Prob-17

Sample	Age	Credit Score	Riskclass
S1	25	610	1
S2	30	640	1
S3	35	620	1
S4	40	680	0
S5	45	700	0
S6	50	720	0
T1	33	660	?

Normalise Function

$$x' = \frac{x - x_{min}}{x_{max} - x_{min}}$$

Applying Normalization

Sample	Age	Credit Score	Age (norm)	Score (norm)
S1	25	610	0.00	0.000
S2	30	640	0.20	0.273
S3	35	620	0.40	0.091
S4	40	680	0.60	0.636
S5	45	700	0.80	0.818
S6	50	720	1.00	1.000
T1	33	660	0.32	0.455

using Euclidean Distance to T_1

Sample	Distance to T_1	Class
S2	0.218	1
S4	0.333	0
S3	0.372	1
S1	0.556	1
S5	0.601	0
S6	0.871	0

K=3 → Nearest Neighbors

Top 3 closest:

Sample	Class
S2	1
S4	0
S3	1

Prob-18

From Question 17 We Can take K=3 Nearest Neighbors

Top 3 closest

Sample	Age	Credit Score	class	Distance of T
S2	30	640	1	0.218
S4	40	680	0	0.333
S3	35	620	1	0.372

Standard k-NN classification

* class 1 (Low Risk) : S2, S3 → 2 votes
* class 0 (High Risk) : S4 → 1 votes

Details

Neighbors	class	vote
S2	1 (Low Risk)	1 vote
S4	0 (High Risk)	1 vote
S3	1 (Low Risk)	1 vote

Prediction class 1 (Low Risk)

Prob: 19.

$S = \{Low, Medium, High\}$

$O = \{710, 650, 680\}$

□ Observation 1 (710)

$$P(710|Low) = \frac{1}{\sqrt{2\pi(50)^2}} \exp\left(-\frac{(710-700)^2}{2(50)^2}\right) \approx 0.00788$$

$$P(710|Medium) = \frac{1}{\sqrt{2\pi(50)^2}} \exp\left(-\frac{(710-660)^2}{2(50)^2}\right) \approx 0.00086$$

$$P(710|High) = \frac{1}{\sqrt{2\pi(50)^2}} \exp\left(-\frac{(710-620)^2}{2(50)^2}\right) \approx 1.50 \times 10^{-5}$$

□ Observation 2 (650)

$$P(650|Low) = \frac{1}{\sqrt{2\pi(50)^2}} \exp\left(-\frac{(650-700)^2}{2(50)^2}\right) \approx 0.00064$$

$$P(650|Medium) = \frac{1}{\sqrt{2\pi(50)^2}} \exp\left(-\frac{(650-660)^2}{2(50)^2}\right) \approx .00788$$

$$P(650|High) = \frac{1}{\sqrt{2\pi(50)^2}} \exp\left(-\frac{(650-620)^2}{2(50)^2}\right) \approx .00344$$

□ Observation 3 (680)

$$P(680|Low) = \frac{1}{\sqrt{2\pi(50)^2}} \exp\left(-\frac{(680-700)^2}{2(50)^2}\right) \approx 0.0072$$

$$P(680|Medium) = \frac{1}{\sqrt{2\pi(50)^2}} \exp\left(-\frac{(680-660)^2}{2(50)^2}\right) \approx 0.00$$

$$P(680|High) = \frac{1}{\sqrt{2\pi(50)^2}} \exp\left(-\frac{(680-620)^2}{2(50)^2}\right) \approx 0.0$$

$$A = \begin{vmatrix} 0.7 & 0.3 & 0 \\ 0 & 0.6 & 0.4 \\ 0 & 0.2 & 0.8 \end{vmatrix} \qquad \pi = [1/3, 1/3, 1/3]$$

t = 1

$\delta_1(\text{Low}) = \pi(\text{Low}) \times P(O_1 | \text{Low}) = (1/3) \times 0.00788 \approx 0.00263$

$\delta_1(\text{Medium}) = \pi(\text{Medium}) \times P(O_1 | \text{Medium}) = (1/3) \times 0.00086 \approx 0.00029$

$\delta_1(\text{High}) = \pi(\text{High}) \times P(O_1 | \text{High}) = (1/3) \times 1.59 \times 10^{-5} = 5.3 \times 10^{-6}$

t = 2

$\delta_2(\text{Low}) = P(O_2 | \text{Low}) \times \max[\delta_1(j) \times P(j \to \text{Low})]$

$= 0.00648 \times \max[0.00263 \times 0.7, 0.00029 \times 0, 5.3 \times 10^{-6} \times 0]$

$= 0.00648 \times 0.001841 \approx 1.19 \times 10^{-5}$

$\psi_2(\text{Low}) = \text{Low}$

$\delta_2(\text{Medium}) = P(O_2 | \text{Medium}) \times \max[\delta_1(j) \times P(j \to \text{Medium})]$

$= 0.00788 \times \max[0.00263 \times 0.3, 0.00029 \times 0.6, 5.3 \times 10^{-6} \times 0$

$= 6.22 \times 10^{-6}$

$\psi_2(\text{Medium}) = \text{Low}$

$\delta_2(\text{High}) = P(O_2 | \text{High}) \times \max[\delta_1(j) \times P(j \to \text{High})]$

$= 0.00344 \times \max[.00263 \times 0, 0.00029 \times 0.4, 5.3 \times 10^{-6} \times .8$

$= 3.99 \times 10^{-7}$

$\psi_2(\text{High}) = \text{Medium}$

T = 3

$\delta_3(\text{Low}) = 0.00725 \times \max[1.19 \times 10^{-5} \times 0.7, 6.22 \times 10^{-6} \times 0, 3.99 \times 10^{-7} \times 0]$

$= 6.04 \times 10^{-8} = \text{Low}$

$\delta_3(\text{Medium}) = 0.00648 \times \max[1.19 \times 10^{-5} \times 0.3, 6.22 \times 10^{-6} \times 0.6, 3.99 \times 10^{-7} \times 0$

$= 2.42 \times 10^{-8} = \text{Medium}$

$\delta_3(\text{High}) = 0.00086 \times \max[1.19 \times 10^{-5} \times 0, 6.22 \times 10^{-6} \times 0.4, 3.99 \times 10^{-7} \times 0.$

$= 2.14 \times 10^{-9} = \text{Medium}$

Sequence Low → Low → Low

20.

State: $S = \{$Low, Medium, High$\}$

Observation: $O = [O_1, O_2] = [705, 645]$

Emission Probabilities

$\mu_{Low} = 700, \sigma = 50$

$\mu_{Medium} = 660\ \sigma = 50$

$\mu_{High} = 620, \sigma = 50$

observation 1 (705)

$$P(705|Low) = \frac{1}{\sqrt{2\pi(50)^2}} \exp\left(-\frac{(705-700)^2}{2(50)^2}\right) \approx .00795$$

$$P(705|Medium) = \frac{1}{\sqrt{2\pi(50)^2}} \exp\left(-\frac{(705-660)^2}{2(50)^2}\right) \approx .00158$$

$$P(705|High) = \frac{1}{\sqrt{2\pi(50)^2}} \exp\left(-\frac{(705-620)^2}{2(50)^2}\right) \approx 3.37\times10^{-5}$$

Observation 2 (645)

$$P(645|Low) = \frac{1}{\sqrt{2\pi(50)^2}} \exp\left(-\frac{(645-700)^2}{2(50)^2}\right) \approx 0.0056$$

$$P(645|Medium) = \frac{1}{\sqrt{2\pi(50)^2}} \exp\left(-\frac{(645-660)^2}{2(50)^2}\right) \approx 0.007$$

$$P(645|High) = \frac{1}{\sqrt{2\pi(50)^2}} \exp\left(-\frac{(645-620)^2}{2(50)^2}\right) \approx 0.004$$

$$\pi = [1/3,\ 1/3,\ 1/3]$$

$t = 1$

$\alpha_1(\text{Low}) = \pi(\text{Low}) \times P(O_1|\text{Low}) = (1/3) \times 0.007965 \approx 0.002655$

$\alpha_1(\text{Medium}) = \pi(\text{Medium}) \times P(O_1|\text{Medium}) = (1/3) \times 0.001158 \approx 0.000386$

$\alpha_1(\text{High}) = \pi(\text{High}) \times P(O_1|\text{High}) = (1/3) \times 3.37 \times 10^{-5} \approx 1.12 \times 10^{-5}$

$t = 2$

$\alpha_2(\text{Low}) = P(O_2|\text{Low}) \times [\alpha_1(\text{Low}) \times P(\text{Low} \rightarrow \text{Low}) + \alpha_1(\text{Medium})$
$\times P(\text{Medium} \rightarrow \text{Low}) + \alpha_1(\text{High}) \times P(\text{High} \rightarrow \text{Low})$

$= 0.005697 \times [0.002655 \times 0.7 + 0.000386 \times 0 + 1.12 \times 10^{-5} \times 0$

$= 1.059 \times 10^{-5}$

$\alpha_2(\text{Medium}) = P(O_2|\text{Medium}) \times [\alpha_1(\text{Low}) \times P(\text{Low} \rightarrow \text{Medium}) + \alpha_1(\text{Med}$
$\times P(\text{Medium} - \text{Medium}) + \alpha_1(\text{High}) \times P(\text{High} \rightarrow \text{Medium})$

$= 0.007250 \times [0.002655 \times 0.3 + 0.000386 \times 0.6 + 1.1$
$\times 10^{-5} \times 0.2]$

$= 7.466 \times 10^{-6}$

$\alpha_3(\text{High}) = P(O_2|\text{High}) \times [\alpha_1(\text{Low}) \times P(\text{Low} \rightarrow \text{High}) + \alpha_1(\text{Med}$
$\times P(\text{Medium} \rightarrow \text{High}) + \alpha_1(\text{High}) \times P(\text{High} \rightarrow \text{High})$

$= 0.004830 \times [0.002655 \times 0 + 0.000386 \times 0.4$
$1.12 \times 10^{-5} \times 0.8]$

$= 7.004 \times 10^{-7}$