

* Ques: 05

Credit Score = y

Age = x_1

Education = x_2

Mean Value = $16 + 14 + 12 + 18 + 14 + 16 + 12 = 14.5$

	x_1	x_2
720	1	35
650	1	28
750	1	45
600	1	31
780	1	52
630	1	29
710	1	42
640	1	33

$$\beta = (\mathbf{x}^T \mathbf{x})^{-1} \mathbf{x}^T \mathbf{y} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}$$

$$\mathbf{x}^T \mathbf{x} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 35 & 28 & 45 & 31 & 52 & 29 & 42 & 33 \\ 16 & 14 & 14 & 12 & 18 & 14 & 16 & 12 \end{bmatrix} \begin{bmatrix} 1 & 35 & 16 \\ 1 & 28 & 14 \\ 1 & 45 & 14 \\ 1 & 31 & 12 \\ 1 & 52 & 18 \\ 1 & 29 & 14 \\ 1 & 42 & 16 \\ 1 & 33 & 12 \end{bmatrix}$$

$$= \begin{vmatrix} 8 & 205 & 116 \\ 205 & 11303 & 4364 \\ 116 & 4364 & 1712 \end{vmatrix}$$

$$X^T Y = \left[\begin{array}{ccccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 720 \\ 35 & 28 & 45 & 31 & 32 & 29 & 42 & 33 \\ 16 & 14 & 14 & 12 & 12 & 14 & 16 & 12 \end{array} \right] \begin{array}{c} x \\ \downarrow \end{array} \begin{array}{c} 650 \\ 750 \\ 600 \\ 780 \\ 630 \\ 710 \\ 640 \end{array}$$

$$= \begin{bmatrix} 5480 \\ 205520 \\ 80220 \end{bmatrix}$$

$$(X^T X)^{-1} = \frac{1}{\det X^T X} \cdot \text{Adjoint of } X^T X$$

Adjoint of $X^T X = (\text{Cofactor of } X^T X)$

$$\begin{aligned} |X^T X| &= \begin{vmatrix} 8 & 11303 & 4364 & -205 & 205 & 4364 & +116 & 205 & 11303 \\ 11303 & 4364 & 1712 & 116 & 1712 & +116 & 116 & 4364 \\ 4364 & 1712 & 116 & 116 & 1712 & 116 & 4364 \end{vmatrix} \\ &= \begin{vmatrix} 8(460320) - 205(1184) + 116(-34208) \\ -634848 \\ 634848 \end{vmatrix} \end{aligned}$$

(Cofactor of $\alpha^T \alpha$)^T

$$= + \begin{vmatrix} 11393 & 4364 \\ 4364 & 1712 \end{vmatrix} - \begin{vmatrix} 295 & 1364 \\ 116 & 1712 \end{vmatrix} + \begin{vmatrix} 295 & 116 \\ 116 & 1712 \end{vmatrix} - \begin{vmatrix} 8 & 116 \\ 116 & 1712 \end{vmatrix} + \begin{vmatrix} 8 & 295 \\ 116 & 4364 \end{vmatrix}$$
$$+ \begin{vmatrix} 295 & 116 \\ 11393 & 4364 \end{vmatrix} - \begin{vmatrix} 8 & 116 \\ 295 & 4364 \end{vmatrix} + \begin{vmatrix} 8 & 295 \\ 295 & 11393 \end{vmatrix}$$

$$= \begin{vmatrix} 460320 & 1184 & -34208 \\ 1184 & 240 & -692 \\ -34208 & -692 & 4119 \end{vmatrix}$$

(Cofactor of $\alpha^T \alpha$)^T = Adjunct of $\alpha^T \alpha$

$$(\alpha^T \alpha)^{-1} = \frac{1}{-634848} \begin{vmatrix} 460320 & 1184 & -34208 \\ 1184 & 240 & -692 \\ -34208 & -692 & 4119 \end{vmatrix}$$

$$\beta = \frac{1}{-634848} \begin{vmatrix} 460320 & 1184 & -34208 \\ 1184 & 210 & -692 \\ -34208 & -692 & 4119 \end{vmatrix} \begin{vmatrix} 5480 \\ 205520 \\ 80220 \end{vmatrix} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}$$

$$= \frac{1}{-634848} \begin{vmatrix} 21723520 \\ 300880 \\ 746500 \end{vmatrix} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}$$

$$\beta = \begin{vmatrix} 34.22 \\ 0.4739 \\ 1.1759 \end{vmatrix} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}$$

$$y = 34.22 + 0.4739x_1 + 1.1759x_2$$

Ques 06:

Mean Education: $\frac{16+14+12+18+14+16+12}{7} = 14.57$

Matrices

$$X = \begin{pmatrix} X_1 & X_2 \\ 35 & 16 \\ 28 & 14 \\ 45 & 14.57 \\ 31 & 12 \\ 52 & 18 \\ 29 & 14 \\ 42 & 16 \\ 33 & 12 \end{pmatrix} \quad Y = \begin{pmatrix} 720 \\ 650 \\ 750 \\ 600 \\ 780 \\ 630 \\ 710 \\ 640 \end{pmatrix}$$

$$\beta = (\alpha^T \alpha)^{-1} \alpha^T y$$

$$\alpha^T \alpha = \begin{pmatrix} 35 & 28 & 45 & 31 & 52 & 29 & 42 & 33 \\ 16 & 14 & 14.57 & 12 & 18 & 14 & 16 & 12 \end{pmatrix} \alpha$$

$$\begin{pmatrix} 35 & 16 \\ 28 & 14 \\ 45 & 14.57 \\ 31 & 12 \\ 52 & 18 \\ 29 & 14 \\ 42 & 16 \\ 33 & 12 \end{pmatrix}$$

$$A = \begin{pmatrix} 1139.3 & 4389.65 \\ 4389.65 & 1728.2840 \end{pmatrix}$$

$$\vec{x}^T y = \begin{pmatrix} 39 & 28 & 45 & 31 & 52 & 29 & 42 & 33 \\ 16 & 14 & 11.97 & 12 & 18 & 14 & 16 & 12 \end{pmatrix} \times \begin{pmatrix} 720 \\ 650 \\ 750 \\ 600 \\ 780 \\ 630 \\ 710 \\ 640 \end{pmatrix}$$

$$= \begin{pmatrix} 205520 \\ 80647.5 \end{pmatrix}$$

$$(\vec{x}^T \vec{x})^{-1} = \frac{1}{\det \vec{x}^T \vec{x}} \cdot \text{Adjoint } \vec{x}^T \vec{x}$$

$$|\vec{x}^T \vec{x}| = \frac{1}{421322.7432}$$

$$\text{Adjoint } \vec{x}^T \vec{x} = (\text{Collection of } \vec{x}^T a)^T$$

$$= \begin{pmatrix} 1728.2840 & -4389.65 \\ -4389.65 & 1139.3 \end{pmatrix}$$

$$(\vec{x}^T \vec{x})^{-1} = \frac{1}{421322.7432} \begin{pmatrix} 1728.2840 & -4389.65 & 1139.3 \\ -4389.65 & -0.0104187 & 0.0270410 \\ 1139.3 & 0.0270410 & -0.0104187 \end{pmatrix}$$

$$= \begin{pmatrix} 4.102 \times 10^{-3} \\ -0.0104187 \end{pmatrix}$$

$$\beta = \begin{vmatrix} 4.102 \times 10^3 & 0.0104187 \\ 0.0104187 & 0.6270410 \end{vmatrix} = \begin{vmatrix} 205520 \\ 80647.5 \end{vmatrix} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}$$

$$= \begin{pmatrix} 2.801 \\ 39.53 \end{pmatrix}$$

$$\begin{aligned} \beta_0 &= \bar{Y} - \beta_1 \bar{X}_1 - \beta_2 \bar{X}_2 & \bar{Y} &= 685 \\ &= 685 - (2.801 \times 36.875) - (39.53 \times 14.57) & \bar{X}_1 &= 36.875 \\ &= 5.76 & \bar{X}_2 &= 14.57 \end{aligned}$$

$$\begin{aligned} \hat{Y} &= \beta_0 + \beta_1 X_1 + \beta_2 X_2 \\ &= 5.76 + (2.801 \times 36.875) + (39.53 \times 14.57) \\ &= 689.999 \end{aligned}$$

Predicted credit score

1st data point (Age: 35, Education: 16)

$$\hat{Y} = 5.76 + (2.801 \times 35) + (39.53 \times 16) = 736.275$$

2nd data point (Age: 28, Education: 14)

$$\hat{Y} = 5.76 + (2.801 \times 28) + (39.53 \times 14) = 637.608$$

3rd data point (Age: 45, Education: 14.57)

$$\hat{Y} = 5.76 + (2.801 \times 45) + (39.53 \times 14.57) = 707.757$$

1st Data point (Age: 31, Education: 12)

$$\hat{y} = 576 + (2.801 \times 31) + (30.53 \times 12) = 566.951$$

2nd Data point (Age: 52, Education: 18)

$$\hat{y} = 862.052$$

3rd Data point (Age: 29, Education: 14)

$$\hat{y} = 640.409$$

4th Data point (Age: 42, Education: 16)

$$\hat{y} = 755.882$$

5th Data point (Age: 33, Education: 12)

$$\hat{y} = 572.553$$

$$MSE = \frac{(720 - 736.275)^2 + (650 - 637.608)^2 + (750 - 707.757)^2 + (680 - 566.951)^2 + (780 - 862.052)^2 + (630 - 640.409)^2 + (710 - 755.882)^2 + (640 - 572.553)^2}{8}$$

$$= \frac{264.88 + 153.56 + 1784.471 + 1602.236 + 6881.034}{8}$$

$$= \frac{108.347 + 2105.158 + 4549.098}{8}$$

8

$$= 2117.348$$

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2} = 1 - \frac{16038.784}{28600} = 0.4077$$

$$\log_{10}(0.0011) = -3 + 0.041 = -2.959$$

$$J = -(-2.959) = 2.959$$

natural logarithm (\ln)

$$J = -\ln(0.0011)$$

$$\ln(0.0011) = \ln(1.1) + \ln(e^{-6.8})$$

$$\ln(1.1) \approx 0.095$$

$$\ln(e^{-6.8}) = -6.8$$

$$\ln(0.0011) \approx -6.8 + 0.095 = -6.705$$

$$J = -(-6.705) \approx 6.705$$

Cost function 6.8

Age	Credit Score	Education	P Probability	Predicted Class High Risk /
35	720	16	.9999	1
28	650	14	1.0000	1
45	790	16	.9999	1
31	600	12	.9999	1
52	780	18	.9999	1
29	630	14	.9999	1
42	710	16	.9999	1
33	640	12	.9999	1

$$h_0(x) - y [0, 1, 0, 1, 0, 1, 0, 1]$$

Result: [0.0000, 0, -0.0000, -0.0001, 0.0000, -0.0001, -0.0000, -0.0001]

$$X = \begin{pmatrix} 1 & 35 & 720 & 16 \\ 1 & 28 & 650 & 14 \\ 1 & 45 & 600 & 16 \\ 1 & 31 & 780 & 12 \\ 1 & 52 & 630 & 18 \\ 1 & 29 & 710 & 16 \\ 1 & 42 & 640 & 12 \end{pmatrix}$$

$$\sum_{i=1}^m (h_0(x_i) - y^i) x_j^{(i)}$$

$$= \begin{bmatrix} 3.0000 \\ 173.0093 \\ 2959.086 \\ 65.0007 \end{bmatrix}$$

$$\frac{1}{m} \sum (h_0 - y) x_j = \begin{bmatrix} 5.0 \\ 21.75 \\ 370.00 \\ 8.25 \end{bmatrix}$$

$$\theta_0 = -1 - (0.01 \times 5.0) = -1.005$$

$$\theta_1 = 0.01 - (0.01 \times 21.75) = -0.2075$$

$$\theta_2 = 0.02 - (0.01 \times 370.00) = -3.68$$

$$\theta_3 = 0.03 - (0.01 \times 8.25) = -0.0525$$