Subject Code: BAS103

PAPER ID-311667

Roll No:

BTECH
(SEM I) THEORY EXAMINATION 2024-25
ENGINEERING MATHEMATICS-I

TIME: 3 HRS **M.MARKS: 70**

Note: Attempt all Sections. In case of any missing data; choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 07 = 14

Q no.	Question	CO	Level
a.	Find the eigen values of the matrix $\begin{bmatrix} cos\theta & -sin\theta \\ -sin\theta & -cos\theta \end{bmatrix}$.	1	K2
b.	If $u = \frac{x^2+y^2}{x+y}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.	2	K3
c.	What is the difference between total derivatives and partial derivatives?	2	K1
d.	What are the applications of Jacobians	3	K4
e.	Write the statement of Liouville's Theorem.	4	K2
f.	Evaluate $\int_1^2 \int_1^3 x^2y^2dxdy$.	4	K3
g.	Prove that $curl\,\vec{r} = 0$.	5	K2

SECTION B

2. Attempt any *three* of the following: 07 x 3 = 07

Q no.	Question	CO	Level
a.	Find two non-singular matrices P and Q such that PAQ is in normal form, Where $A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$	1	K2
b.	Find the n^{th} derivative of $tan^{-1}\left(\frac{x}{a}\right)$	2	K3
c.	Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.	3	K4
d.	Apply Dirichlet's theorem to evaluate $\iiint xyzdxdxdz$ taken throughout the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \le 1$	4	K3
e.	Show that the vector $f(r)\vec{r}$ is irrotational. Where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$	5	K5

SECTION C

3. Attempt any *one* part of the following: 07 x 1 = 07

Q no.	Question	CO	Level
a.	Find the eigen values and eigen vectors of the following matrices: $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.	1	K4
b.	Discuss for all values of K for the system of equations	1	K2

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	$x+y+4z=6, x+2y-2z=6, Kx+y+z=6$ as regards existence and nature of solution.		

4. Attempt any *one* part of the following:

Q no.	Question	CO	Level
a.	Trace the curve $y^2(a+x)=x^2(3a-x)$.	2	K1
b.	If $u=f(r), where\ r^2=x^2+y^2$,prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r)$.	2	K1

5. Attempt any *one* part of the following: **07 x 1 = 07**

Q no.	Question	CO	Level
a.	If $u=xyz, v=x^2+y^2+z^2 and$ $w=x+y+z. Find\ the\ jacobian\ \frac{\partial(x,y,z)}{(u,v,w)}$	3	K3
b.	Find the maxima and minima of the function $sinx+siny+\sin(x+y)$.	3	K3

6. Attempt any *one* part of the following: **07 x 1 = 07**

Q no.	Question	CO	Level
a.	Find the area inside the circle $r=2acos\theta$ and outside the circle $r=a$	4	K4
b.	Change the order of integration and then evaluate $\int_0^{2a}\int_{\frac{x^4}{4a}}^{3a-x}(x^2+y^2)\,dydx$	4	K2

7. Attempt any *one* part of the following: **07 x 1 = 07**

Q no.	Question	CO	Level
a.	Show that div (gradr^n) $=n(n+1)r^{n-2}$. Where $\vec{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$	5	K4
b.	Verify Stokes theorem for $\vec{F}=(x^2+y^2)\hat{\imath}-2xy\hat{\jmath}$ taken round the rectangle bounded by the lines$x=0, x=a, y=0, y=b$.	5	K5

QP25DP2_145 | 03-Mar-2025 9:15:36 AM | 115.244.153.242

Subject Code: BAS103

PAPER ID-311443

Roll No:

BTECH
(SEM I) THEORY EXAMINATION 2023-24
ENGINEERING MATHEMATICS-I

TIME: 3HRS **M.MARKS: 70**

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. **2 x 7 = 14**

Q no.	Question	Marks	CO
a.	Find the product and sum of the eigen values for $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$.	2	1
b.	Find all symmetry in the curve $y^2(a^2 + x^2) = x^2(a^2 - x^2)$.	2	2
c.	Calculate the error in R if $E = RI$ and possible errors in E and I are 30% and 20% respectively.	2	3
d.	Determine the value of $\Gamma\frac{1}{4}\ \Gamma\frac{3}{4}$.	2	4
e.	Prove that $B(p,q) = B(p+1,q) + B(p,q+1)$	2	4
f.	Prove that $\vec{A} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational.	2	5
g.	Find a unit normal vector to the surface $xy^3z^2 = 4$ at the point $(-1,-1,2)$.	2	5

SECTION B

2. Attempt any *three* of the following: **7 x 3 = 21**

a.	Solve the system of homogenous equations: $x_1 + x_2 + x_3 + x_4 = 0,\ \ x_1 + 3x_2 + 2x_3 + 4x_4 = 0,$ $2x_1 + x_3 - x_4 = 0$	7	1
b.	If $u = y^2 e^{y/x} + x^2 \tan^{-1}\left(\frac{x}{y}\right)$, show that (i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$ (ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2u$.	7	2
c.	Expand $f(x,y) = e^x \cos y$ about the point $\left(1,\frac{\pi}{4}\right)$ by Taylor's sereies.	7	3
d.	Evaluate the integral $\iint_D (y-x)dxdy$; by changing the variables, D: Region in xy-plane bounded by the lines $y - x = -3, y - x = 1, y + \frac{1}{3}x = \frac{7}{3}, y + \frac{1}{3}x = 5$.	7	4
e.	Find the directional derivative of $f(x,y,z) = e^{2x}\cos yz$ at $(0,0,0)$ in the direction of the tangent to the curve $x = a\sin\theta, y = a\cos\theta,\quad z = a\theta$ at $\theta = \pi/4$.	7	5

SECTION C

3. Attempt any *one* part of the following: **7 x 1 = 7**

a.	Determine eigen vectors for the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$.	7	1
b.	Determine A^{-1}, A^{-2} and A^{-3} if $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$ using Cayley-Hamilton theorem.	7	1

QP24DP2_290 | 18-03-2024 08:55:02 | 117.55.242.132

Subject Code: BAS103

PAPER ID-311443

Roll No:

BTECH
(SEM I) THEORY EXAMINATION 2023-24
ENGINEERING MATHEMATICS-I

TIME: 3HRS **M.MARKS: 70**

4. Attempt any *one* part of the following: **7 x 1 = 7**

a.	If $y = \cos(m\sin^{-1}x)$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$. Also find $(y_n)_0$.	7	2
b.	If $z = f(x,y), x = e^u + e^{-v}, y = e^{-u} - e^v$ then show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$	7	2

5. Attempt any *one* part of the following: **7 x 1 = 7**

a.	If $u^3 + v^3 = x + y, u^2 + v^2 = x^3 + y^3$, then show $\frac{\partial(u,v)}{\partial(x,y)} = \frac{y^2-x^2}{2uv(u-v)}$.	7	3
b.	The pressure P at any point (x,y,z) in space is $P = 400\,xyz^2$. Find the highest pressure at the surface of a unit sphere $x^2+y^2+z^2 = 1$ using Lagrange's method.	7	3

6. Attempt any *one* part of the following: **7 x 1 = 7**

a.	Find the volume of the solid bounded by the coordinate planes and the surface $\left(\frac{x}{a}\right)^{1/2} + \left(\frac{y}{b}\right)^{1/2} + \left(\frac{z}{c}\right)^{1/2} = 1$.	7	4
b.	Prove that $B(m,n) = \frac{\Gamma m \Gamma n}{\Gamma m+n}$.	7	4

7. Attempt any *one* part of the following: **7 x 1 = 7**

a.	Applying Gauss Divergence theorem, evaluate $\iint_S [e^x dydz - ye^x dzdx + 3zdxdy]$, where S is the surface of the cylinder $x^2+y^2 = c^2, 0 \le z \le h$.	7	5
b.	Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and hence show that $\nabla^2\left(\frac{1}{r}\right) = 0$.	7	5

Printed Pages:04 | Sub Code: BAS-103

Paper Id: 233418 | Roll No.

B. TECH.
(SEM I) THEORY EXAMINATION 2022-23
ENGINEERING MATHEMATICS I

Time: 3 Hours | ***Total Marks: 70***

समयः 03 घण्टे | पूर्णांकः 70

Note:

1. Attempt all Sections. If require any missing data; then choose suitably.
2. The question paper may be answered in Hindi Language, English Language or in the mixed language of Hindi and English, as per convenience.

नोटः 1. सभी प्रश्नो का उत्तर दीजिए। किसी प्रश्न में, आवश्यक डेटा का उल्लेख न होने की स्थिति में उपयुक्त डेटा स्वतः मानकर प्रश्न को हल करें।

2. प्रश्नों का उत्तर देने हेतु सुविधानुसार हिन्दी भाषा, अंग्रेजी भाषा अथवा हिंदी एवं अंग्रेजी की मिश्रित भाषा का प्रयोग किया जा सकता है।

SECTION A

1. **Attempt *all* questions in brief.** **2 x 7 = 14**

निम्न सभी प्रश्नों का संक्षेप में उत्तर दीजिए।

a. If A is a Hermitian matrix, then show that iA is Skew-Hermitian matrix.
यदि A एक हर्मिटियन (Hermitian) मैट्रिक्स है, तो दिखाएँ कि iA यह स्क्यू-हर्मिटियन (Skew-Hermitian) मैट्रिक्स है।

b. Find the eigen value of the matrix $A=\begin{bmatrix}4 & 2\\ 2 & 4\end{bmatrix}$ corresponding to the eigen vector $\begin{bmatrix}51\\ 51\end{bmatrix}$.
आइजेन मान $\begin{bmatrix}51\\ 51\end{bmatrix}$ के संगत मैट्रिक्स $A=\begin{bmatrix}4 & 2\\ 2 & 4\end{bmatrix}$ का आइजेन वेक्टर ज्ञात करें।

c. If $y=\cos^{-1}x$, prove that $(1-x^2)\,y_2-xy_1=0$.
यदि, $y=\cos^{-1}x$ तो सिद्ध कीजिए कि $(1-x^2)\,y_2-xy_1=0$.

d. If $u=\sin^{-1}\left(\dfrac{x^3+y^3}{\sqrt{x}+\sqrt{y}}\right)$, then show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\dfrac{5}{2}\tan u$.
यदि, $u=\sin^{-1}\left(\dfrac{x^3+y^3}{\sqrt{x}+\sqrt{y}}\right)$ तो सिद्ध कीजिए कि $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\dfrac{5}{2}\tan u$.

e. Find the percentage error in measuring the volume of a rectangular box when the error of 1% is made in measuring each side.
यदि प्रत्येक भुजा को मापने में 1% की त्रुटि होती है तो एक आयताकार बॉक्स के आयतन को मापने में कितनी प्रतिशत त्रुटि होगी?

f. Evaluate $\iint y\,dx\,dy$ over the part of the plane bounded by the line $y=x$ and the parabola $y=4x-x^2$.
रेखा $y=x$ और परवलय $y=4x-x^2$ से घिरे क्षेत्र के भाग के लिए $\iint y\,dx\,dy$ की गणना कीजिए।

g. Find curl of a vector field given by $\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2 y)\hat{j}$.

$\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2 y)\hat{j}$ द्वारा परिभाषित वेक्टर फ़ील्ड $\vec{F}$ का कर्ल (curl) ज्ञात करें।

SECTION B

2. Attempt any *three* of the following: **7 x 3 = 21**

निम्न में से किसी तीन प्रश्नों का उत्तर दीजिए।

a. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find its inverse.

मैट्रिक्स $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ के लिए केली-हैमिल्टन प्रमेय को सत्यापित करें और इसका व्युत्क्रम ज्ञात करें।

b. If $y\sqrt{x^2 - 1} = \log_e(x + \sqrt{x^2 - 1})$, prove that
$(x^2 - 1)y_{n+1} + (2n+1)xy_n + n^2 y_{n-1} = 0$.

यदि $y\sqrt{x^2 - 1} = \log_e(x + \sqrt{x^2 - 1})$, तो सिद्ध कीजिए कि
$(x^2 - 1)y_{n+1} + (2n+1)xy_n + n^2 y_{n-1} = 0$.

c. Expand $f(x, y) = y^x$ about $(1,1)$ up to second degree terms and hence evaluate $(1.02)^{1.03}$.

$(1,1)$ के सापेक्ष $f(x, y) = y^x$ का द्वितीय डिग्री के पदों तक विस्तार करें और तदोपरान्त $(1.02)^{1.03}$ की गणना कीजिए।

d. Evaluate the double integral $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{(y^4 - a^2 x^2)}}\,dxdy$ by changing the order of integration.

समाकलन के क्रम को बदलकर डबल इंटीग्रल $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{(y^4 - a^2 x^2)}}\,dxdy$ का मान ज्ञात कीजिए।

e. Find the directional derivative of scalar function $f(x, y, z) = xyz$ at point $P(1,1,3)$ in the direction of the outward drawn normal to the sphere $x^2 + y^2 + z^2 = 11$ through the point P.

गोले $x^2 + y^2 + z^2 = 11$ पर बिंदु P से गुजरते हुए बाहर की ओर खीचें गये नार्मल की दिशा में अदिश फलन $f(x, y, z) = xyz$ का बिन्दु $P(1,1,3)$ पर दिशात्मक अवकलज (directional derivative) ज्ञात कीजिए।

SECTION C

3. Attempt any *one* part of the following: **7 x 1 = 7**

निम्न में से किसी एक प्रश्न का उत्तर दीजिए।

(a) Test the consistency for the following system of equations and if system is consistent, solve them:

समीकरणों की निम्नलिखित निकाय के लिए संगतता (consistency) का परीक्षण करें और यदि निकाय सुसंगत है, तो उन्हें हल करें:

$$x + y + z = 6,$$
$$x + 2y + 3z = 14,$$
$$x + 4y + 7z = 30.$$

(b) Find the eigen values and corresponding eigen vectors of the matrix ***A*.**

मैट्रिक्स A के आइजेन मान और संगत आइजेन वेक्टर ज्ञात कीजिए।

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}.$$

4. Attempt any *one* part of the following: 7 x 1 = 7

निम्न में से किसी एक प्रश्न का उत्तर दीजिए।

(a) Trace the curve $x^2y^2 = (a^2 + y^2)(a^2 - y^2)$ in xy-plane, where a is constant.

xy-तल में वक्र $x^2y^2 = (a^2 + y^2)(a^2 - y^2)$ जहां a एक नियतांक है, का अनुरेखण करें।

(b) If $u = \dfrac{x^2y^2}{x^2+y^2} + \cos\left(\dfrac{xy}{x^2+y^2}\right)$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\frac{x^2y^2}{x^2+y^2}.$$

यदि $u = \dfrac{x^2y^2}{x^2+y^2} + \cos\left(\dfrac{xy}{x^2+y^2}\right)$ तो सिद्ध कीजिए कि:

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\frac{x^2y^2}{x^2+y^2}$$

5. Attempt any *one* part of the following: 7 x 1 = 7

निम्न में से किसी एक प्रश्न का उत्तर दीजिए।

(a) Find the Jacobian of the functions $y_1 = (x_1 - x_2)(x_2 + x_3)$, $y_2 = (x_1 + x_2)(x_2 - x_3)$, $y_3 = x_2(x_1 - x_3)$, hence show that the functions are not independent. Find the relation between them.

फलन $y_1 = (x_1 - x_2)(x_2 + x_3)$, फलन $y_2 = (x_1 + x_2)(x_2 - x_3)$, फलन $y_3 = x_2(x_1 - x_3)$, का जेकोबियन (Jacobian) ज्ञात कीजिए। दिखाएं कि फलन स्वतंत्र नहीं हैं। उनके बीच संबंध ज्ञात कीजिए।

(b) A rectangular box, which is open at the top, has a capacity of 32 cubic feet. Determine, using Lagrange's method of multipliers, the dimensions of the box such that the least material is required for the construction of the box.

एक आयताकार बॉक्स, जो शीर्ष पर खुला है, की क्षमता 32 घन फीट है। लाग्रेंज की मल्टीप्लायर विधि (Lagrange's method of multipliers) का उपयोग करते हुए, बॉक्स के आयामों का इसप्रकार निर्धारण करें कि बॉक्स के निर्माण के लिए कम से कम सामग्री की आवश्यकता हो।

6. Attempt any *one* part of the following: 7 x 1 = 7

निम्न में से किसी एक प्रश्न का उत्तर दीजिए।

(a) Evaluate $\iiint_R (x-2y+z)\,dz\,dy\,dx,$ where R is the region determined by $0 \le x \le 1, 0 \le y \le x^2, 0 \le z \le x+y$.

$\iiint_R (x-2y+z)\,dz\,dy\,dx,$ को ज्ञात कीजिए, जहां R क्षेत्र $0 \le x \le 1, 0 \le y \le x^2, 0 \le z \le x+y$ द्वारा निर्धारित है।

(b) Use Dirichlet's integral to evaluate $\iiint xyz\,dx\,dy\,dz$ throughout the volume bounded by $x=0, y=0, z=0$ and $x+y+z=1$.

Dirichlet's integral की सहायता से $x=0, y=0, z=0$ और $x+y+z=1$ से घिरे हुए आयतन के लिए $\iiint xyz\,dx\,dy\,dz$ को ज्ञात कीजिए।

7. Attempt any *one* part of the following: 7 x 1 = 7

निम्न में से किसी एक प्रश्न का उत्तर दीजिए।

(a) Apply Gauss divergence theorem to evaluate $\iint_S \vec{F}.\hat{n}\,ds$, where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface of the region bounded by the cylinder $x^2 + y^2 = 4, z = 0, z = 3$.

गॉस डाइवर्जेंस प्रमेय (Gauss divergence theorem) का प्रयोग करते हुए $\iint_S \vec{F}.\hat{n}\,ds$ आकलन कीजिए, जहां $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ और S, बेलन $x^2 + y^2 = 4, z = 0$ $z = 3$. से घिरे क्षेत्र, की सतह है ।

(b) Evaluate $\oint_C \vec{F}.\vec{dr}$ by Stoke's theorem, where $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$ and C is the boundary of the triangle with vertices at $(0,0,0), (1,0,0)$ and $(1,1,0)$.

स्टोक के प्रमेय द्वारा $\oint_C \vec{F}.\vec{dr}$ का आकलन कीजिए, जहां $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$ और C ऐसे त्रिभुज की सीमा है जिसके शीर्ष $(0,0,0), (1,0,0)$ और $(1,1,0)$ है।