

# Some setting

## ■ Data

```
xn = {-27.02, 3.57, 8.191, 9.898, 9.603, 9.945, 10.056}  
{-27.02, 3.57, 8.191, 9.898, 9.603, 9.945, 10.056}
```

## ■ Sample mean

```
Mean[xn]  
3.46329
```

## ■ Normalizing constant

$$\int_{-\infty}^{\infty} \mathbf{Exp}\left[\frac{-\mathbf{N}(\mu - \mathbf{x})^2}{2\sigma^2}\right] d\mu$$
$$\text{ConditionalExpression}\left[\frac{\sqrt{2\pi}}{\sqrt{\frac{\mathbf{N}}{\sigma^2}}}, \text{Re}\left[\frac{\mathbf{N}}{\sigma^2}\right] \geq 0\right]$$

# Posterior distribution function w.o. normalizing constant

$$\int_0^{\infty} \sigma^c \mathbf{Exp}\left[\frac{-(\mathbf{x} - \mu)^2}{2\sigma^2} - \frac{\sigma}{\mathbf{s}}\right] d\sigma$$

```

ConditionalExpression[11.976796597153522` HypergeometricPFQ[{},
  {-0.0500000000000000044`, 0.44999999999999996`, - $\frac{1}{800} (x - \mu)^2$ ] -  $\frac{1}{\left(\frac{1}{(x - \mu)^2}\right)^{0.55}}$ 
  1.2220697972757049` HypergeometricPFQ[{}, {1.55`, 0.5`, - $\frac{1}{800} (x - \mu)^2$ ] -
   $\frac{1}{\left(\frac{1}{(x - \mu)^2}\right)^{1.05}}$  0.47443725266903375` HypergeometricPFQ[{},
    {2.05`, 1.4999999999999998`, - $\frac{1}{800} (x - \mu)^2$ ], Re[(x - μ)2] > 0] /. x -> 10

ConditionalExpression[11.9768 HypergeometricPFQ[{}, {-0.05, 0.45}, - $\frac{1}{800} (10 - \mu)^2$ ] -
   $\frac{1}{\left(\frac{1}{(10 - \mu)^2}\right)^{0.55}}$  1.22207 HypergeometricPFQ[{}, {1.55, 0.5}, - $\frac{1}{800} (10 - \mu)^2$ ] -  $\frac{1}{\left(\frac{1}{(10 - \mu)^2}\right)^{1.05}}$ 
  0.474437 HypergeometricPFQ[{}, {2.05, 1.5}, - $\frac{1}{800} (10 - \mu)^2$ ], Re[(10 - μ)2] > 0]

Assuming[Re[(x - μ)2] > 0,  $\int_0^\infty \sigma^{0.1} \text{Exp}\left[-\frac{(x - \mu)^2}{2 \sigma^2}\right] \text{Exp}\left[-\frac{\sigma}{10}\right] d\sigma$ ]

```

## ■ Define function for posterior distribution of one data

```

Clear[plot]

plot[x_] := 11.976796597153522` HypergeometricPFQ[{},
  {-0.0500000000000000044`, 0.44999999999999996`, - $\frac{1}{800} (x - \mu)^2$ ] -  $\frac{1}{\left(\frac{1}{(x - \mu)^2}\right)^{0.55}}$ 
  1.2220697972757049` HypergeometricPFQ[{}, {1.55`, 0.5`, - $\frac{1}{800} (x - \mu)^2$ ] -
   $\frac{1}{\left(\frac{1}{(x - \mu)^2}\right)^{1.05}}$  0.47443725266903375`
  HypergeometricPFQ[{}, {2.05`, 1.4999999999999998`, - $\frac{1}{800} (x - \mu)^2$ ]

HypergeometricPFQ[{}, {-0.0500000000000000044`, 0.44999999999999996`, - $\frac{1}{800} (0.1)^2$ ]
1.00056

 $\int \sigma^{0.1} \text{Exp}\left[-\frac{(x - \mu)^2}{2 \sigma^2}\right] \text{Exp}\left[\frac{\sigma}{10}\right] d\sigma$ 
 $\int e^{-\frac{(x - \mu)^2}{2 \sigma^2} + \frac{\sigma}{10}} \sigma^{0.1} d\sigma$ 

```

$$\int \sigma^{0.1} \mathbf{Exp}[\sigma] d\sigma$$

$$- \frac{1. \sigma^{1.1} \text{Gamma}[1.1, -1. \sigma]}{(-1. \sigma)^{1.1}}$$

## Plot

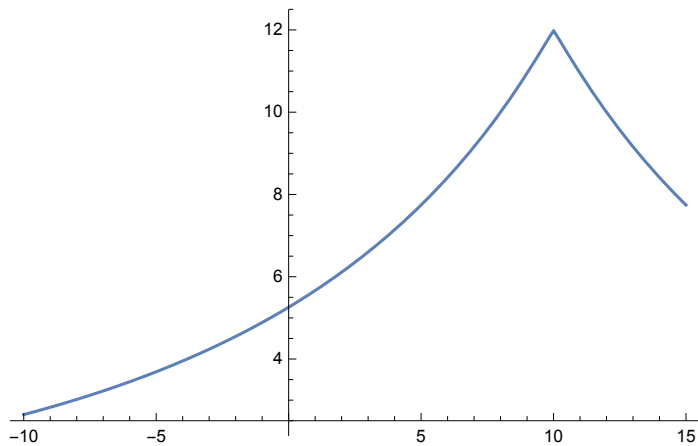
### ■ Parameters

```
Clear[x, μ]
```

```
s = 10; c = 0.1; x = 12.1; μ = 10;
```

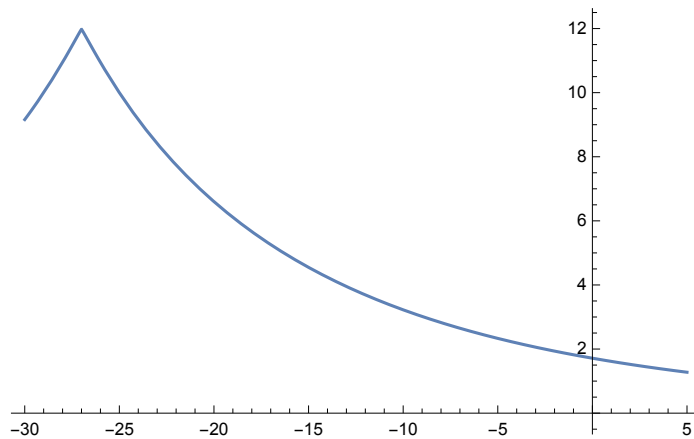
### ■ Posterior $P(\mu|x)$ for one datum from $X_n = -10$

```
Plot[plot[10], {μ, -10, 15}]
```



## ■ Posterior $P(\mu|x)$ for one datum from $X_n = -27$

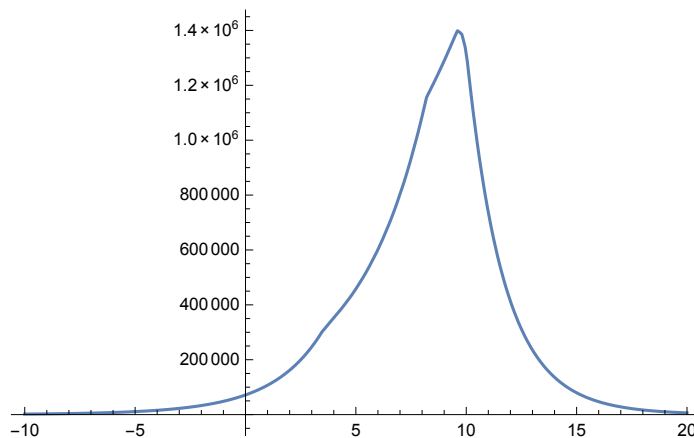
```
Plot[plot[-27], {μ, -30, 5}]
```



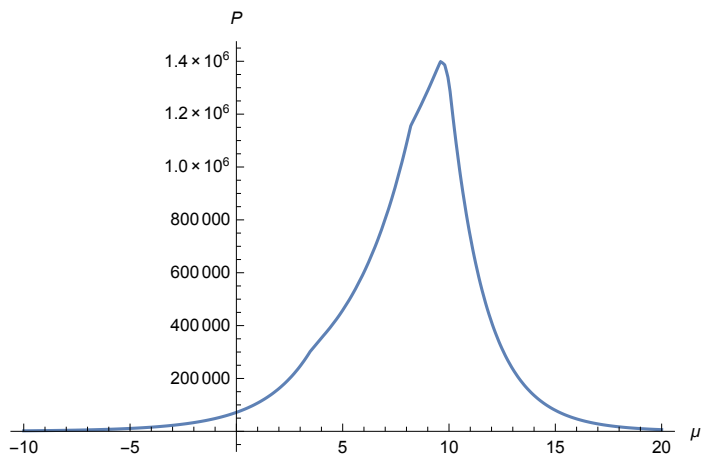
## ■ Posterior $P(\mu|x)$ for all 7 data

```
Plot[plot[-27] plot[3.6] plot[8.2]
```

```
plot[9.8] plot[9.6] plot[9.95] plot[10.05], {μ, -10, 20}]
```

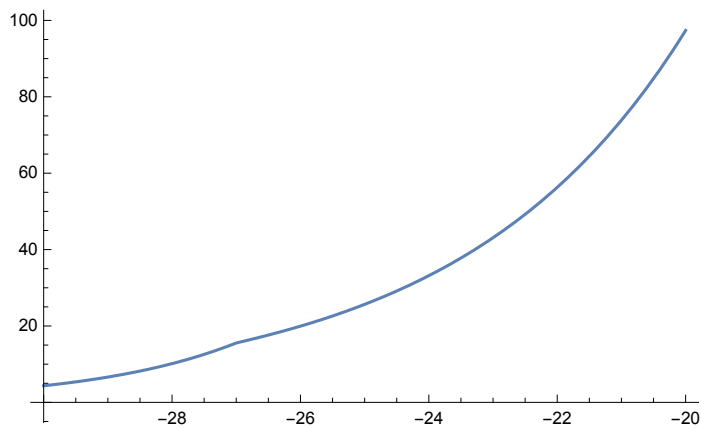


```
Show[%67, AxesLabel -> {HoldForm[μ], HoldForm[P]},
PlotLabel -> None, LabelStyle -> {GrayLevel[0]}]
```



■ check a small bump around  $\mu = -27$

```
Plot[plot[-27] plot[3.6] plot[8.2]
plot[9.8] plot[9.6] plot[9.95] plot[10.05], {μ, -30, -20}]
```



```
Show[%64, AxesLabel -> {HoldForm[ $\mu$ ], HoldForm[P]},  
PlotLabel -> None, LabelStyle -> {GrayLevel[0]}]
```

