Neural networks and Chinese character recognition

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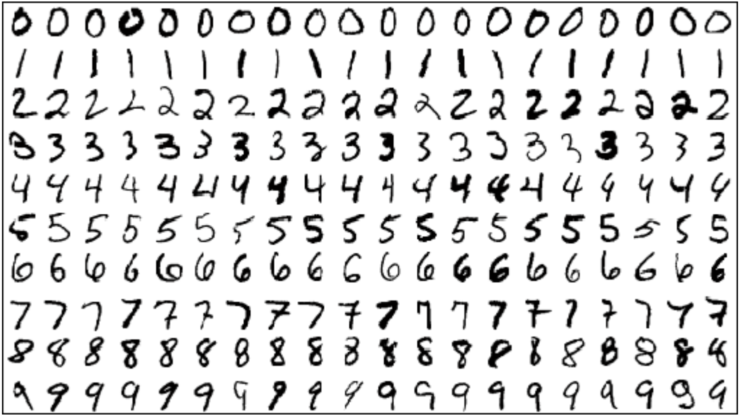
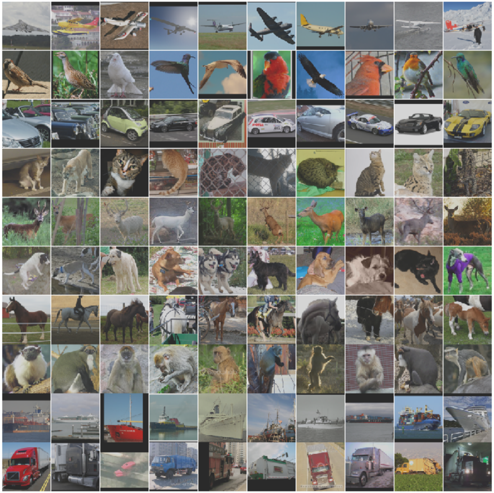
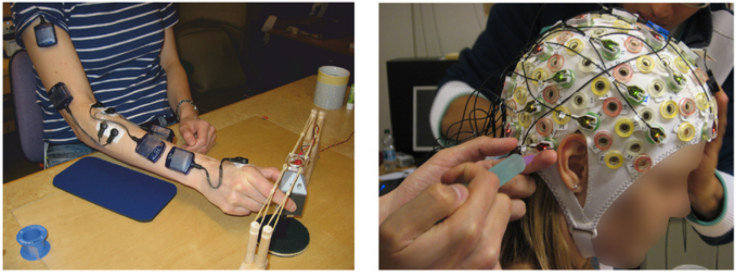
Example: Mnist

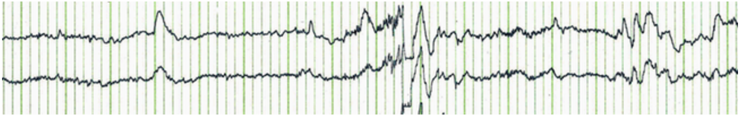


Example: Imagenet



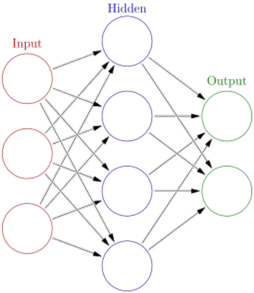
Example: EEG

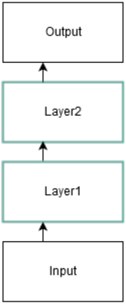

[1]



[1] Multi-channel EEG recordings during 3,936 grasp and lift trials, Luciw, Jarocka and Edin, Scientific Data 1, 2014; kaggle 2014

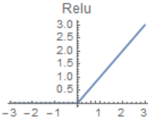
Neural Networks

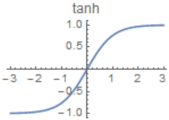




$$H = \sigma(\mathbf{W}_1 I + \mathbf{b}_1)$$

$$O = \sigma(\mathbf{W}_2 H + \mathbf{b}_2)$$





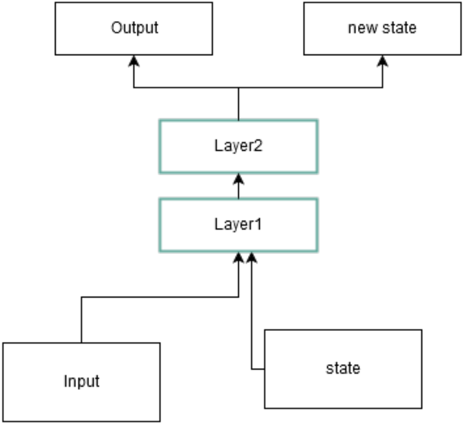
Typical approach to supervised machine learning

$$\text{input} \xrightarrow{\quad\text{representation}\quad} \mathbb{R}^k \xrightarrow{\quad\text{classification}\quad} \text{labels}$$

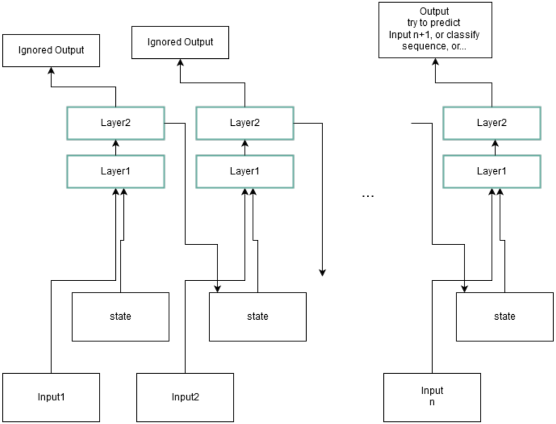
Timeseries and paths

Data which varies in time doesn't live in a fixed $\mathbb{R}^k$.
Time-structure of the data matters

Recurrent Neural Networks

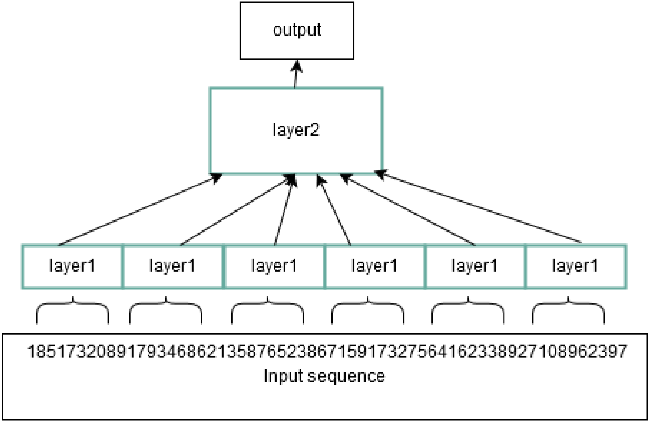


Recurrent Neural Networks

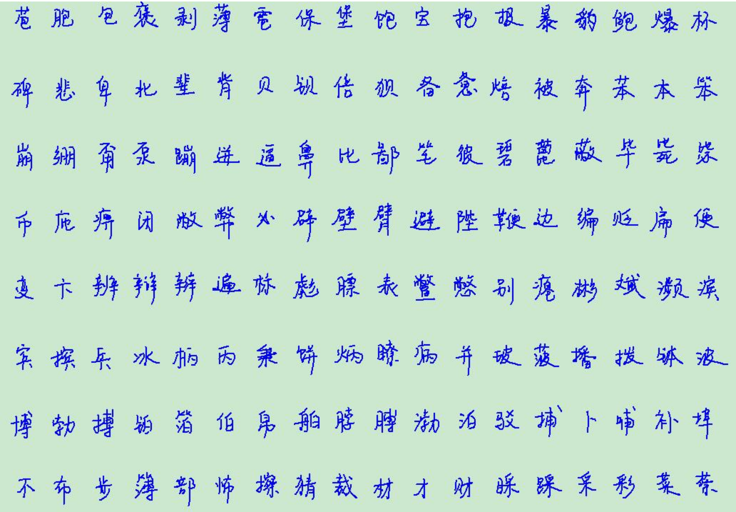


http://karpathy.github.io/2015/05/21/rnn-effectiveness/

Convolutional Neural Networks

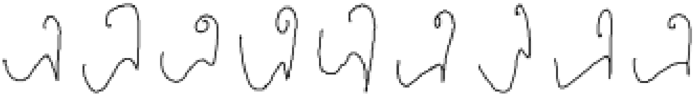


Chinese - the Casia online dataset



http://www.nlpr.ia.ac.cn/databases/handwriting/Home.html

Online handwriting



Typical approaches

$$\text{input} \xrightarrow{\text{representation}} \mathbb{R}^k \xrightarrow{\text{classification}} \text{labels}$$

$$\text{input} \times t \xrightarrow{\text{representation}} \mathbb{R}^{k\times t} \xrightarrow{\text{RNN}} \text{labels}$$

Signatures

The signature of a path is a set of iterated integrals.
Consider a path in $\mathbb{R}^3$ parameterised by the variable t ranging from 0 to 1, given by

$$t \mapsto (f_1(t), f_2(t), f_3(t))$$

Then, for example, the element 2,3 of the signature is

$$\int_0^1 \left[\int_0^t f_2'(s)\,ds\right] f_3'(t)\,dt = \int_0^1 \int_0^t df_2(s)\,df_3(t)$$

and element 2,1,2 of the signature is

$$\int_0^1 \int_0^t \int_0^s df_2(r)\,df_1(s)\,df_2(t).$$

Signatures

The mth level of the signature of a path in $\mathbb{R}^d$ is the d^m values of the elements with m integrated integrals. It takes values in $(\mathbb{R}^d)^{\otimes m}$. Given a piecewise linear path, it is easy to compute the first m levels of its signature using a theorem called Chen's identity.

Log-Signature demonstration

There is redundancy in the signature. The log signature is a transformation of the same information as the signature which is not redundant. For example, in $\mathbb{R}^2$, the first four levels of the signature look like this

$$(\cdot\,\cdot) + \begin{pmatrix}(\cdot\,\cdot)\\(\cdot\,\cdot)\end{pmatrix} + \left(\begin{pmatrix}(\cdot\,\cdot)\\(\cdot\,\cdot)\end{pmatrix}\;\begin{pmatrix}(\cdot\,\cdot)\\(\cdot\,\cdot)\end{pmatrix}\right) + \begin{pmatrix}\left(\begin{pmatrix}(\cdot\,\cdot)\\(\cdot\,\cdot)\end{pmatrix}\;\begin{pmatrix}(\cdot\,\cdot)\\(\cdot\,\cdot)\end{pmatrix}\right)\\\left(\begin{pmatrix}(\cdot\,\cdot)\\(\cdot\,\cdot)\end{pmatrix}\;\begin{pmatrix}(\cdot\,\cdot)\\(\cdot\,\cdot)\end{pmatrix}\right)\end{pmatrix}$$

- that is $2+4+8+16=30$ numbers while the log signature is only $2+1+2+3=8$ numbers.

Questions

A signature is a nice representation of a path of arbitrary length. When is it good enough? What properties of a complicated path can be derived from some levels of its signature? How to balance a representation using more levels of the signature versus chopping the path up and looking at signatures of the chunks?

Thanks!

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Dr Ben Graham