Linear Regression

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Prerequisite for the video

Bit of Mathematics in here

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1. Basic algebra

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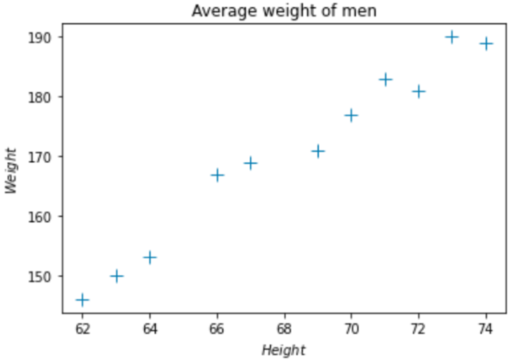
2. Basic calculus

Overview

Suppose we have a data set giving a weight in pounds and height in inches;

Height(inches)	Weight(Pounds)
62	146
63	150
64	153
70	‘177
71	183
72	181
73	190
74	189
⋮	⋮

Plot



Source of data: https://www.cdc.gov/nchs/data/series/sr_11/sr11_014acc.pdf

Figure: Average weight of men between 18-79 years from 1960-1962 in US

Variables

1. **Independent:** Height X
2. **Dependent:** Weight y

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Objective of Linear Regression

To determine the extent to which there is a linear relationship between a dependent variable and one or more independent variable.

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To predict the value of the dependent variable based on independent variable. Hence, finding the best fit line.

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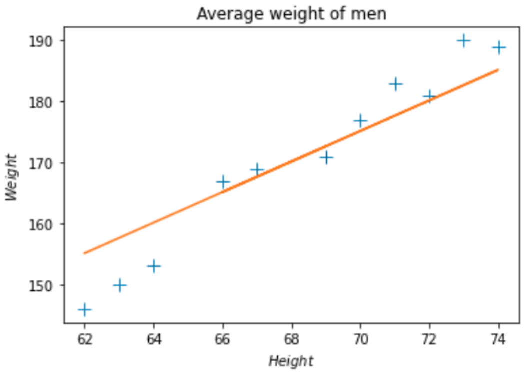
Equation of a Line

$$y_i = \theta X_i + \epsilon_i$$

Best fit line



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Goal Linear Regression

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How do we do this??

MLE

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But $\mu = 0$; $x = y_i - \theta^T X_i$

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$$\frac{1}{\sigma\sqrt{2\pi}}e^{\left(\frac{-(y_i-\theta^T X_i)^2}{2\sigma^2}\right)} \tag{5}$$

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$$\mathbb{P}(y|X;\theta) = \prod_{i=1}^{n} \mathbb{P}(y|X;\theta) = \mathbb{L}(\theta) \tag{6}$$

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Goal: $\max \mathbb{P}(y|X;\theta)$. How?

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$$\log \mathbb{L}(\theta) = \log \left[\prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{\left(\frac{-(y_i - \theta^T X_i)^2}{2\sigma^2} \right)} \right] \tag{8}$$

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Rule: $e^a . e^b = e^{a+b}$

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$$\underset{\theta}{\operatorname{argmax}} \log \mathbb{L}(\theta) = \underset{\theta}{\operatorname{argmax}} \left(- \sum_{i=1}^{n} \frac{(y_i - \theta^T X_i)^2}{2\sigma^2} \right) \tag{12}$$

Rule: $\underset{\theta}{\operatorname{argmax}}\, f(\theta) = \underset{\theta}{\operatorname{argmax}}\, \alpha\, f(\theta)\ , \forall\, \alpha \in \mathbb{R}_+^*$

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$$\operatorname*{argmax}_{\theta} \log \mathbb{L}(\theta) = \operatorname*{argmax}_{\theta} \left(- \sum_{i=1}^{n} \frac{(y_i - \theta^T X_i)^2}{2\sigma^2} \right) \tag{12}$$

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Note $\max f(\theta) = -\min(-f(\theta))$

This then produces the mean square error and minimizing this function is the interest.

$$\boxed{\ell(\theta) = \frac{1}{2}\sum_{i=1}^{n}(y_i - \theta^T X_i)^2}$$