977096459527`}}
```

In[53]:= $(\texttt{w0 - 0.8129608926369966`}\quad \texttt{w1 - 3.16977096459527`}).A.\begin{pmatrix} \texttt{w0 - 0.8129608926369966`} \\ \texttt{w1 - 3.16977096459527`} \end{pmatrix}$

Out[53]= {{(-0.812961 + w0) (-0.147917 (-0.812961 + w0) - 0.571291 (-3.16977 + w1)) +
(-0.571291 (-0.812961 + w0) - 3.81237 (-3.16977 + w1)) (-3.16977 + w1)}}

Full approximation

In[54]:=
```
G = G0
  Exp[(-0.8129608926369966` + w0) (-0.1479173249248259` (-0.8129608926369966` + w0) -
        0.5712906972305959` (-3.16977096459527` + w1)) +
     (-0.5712906972305959` (-0.8129608926369966` + w0) -
        3.812366606111279` (-3.16977096459527` + w1)) (-3.16977096459527` + w1)]
```

Out[54]= 1.93281×10^{-11}
$e^{(-0.812961+w0)\ (-0.147917\ (-0.812961+w0)-0.571291\ (-3.16977+w1))+(-0.571291\ (-0.812961+w0)-3.81237\ (-3.16977+w1))\ (-3.}$

Contour plot of the Laplace approximation

In[56]:=
```
ContourPlot[G, {w0, -5, 10}, {w1, -1, 7},
 PlotLegends → Automatic, PlotRange → {2 * 10^-11, 0}]
```

Out[56]=