Unit 3 : INTERFERENCE AND DIFFRACTION OF LIGHT

Coherence& Coherent Sources:

When two or more waves travel in a region of space maintaining a constant phase relationship among themselves then they are said to have coherence with each other.

Coherent Sources *: If two sources emit waves with a constant phase difference between each other then they are called coherent sources.*

INTERFERENCE OF LIGHT

Interference is defined *as "the phenomenon of redistribution of energy of the light waves having a constant phase difference among themselves, in the region of their superimposition."*

- **Constructive Interference:** When the intensity of the resultant at a certain point in the region of superimposition is **more** than the algebraic sum of the interfering waves, the point becomes a site of Constructive interference or Reinforcement or maxima.
- **Destructive Interference:** When at a particular point in the region of superimposition the intensity of the resultant is found to have a value **less** than that of the algebraic sum of the interfering waves then the point is said to be a site of destructive interference or minima.

Principle of Superposition

The theory of interference is based upon the principle of superposition of waves which states that, ***"When two waves of same frequency act simultaneously on a particle in a medium, the resultant displacement of the particle will be the algebraic sum of the displacement that occurs because of individual waves."***

Theory of Interference

Let the equation of two waves that have same amplitude and frequency, be

$$y_1 = a_1 \sin \omega t$$

$$y_2 = a_2 \sin(\omega t + \delta)$$

where δ is the path difference between the two waves.

Now according to the principle of super position, the resultant amplitude is

$$Y = y_1 + y_2$$

$$Y = a_1 \sin \omega t + a_2 \sin(\omega t + \delta)$$

$$Y = a_1 \sin \omega t + a_2 \sin \omega t \cos \delta + a_2 \cos \omega t \sin \delta$$

$$Y = (a_1 + a_2 \cos \delta) \sin \omega t + a_2 \sin \delta \cos \omega t$$

$$[\because \sin(A+B) = \sin A \cos B + \cos A \sin B]$$

Let us take, $(a_1 + a_2 \cos \delta) = A \cos \theta$ ---------------1(a)

and $a_2 \sin \delta = A \sin \theta$ ---------------1(b)

Then, $Y = A \cos \theta \sin \omega t + A \sin \theta \cos \omega t$

$$\boldsymbol{Y = A \sin(\omega t + \theta)}$$

This is the equation of the resultant wave.

Now squaring and adding equations 1(a) &(b), we get

$$(a_1 + a_2 \cos \delta)^2 + {a_2}^2 \sin^2 \delta = A^2(\cos^2 \theta + \sin^2 \theta)$$

$$A^2 = {a_1}^2 + 2a_1a_2 \cos \delta + {a_2}^2 \cos^2 \delta + {a_2}^2 \sin^2 \delta$$

$$\boldsymbol{A^2 = {a_1}^2 + 2a_1a_2 \cos \delta + {a_2}^2}$$

[From the above equation it is obvious that the intensity of the resultant ($I = A^2$) is not equal to the sum of the intensities of the interfering waves (${a_1}^2 + {a_2}^2$).]

Further, I=$A^2 = {a_1}^2 + 2a_1a_2 \cos \delta + {a_2}^2$

Here it is observed that I depends upon the value of δ.

Condition of Maxima:

If $\boldsymbol{\delta = 0, 2\pi, 4\pi, \dots 2n\pi}$ then $\cos \delta = +1$

Therefore, $\mathbf{I} = \boldsymbol{{a_1}^2 + 2a_1a_2 + {a_2}^2 = (a_1 + a_2)^2}$ is the maximum value of intensity.

*If $a_1 = a_2 = a$, then maximum intensity is $I = 4a^2$

Condition of Minima

If $\boldsymbol{\delta = \pi, 3\pi, 5\pi \dots (2n+1)\pi}$ then $\cos \delta = -1$

Therefore, $\mathbf{I} = \boldsymbol{{a_1}^2 - 2a_1a_2 + {a_2}^2 = (a_1 - a_2)^2}$ is the minimum value of intensity.

*If $a_1 = a_2 = a$, then minimum intensity is $I = 0$

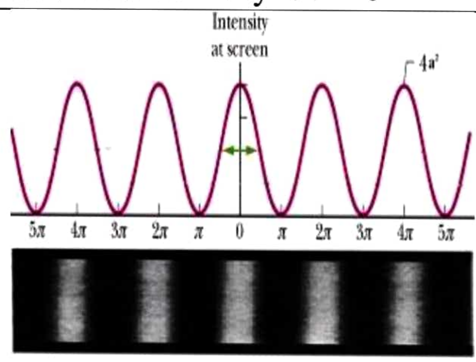
The average intensity is

$$I_{average} = \frac{I_{max} + I_{min}}{2} = \frac{4a^2 + 0}{2} = 2a^2$$

Hence, intensity is maximum when the phase difference is an even multiple of π, and it is minimum when the phase difference is an odd multiple of π.

From here it can be concluded that the intensity is maximum or the points are bright where $\delta = 0, \pm 2\pi, \pm 4\pi, \dots$ ...etc and is minimum i.e. zero where $\delta = \pm\pi, \pm 3\pi, \pm 5\pi \dots$ ...etc.

Here the energy is not destroyed but is transferred from the points of minimum displacement to the points of maximum displacement.



Conditions of Sustained Interference

1. The beams from the two sources must be coherent i.e. they must have a constant/invariable phase difference between each other. This is the only essential condition for two light waves to produce interference.
2. The beams from the two sources must be propagated along the same line, otherwise the vibrations won't superimpose on each other.
3. The two sources should emit light of equal amplitudes (intensities) or the intensity of the dark bands won't be zero & the pattern won't have differentiable fringes.
4. The two sources must be monochromatic i.e. must have equal frequency to avoid overlapping of patterns of different frequencies.
5. The two sources must be as narrow as possible because a broader source behaves as a combination of large number of small sources placed side by side, producing different interference patterns for different pairs of such sources which overlap and make the pattern look like a generally illuminated region.
6. The separation between the two sources must be as small as possible so that the fringes of max. & min. intensity do not fall too close to each other, which happens because the fringe width is directly proportional to the separation 2d between the sources.

Classification of Interference Phenomenon

In general there are two methods basic of methods of obtaining coherent sources giving rise to two different classes of interference phenomenon.

(i) **Division of Wave front**: In this case the wave front originating from a common source is divided into two parts by employing mirrors, prisms or lenses to serve the purpose of two coherent sources. The two wave fronts thus separated travel unequal distances and are finally brought together to produce interference.
The systems belonging to this class are Fresnel's biprism, Lloyd's mirror, Billet's split lens etc.

(ii) **Division of Amplitude:** In this class the amplitude of the initial beam of light is divided into two parts by partial reflection or refraction methods. The waves corresponding to the divided parts travel different paths and are brought together to produce interference.
The systems belonging to this class are thin films, Newton's rings and Michelson's interferometer.

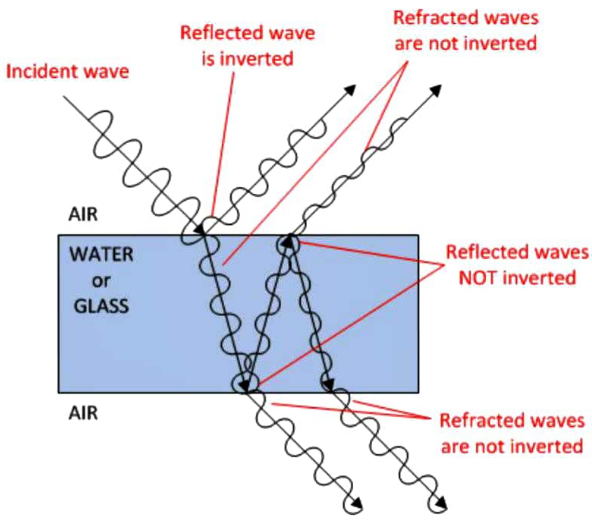
What happens if white light is used in place of monochromatic light?

Since white light consists of seven colours, the central maxima would be white as light of all the wavelengths will meet there in phase. On either side of the centre of the fringe system, the path difference gradually increases from zero and will become equal to half the wave length of the component corresponding to the smallest wavelength, i.e. violet at some point. This corresponds to the first dark fringe for the violet light. Beyond this point, the first dark fringe from various colours like indigo, blue, green etc., and lastly for red will lie. So figure will be violet on inner side and red on the outer side. As $\omega \propto \lambda$, so, the fringe width of red will be nearly double then that of violet. As a result there may be some ***overlapping of the various fringes and this overlapping will increase with the order of the fringes till we get a general illumination.***

Stoke's Statement:

When a ray of light which is incident from a rare medium is reflected from the surface of an optically denser medium back into the rare medium, the ray suffers a phase change of π or a path difference of $\lambda/2$.

No such phase or path difference is introduced if the ray is reflected from a rare medium back into the denser medium.



Unit - 3 Wave Optics — Interference & Diffraction

Coherence : When two or more waves travel in a region of a space maintaining a constant phase difference between them, they are said to have coherence with each other.

Coherent Sources : If two sources emit waves with a constant phase difference between each other then they are called coherent.

Interference : It is the phenomenon of Re-distribution of energy of light waves having a constant phase difference (coherence) among themselves in the region of their superimposition.

- **Constructive Interference** : When the intensity of the resultant at a certain point in the region of superimposition is more than the algebric sum of the intensities of the interfering waves, the point becomes a sight of constructive interference or reinforcement or maximum.
- **Destructive Interference** : When at a particular point in the region of superimposition, the intensity of the resultant has found to have a value less than the algebric sum of the intensities of the interfering waves, the point becomes a site of destructive interference or minimum.

Principle of Superposition

" When two waves of same frequency act simultaneously on a particle in a medium the resultant displacement of the particle will be the algebric sum of the displacement that occurs because of individual waves ".

Theory of Interference

Let there be two coherent waves given as :-

$$y_1 = a_1 \sin \omega t \quad \text{———} (1)$$

$$y_2 = a_2 \sin(\omega t + \delta) \quad \text{———} (2)$$

where, δ = phase difference

If the two waves superimpose then, the resultant will be

$$y = y_1 + y_2$$
$$= a_1 \sin\omega t + a_2 \sin(\omega t + \delta)$$
$$= a_1 \sin\omega t + a_2 \sin\omega t \cos\delta + a_2 \cos\omega t \sin\delta$$
$$= (a_1 + a_2\cos\delta)\sin\omega t + (a_2 \sin\delta)\cos\omega t$$

Let $a_1 + a_2\cos\delta = A\cos\theta$ ——— 3(a)

$a_2 \sin\delta = A\sin\theta$ ——— 3(b)

placing the values from eqn 3(a) & 3(b) in eqn the above

$$y = A\cos\theta \sin\omega t + A\sin\theta\cos\omega t$$

$$\boxed{y = A\sin(\omega t + \theta)}$$

This is the eqn of resultant wave.

Here, A is the amplitude of resultant wave

Let us square & then add eqn 3(a) & 3(b)

$$\Rightarrow A^2 = (a_1 + a_2\cos\delta)^2 + a_2^2\sin^2\delta$$
$$= a_1^2 + 2a_1a_2\cos\delta + a_2^2\cos^2\delta + a_2^2\sin^2\delta$$

$$\boxed{I = A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta}$$

This is the intensity of Resultant.

* Condition of Maxm Intensity

at $\delta = 2\pi n$ where $n = 0, 1, 2 \ldots$

$\cos\delta = 1$

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2 \Rightarrow \boxed{I_{max} = (a_1 + a_2)^2}$$

* Condition of Minimum Intensity

at $\delta = (2n-1)\pi$ where $n = 1, 2, 3, \ldots$

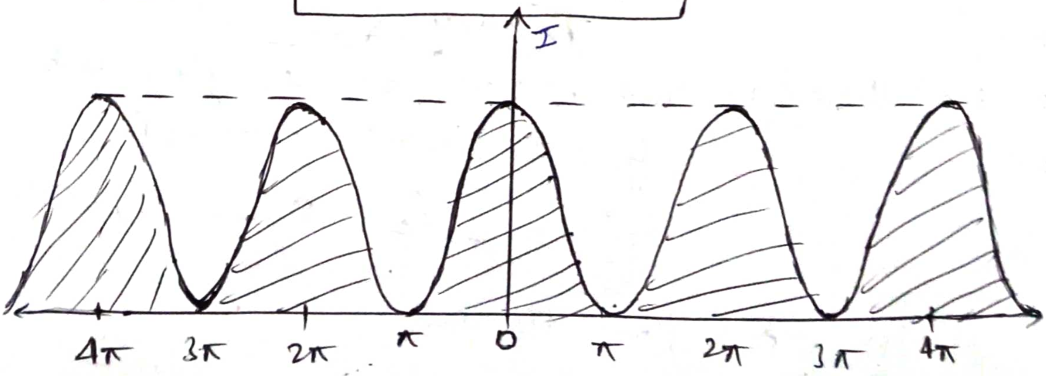
$\cos\delta = -1$

$$I_{min} = a_1^2 + a_2^2 - 2a_1a_2 \Rightarrow \boxed{I_{min} = (a_1 \sim a_2)^2}$$

* If $a_1 = a_2 = a$

$I_{max} = 4a^2$

$I_{min} = 0$



Difference between Interference and Diffraction

Interference	Diffraction
1. The interference occurs between two separate wave fronts origination from two coherent sources.	1. It is the interference that occurs between the secondary wavelets originating from the exposed part of the same wave front.
2. The interference fringes are equally spaced.	2. The diffraction fringes are never equally spaced.
3. The maxima are of same intensity.	3. The intensity of the central maximum is maximum and decreases on either side as the order of the maxima increases.
4. The minima are perfectly dark with light waves having equal amplitudes.	4. The minima is never completely dark.

Classification of Interference Phenomenon

In general there are two methods basic of methods of obtaining coherent sources giving rise to two different classes of interference phenomenon.

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The systems belonging to this class are Fresnel's biprism, Lloyd's mirror, Billet's split lens etc.

(ii) **Division of Amplitude:** In this class the amplitude of the initial beam of light is divided into two parts by partial reflection or refraction methods. The waves corresponding to the divded parts travel different paths and are brought together to produce interference.

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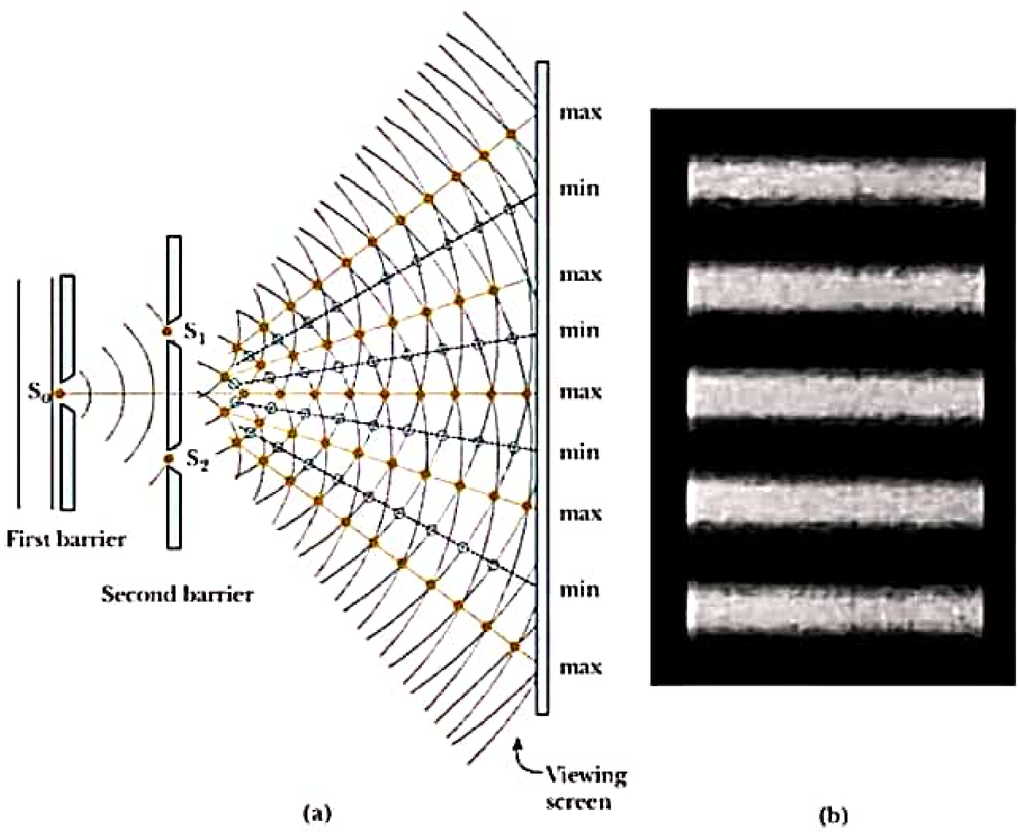

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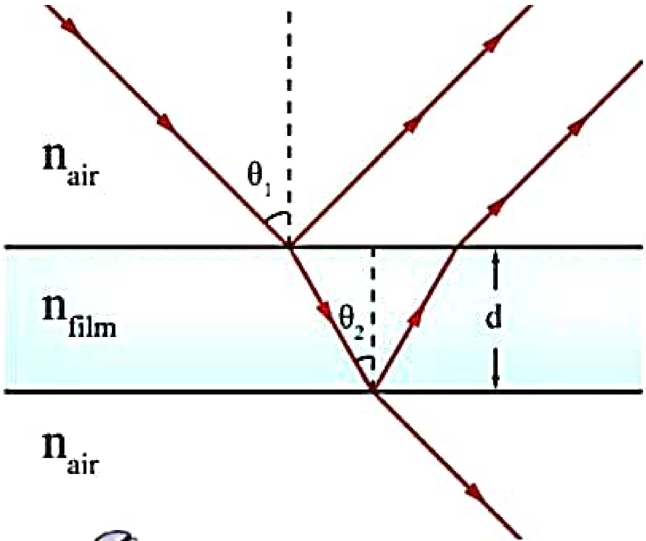
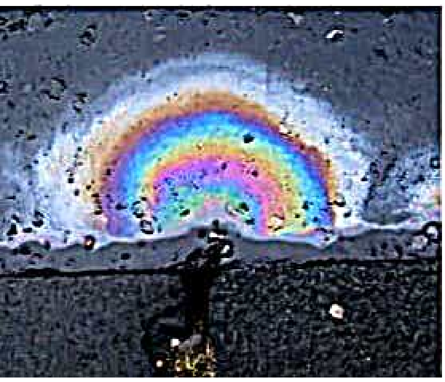
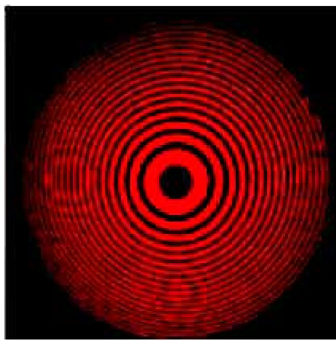
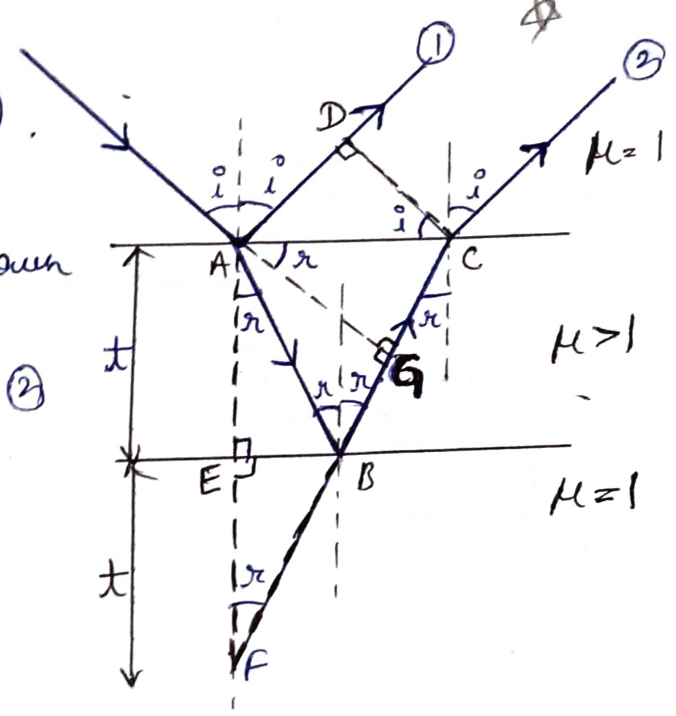






Interference due to thin film of Uniform thickness : Reflected System



Let there be a transparent film of uniform thickness 't' & Refractive Index (μ).

A monochromatic beam of light produces interference in its reflected system, as shown in the figure.

The path difference between rays ① & ② is

$$\Delta = \mu(AB + BC) - AD \quad \text{—— ①}$$

Now at point A, Snell's Law gives

$$\mu = \frac{\sin i}{\sin r} \quad \text{—— ②}$$

In $\angle\Delta$ ADC, $\sin i = AD/AC$

In $\angle\Delta$ AGC, $\sin r = GC/AC$

$\therefore$ Eqn ② gives $\mu = \dfrac{\sin i}{\sin r} = \dfrac{AD/AC}{GC/AC} = \dfrac{AD}{GC} \Rightarrow AD = \mu GC$

placing these values in eqn ①

$$\Delta = \mu(AB + BC - GC) \Rightarrow \Delta = \mu(AB + BG) \quad \text{—— ③}$$

$\angle\Delta$s AEB & BEF are congruent by Angle-Side Angle Rule (A S A rule)

$\Rightarrow AB = FB$

& $AE = EF = t$

using above in eqn ③ $\Delta = \mu(FB + BG) = \mu(FG)$ —— ④

Now in $\angle\Delta$ FGA

$\cos r = \dfrac{FG}{AF} \Rightarrow FG = (AF)\cos r \Rightarrow FG = 2t\cos r$

$\therefore$ eqn ④ becomes $\Delta = 2\mu t\cos r$

Including Stoke's treatement the effective path difference is

$$\boxed{\Delta = 2\mu t\cos r \pm \lambda/2}$$

* Condition of Maxima

Path difference, $\Delta = 2n\dfrac{\lambda}{2}$

$$2\mu t\cos r \pm \frac{\lambda}{2} = 2n\frac{\lambda}{2}$$

$$\boxed{2\mu t\cos r = (2n \mp 1)\frac{\lambda}{2}}$$

* Condition of Minima

Path difference, $\Delta = (2n-1)\lambda/2$

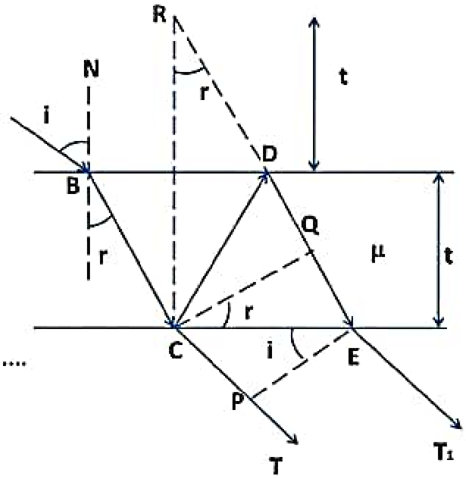
$$2\mu t\cos r \pm \lambda/2 = (2n-1)\lambda/2$$

$$\boxed{2\mu t\cos r = 2n\,\lambda/2}$$

Analytical theory of Thin film Interference: Transmitted System

Consider a thin film of uniform thickness t and refractive index μ. When a ray AB after refraction goes along BC. At the lower surface of the film it is partially reflected along CD and partially transmitted along CT_2. The ray CD will again be partially reflected and go along DE and then partially transmitted along ET_1.

The interference takes place between the transmitted rays going along CT_2 and ET_1. CQ and EP are the normals drawn on DE and CT_2 respectively. i is the angle of incidence and r is the angle of refraction.



The optical path difference is given as

$\Delta = \mu\ (CD + DE)\ \text{-}\ CP$

But $\qquad \mu = \frac{\sin i}{\sin r} = \frac{CP/CE}{EQ/CE}$

$\therefore \qquad CP = \mu . EQ$

So the path difference $\qquad \Delta = \mu(CD + DE) - \mu EQ$

$$\Delta = \mu(RD + DE - EQ) = \mu(RD + DQ) \qquad [\because CD = RD$$

$\Delta = \mu(RQ)$

In ΔCQR, $\cos r = \frac{RQ}{CR}$

$$\text{RQ} = \text{CR} \cos r = 2t \cos r$$

hence the effective path difference,

$$\boxed{\Delta = 2\mu t \cos r}$$

For constructive interference the path difference should be an even multiple of $\lambda/_2$.

So for maximum intensity $\quad 2\mu t \cos r = n\lambda$

For destructive interference the path difference should be an odd multiple of $\lambda/_2$.

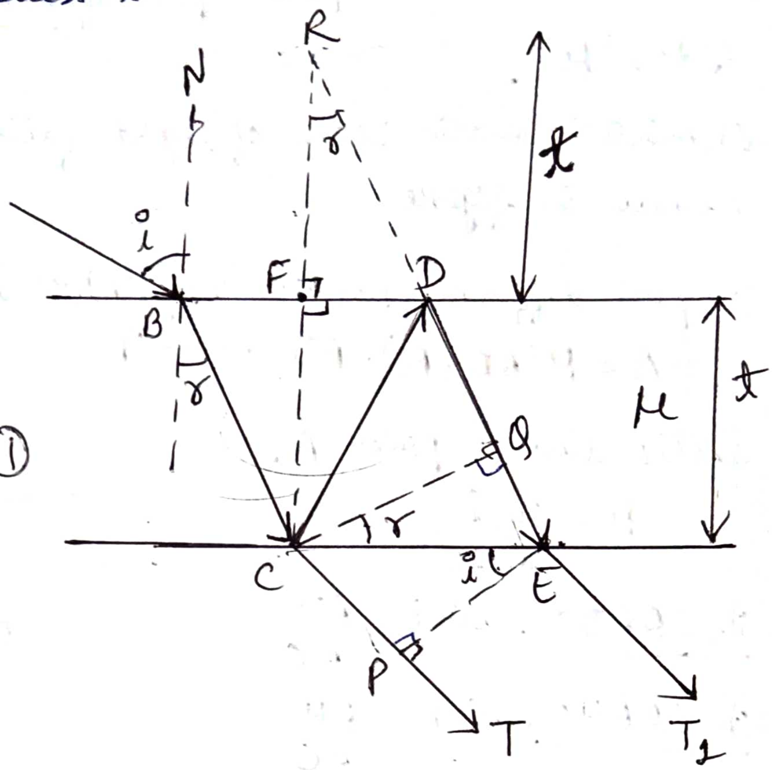
So for minimum intensity $\quad 2\mu t \cos r = (2n \pm 1)\lambda/_2$

Where n = 0, 1, 2, 3, ...

Since in this case the light is transmitted, not reflected from a surface of a denser medium, then according to Stoke's treatment no extra path difference is introduced.

(B) Analytical theory of Thin Film Interference: Transmitted System

Let us consider a thin film of thickness 't' and refractive index μ.
A monochromatic beam of light produces interference in its transmitted system as shown in figure.



The path difference is given as

$$\Delta = \mu(CD + DE) - CP \quad \text{—— (1)}$$

But Snell's law at point E

$$\mu = \frac{\sin i}{\sin r} \quad \text{—— (2)}$$

In $\angle\Delta$ CEP, $\sin i = CP/CE$

In $\angle\Delta$ ECQ, $\sin r = \frac{EQ}{CE}$

$\therefore$ Eq^n (2) gives $\mu = \frac{\sin i}{\sin r} = \frac{CP/CE}{EQ/CE} = \frac{CP}{EQ} \Rightarrow CP = \mu . EQ$

placing these values in eq^n (1)

$$\Delta = \mu(CD + DE - EQ) \Rightarrow \Delta = \mu(CD + DQ) \quad \text{—— (3)}$$

$\angle\Delta$s RFD and $\angle\Delta$ CFD are congruent by ASA rule

$\Rightarrow RD = CD$ & $RF = CF = t$

using above in eq^n (3) $\Delta = \mu(RD + DQ) = \mu(RQ)$ —— (4)

Now in $\angle\Delta$ RQC

$\cos r = \frac{RQ}{RC} \Rightarrow RQ = (RC)\cos r \Rightarrow RQ = 2t\cos r$

$\therefore$ eq^n (4) becomes $\boxed{\Delta = 2\mu t\cos r}$

* For constructive interference / Maxima

Path difference, $\Delta = 2n . \lambda/2$

for max^m intensity $\boxed{2\mu t\cos r = 2n\, \lambda/2}$

* For destructive interference / minima

Path difference, $\Delta = (2n - 1)\, \lambda/2$

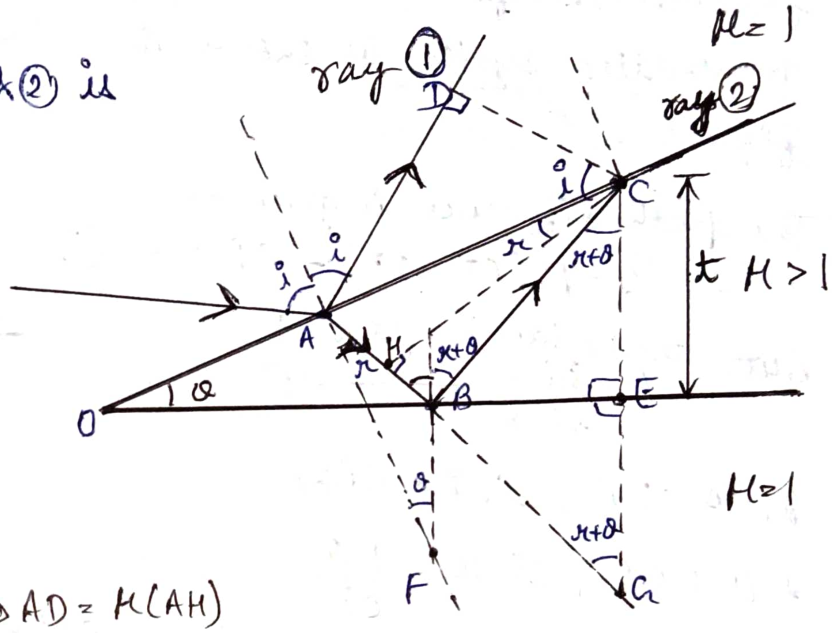
for min^m intensity $\boxed{2\mu t\cos r = (2n-1)\, \lambda/2}$

where $n = 0, 1, 2, \ldots\ldots$

Since in this case light is transmitted not reflected from a surface of a denser medium, then according to Stoke's treatment no extra path difference is introduced.

(4) Interference due to Wedge Shaped Film.

Let there be a transparent film of Non-uniform (variable) thickness having refractive index 'μ'.
A monochromatic beam of light falls on it & produces interference as shown in figure.



The path difference between ray (1) & (2) is

$$\Delta = \mu(AB+BC) - AD \quad \text{——— (1)}$$

Snells law at point A.

$$\mu = \frac{\sin i}{\sin r}$$

In $\angle\Delta ADC$, $\sin i = \frac{AD}{AC}$

In $\angle\Delta AHC$, $\sin r = \frac{AH}{AC}$

$$\therefore \mu = \frac{\sin i}{\sin r} = \frac{AD/AC}{AH/AC} = \frac{AD}{AH} \Rightarrow AD = \mu(AH)$$

placing this value in eq^n (1) $\quad \Delta = \mu(AB + BC - AH) \Rightarrow \Delta = \mu(HB + BC)$ ——— (2)

$\angle\Delta_s$ BEC & BEG are congruent by A.S.A rule

$BC = BG$

$CE = EG = t$

using the above in eq^n (2) we get $\quad \Delta = \mu(HB + BG) \Rightarrow \Delta = \mu(HG)$ ——— (3)

Now in $\angle\Delta CHG$ $\quad \cos(r+\theta) = \frac{HG}{GC} \Rightarrow HG = (GC)\cos(\theta + r) \Rightarrow HG = 2t\cos(r+\theta)$

$\therefore$ eq^n (3) becomes $\quad \Delta = 2\mu t\cos(r+\theta)$

Including Stoke's treatement at point A in ray (1) iff path diff. is

$$\boxed{\Delta = 2\mu t\cos(r+\theta) \pm \lambda/2} \quad \text{——— (4)}$$

* Condition of maximum Intensity

Path difference, $\Delta = 2n.\frac{\lambda}{2}$

$$2\mu t\cos(r+\theta) \pm \lambda/2 = 2n\,\lambda/2$$

$$\boxed{2\mu t\cos(r+\theta) = (2n \pm 1)\,\lambda/2} \quad \text{——— (5)}$$

* Condition of Minimum Intensity

Path difference, $\Delta = (2n-1)\,\lambda/2$.

$$2\mu t\cos(r+\theta) \pm \lambda/2 = (2n-1)\,\lambda/2$$

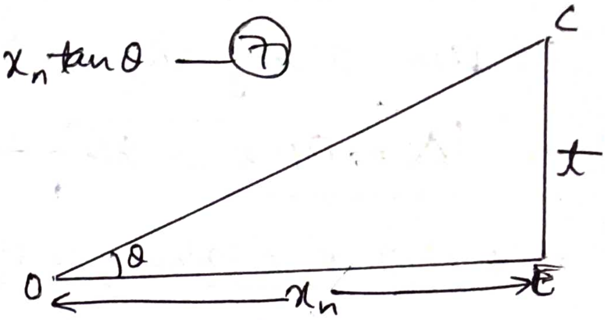
$$\boxed{2\mu t\cos(r+\theta) = 2n\,\lambda/2} \quad \text{——— (6)}$$

* The fringe pattern contains alternatively bright & dark fringes (straight line), parallel to the edge of the wedge.

Fringe Width

Let us suppose that n^{th} dark fringe is found at thickness 't'.

$\therefore$ from the figure. $\tan\theta = \frac{t}{x_n} \Rightarrow t = x_n \tan\theta$ —— (7)



This should satisfy eqn (6)

$\therefore 2\mu x_n \tan\theta \cos(r+\theta) = 2n \cdot \lambda/2$ —— (8)

for (n+1)th dark fringe

$2\mu x_{n+1} \tan\theta \cos(r+\theta) = 2(n+1)\lambda/2$ —— (9)

Subtracting eqn (8) from (9)

$\Rightarrow 2\mu[x_{n+1} - x_n]\tan\theta \cos(r+\theta) = (\cancel{2n} + 2 - \cancel{2n})\lambda/\cancel{2}$

$$\Rightarrow \boxed{x_{n+1} - x_n = \frac{\lambda}{2\mu\tan\theta\cos(r+\theta)}}$$

* For normal incidence

$r = 0$

$$x_{n+1} - x_n = \frac{\lambda}{2\mu\sin\theta}$$

* for very thin film

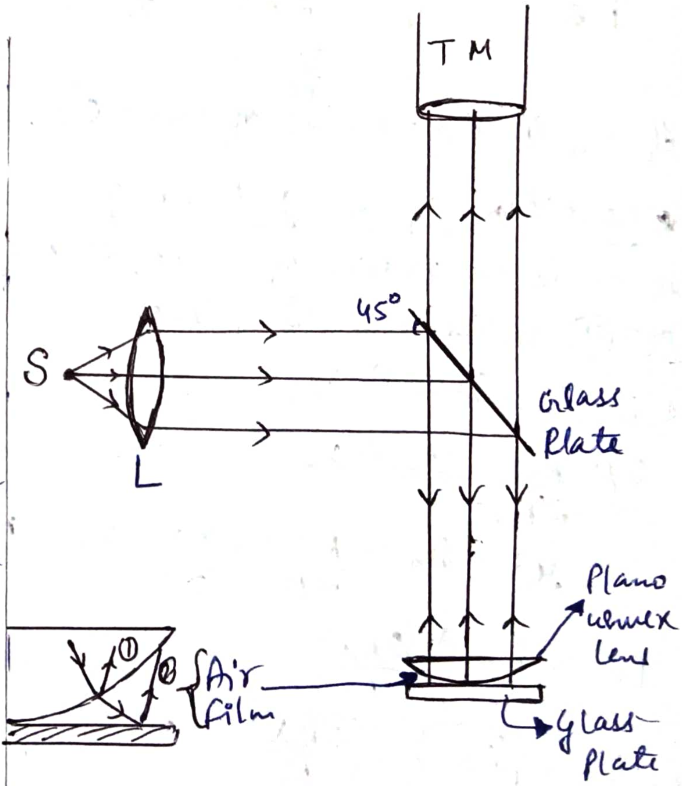
$\theta \simeq 0$, $\sin\theta \simeq \theta$

$$x_{n+1} - x_n = \frac{\lambda}{2\mu\theta}$$

* (A)(A) // Nu. o

Newtons Rings

- When a plano convex lens is placed on a glass plate with its curved surface in contact with a plate, an air film is formed between the two surfaces with gradually increasing thickness.
- If a monochromatic beam of light falls normally upon plate & the reflected system is view, an interference pattern with bright & dark cocentric rings is formed around the point of contact of lens & plate.
- This pattern is called "Newton's Rings".
- It is a special case of interference in wedge shaped film.



- We know that the path difference in the case of wedge shaped film is $\Delta = 2\mu t\cos(r+\theta) \pm \lambda/2$

Na lamp
$\lambda = 5893 Å$

For Newton's Rings

(i) Incidence is normal $\Rightarrow r = 0$

(ii) Film is very thin $\Rightarrow \theta \simeq 0$

$$\Rightarrow \boxed{\Delta = 2\mu t \pm \lambda/2} \quad \text{—— (1)}$$

*** Condition of Maximum Intensity**

Path diff, $\Delta = 2n \cdot \lambda/2$

$$2\mu t \pm \lambda/2 = 2n \lambda/2$$

$$\boxed{2\mu t = (2n \pm 1)\lambda/2} \quad \text{—— (2)}$$

*** Condition of minimum Intensity**

Path diff, $\Delta = (2n-1)\lambda/2$

$$2\mu t \pm \lambda/2 = (2n-1)\lambda/2$$

$$\boxed{2\mu t = 2n \cdot \lambda/2} \quad \text{—— (3)}$$

*** Diameter of the Rings**

Let us consider the geometry in the adjoining figure.

$CN \times ND = BN \times NA$ (By property of circle)

$$r_n^2 = (2R - t) \times t$$

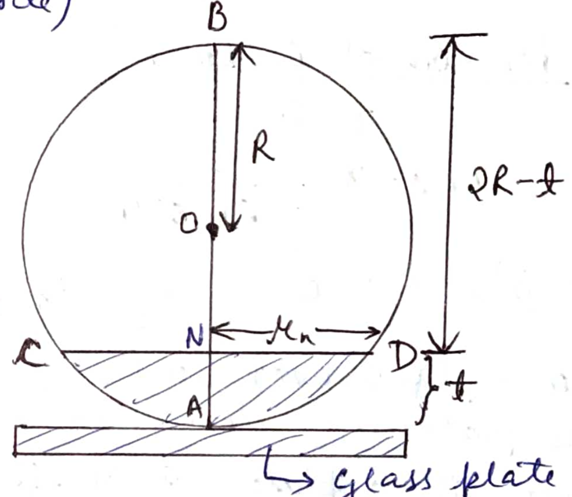
$$r_n^2 = 2Rt - t^2$$

$\because R \gg t$ ignoring 't'

i.e $\quad r_n^2 = 2Rt \quad$ or $\quad \boxed{t = \dfrac{r_n^2}{2R}} \quad$ —— (4)

$$t = \frac{D^2}{8R}$$

If r_n is the radius of chord CD.



*** Condition of Bright Rings**

If r_n is the radius of nth bright ring then eqn (4) satisfies eqn (2), then

i.e $2\mu t = (2n-1)\lambda/2$

$$2\mu \frac{r_n^2}{2R} = (2n-1)\lambda/2$$

$$r_n^2 = (2n-1)\frac{R\lambda}{2\mu} \Rightarrow \left(\frac{D_n}{2}\right)^2 = (2n-1)\frac{R\lambda}{2}$$

$$D_n^2 = (2n-1)\left(\frac{2R\lambda}{\mu}\right)$$

or, $D_n = \sqrt{(2n-1)} \cdot \sqrt{\dfrac{2R\lambda}{\mu}}$

$$\Rightarrow \boxed{[D_n]_b \propto \sqrt{(2n-1)}} \quad \text{—— (5)}$$

This shows that the diameter of Bright Rings is proportional to square root of odd no.s.

*** Condition of Dark Rings**

If r_n is the radius of nth dark ring, then eqn (4) satisfies eqn (3)

i.e $\quad 2\mu t = 2n\lambda/2$

$$2\mu \frac{r_n^2}{2R} = 2n\lambda/2$$

$$r_n^2 = n R\lambda/\mu$$

$$\left(\frac{D_n}{2}\right)^2 = n\frac{R\lambda}{\mu}$$

or, $D_n^2 = \dfrac{4n\lambda R}{\mu}$

$$\Rightarrow D_n = \sqrt{n}\sqrt{\frac{4\lambda R}{\mu}} \Rightarrow \boxed{D_n \propto \sqrt{n}} \quad \text{—— (6)}$$

This shows that the diameter of Dark rings are proportional to the square of natural no's.

Determination of Wavelength

we know that the diameter of nth Dark ring is given as –

$$D_n^2 = \frac{4n\lambda R}{\mu}$$

For air film, $\mu = 1$

$\therefore$ $D_n^2 = 4n\lambda R$ —— ①

similarly diameter of $(n+p)$th dark ring is $D_{n+p}^2 = 4(n+p)\lambda R$ —— ②

Subtracting eqn ① from ②, we get

$$D_{n+p}^2 - D_n^2 = 4(n+p)\lambda R - 4n\lambda R = 4p\lambda R$$

$$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}}$$

radius of convex lens

gap b/w ring

Determination of Refractive Index

For this purpose experiment is conducted for air first & then repeated similarly with a drop of transparent liquid between the lens and a plate.

Diameter of the dark ring when air is there between glass and plate is given as $D_n^2 = 4n\lambda R$

diameter of $(n+p)$th dark ring

$$D_{n+p}^2 = 4(n+p)\lambda R$$

$\Rightarrow$ $\left[D_{n+p}^2 - D_n^2\right]_A = 4p\lambda R$ —— ①

when experiment is repeated with transparent liquid of R.I (μ), we get

$$\left[D_{n+p}^2 - D_n^2\right]_L = \frac{4p\lambda R}{\mu} \quad \text{—— ②}$$

Divide eqn ① by ②

$$\boxed{\frac{\left[D_{n+p}^2 - D_n^2\right]_{Air}}{\left[D_{n+p}^2 - D_n^2\right]_{liquid}} = \mu}$$

Q) Why Newton's Rings are Circular?

Ans As the lens is said to be a symmetric along its axis the thickness is said to be constant along the circumference which is of a given radius.

Diffraction

~~When light falls on small~~

"Bending of light round the corners of a small obstacle or an aperture the dimension of whose edges are comparable to the wavelength of incident light is known as diffraction."

- Bent light enters the region of geometrical shadow.
- It was first discovered by Grimaldy in 1665.
- Modern theory of the phenomenon is based on Hygens principle according to which "every point in any surface through which light passes may be considered as a secondary source and subsequent effects beyond the surface may be considered due to action of secondary wavelets alone."

Diffraction Classes

(a) Frennel's class – It is said to occurs when the source & the observation point are at finite distances from the diffracting element.
In this class of diffraction the incident wavefront is generally spherical or cylindrical.

(b) Fraunhofer's class – It consists of all those cases in which either the source or the observation point or both are situated at infinite distances from the diffracting element.
In this class of diffraction the incident wavefront is always plane.

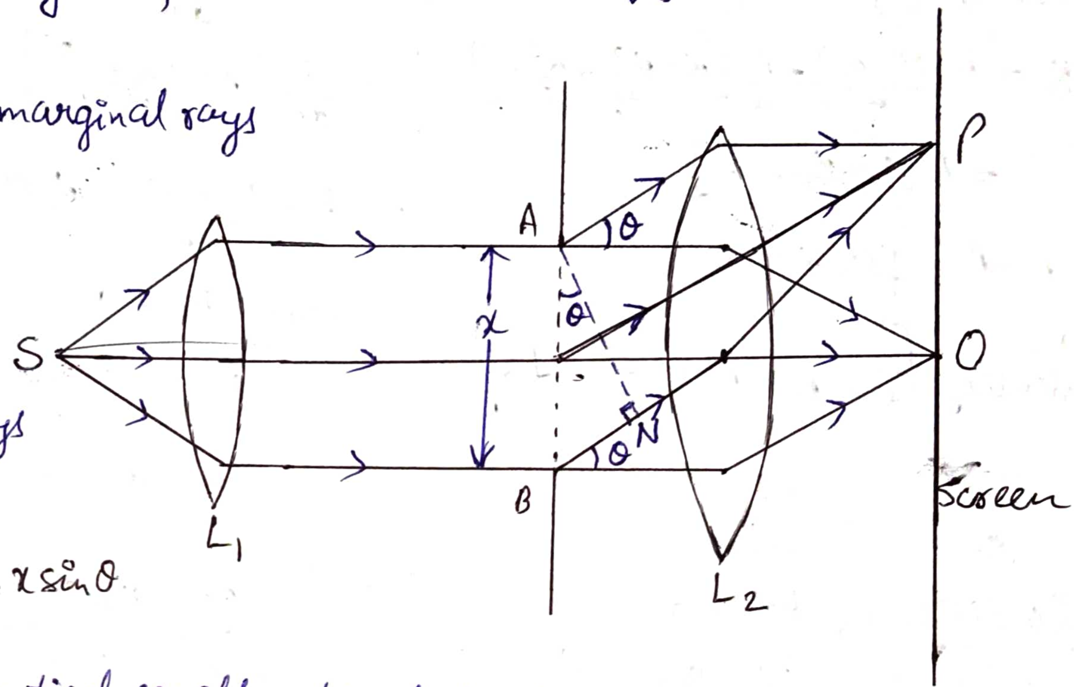
Difference between Interference and Diffraction

Interference	Diffraction
1. The interference occurs b/w two separate wave-fronts origination from two coherent sources.	1. It is the intensity that occurs b/w the secondary wavelets originating from the exposed part of the same wavefront.
2. The interference fringes are equally spaced.	2. The diffraction fringes are never equally spaced.
3. The maxima are of same intensity.	3. The intensity of the central maximum is maximum and decreases on either side as the order of maxima increases.
4. The minima are perfectly dark with light waves having equal amplitudes.	4. The minima is never completely dark.

Diffraction at Single Slit (Fraunhofer diff^n at a single slit)

Let us suppose that a plane wavefront of monochromatic light falls on a single slit AB of width x and produces diffraction on the screen.

We consider only two points on the screen O at diffraction angle 0 & P at diffraction angle θ, as shown in the figure



Path difference b/w marginal rays at point P.

$$\Delta = BN = x \sin\theta$$

Then phase difference b/w the marginal rays at point P

$$\delta_T = \frac{2\pi}{\lambda} \cdot \Delta = \frac{2\pi}{\lambda} x \sin\theta$$

If there are 'n' identical equally spaced light components b/w marginal rays, then phase difference b/w consecutive component is given as

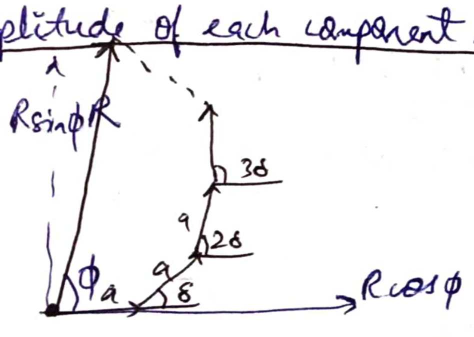
$$\delta = \frac{\delta_T}{n} = \frac{2\pi}{n\lambda} x \sin\theta \quad \text{———} \quad (1)$$

The amplitude of the resultant at point P is given as

$$R = a \frac{\sin\left(\frac{n\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)} \quad \text{———} \quad (2)$$

where a = amplitude of each component.

for reference



$R\cos\phi = a + a\cos\delta + a\cos 2\delta + \dots a\cos(n-1)\delta$

$= a[1 + \cos\delta + \cos 2\delta + \dots \cos(n-1)\delta]$

$= a[1 +$

placing the value from eq^n (1) in (2)

$$R = a \frac{\sin\left(\frac{\cancel{n}}{\cancel{2}} \times \frac{\cancel{2}\pi}{\cancel{n}\lambda} x \sin\theta\right)}{\sin\left(\frac{1}{\cancel{2}} \times \frac{\cancel{2}\pi}{n\lambda} x \sin\theta\right)}$$

if $\boxed{\alpha = \frac{\pi x \sin\theta}{\lambda}}$ ——— (3)

$$\therefore \quad R = a \frac{\sin\alpha}{\sin(\alpha/n)}$$

$\because n >>> \alpha$; $\sin(\alpha/n) \simeq \alpha/n$

$$\Rightarrow R = (na) \frac{\sin\alpha}{\alpha} \quad \text{———} \quad (4)$$

Intensity is given by $I = R^2 = A^2 \left(\frac{\sin\alpha}{\alpha}\right)^2$ — (5)

where, $A = na \rightarrow$ algebric sum of all the amplitudes.

* Condition for Central Maxima.

At C.M. $\theta = 0$, $\alpha = 0$

from eqn (4)

$$R = A\left(\frac{\sin\alpha}{\alpha}\right) = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} \cdots\right]$$

$$= A\left[1 - \frac{\alpha^2}{3!} \cdots\right]$$

$\Rightarrow R = A$ for $\alpha = 0$

or,

Intensity $\boxed{I = R^2 = A^2 = (na)^2 = I_0}$

* Condition for Maxima & Minima

for either maxima or minima

$$\frac{dI}{d\alpha} = 0$$

$$\Rightarrow \frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

$$\Rightarrow A^2\left(\frac{2\sin\alpha}{\alpha}\right)\left(\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right) = 0$$

here $A^2 \neq 0$

hence either $\frac{\sin\alpha}{\alpha} = 0$ — (6)

or $\alpha\cos\alpha - \sin\alpha = 0$

* Condition of Minima

from the discussion on eqn (6) it is clear that minimas are obtained where

$\frac{\sin\alpha}{\alpha} = 0 \Rightarrow \sin\alpha = 0$

$\Rightarrow \alpha = \pm n\pi \ (n = 1, 2, 3 \ldots)$

zero not considered because of central maxima

$\Rightarrow \frac{\cancel{\pi}}{\lambda} x \sin\theta = n\cancel{\pi}$

$\boxed{x\sin\theta = n\lambda}$

Condition of Maxima

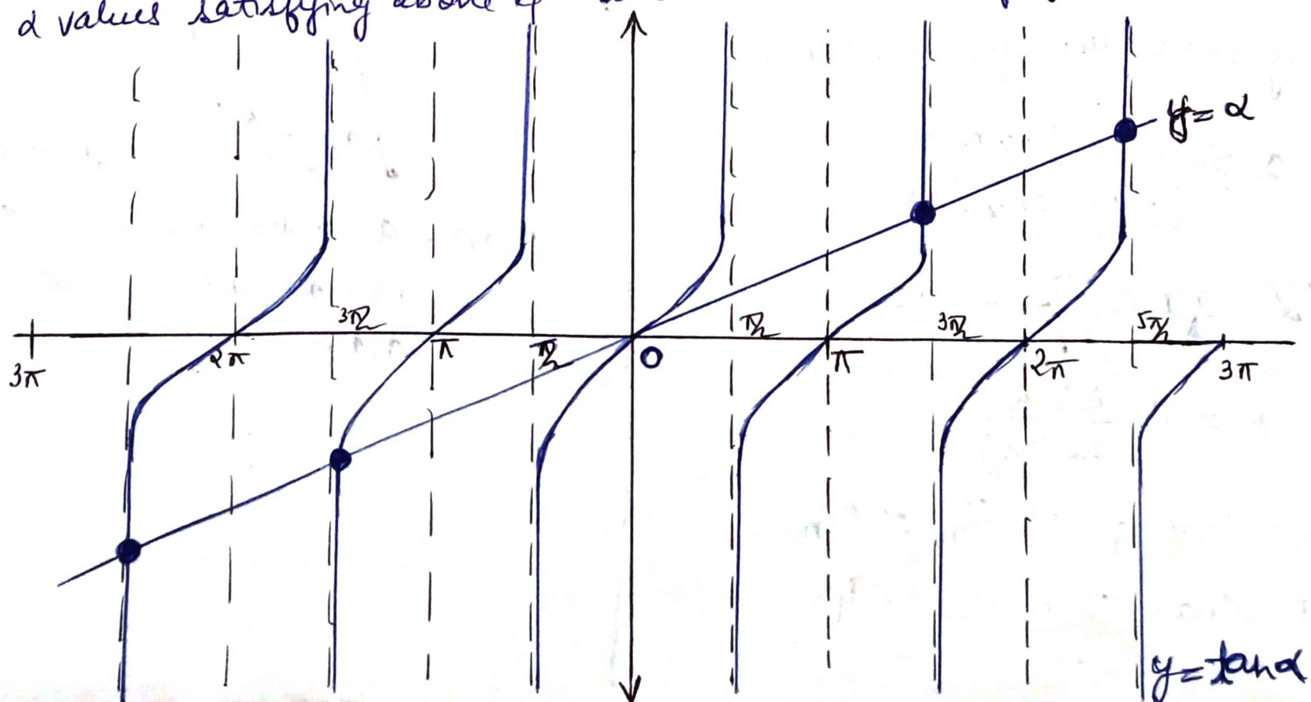
from the above discussion maxima are obtained where

$\alpha\cos\alpha - \sin\alpha = 0$

$\Rightarrow \alpha\cos\alpha = \sin\alpha$

$\Rightarrow \boxed{\alpha = \tan\alpha}$

The α values satisfying above eqn are the intersections of $y = \alpha$ & $y = \tan\alpha$



$\Rightarrow \alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}$

here $\begin{cases} \text{at } \alpha = 0 \\ \text{at } \alpha = (2n+1)\pi/2 \quad \{n = 1, 2, 3 \ldots \end{cases}$

secondary max

Intensity Ratio of Maxima

Maximas are obtained as under –

- At $n = 0$, $\alpha = 0$

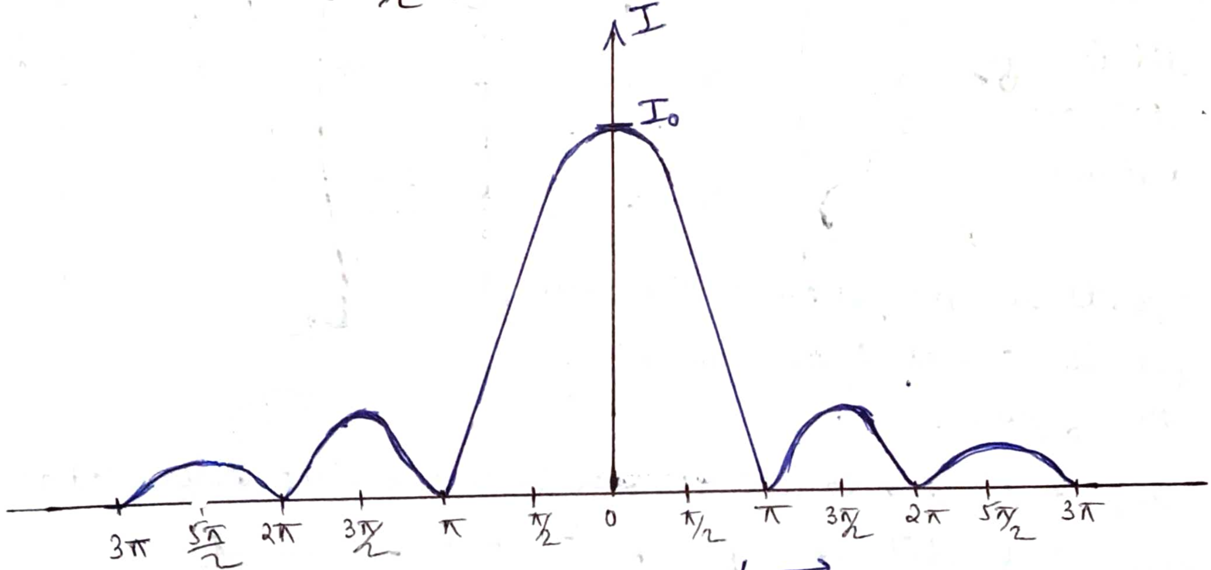
$$I_{cm} = A^2 = (na)^2 = I_0$$

- At $n = 1$, $\alpha = 3\pi/2$

$$I_1 = A^2 \times \left(\frac{\sin 3\pi/2}{3\pi/2}\right)^2 = \frac{4}{9\pi^2} I_0$$

- At $n = 2$, $5\pi/2$

$$I_2 = A^2 \left(\frac{\sin 5\pi/2}{5\pi/2}\right)^2 = \frac{4}{25\pi^2} I_0$$



Width of the Central Maxima

Central max^ma lies b/w 1st minimas

Condition of minimas

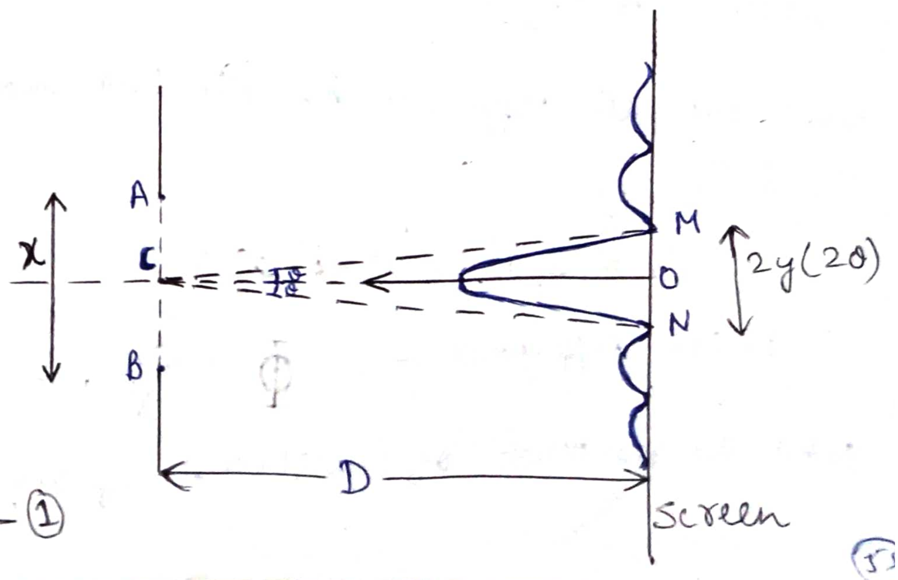
$$x \sin\theta = \pm n\lambda \quad (n = 1, 2, 3 \ldots)$$

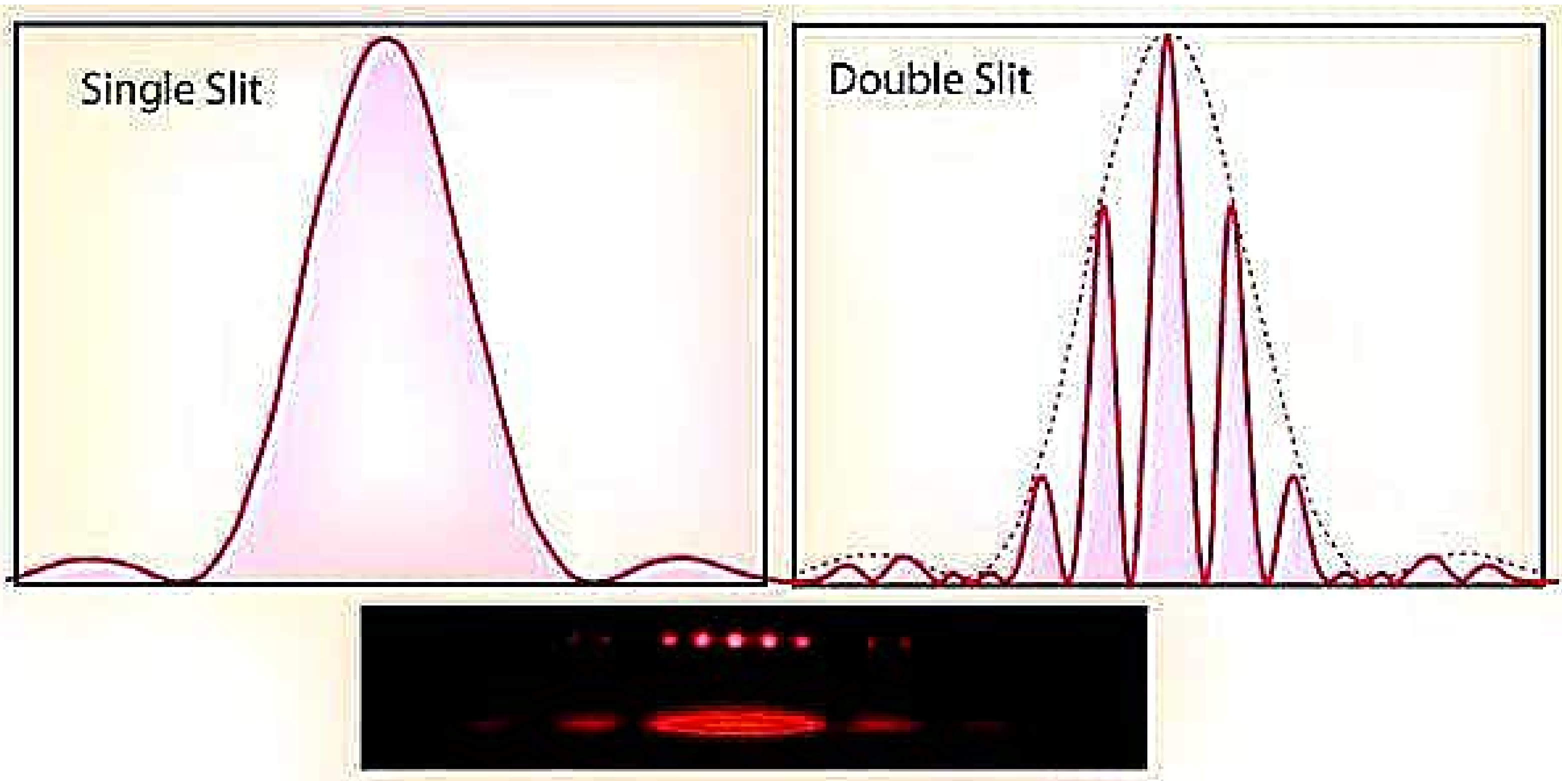
For 1st minimas, $n = 1$

$$x \sin\theta = \pm\lambda$$

or $\boxed{\sin\theta = \lambda/x}$

for $\theta \simeq 0$ (very small) $\quad \sin\theta \simeq \theta = \frac{\lambda}{x}$ —— ①




Single Slit
Double Slit

Now from $\triangle MOC$

$\sin\theta = \frac{OM}{CM} \simeq \frac{OM}{CO} \Rightarrow \sin\theta = \frac{y}{D}$ Or, $\sin\theta \simeq \theta = y/D$ — ②

from eqn ① & ②

$$\frac{y}{D} = \lambda/x$$

$$y = \frac{D\lambda}{x} \Rightarrow \boxed{2y = \frac{2D\lambda}{x}}$$

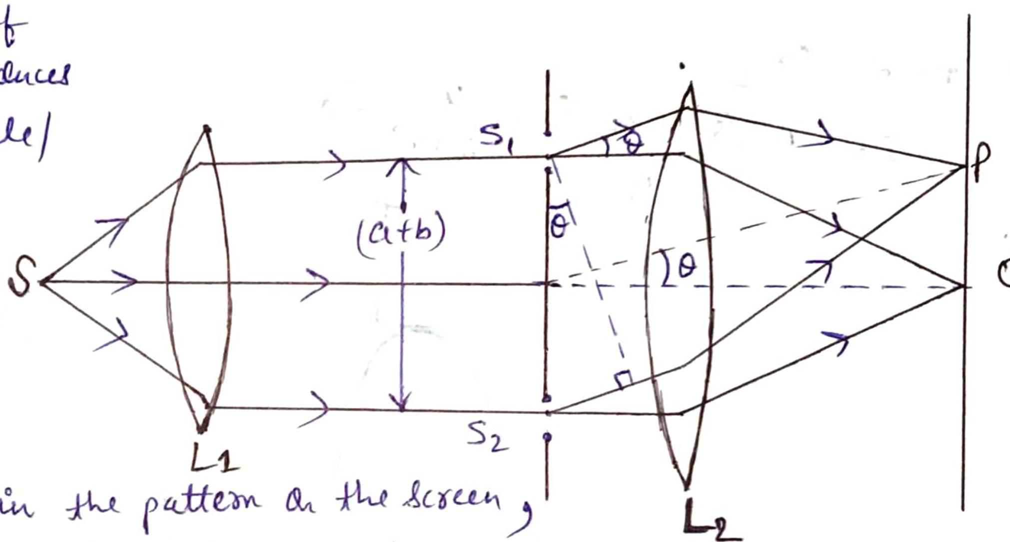
This is the Linear Width of CM (central max.)

<u>Angular Width of CM</u> $\boxed{2\theta = \frac{2\lambda}{x} = \frac{2y}{D}}$

<u>Diffraction at Double / Two slits.</u>

Let a plane wavefront of monochromatic light produces diffraction from a double / two slit arrangement.

Width of each slit is 'a' & both the slits are separated by a distance 'b'.



We consider two points in the pattern on the screen, P & O at diffraction angles θ & 0 respectively, as shown in the figure.

The resultant amplitude due to all the ~~wavefront~~ wavelets diffracted at angle 'θ' from <u>each</u> slit is $R = A\left(\frac{\sin\alpha}{\alpha}\right)$ ——— ①

where $\alpha = \frac{\pi}{\lambda}(a\sin\theta)$ —— ②

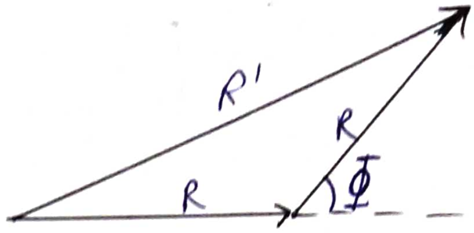
Now the path difference b/w the two individual resultants from slits S_1 & S_2

$$\Delta = S_2N = (a+b)\sin\theta$$

∴ Phase difference is $\Phi = \frac{2\pi}{\lambda}\cdot\Delta = \frac{2\pi}{\lambda}(a+b)\sin\theta$ — ③

Now the ~~resultant~~ of amplitudes of wavelets at point P is given by

$$(R')^2 = R^2 + R^2 + 2RR\cos\Phi$$
$$= 2R^2 + 2R^2\cos\Phi = 2R^2[1+\cos\Phi]$$
$$(R')^2 = 4R^2\cos^2\left(\frac{\Phi}{2}\right)$$



Placing values of R & Φ from eqⁿ ① & ③ in the above, we get.

$$(R')^2 = 4A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\cos^2\left[\frac{1}{2}\times\frac{2\pi}{\lambda}(a+b)\sin\theta\right]$$

$$\boxed{I = (R')^2 = 4A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\cos^2\beta}$$ where $\beta = \frac{\pi}{\lambda}(a+b)\sin\theta$

Here, $A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \rightarrow$ is the diffraction part due to each slit

$\cos^2\beta \rightarrow$ is the interference part due to superposition of equal resultant amplitudes from S_1 & S_2

Directions of Maxima & Minima

(i) Diffraction : Pertaining to the term $A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$

* Maximas –
- At $\theta = 0$, $\alpha = 0 \rightarrow$ Central Max
- At $\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots$ (m represents order of fringes)

$= (2m+1)\pi/2 \quad (m = 1, 2, 3, \ldots)$ Secondary Max

$$\boxed{\frac{\pi}{\lambda}(a\sin\theta) = (2m+1)\frac{\pi}{2}} \Rightarrow \boxed{a\sin\theta = (2m+1)\frac{\lambda}{2}}$$

* Minimas – are obtained where

$\alpha = \pm\pi, \pm 2\pi, \pm 3\pi \ldots$

$\alpha = \pm m\pi \quad (m = 1, 2, 3 \ldots)$

$$\Rightarrow \boxed{a\sin\theta = m\lambda}$$

(ii) Interference : Pertaining to the term $\cos^2\beta$

* Maximas – are obtained where $\cos^2\beta = 1 \Rightarrow \beta = \pm n\pi \quad (n = 0, 1, 2, 3 \ldots)$

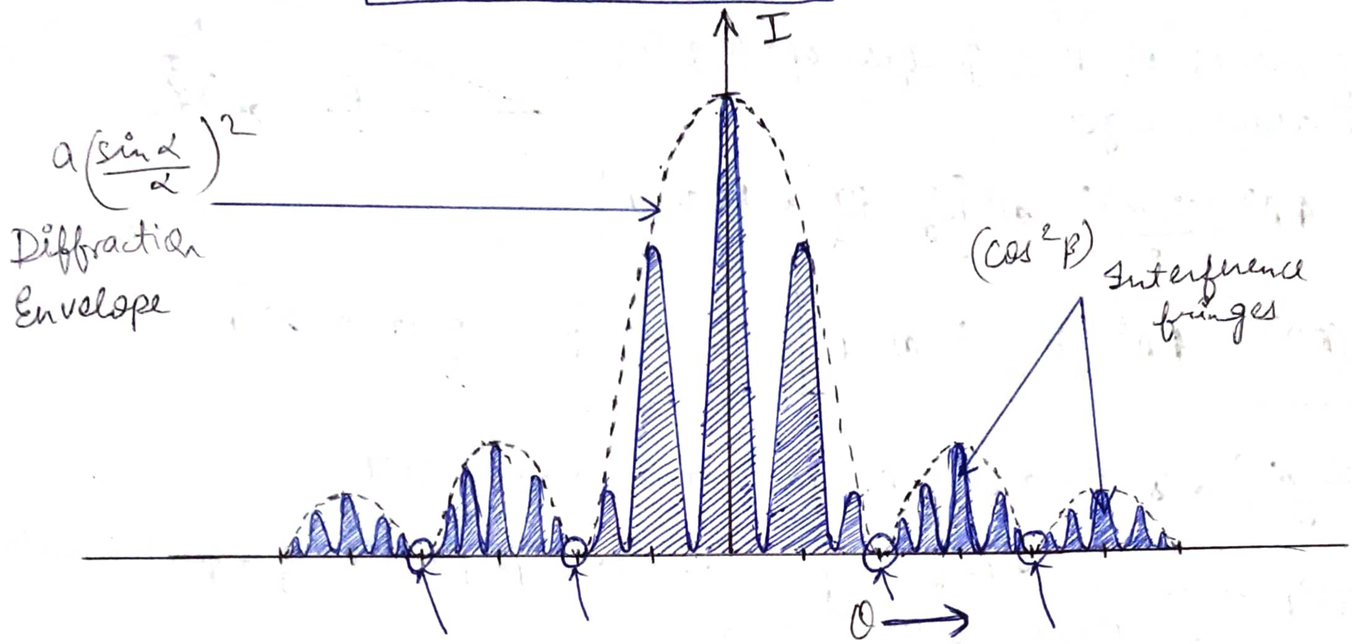
$$\frac{\pi}{\lambda}(a+b)\sin\theta = n\pi \qquad \boxed{(a+b)\sin\theta = n\lambda}$$

- At $n = 0$, $\theta = 0 \rightarrow$ Central Max
- At $n = 1, 2, 3 \ldots \rightarrow$ Secondary Max

* Minimas : are obtained where $\cos^2\beta = 0$

$$\Rightarrow \beta = \pm (2n-1)\frac{\pi}{2} \quad (n = 1,2,3\ldots)$$

$$\Rightarrow \boxed{(a+b)\sin\theta = (2n-1)\frac{\lambda}{2}}$$



* Absent Spectra or Missing Order (Also applicable for N slits as well)

In the pattern obtained on the screen the points where interference maximas coincide with diffraction minimas, interference maximas are supressed and are absent/missing from the pattern. These are called missing orders or absent spectra.

We known that interference maximas are obtained where,

$$(a+b)\sin\theta = n\lambda \quad \text{——— (1)}$$

& diffraction minimas are formed where, $a\sin\theta = m\lambda$ ——— (2)

At the points where the above two coincide, dividing (1) by (2)

$$\boxed{\frac{a+b}{a} = \frac{n}{m}}$$

($n \rightarrow$ order of interference, $m \rightarrow$ order of diffraction)

$$\Rightarrow \boxed{n = m\left(\frac{a+b}{a}\right)}$$

This represents the interference orders missing from the pattern.

- Case I : If $b = a$, $n = 2m$

i.e $m = 1, 2, 3 \ldots$

$n = 2, 4, 6 \ldots$

are missing

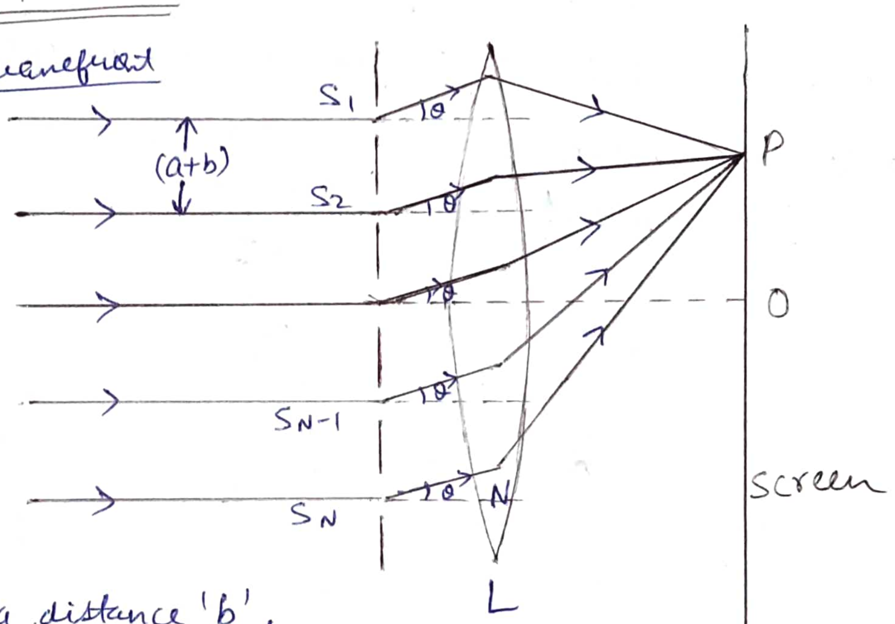
- Case-II if $b = 2a$, $n = 3m$
  i.e for $m = 1, 2, 3 \ldots$
  $n = 3, 6, 9 \ldots$
  are missing

- Case-III if $(a+b) = a$ i.e $b = 0$
  Double slit arrangement becomes single slit
  interference vanishes.

- Case-IV: As the slit separation b increases the no. of interference maxima in the region of diffraction central maxima increases.

* Diffraction due to N slits

Let there be a plane wavefront of monochromatic light which produces diffraction from a combination of N parallel identical slits.



Width of each slit is a 'a' and slits are separated equally by a distance 'b'.

The intensity at any point on screen in the pattern is given by

$$I = (R')^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

where, $\alpha = \frac{\pi}{\lambda} a\sin\theta$

$\beta = \frac{\pi}{\lambda}(a+b)\sin\theta$

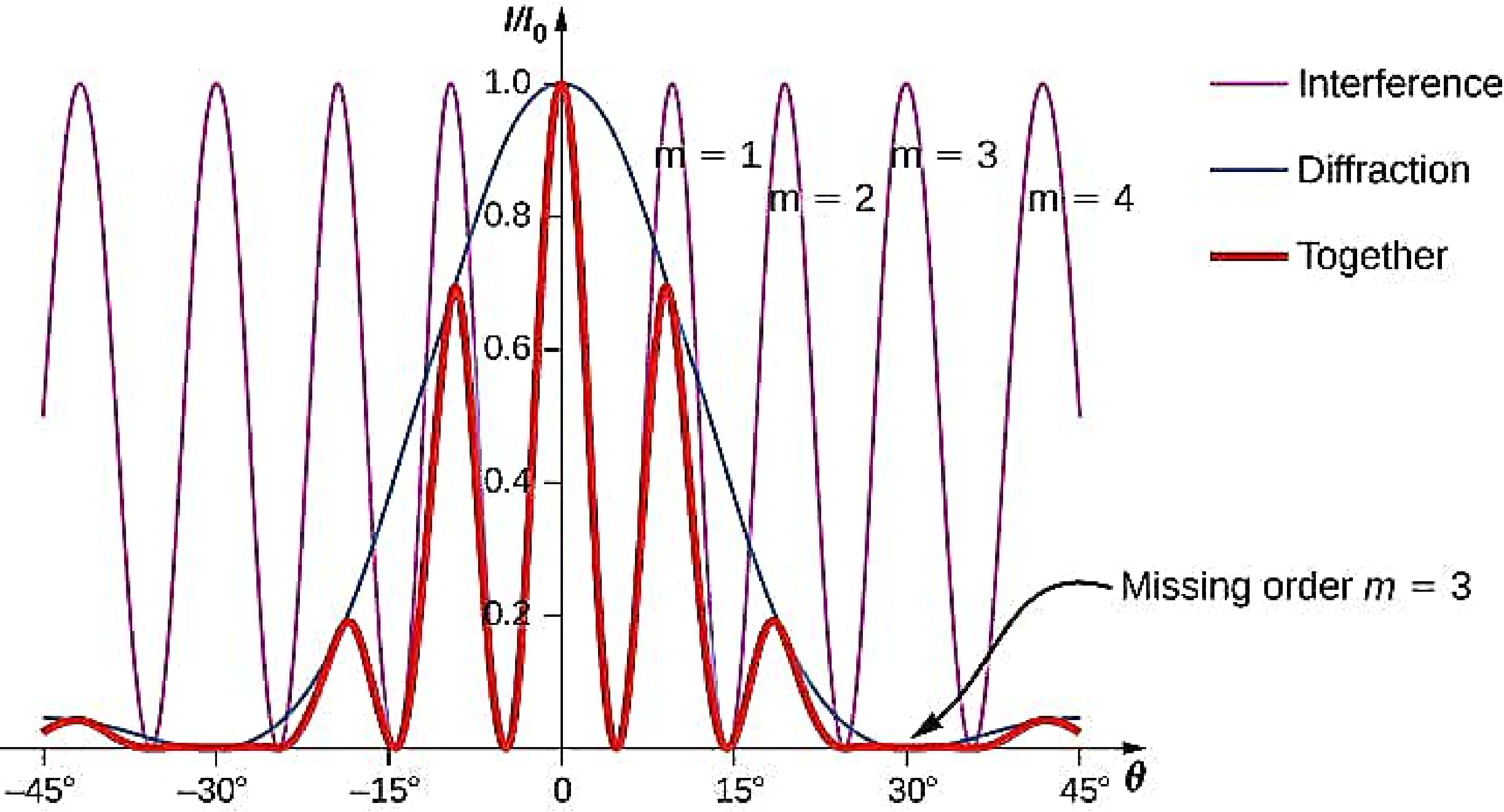
* Directions of Principle Max

Principle maximas are obtained where $\sin\beta = 0 \Rightarrow \beta = \pm n\pi \; (n = 0, 1, 2, 3 \ldots)$

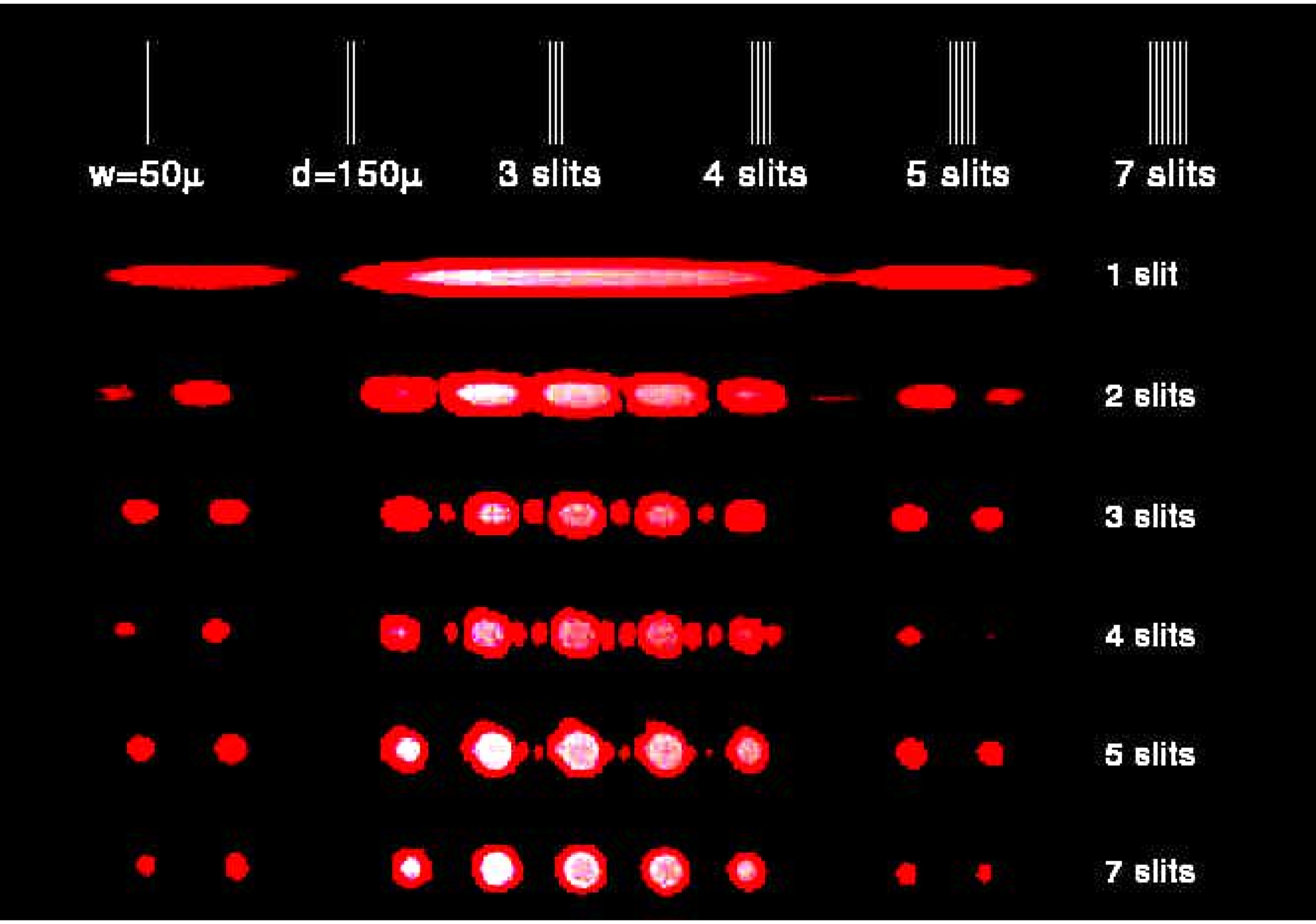
$$\Rightarrow \boxed{(a+b)\sin\theta = \pm n\lambda}$$

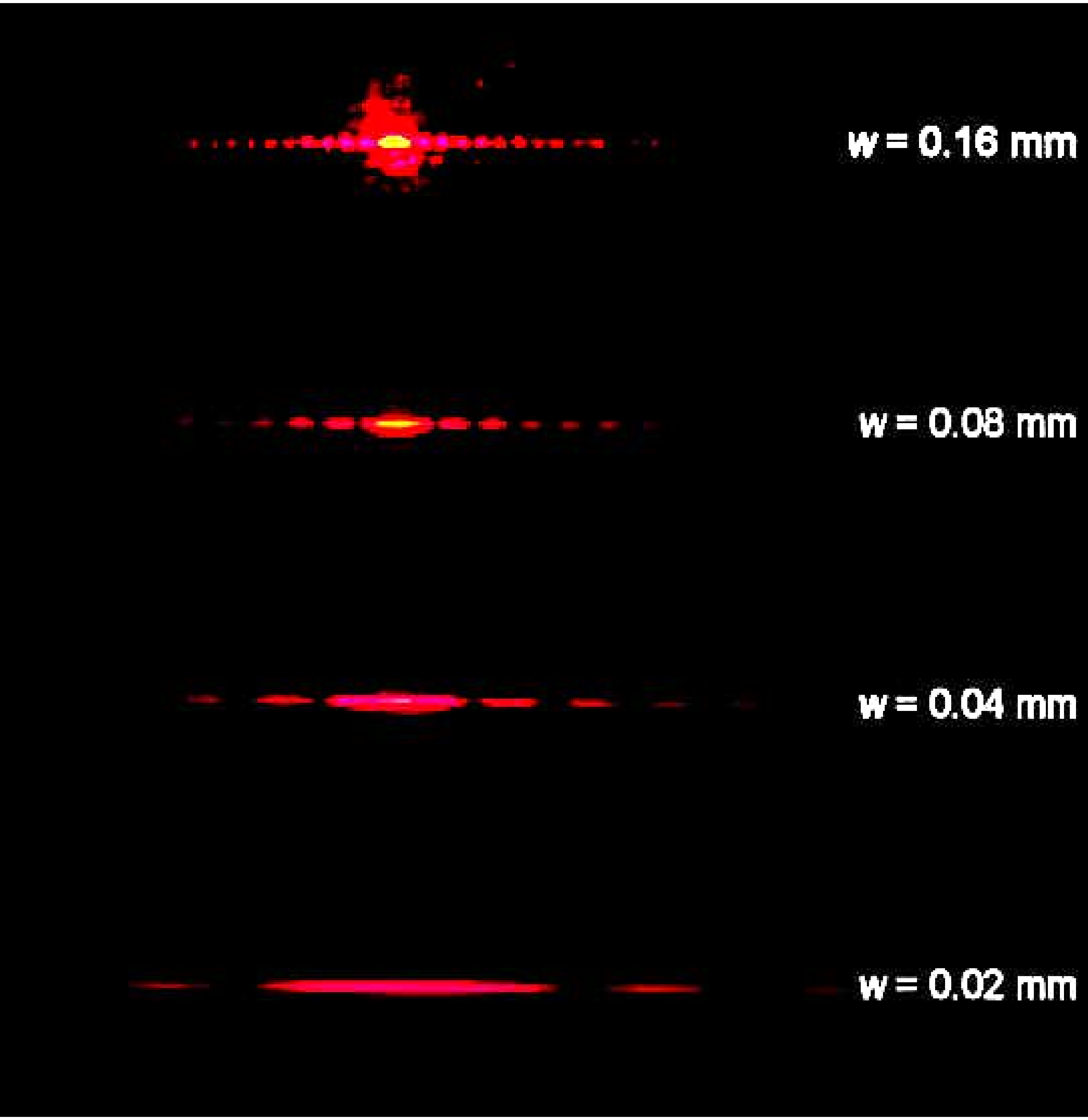
- At $n = 0 \longrightarrow$ Central max
- At $n = 1, 2, 3 \ldots \longrightarrow$ Higher order Principle Max.
- Intensity of Central Max $\boxed{I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 N^2}$ where $N = \lim_{\beta\to n\pi}\left(\frac{\sin N\beta}{\sin\beta}\right)$
- Width of n^{th} order principle max

$$\boxed{2d\theta_n = \frac{2\lambda}{N(a+b)\cos\theta}}$$

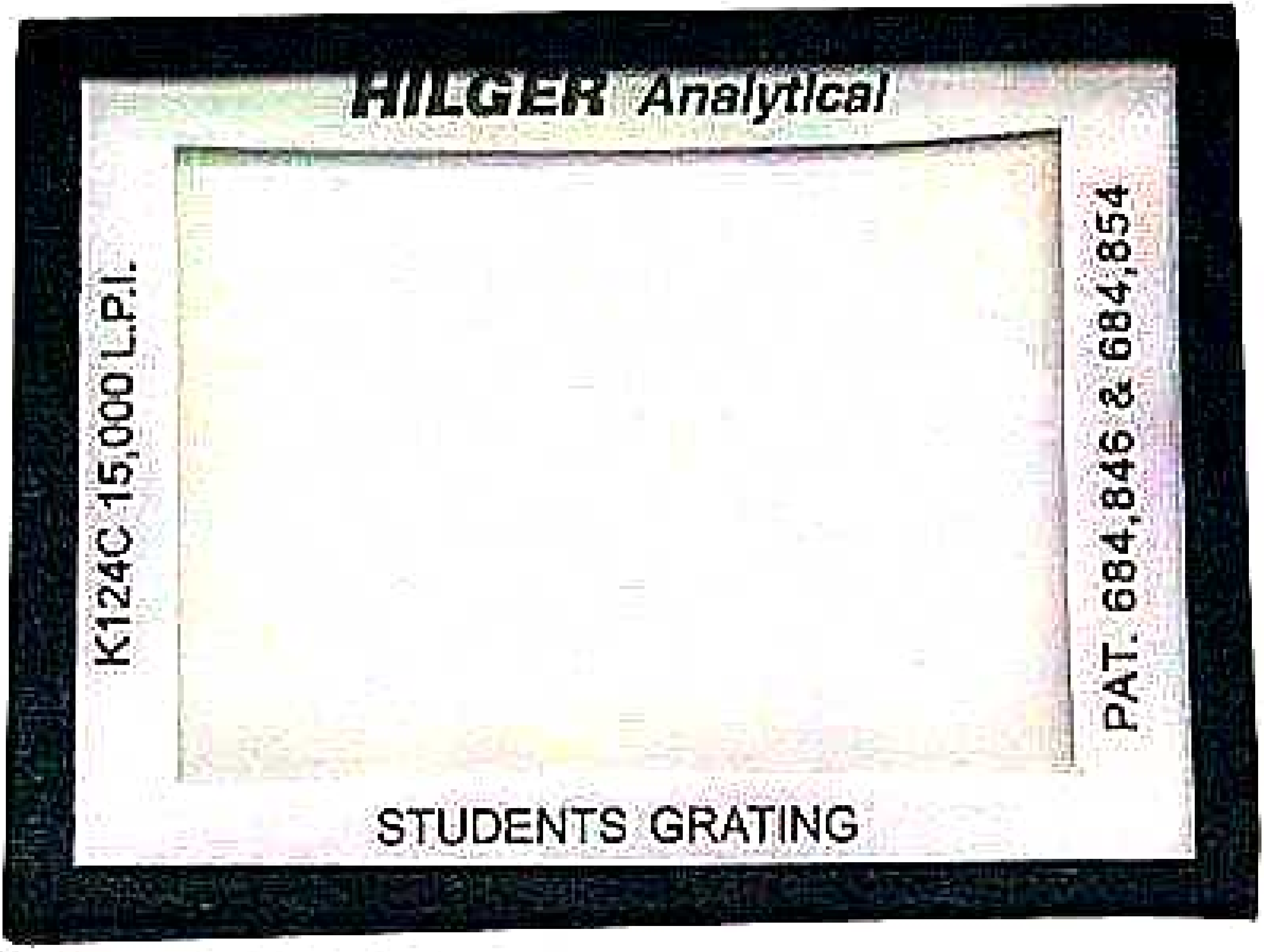


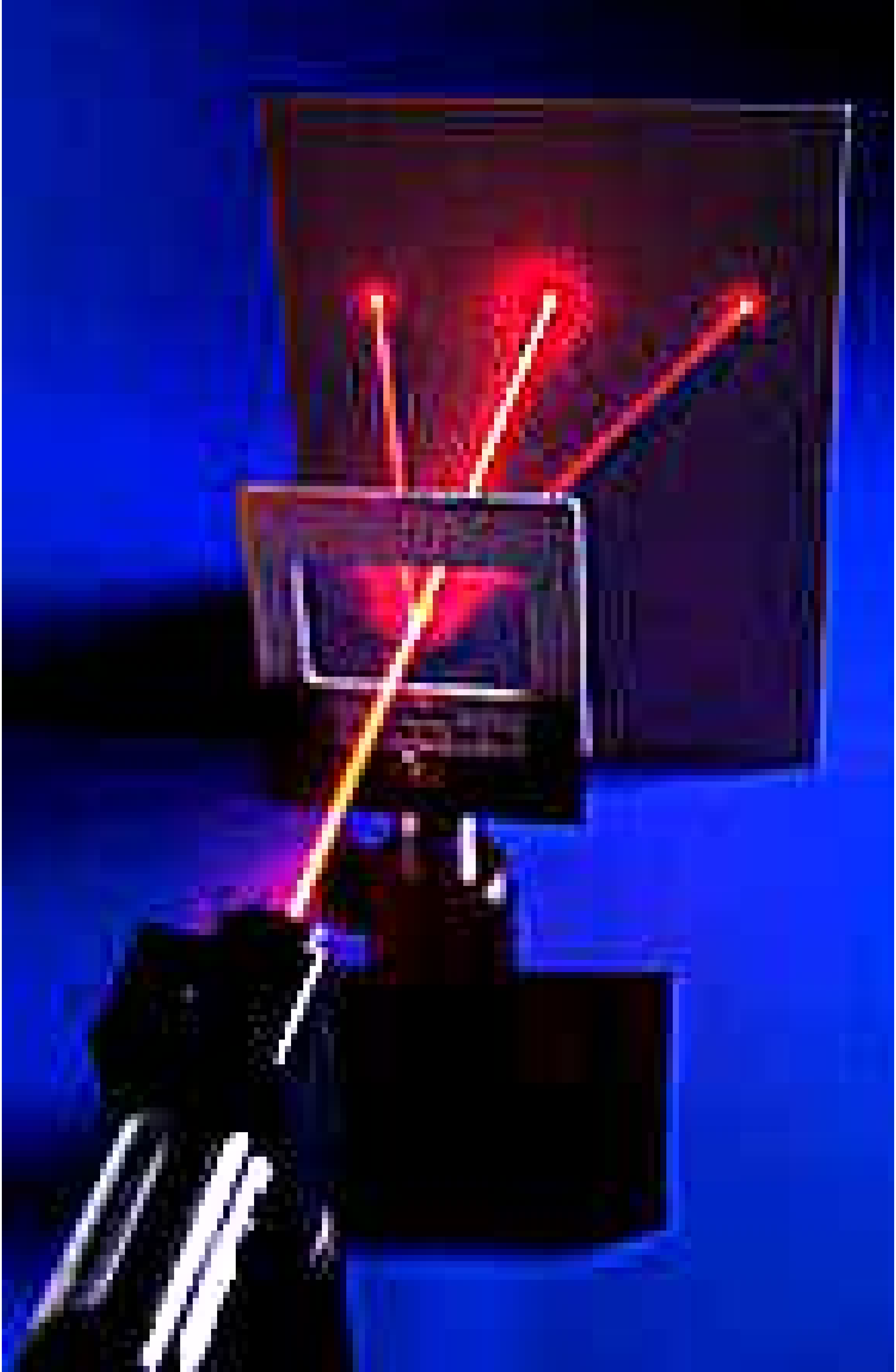
I/I0
1.0
0.8
0.6
0.4
0.2
m = 1
m = 2
m = 3
m = 4
Interference
Diffraction
Together
Missing order m = 3
–45°
–30°
–15°
0
15°
30°
45°
θ


w=50μ
d=150μ
3 slits
4 slits
5 slits
7 slits
1 slit
2 slits
3 slits
4 slits
5 slits
7 slits



w = 0.16 mm
w = 0.08 mm
w = 0.04 mm
w = 0.02 mm


HILGER Analytical
K124C 15,000 L.P.I.
PAT. 684,846 & 684,854
STUDENTS GRATING



* **Directions of Secondary Maxima**

These are obtained where

$$\boxed{\tan N\beta = N \tan\beta}$$

• Intensity/ Intensities of Secondary Max.

$$I = A^2 \left(\frac{\sin\alpha}{\alpha}\right)^2 \left[\frac{N^2}{1+(N^2-1)\sin^2\beta}\right]$$

* **Directions of Minimas**

These are obtained where

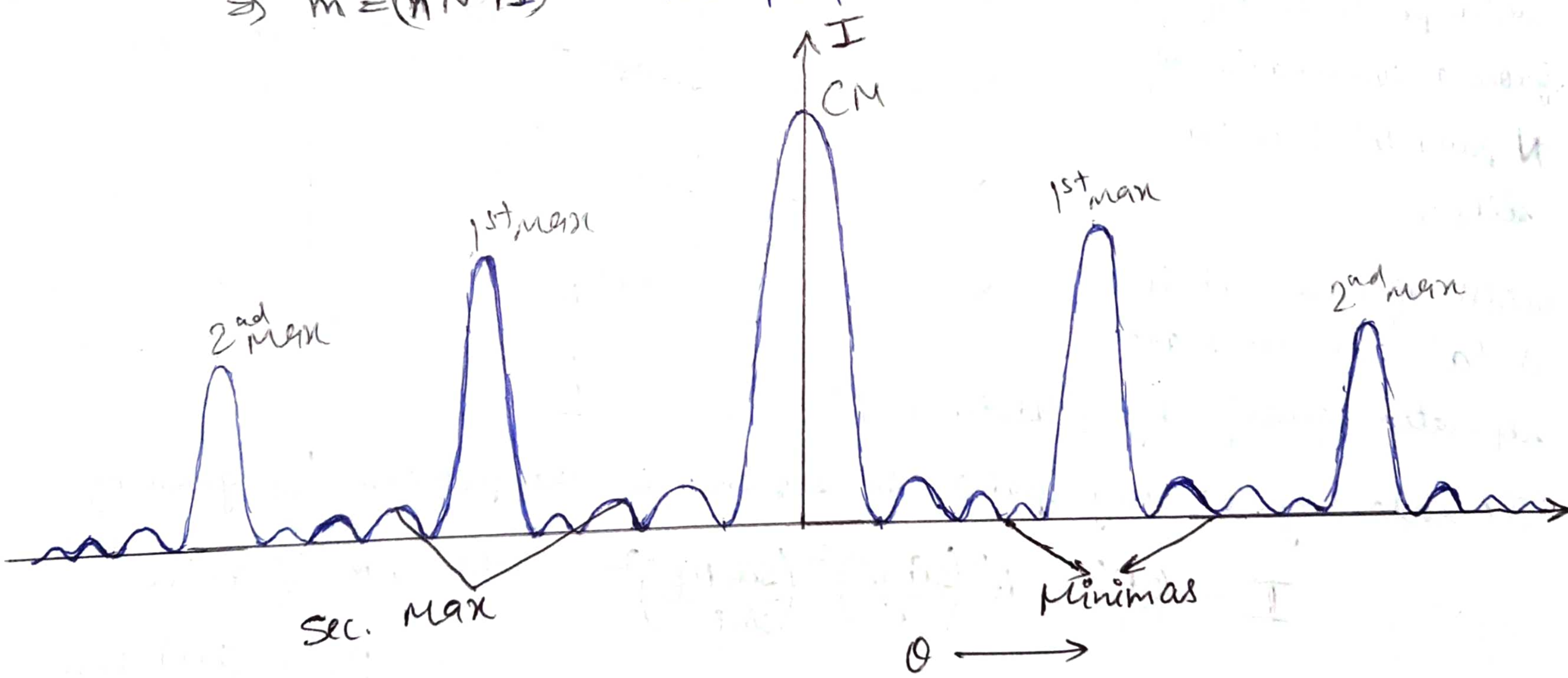
$$\sin N\beta = 0$$

$$N\beta = \pm m\pi$$

$$\Rightarrow \boxed{N(a+b)\sin\theta = m\lambda}$$

where 'm' can take all integral values except $m = 0, N, 2N, 3N \ldots$ because these values give the positions of maximas

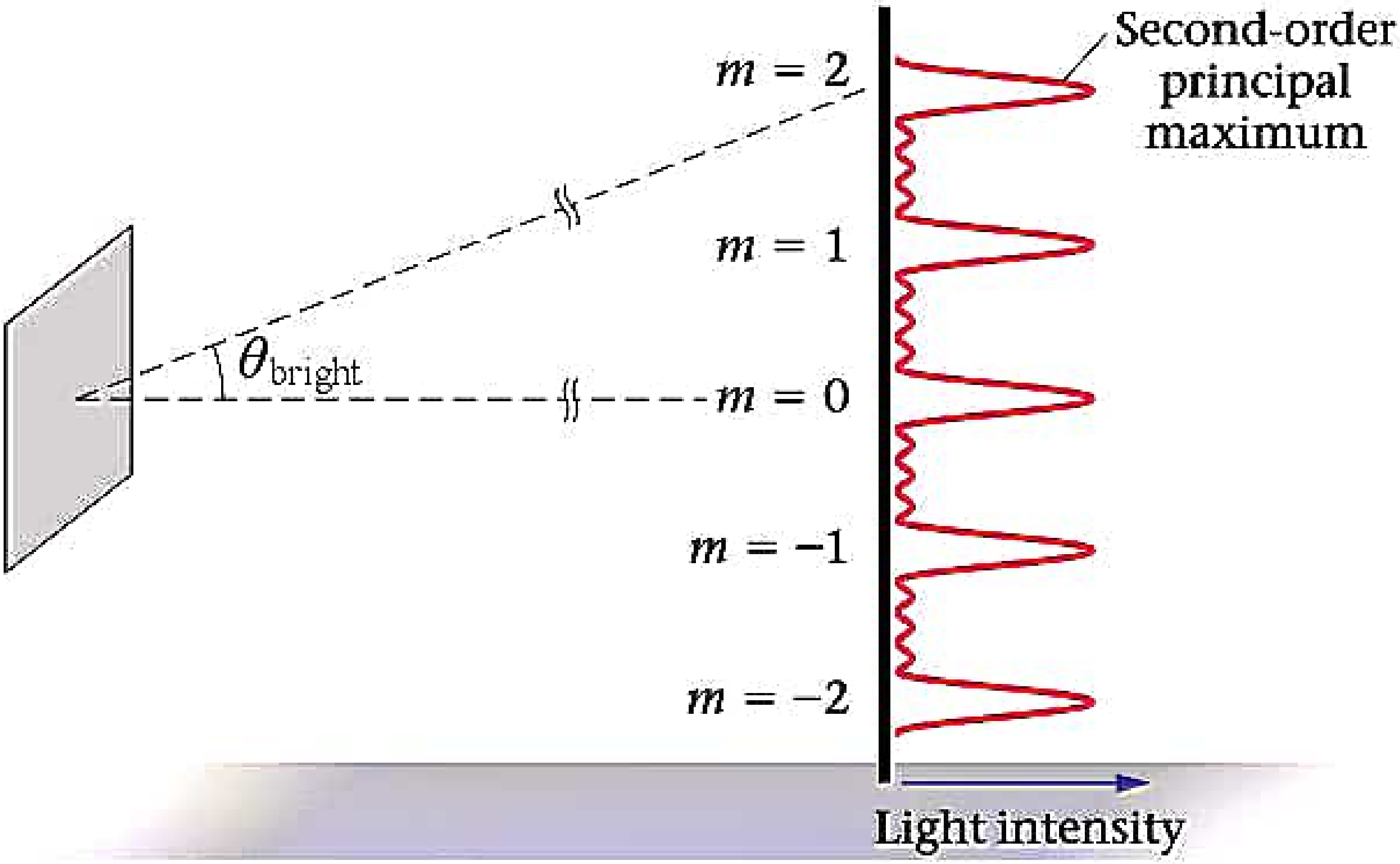
$\Rightarrow m = (nN+1)$ $\quad n = 1, 2, 3 \ldots$



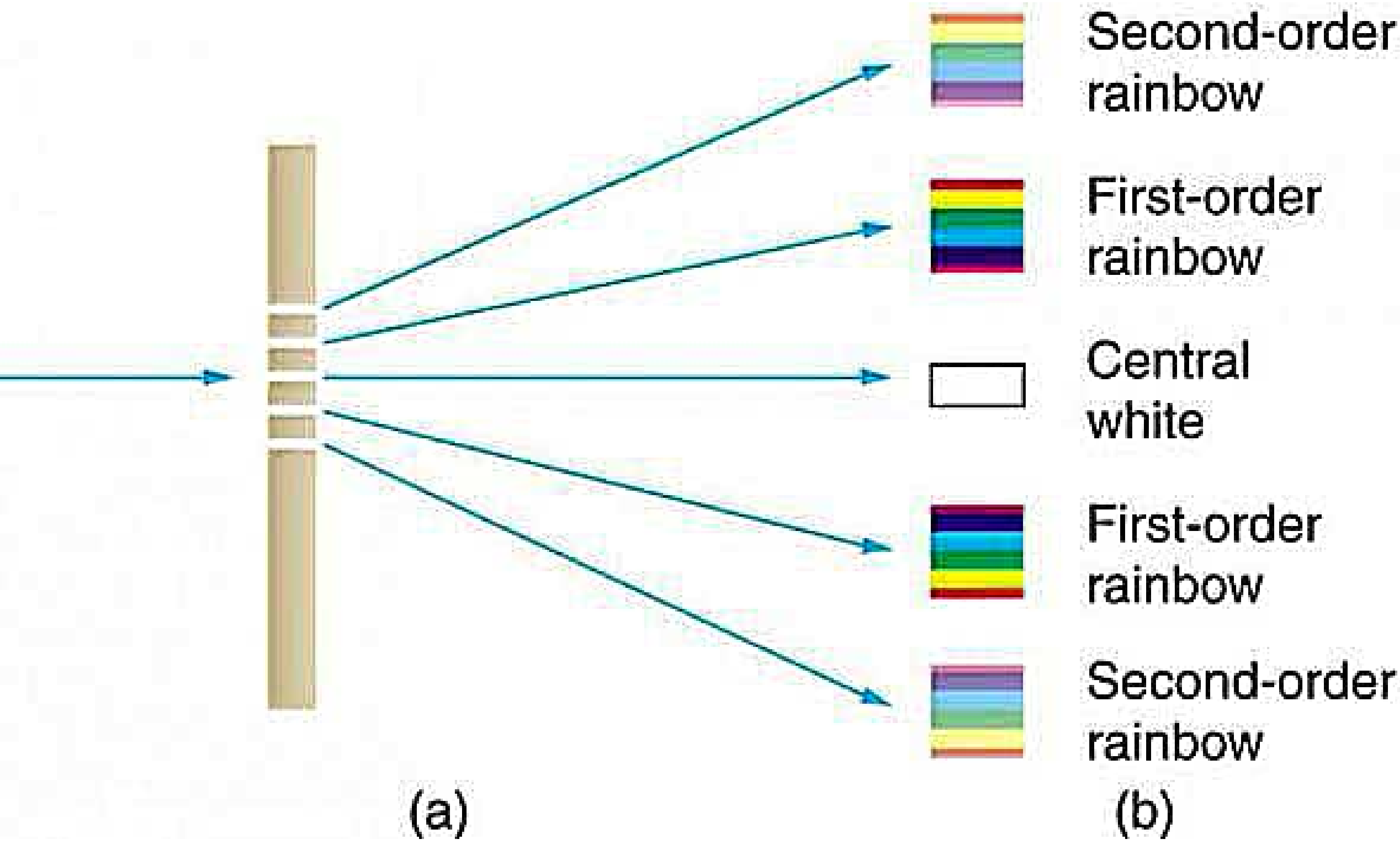
• **Diffraction grating**

An arrangement consisting of large no. of parallel slits of equal width and separated from each other by equal opaque spaces is called diffraction grating.

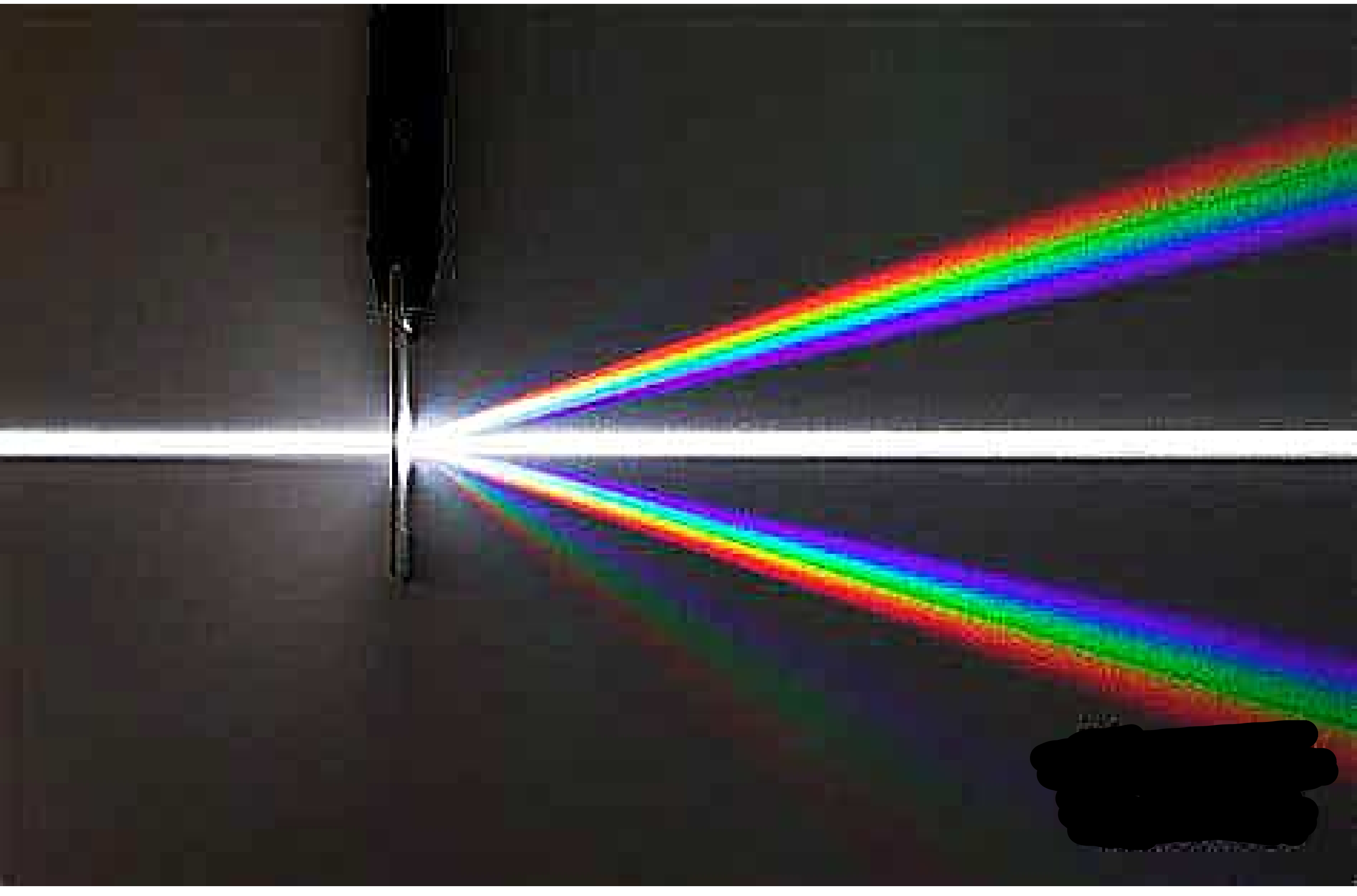
There are about 15000 lines per inch (L.P.I) in a regular students grating

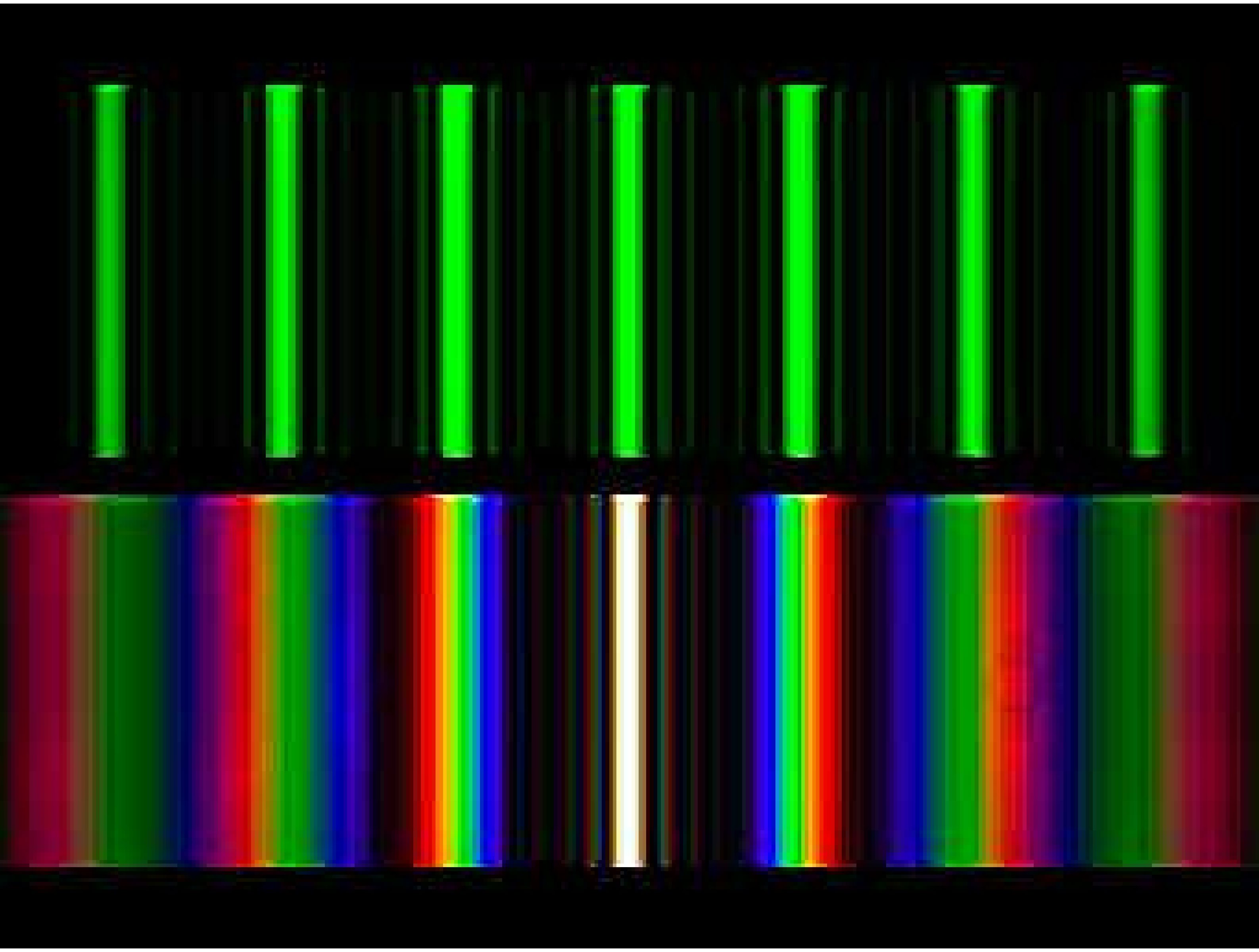


Second-order principal maximum
$m = 2$
$m = 1$
θ_{bright}
$m = 0$
$m = -1$
$m = -2$
Light intensity

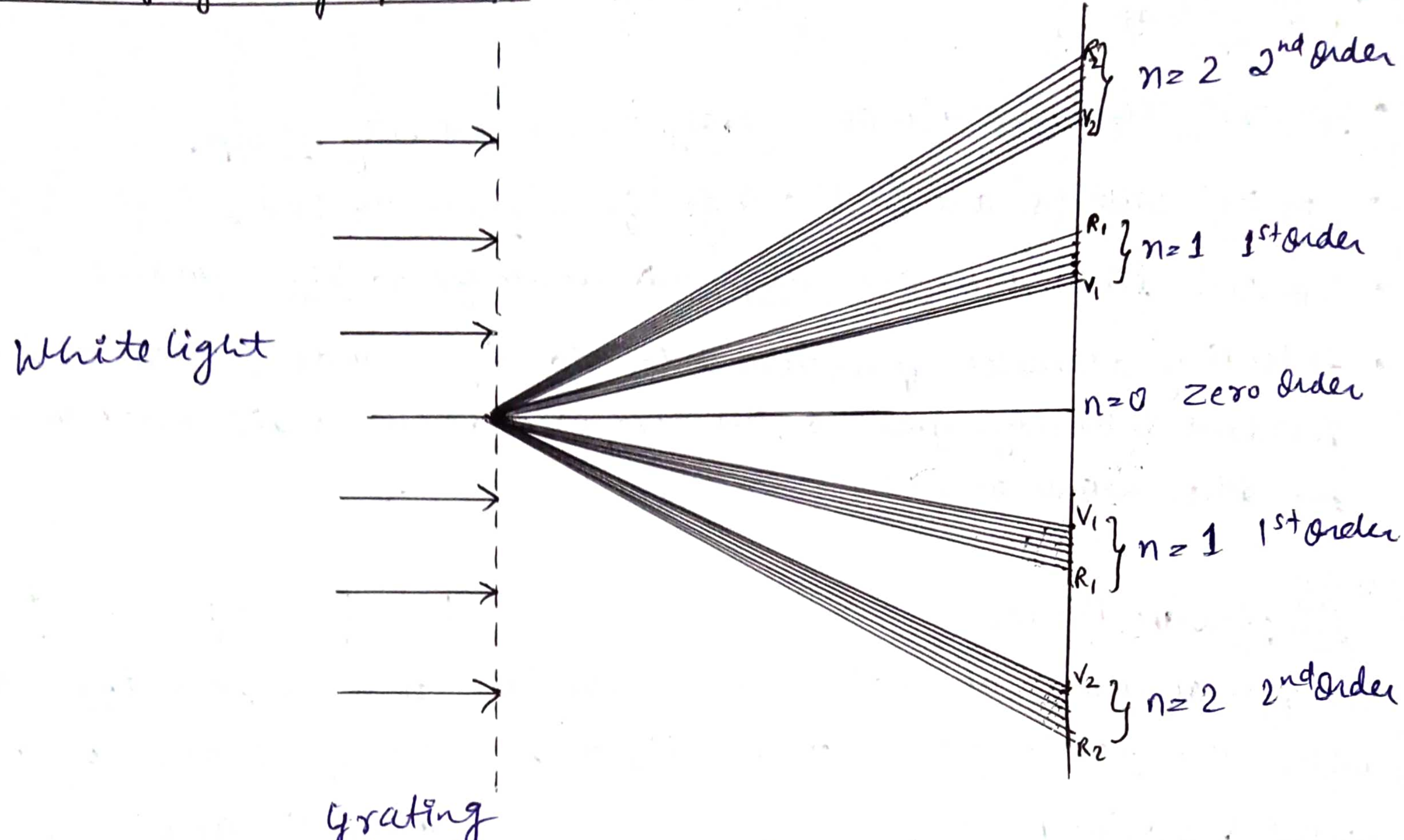


Second-order rainbow
First-order rainbow
Central white
First-order rainbow
Second-order rainbow
(a)
(b)


[redacted]



Features of Grating Spectra



- The direction of principle max is given by

$\boxed{(a+b)\sin\theta = n\lambda}$ where a = slit width (Transparent)
b = slit separation (opaque)
θ = angle of diffraction
$(a+b)$ = Grating element.

- For a given wavelength λ, the direction of principle maxima of different orders are different.
- For a particular value of order (n) the angle of diffraction θ varies with the wavelength.
- The angle of diffraction increases with increase in wavelengths. That's the reason why we obtain coloured spectra for white light.
- At the centre of the pattern the n is equal to zero, we have θ is equal to zero for all the wavelength. This is called the zeroth order of Principle maxima which lies in the same direction for all the wavelengths. That's the reason why it is white (not coloured) for white light.
- Similarly the principle maximas of all wavelengths corresponding to $n=1$ form 1st order spectrum, $n=2$ form 2nd order spectrum, $n=3$ form 3rd order spectrum & so on.

- Spectra of different orders are situated symmetrically on both sides of zero order.
- Spectral lines are almost straight & reasonably sharp.
- Spectral colours are in the order from violet to Red.
- Spectral lines are more dispersed as we go to higher orders.
- Intensity decreases from zero order to higher order. Most of the incident intensity goes to the zero order & rest is distributed among the other higher orders.

Dispersive Power

The rate of change of angle of diffraction 'θ' with the wavelength λ of the light is called dispersive power.

It is represented by $\frac{d\theta}{d\lambda}$

Grating eqn is

$$(a+b)\sin\theta = n\lambda$$

diffg w.r.t 'λ' we get

$$(a+b)\cos\theta \frac{d\theta}{d\lambda} = n$$

$$\Rightarrow \boxed{\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}}$$

Grating Element

It is an identifying parameter or specification of a diffraction grating.

It is given by –

$$N(a+b) = 2.54$$

$$\Rightarrow (a+b) = \frac{2.54}{N} \text{ cm}$$

where N = No. of lines/inch

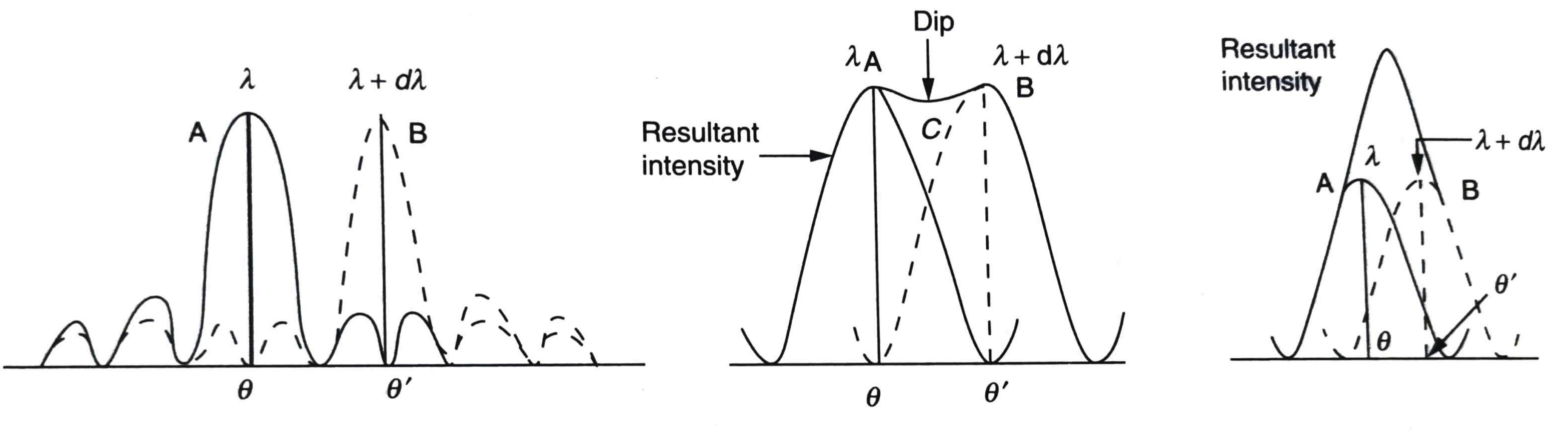
Resolving Power

The ability of an optical instrument to just resolve the images of two nearby point sources or objects or spectral lines is called the resolving power of that instrument.

$$\boxed{R.P = \frac{\lambda}{d\lambda} = nN}$$

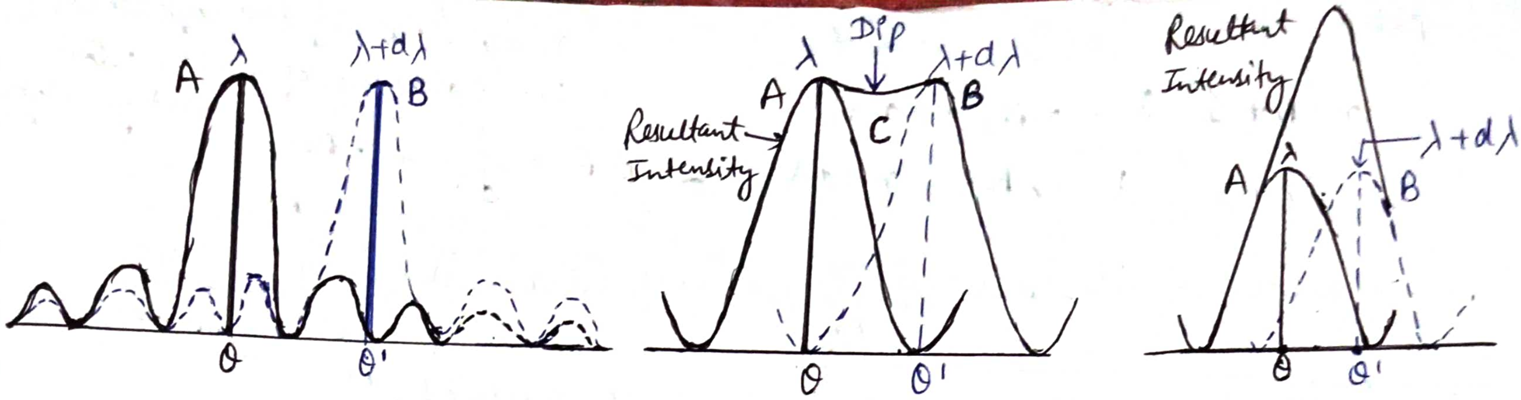
Rayleigh's Criteria of Resolution

"Two point sources or objects or spectral lines of equal intensity are said to be just resolved by an optical instrument when the central maxima of diffraction pattern due to one falls on the first/ Ist minima of the diffraction pattern on the other."



λ
λ + dλ
A
B
θ
θ′
Dip
λ A
λ + dλ
B
C
Resultant intensity
θ
θ′
Resultant intensity
λ
λ + dλ
A
B
θ′
θ

(a) (b) (c)

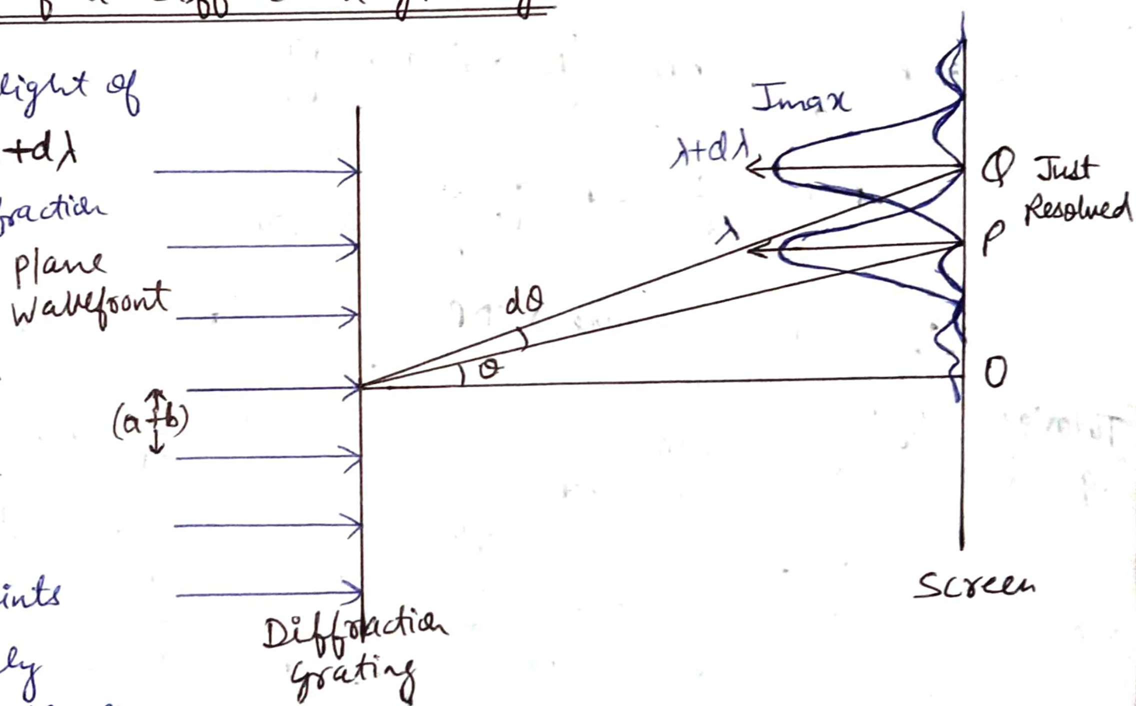


"Two spectral lines/images are said to be just resolved when the intensity at the dip in the middle is 81% of the intensity at either maxima."

i.e $I_{Dip} = \frac{8}{\pi^2} \times I_{max}$

Resolving Power of a Diffraction Grating

Let us suppose that light of wavelength λ and $\lambda+d\lambda$ is incident on a diffraction grating.



The principle maxima of two wavelengths are just resolved at angles θ and $\theta+d\theta$ on the screen at points P and Q respectively as shown in the figure.

At point Q we have -

(i) 1st Minimum of wavelength λ

The condition is $N(a+b)\sin(\theta+d\theta) = m\lambda$ where $m = (nN+1)$ $n = 1, 2, 3$

$\Rightarrow N(a+b)\sin(\theta+d\theta) = (nN+1)\lambda$ —— (1)

(ii) Principle Maximum of wavelength $(\lambda+d\lambda)$

The condition is $(a+b)\sin(\theta+d\theta) = n(\lambda+d\lambda)$

multiplying throughout by N

$$N(a+b)\sin(\theta+d\theta) = n(\lambda+d\lambda)N \quad \text{—— (2)}$$

Comparing eqn ① & ②, we get

$$(n(N)+1)\lambda = nN(\lambda + d\lambda)$$

$$nN\lambda + \lambda = nN\lambda + nNd\lambda$$

$n \rightarrow$ order of the spectrum
$N \rightarrow$ total no. of rulings on the grating.

$$\Rightarrow \boxed{RP = \frac{\lambda}{d\lambda} = nN}$$

"Resolving Power of diffraction grating is defined as the ratio of the wavelength of either spectral line with smallest wavelength difference of two close spectral lines which can be just resolved."

It is given by $\boxed{R.P = \frac{\lambda}{d\lambda}}$

* Dispersive Power of the grating is given by

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} \quad \Rightarrow \quad n = (a+b)\cos\theta \frac{d\theta}{d\lambda}$$

placing this value in the expression of R.P we get

$$R.P = \frac{\lambda}{d\lambda} = nN = \left[N(a+b)\cos\theta\right]\frac{d\theta}{d\lambda}$$

Or,

$$\boxed{R.P = \frac{\lambda}{d\lambda} = A \cdot \frac{d\theta}{d\lambda}}$$

where $A = N(a+b)\cos\theta$ is called the Aperture of grating

$\therefore$ RP = Aperture × DP