

Edition 2021 - 22

# Data Structure

PEN-Drive / G-Drive Course / VOD & Tablet Users

Workbook

Computer Science Engineering  
Information Technology

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# Data Structure

PEN-Drive / G-Drive Course / VOD & Tablet Users

Workbook

CS / IT

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- ✎ Arrays
- ✎ Stacks
- ✎ Queues
- ✎ Linked lists
- ✎ Trees
- ✎ Binary search trees
- ✎ Binary heaps
- ✎ Graphs

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# 1

# ARRAY



## Classroom Practice Questions :

- Q.1** Let  $A[1..8, -5..5, -10..5]$  be a 3D array ? What is the number of elements in the array ?  
(A) 1200  
(B) 1408  
(C) 33  
(D) 1050
- Q.2** A One Dimensional array A has indices  $1..75$ . Each element is a string and takes up 3 memory words. The array is stored at location 1120 . The starting address of  $A[49]$  is :  
(A) 1267  
(B) 1164  
(C) 1264  
(D) 1169
- Q.3** Let  $A[-5..5, -10..10]$  be an array of integers where each element takes 4 bytes in memory and the base address of the array is 1000 .The starting address of  $A[0,0]$  is \_\_\_\_\_?
- Q.4** Let  $A[-5..5, -20..20]$  be an array of integers where each element takes 4 bytes in memory and the base address of the array is 1000 .The starting address of  $A[2,3]$  is \_\_\_\_\_?
- Q.5** Let  $A[-5..5, -5..5, -10..10]$  be an array of integers where each element takes 4 bytes in memory and the base address of the array is 1000 .The starting address of  $A[1,1,1]$  is \_\_\_\_\_?
- Q.6** Let  $A[-5..5, -10..10, -10..10, -5..5]$  be an array of integers where each element takes 4 bytes in memory and the base address of the array is 1000 .The starting address of  $A[0][0][0][0]$  is \_\_\_\_\_?
- Q.7** Let  $A[-25..25, -20..20, -10..10]$  be an array of integers where each element takes 2 bytes in memory and the base address of the array is 1062 .The starting address of  $A[0][0][0]$  is \_\_\_\_\_?
- Q.8** Let A be a two dimensional array declared as follows: **GATE 1998**  
A: array [1 ... 10] [1 ... 15] of integer;  
Assuming that each integer takes one memory location, the array is stored in row-major order and the first element of the array is stored at location 100, what is the address of the element  $a[i][j]$  ?  
(A)  $15i + j + 84$   
(B)  $15j + i + 84$   
(C)  $10i + j + 89$   
(D)  $10j + i + 89$

**Q.9** Consider the following declaration of a two-dimensional array in C :

`char a[100][100];`

Assuming that the main memory is byte-addressable and that the array is stored starting from memory address 0, the address of `a[40][50]` is:

- (A) 4040
- (B) 4050
- (C) 5040
- (D) 5050

**Q.10** Let `A[-5..5,-5..5]` be an array of integers where each element takes 4 bytes in memory and the base address of the array is 1000. The starting address of `A[2,3]` (assuming that array is stored in Column Major order) is \_\_\_\_\_?

**Q.11** Let `A[-5..5,-10..10,-10..10]` be an array of integers where each element takes 4 bytes in memory and the base address of the array is 1000. The starting address of `A[0,0,0]` (assuming that array is stored in Column Major order) is \_\_\_\_\_?

**Q.12** Let `A[-5..5,-10..10,-10..10,-5..5]` be an array of integers where each element takes 2 bytes in memory and the base address of the array is 1000. The starting address of `A[1,1,1,1]` (assuming that array is stored in Column Major order) is \_\_\_\_\_?

**Q.13** In a compact single dimensional array representation for lower triangular matrices (i.e all the elements above the diagonal are zero) of size  $n \times n$ , non-zero elements, (i.e elements of lower triangle) of each row are stored one after another, starting from the first row, the index of the  $(i,j)$ th element of the lower triangular matrix in this new representation is: 1994 2 marks

- (A)  $i+j$
- (B)  $i+j-1$
- (C)  $(j-1)+i(i-1)/2$
- (D)  $i+j(j-1)/2$

**Q.14** A tridiagonal matrix `[-2..2,5..9]` is stored in row major order with base address 301. what is the address of data `[0][8]` if nonzero elements are stored?

**Q.15** If the address of `A[1][1]` and `A[2][1]` are 1000 and 1010 and each element occupies 2 bytes , then the array has been stored in \_\_\_\_\_ order .

- (A) Row Major order
- (B) Column major order
- (C) Matrix Order
- (D) Simple

**Q.16** Array `A[8,15]` is stored in row major order with first element `A[1,1]` stored as location `A0` . Assuming that each element of the array is of size  $s$  bytes , element `A[I,j]` can be accessed at :

- (A)  $A0 + 15s(i-1) + s(j-1)$
- (B)  $A0 + 8s(i-1) + s(j-1)$
- (C)  $A0 + 15s(j-1) + s(i-1)$
- (D)  $A0 + 8s(j-1) + s(i-1)$

**Q.17** The elements of the triangular array are stored as a vector in the

$$A[1,1], A[2,1], A[2,2], A[3,1], A[3,2], A[3,3], \dots, A[n,n]$$

Assuming that  $A[1,1]$  is stored at location 1, addressing function for  $A[i,j]$  is given by :

- (A)  $(i-1)/2 + j$
- (B)  $((i-1)*i)/2 + j$
- (C)  $(i*i) + j$
- (D)  $(i*j-1)/2$

**Q.18** Consider a 2D array  $A[40..95, 40..95]$  in lower triangular matrix representation. The size of each element in the array is 1 byte. If the array is implemented in the form of row major order and base address of the array is 1000, the address of  $A[66][50]$  will be \_\_\_\_\_

**Q.19** Consider the integer array  $A[1..100, 1..100]$  in which the elements are stored in Z representation. An example of a 5 X 5 array in Z representation is shown below :

A11	A12	A13	A14	A15
			A24	
		A33		
	A42			
A51	A52	A53	A54	A55

If the base address of A is starting from 1000 onwards, size of each element is 1 byte and A is stored in Row Major Order, then the address corresponding to  $A[100][55]$  is \_\_\_\_\_

**Q.20** In a lower triangular matrix (size  $15 \times 15$ ) representation of compact single dimension array, non zero elements (i.e. the elements of the lower triangle) of each row are stored one after another, starting from the first row. Assume each integer takes 1 Byte and the array is stored in row major order and first element of the array is stored at location 1000, then the address of the element  $a[10][6]$  is \_\_\_\_\_

**Q.21** An  $n \times n$  matrix A where n ranging from 1 to n is defined as follows :

$$A[i,j] = i \text{ if } (i=j)$$

$$A[i,j] = i*j - j*j \text{ if } (i < j)$$

$$A[i,j] = j*j - i*i \text{ if } (i > j)$$

The sum of elements of the array A is

- (A) 0
- (B)  $n^2$
- (C)  $n(n+1)/2$
- (D) None of these

**Q.22** Consider a 3D array  $A[90][30][40]$  stored in linear array in column major order. If the base address starts at 10, then the location of  $A[20][20][30]$  is \_\_\_\_\_.

Assume that the first element is stored at  $A[1][1][1]$  and each element takes 1 Byte of memory.

**Q.23** An  $n \times n$  array v is defined as follows:

**2000 1 marks**

$$v[i, j] = i-j \text{ for all } i, j, 1 \leq i \leq n, 1 \leq j \leq n$$

The sum of the elements of the array v is

- (A) 0
- (B)  $n - 1$
- (C)  $n^2 - 3n + 2$
- (D)  $n^2 (n+1)/2$

**Q.24** An  $n \times n$  array  $v$  is defined as follows:

$$v[i, j] = i + j \text{ for all } i, j, 1 \leq i \leq n, 1 \leq j \leq n$$

The sum of the elements of the array  $v$  is \_\_\_\_\_

**Q.25** An  $n \times n$  array  $v$  is defined as follows:

$$v[i, j] = i + j - 2 \text{ for all } i, j, 1 \leq i \leq n, 1 \leq j \leq n$$

The sum of the elements of the array  $v$  is \_\_\_\_\_

**Q.26** Suppose you are given an array  $s[1..n]$  and a procedure  $\text{reverse}(s, i, j)$  which reverses the order of elements in a between positions  $i$  and  $j$  (both inclusive). What does the following sequence do, where  $1 \leq k \leq n$ :

$\text{reverse}(s, 1, k)$  ;

$\text{reverse}(s, k + 1, n)$ ;

$\text{reverse}(s, 1, n)$ ;

[GATE 2000]

(A) Rotates  $s$  left by  $k$  positions

(B) Leaves  $s$  unchanged

(C) Reverses all elements of  $s$

(D) None of the above

### Self-Practice Questions :

**Q.1**  $A$  is an array of range  $[1 \dots 25, 1 \dots 25]$ . It is stored at starting location of 300 and the size of each element is 2 bytes. If RMO is considered for storing elements, then the address of  $A[13, 22]$  is : \_\_\_\_\_

**Q.2**  $A$  is an array of range  $[-5 \dots 5, -10 \dots 10, -10 \dots 10]$ . It is stored at starting location of 300 and the size of each element is 2 bytes. If RMO is considered for storing elements, then the address of  $A[0, 0, 0]$  is \_\_\_\_\_

**Q.3**  $A$  is an array of range  $[-5 \dots 5, -10 \dots 10, -10 \dots 10]$ . It is stored at starting location of 300 and the size of each element is 2 bytes. If CMO is considered for storing elements, then the address of  $A[0, 0, 0]$  is \_\_\_\_\_

**Q.4**  $A$  is an array of range  $[-5 \dots 5, -10 \dots 10]$ . It is stored at starting location of 300 and the size of each element is 2 bytes. If RMO is considered for storing elements, then the address of  $A[0, 0]$  is \_\_\_\_\_

**Q.5** If the TOEPLITZ matrix of order  $20 \times 20$  is stored in One Dimensional array, the optimum number of elements stored is : \_\_\_\_\_

**Q.6** A tridiagonal Matrix  $A[-4 \dots 1, 4 \dots 9]$  is stored in RMO with base address 100. The address of  $A[0][7]$  (if non zero elements are stored and the size of each element is 1 byte) is \_\_\_\_\_

**Q.7** Consider the integer array  $A[1..200, 1..200]$  in which the elements are stored in Z representation. An example of a  $5 \times 5$  array in Z representation is shown below :

A11	A12	A13	A14	A15
			A24	
		A33		
	A42			
A51	A52	A53	A54	A55

If the base address of  $A$  is starting from 1000 onwards, size of each element is 1 byte and  $A$  is stored in Row Major Order, then the address corresponding to  $A[100][23]$  is \_\_\_\_\_

- Q.8** In a lower triangular matrix  $A[1..20][1..20]$  representation of compact single dimension array ,non zero elements (i.e. the elements of the lower triangle) of each row are stored one after another ,starting from the first row .Assume each integer take 1 Byte and the array is stored in row major order and first element of the array is stored at location 1000, then the address of the element  $a[19][4]$  is \_\_\_\_\_
- Q.9** Consider a 3D array  $A[-25..25][-30..30][-40..40]$  of character where each element is of 1 byte is stored in linear array in column major order. If the base address starts at 1000,then the location of  $A[20][20][30]$  is \_\_\_\_\_.
- Q.10** In a upper triangular matrix  $A[1..20][1..20]$  representation of compact single dimension array ,non zero elements (i.e the elements of the lower triangle) of each row are stored one after another ,starting from the first row .Assume each integer take 1 Byte and the array is stored in row major order and first element of the array is stored at location 1000, then the address of the element  $a[10][10]$  is \_\_\_\_\_

### Answers

Classroom Practice Questions									
1	B	2	C	3	1600	4	1340	5	7092
6	107720	7	44972	8	A	9	B	10	1380
11	10700	12	63548	13	C	14	308	15	A
16	A	17	B	18	1361	19	1252	20	1061
21	26	22	23699	23	A	24	$n^2(n+1)$	25	$2n^3$
26	A								
Self-Practice Questions									
1	9042	2	5150	3	5150	4	530	5	39
6	111	7	1420	8	1174	9	221637	10	1144

G A T E



Since 2004

# 2

## LIST

### Classroom Practice Questions :

**Q.1** In a circular linked list organization, insertion of a record involves modification of

- (A) One pointer.
- (B) Two pointers.
- (C) Multiple pointers.
- (D) No pointer.

[1987 : 2 Marks]

**Q.2** What is the functionality of the following piece of code?

```
int fun(int x)
{
    node *temp=head;
    int y=0;
    while(temp!=NULL)
    {
        if(temp->data==x)
            return y;
        y++;
        temp=temp->next;
    }
    return -10000;
}
```

Assuming node template as :

```
struct node
{
    int data;
    struct node *next;
};
```

- (A) Find and delete a given element in the list if it exists otherwise returns -10000.
- (B) Find and return the given element in the list otherwise return -10000.
- (C) Find and return the position of the given element in the list otherwise return -10000.
- (D) Find and insert a new element in the list.



- Q.3** In the worst case, the number of comparisons needed to search a singly linked list of length  $n$  for a given element is
- (A)  $\log_2 n$
  - (B)  $n/2$
  - (C)  $\log_2 n - 1$
  - (D)  $n$
- [2002 : 1 Mark]**

- Q.4** Consider the function  $f$  defined below.

```
struct item
{
    int data;
    struct item * next;
};
int f(struct item *p)
{
    return ( (p == NULL) || (p->next == NULL) || ((P->data <= p-
    >next->data) && f(p->next))
    );
}
```

For a given linked list  $p$ , the function  $f$  returns 1 if and only if

- (A) The list is empty or exactly one element.
  - (B) the elements in the list are sorted in non-decreasing order of data value
  - (C) the elements in the list are sorted in non-increasing order of data value
  - (D) not all elements in the list have the same data value.
- [2003 : 2 Marks]**

- Q.5** What does the following function do for a given Linked List with first node as *head*?

```
void fun1(struct node* head)
{
    if(head == NULL)
        return;
    fun1(head->next);
    printf("%d ", head->data);
}
```

- (A) Prints all nodes of linked lists
  - (B) Prints all nodes of linked list in reverse order
  - (C) Prints alternate nodes of Linked List
  - (D) Prints alternate nodes in reverse order
- Q.6** Which of the following points is/are true about Linked List data structure when it is compared with array
- (A) Arrays have better cache locality that can make them better in terms of performance.
  - (B) It is easy to insert and delete elements in Linked List
  - (C) Random access is not allowed in a typical implementation of Linked Lists
  - (D) The size of array has to be pre-decided, linked lists can change their size any time.
  - (E) All of the above

- Q.7** Consider the following function that takes reference to head of a Doubly Linked List as parameter. Assume that a node of doubly linked list has previous pointer as *prev* and next pointer as *next*.

```
void fun(struct node **head_ref)
{
    struct node *temp = NULL;
    struct node *current = *head_ref;
    while (current != NULL)
    {
        temp = current->prev;
        current->prev = current->next;
        current->next = temp;
        current = current->prev;
    }
    if(temp != NULL )
        *head_ref = temp->prev;
}
```

Assume that reference of head of following doubly linked list is passed to above function 1 2 3 4 5 6. What should be the modified linked list after the function call?

- (A) 2 1 4 3 6 5
- (B) 5 4 3 2 1 6.
- (C) 6 5 4 3 2 1.
- (D) 6 5 4 3 1 2

- Q.8** The following function reverse() is supposed to reverse a singly linked list. There is one line missing at the end of the function.

```
struct node
{
    int data;
    struct node* next;
};
/* head_ref is a double pointer which points to head (or start) pointer of linked list */
static void reverse(struct node** head_ref)
{
    struct node* prev = NULL;
    struct node* current = *head_ref;
    struct node* next;
    while (current != NULL)
    {
        next = current->next;
        current->next = prev;
        prev = current;
        current = next;
    }
}
```

/\*ADD A STATEMENT HERE\*/

}

What should be added in place of “/\*ADD A STATEMENT HERE\*/”, so that the function correctly reverses a linked list.

- (A) \*head\_ref = prev;
- (B) \*head\_ref = current;
- (C) \*head\_ref = next;
- (D) \*head\_ref = NULL;

**Q.9** Which among the following segment of code counts the number of elements in a linked list, if it is assumed that ptr is pointing to the first node and cnt is the variable which counts the number of elements in the list ?

- (A) for(count=1;ptr!=NULL;count++) ptr=ptr->next
- (B) for(count=0;ptr->next!=NULL;count++) ptr=ptr->next
- (C) for(count=1;ptr->next!=NULL;count++) ptr=ptr->next
- (D) for(count=0;ptr!=NULL;count++) ptr=ptr->next

**Q.10** In a circular linked list

- (A) Components are linked together in some sequential manner.
- (B) There is no beginning and no end.
- (C) Components are arranged hierarchically.
- (D) Forward and backward traversal within the list is permitted.

**Q.11** Given pointer to a node X in a singly linked list. Only one pointer is given, pointer to head node is not given, can we delete the node X from given linked list?

- (A) Possible if X is not last node. Use following two steps (a) Copy the data of next of X to X. (b) Delete next of X.
- (B) Possible if size of linked list is even.
- (C) Possible if size of linked list is odd
- (D) Possible if X is not first node. Use following two steps (a) Copy the data of next of X to X. (b) Delete next of X.

**Q.12** You are given pointers to first and last nodes of a singly linked list, which of the following operations are dependent on the length of the linked list?

- (A) Delete the first element
- (B) Insert a new element as a first element
- (C) Delete the last element of the list
- (D) Add a new element at the end of the list

**Q.13** In a doubly linked list, the number of pointers affected for an insertion operation will be :

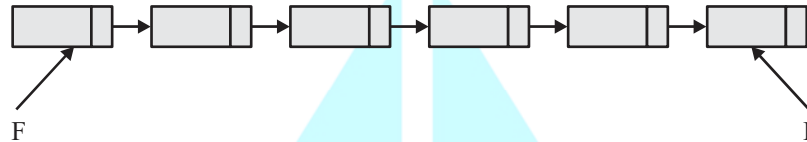
[ISRO CS 2017]

- (A) 4
- (B) 0
- (C) 1
- (D) None of these

**Q.14** Consider an implementation of unsorted single linked list. Suppose it has its representation with a head and a tail pointer (i.e. pointers to the first and last nodes of the linked list). Given the representation, which of the following operation can not be implemented in  $O(1)$  time ?

- (A) Insertion at the front of the linked list.
- (B) Insertion at the end of the linked list.
- (C) Deletion of the front node of the linked list.
- (D) Deletion of the last node of the linked list.

**Q.15** Consider a single linked list where F and L are pointers to the first and last elements respectively of the linked list. The time for performing which of the given operations depends on the length of the linked list? [ISRO CS 2018]



- (A) Delete the first element of the list
- (B) Interchange the first two elements of the list
- (C) Delete the last element of the list
- (D) Add an element at the end of the list

**Q.16** Which of the following operations is performed more efficiently by doubly linked list than by linear linked list? [ISRO CS 2008]

- (A) Deleting a node whose location is given
- (B) Searching an unsorted list for a given item
- (C) Inserting a node after the node with a given location
- (D) Traversing the list to process each node

**Q.17** The following C function takes a singly-linked list of integers as a parameter and rearranges the elements of the list. The list is represented as pointer to a structure. The function is called with the list containing the integers 1, 2, 3, 4, 5, 6, 7 in the given order. What will be the contents of the list after the function completes execution? [GATE 2005]

```
struct node
{
    int value;
    struct node *next;
};

void rearrange (struct node *list)
{
    struct node *p, *q;
    int temp;
    if (!list || !list -> next)
        return;
    p = list;
    q = list -> next;
    while (q)
```

```

{
    temp = p -> value;
    p -> value = q -> value;
    q -> value = temp;
    p = q -> next;
    q = p ? p -> next : 0;
}
}

```

- (A) 1, 2, 3, 4, 5, 6, 7  
 (B) 2, 1, 4, 3, 6, 5, 7  
 (C) 1, 3, 2, 5, 4, 7, 6  
 (D) 2, 3, 4, 5, 6, 7, 1

**Q.18** A singly linked list is declared as follows :

```

struct list
{
    struct list* next;
    int data;
};

```

Where next represents links to adjacent elements of the list. Which of the following code segments deletes the element pointed by ptr from linked list, if it is assumed that ptr points to neither the first node nor the last element of the list ?

prev pointer points to previous node

- (A) prev->next = ptr->next ;free(ptr) ;  
 (B) ptr->next =prev->next ;free(ptr);  
 (C) prev->next =ptr->next ;free(ptr);  
 (D) ptr->next=prev->next ;free(prev);

**Q.19** The following C function takes a simply-linked list as input argument. It modifies the list by moving the last element to the front of the list and returns the modified list. Some part of the code is left blank.

[GATE 2010]

```

typedef struct node

```

```

{
    int value;
    struct node *next;

```

```

}Node;

```

```

Node *move_to_front(Node *head)

```

```

{
    Node *p, *q;
    if ((head == NULL || (head->next == NULL))
        return head;
    q = NULL; p = head;
    while (p-> next !=NULL)

```

```

{
    q = p;
    p = p->next;
}

```

---

```

return head;

```

```

}

```

Choose the correct alternative to replace the blank line.

- (A) `q = NULL; p->next = head; head = p;`
- (B) `q->next = NULL; head = p; p->next = head;`
- (C) `head = p; p->next = q; q->next = NULL;`
- (D) `q->next = NULL; p->next = head; head = p;`

**Q.20** Consider the C code fragment given below.

```

typedef struct node
{
    int data;
    node* next ;
} node;
void join(node* m, node* n)
{
    node* p = n;
    while (p->next != NULL)
    {
        p = p->next;
    }
    p->next = m;
}

```

Assuming that `m` and `n` point to valid NULL-terminated linked lists, invocation of `join` will

- (A) append list `m` to the end of list `n` for all inputs
- (B) either cause a null pointer dereference or append list `m` to the end of list `n`
- (C) cause a null pointer dereference for all inputs.
- (D) append list `n` to the end of list `m` for all inputs.

[GATE 2017]

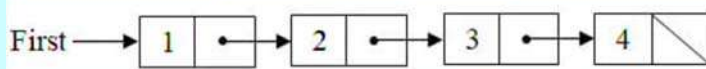
### Self-Practice Questions :

**Q.1** What is the run-time complexity of inserting a new element at the beginning of a circular, doubly-linked list with a sentinel head?

- (A)  $O(1)$
- (B)  $O(\log N)$
- (C)  $O(N)$
- (D)  $O(N^2)$



- Q.2** Given a circular, doubly-linked list whose contents are sorted in ascending order, what is the run-time complexity for inserting a new element into the list so that it remains correctly sorted? (Including the time required to search for the element's correct position.)  
 (A)  $O(1)$  (B)  $O(\log N)$   
 (C)  $O(N)$  (D)  $O(N^2)$
- Q.3** What is the best data structure to solve the following problem? A list needs to be built dynamically. Data must be easy to find, preferably in  $O(1)$ . The user does not care about any order statistics such as finding max or min or median.  
 (A) Use an Array (B) Use a Singly LL  
 (C) Use a Stack (D) Use a Queue
- Q.4** Which of the following application makes use of a circular linked list?  
 (A) Undo operation in a text editor (B) Recursive function calls  
 (C) Allocating CPU to resources (D) All of the mentioned
- Q.5** Let P be a singly linked list. Let Q be the pointer to an intermediate node x in the list. What is the worst-case time complexity of the best known algorithm to delete the node x from the list?  
 (A)  $O(n)$  (B)  $O(\log_2 n)$   
 (C)  $O(\log n)$  (D)  $O(1)$
- Q.6** The concatenation of two lists is to be performed in  $O(1)$  time. Which of the following implementations of a list should be used?  
 (A) singly linked list (B) doubly linked list  
 (C) circular doubly linked list (D) array implementation of lists
- Q.7** Here is the code for this single linked list:

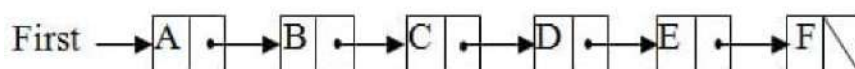


```
void changelist (struct node * x)
```

```
{
    struct node *p, *q;
    p = x → next;
    q = p → next;
    p → next = q → next;
    x → next = q;
    q → next = p;
}
```

What is the output displayed if you traverse the list after invoking changelist(first)?

- (A) 4 3 2 1 (B) 4 2 3 1  
 (C) 13 2 4 (D) 2 1 3 4
- Q.8**

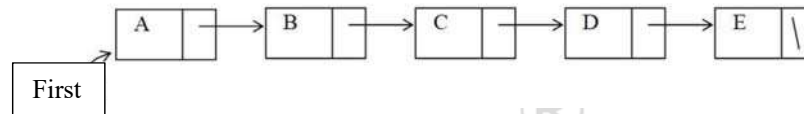


What is the output after the following sequence of steps?

```
struct Node * P ;
```

- (i)  $P = \text{first} \rightarrow \text{link} \rightarrow \text{link} \rightarrow \text{link} \rightarrow \text{link};$
- (ii)  $P \rightarrow \text{link} \rightarrow \text{link} = \text{first};$
- (iii)  $\text{first} \rightarrow \text{link} \rightarrow \text{link} \rightarrow \text{link} = P \rightarrow \text{link};$
- (iv)  $\text{printf}("%C", \text{first} \rightarrow \text{link} \rightarrow \text{link} \rightarrow \text{link} \rightarrow \text{link} \rightarrow \text{link} \rightarrow \text{data});$
- (A) A (B) B
- (C) C (D) D

**Q.9** What is the output after the following steps executed?



- (i)  $P = \text{First} \rightarrow \text{Link} \rightarrow \text{Link}$
- (ii)  $P \rightarrow \text{Link} \rightarrow \text{Link} \rightarrow \text{Link} = \text{first}$
- (iii)  $P = P \rightarrow \text{Link} \rightarrow \text{Link} \rightarrow \text{LINK}$
- (iv)  $\text{printf}("%C", P \rightarrow \text{data});$
- (A) A (B) B
- (C) C (D) D

**Q.10** What does the following function do for a given Linked List with first node as head ?

```
void fun1 (struct node* head)
{
    if(head == NULL)
        return;

    fun1(head->next);
    printf("%d head->data);
}
```

- (A) Prints all nodes of linked lists
- (B) Prints all nodes of linked list in reverse order
- (C) Prints alternate nodes of Linked List
- (D) Prints alternate nodes in reverse order

### Answers

#### Classroom Practice Questions

1	B	2	C	3	B	4	B	5	E
6	E	7	C	8	A	9	C	10	B
11	A	12	C	13	D	14	D	15	C
16	A	17	B	18	A	19	D	20	B

#### Self-Practice Questions

1	A	2	C	3	A	4	C	5	D
6	C	7	C	8	B	9	C	10	D





# 3

## STACKS AND QUEUES

### Classroom Practice Questions :

- Q.1** The following operations are performed on a stack: push(10), push(20), pop, push(10), push(20), pop, pop, pop, push(20), pop. The sequence of values popped out is  
(A) 20, 10, 20, 10, 20  
(B) 20, 20, 10, 10, 20  
(C) 10, 20, 20, 10, 20  
(D) 20, 20, 10, 20, 10  
[1991 : 2 Marks]
- Q.2** Which of the following permutations can be obtained in the output (in the same order) using a stack assuming that the input is the sequence 1, 2, 3, 4, 5 in that order?  
(A) 3, 4, 5, 1, 2  
(B) 3, 4, 5, 2, 1  
(C) 1, 5, 2, 3, 4  
(D) 5, 4, 3, 1, 2  
[1994 : 2Marks]
- Q.3** A program attempts to generate as many permutations as possible of the string, 'abcd' by pushing the characters a, b, c, d in the same order onto a stack, but it may pop off the top character at any time. Which one of the following strings CANNOT be generated using this program?  
(A) abcd  
(B) dcba  
(C) cbad  
(D) cabd  
[2004 : 2Marks]
- Q.4** The postfix expression for the infix expression  $A+B*(C+D)/F+D*E$  is:  
(A)  $AB+CD+*F/D+E*$   
(B)  $ABCD+*F/+DE*+$   
(C)  $A*B+CD/F*DE++$   
(D)  $A+*BCD/F*DE++$   
[1995 : 2Marks]
- Q.5** Which of the following is essential for converting an infix expression to the postfix form efficiently?  
(A) An operator stack  
(B) An operand stack  
(C) An operand stack and an operator stack  
(D) A parse tree  
[1997 : 1Mark]

- Q.6** Assume that the operators  $+$ ,  $-$ ,  $\times$  are left associative and  $^$  is right associative. The order of precedence (from highest to lowest) is  $^$ ,  $\times$ ,  $+$ ,  $-$ . The postfix expression corresponding to the infix expression  $a + b \times c - d \wedge e \wedge f$  is
- (A)  $abc \times + def \wedge \wedge -$   
 (B)  $abc \times + de \wedge f \wedge -$   
 (C)  $ab + c \times d - e \wedge f \wedge$   
 (D)  $- + a \times bc \wedge \wedge def$
- [2004 : 2Marks]

- Q.7** Compute the post fix equivalent of the following expression

$$3 * \log(x+1) - a/2$$

[1998 : 2Marks]

- Q.8** The following postfix expression with single digit operands is evaluated using a stack:

$$8 \ 2 \ 3 \wedge / 2 \ 3 \ * + 5 \ 1 \ * -$$

Note that  $^$  is the exponentiation operator. The top two elements of the stack after the first  $*$  is evaluated are :

- (A) 6, 1  
 (B) 5, 7  
 (C) 3, 2  
 (D) 1, 5
- [2007 : 2Marks]

- Q.9** The result evaluating the postfix expression  $10 \ 5 + 60 \ 6 / * 8 -$  is

- (A) 284  
 (B) 213  
 (C) 142  
 (D) 71

[2015 : 1Mark]

- Q.10** The attributes of three arithmetic operators in some programming language are given below.

[2016 : 2Marks]

Operator	Precedence	Associativity	Arity
$+$	High	Left	Binary
$-$	Medium	Right	Binary
$*$	Low	Left	Binary

The value of the expression  $2 - 5 + 1 - 7 * 3$  in this language is \_\_\_\_\_

- Q.11** A single array  $A[1..MAXSIZE]$  is used to implement two stacks. The two stacks grow from opposite ends of the array. Variables  $top1$  and  $top2$  ( $top1 < top2$ ) point to the location of the topmost element in each of the stacks. If the space is to be used efficiently, the condition for —stack full is
- (A)  $(top1 = MAXSIZE/2 \text{ and } top2 = MAXSIZE/2 + 1)$   
 (B)  $top1 + top2 = MAXSIZE$   
 (C)  $(top1 = MAXSIZE/2) \text{ or } (top2 = MAXSIZE)$   
 (D)  $top1 = top2 - 1$
- [2004 : 1Mark]

- Q.12** The best data structure to check whether an arithmetic expression has balanced parentheses is a

- (A) queue  
 (B) stack  
 (C) tree  
 (D) list

[2004 : 1Mark]

**Q.13** Consider the following statements:

- First-in-first out types of computations are efficiently supported by STACKS.
- Implementing LISTS on linked lists is more efficient than implementing LISTS on an array for almost all the basic LIST operations.
- Implementing QUEUES on a circular array is more efficient than implementing QUEUES on a linear array with two indices.
- Last-in-first-out type of computations are efficiently supported by QUEUES.

Which of the following is correct?

- (ii) and (iii) are true
- (i) and (ii) are true
- (iii) and (iv) are true
- (ii) and (iv) are true

[1996 : 1Mark]

**Q.14** A priority queue Q is used to implement a stack S that stores characters. PUSH(C) is implemented as INSERT(Q, C, K) where K is an appropriate integer key chosen by the implementation. POP is implemented as DELETETMIN(Q). For a sequence of operations, the keys chosen are in

- Non-increasing order
- Non-decreasing order
- Strictly increasing order
- Strictly decreasing order

[1997 : 2Marks]

**Q.15** What is the minimum number of stacks of size n required to implement a queue of size n?

[2001 : 2 marks]

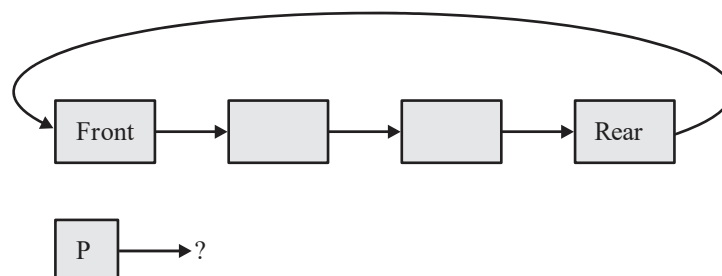
- One
- Two
- Three
- Four

**Q.16** Let S be a stack of size  $n \geq 1$ . Starting with the empty stack, suppose we push the first n natural numbers in sequence, and then perform n pop operations. Assume that Push and pop operation take X seconds each, and Y seconds elapse between the end of one such stack operation and the start of the next operation. For  $m \geq 1$ , define the stack-life of m as the time elapsed from the end of Push(m) to the start of the pop operation that removes m from S. The average stack-life of an element of this stack is

- $n(X + Y)$
- $3Y + 2X$
- $n(X + Y) - X$
- $Y + 2X$

[2003 : 2Marks]

**Q.17** A circularly linked list is used to represent a Queue. A single variable p is used to access the Queue. To which node should p point such that both the operations enqueue and dequeue can be performed in constant time?



- (A) rear node
- (B) front node
- (C) not possible with a single pointer
- (D) node next to front

[2004 : 2Marks]

**Q.18** A function  $f$  defined on stacks of integers satisfies the following properties.  $f(\emptyset) = 0$  and  $f(\text{push}(S, i)) = \max(f(S), 0) + i$  for all stacks  $S$  and integers  $i$ . If a stack  $S$  contains the integers 2, -3, 2, -1, 2 in order from bottom to top, what is  $f(S)$ ?

[2005 : 1Mark]

- (A) 6
- (B) 4
- (C) 3
- (D) 2

**Q.19** Consider the following C program:

[2007 : 2Marks]

```
#include
#define EOF -1
void push (int); /* push the argument on the stack */
int pop (void); /* pop the top of the stack */
void flagError ();
int main ()
{
    int c, m, n, r;
    while ((c = getchar ()) != EOF)
    {
        if (isdigit (c) )
            push (c);
        else if ((c == '+') || (c == '*'))
        {
            m = pop ();
            n = pop ();
            r = (c == '+') ? n + m : n*m;
            push (r);
        }
        else if (c != ' ')
            flagError ();
    }
    printf("%d", pop ());
}
```

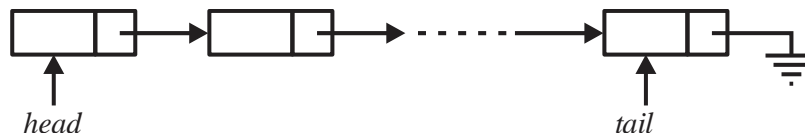
What is the output of the program for the following input ? 5 2 \* 3 3 2 + \* +

- (A) 15
- (B) 25
- (C) 30
- (D) 150

- Q.20** Suppose a circular queue of capacity  $(n - 1)$  elements is implemented with an array of  $n$  elements. Assume that the insertion and deletion operation are carried out using REAR and FRONT as array index variables, respectively. Initially, REAR = FRONT = 0. The conditions to detect queue full and queue empty are
- (A) Full:  $(\text{REAR}+1) \bmod n == \text{FRONT}$ , empty:  $\text{REAR} == \text{FRONT}$   
 (B) Full:  $(\text{REAR}+1) \bmod n == \text{FRONT}$ , empty:  $(\text{FRONT}+1) \bmod n == \text{REAR}$   
 (C) Full:  $\text{REAR} == \text{FRONT}$ , empty:  $(\text{REAR}+1) \bmod n == \text{FRONT}$   
 (D) Full:  $(\text{FRONT}+1) \bmod n == \text{REAR}$ , empty:  $\text{REAR} == \text{FRONT}$  [2012 : 2Marks]
- Q.21** Suppose a stack implementation supports an instruction REVERSE, which reverses the order of elements on the stack, in addition to the PUSH and POP instructions. Which one of the following statements is TRUE with respect to this modified stack?
- (A) A queue cannot be implemented using this stack  
 (B) A queue can be implemented where ENQUEUE takes a single instruction and DEQUEUE takes a sequence of two instructions  
 (C) A queue can be implemented where ENQUEUE takes a sequence of three instructions and DEQUEUE takes a single instruction  
 (D) A queue can be implemented where both ENQUEUE and DEQUEUE take a single instruction each [2014 : 2Marks]
- Q.22** Let  $Q$  denote a queue containing sixteen numbers and  $S$  be an empty stack.  $\text{Head}(Q)$  returns the element at the head of the queue  $Q$  **without** removing it from  $Q$ . Similarly  $\text{Top}(S)$  returns the element at the top of  $S$  **without** removing it from  $S$ . Consider the algorithm given below.
- ```

while  $Q$  is not Empty do
    if  $S$  is Empty OR  $\text{Top}(S) \leq \text{Head}(Q)$  then
         $x := \text{Dequeue}(Q)$ ;
        Push( $S, x$ );
    else
         $x := \text{Pop}(S)$ ;
        Enqueue( $Q, x$ );
    end
end
  
```
- The maximum possible number of iterations of the while loop in the algorithm is \_\_\_\_\_ [2016 : 2Marks]
- Q.23** A Circular queue has been implemented using singly linked list where each node consists of a value and a pointer to next node. We maintain exactly two pointers **FRONT** and **REAR** pointing to the front node and rear node of queue. Which of the following statements is/are correct for circular queue so that insertion and deletion operations can be performed in  $O(1)$  time?
- I. Next pointer of front node points to the rear node.  
 II. Next pointer of rear node points to the front node.
- (A) I only  
 (B) II only  
 (C) Both I and II  
 (D) Neither I nor II [2017 : 1Mark]

- Q.24** A queue is implemented using a non-circular singly linked list. The queue has a head pointer and a tail pointer, as shown in the figure. Let  $n$  denote the number of nodes in the queue. Let `_enqueue` be implemented by inserting a new node at the head, and `_dequeue` be implemented by deletion of a node from the tail.



Which one of the following is the time complexity of the most time-efficient implementation of 'enqueue' and 'dequeue', respectively, for this data structure?

- (A)  $\Theta(1)$ ,  $\Theta(1)$   
 (B)  $\Theta(1)$ ,  $\Theta(n)$   
 (C)  $\Theta(n)$ ,  $\Theta(1)$   
 (D)  $\Theta(n)$ ,  $\Theta(n)$

[2018 : 1 Mark]

- Q.25** Suppose you are given an implementation of a queue of integers. The operations that can be performed on the queue are:

- I. `isEmpty (Q)` — returns true if the queue is empty, false otherwise.
- II. `delete (Q)` — deletes the element at the front of the queue and returns its value.
- III. `insert (Q, i)` — inserts the integer  $i$  at the rear of the queue.

Consider the following function:

```
void f(queue Q)
```

```
{
    int i;
    if (!isEmpty(Q))
    {
        i = delete(Q);
        f(Q);
        insert(Q, i);
    }
}
```

What operation is performed by the above function  $f$ ?

- (A) Leaves the queue  $Q$  unchanged  
 (B) Reverses the order of the elements in the queue  $Q$   
 (C) Deletes the element at the front of the queue  $Q$  and inserts it at the rear keeping the other elements in the same order  
 (D) Empties the queue  $Q$

[2007 : 2Marks]



**Self-Practice Questions :**

- Q.1** Let  $n$  insert and  $m$  ( $\leq n$ ) delete operations be performed in an arbitrary order on an empty queue  $Q$ . Let  $x$  and  $y$  be the number of push and pop operations performed respectively in the process. Which one of the following is true for all  $m$  and  $n$ ?
- (A)  $n+m \leq x < 2n$  and  $2m \leq y \leq n+m$  (B)  $n+m \leq x < 2n$  and  $2m \leq y \leq 2n$   
 (C)  $2m \leq x < 2n$  and  $2m \leq y \leq n+m$  (D)  $2m \leq x < 2n$  and  $2m \leq y \leq 2n$
- Q.2** Which of the following option is not correct?
- (A) If the queue is implemented with a linked list, keeping track of a front pointer, Only rear pointer  $s$  will change during an insertion into a non-empty queue.  
 (B) Queue data structure can be used to implement least recently used (LRU) page fault algorithm and Quick short algorithm.  
 (C) Queue data structure can be used to implement Quick short algorithm but not least recently used (LRU) page fault algorithm.  
 (D) Both (A) and (C)
- Q.3** Consider a standard Circular Queue 'q' implementation (which has the same condition for Queue Full and Queue Empty) whose size is 11 and the elements of the queue are  $q[0], q[1], q[2], \dots, q[10]$ . The front and rear pointers are initialized to point at  $q[2]$ . In which position will the ninth element be added?
- (A)  $q[0]$  (B)  $q[1]$  (C)  $q[9]$  (D)  $q[10]$
- Q.4** Which one of the following is an application of Queue Data Structure?
- (A) When a resource is shared among multiple consumers.  
 (B) When data is transferred asynchronously (data not necessarily received at same rate as sent) between two processes  
 (C) Load Balancing  
 (D) All of the above
- Q.5** A priority queue can efficiently implemented using which of the following data structures? Assume that the number of insert and peek (operation to see the current highest priority item) and extraction (remove the highest priority item) operations are almost same.
- (A) Array  
 (B) Linked List  
 (C) Heap Data Structures like Binary Heap, Fibonacci Heap  
 (D) None of the above
- Q.6** A stack permutation is defined as the permutation that can be obtained by removing elements from a stack with given order of insertion for ex- if the order of insertion is 1,2,3 then possible permutations are :
- (i) 1,2,3  
 (ii) 1,3,2  
 (iii) 2,1,3  
 (iv) 2,3,1  
 (v) 3,2,1
- The number of stack permutations possible for  $n$  elements is :
- (A)  $n!$  (B)  $2n-1$  (C)  $C(2n,n)/(n+1)$  (D)  $2n + 3$

**Q.7** Let X be the result when the postfix expression below is evaluated :

8 4 3 + - 2 4 2 / + \* 3 ^ 2 +

Let Y be the result of the following expression :

2 X \* 7 -

Then the value of  $Y^{1/3}$  is \_\_\_\_\_

**Q.8** Consider the following code

```
Void Fun(char * x)
{
    if( (*x)!='\0')
    {
        Fun(x+1);
        Fun(x+1);
        Printf("%c",*x);
    }
}

void main()
{
    Fun("RAM");
}
```

The output is :

(A) MMAMMAR      (B) MARRAM      (C) RAMMRAM      (D) RAMMAR

**Q.9** An arithmetic tree is generated for the expression  $(-a + (b * c)) / (-d)$ . Which operator is placed at the root node ?

(A) +      (B) -      (C) \*      (D) /

**Q.10** Consider the following pseudocode on stack1 and stack2, initially stack1 contains 4 elements and stack2 contains no elements.

Algo stackfun

while(stack1.pop(data))

stack2.push(data)

while(stack2.pop(data))

stack1.push(data)

(A) stack1 remains unchanged      (B) Reversing the contents of stack1  
(C) stack1 becomes empty      (D) None of these

**Q.11** Which one of the following is an application of Queue Data Structure?

(A) When a resource is shared among multiple consumers.  
(B) When data is transferred asynchronously (data not necessarily received at same rate as sent) between two processes  
(C) Load Balancing  
(D) All of the above



- Q.12** In linked list implementation of queue, if only front pointer is maintained, which of the following operation take worst case linear time?
- (A) Insertion (B) Deletion  
(C) To empty a queue (D) Both Insertion and to empty a queue
- Q.13** If the MAX\_SIZE is the size of the array used in the implementation of circular queue. How is rear manipulated while inserting an element in the queue?
- (A)  $\text{rear} = (\text{rear} \% 1) + \text{MAX\_SIZE}$  (B)  $\text{rear} = \text{rear} \% (\text{MAX\_SIZE} + 1)$   
(C)  $\text{rear} = (\text{rear} + 1) \% \text{MAX\_SIZE}$  (D)  $\text{rear} = \text{rear} + (1 \% \text{MAX\_SIZE})$

### Answers

| Classroom Practice Questions |   |    |     |    |   |    |   |    |   |
|------------------------------|---|----|-----|----|---|----|---|----|---|
| 1                            | B | 2  | B   | 3  | D | 4  | B | 5  | A |
| 6                            | A | 7  | *   | 8  | A | 9  | C | 10 | 9 |
| 11                           | D | 12 | B   | 13 | A | 14 | D | 15 | B |
| 16                           | C | 17 | A   | 18 | C | 19 | B | 20 | A |
| 21                           | C | 22 | 256 | 23 | B | 24 | B | 25 | B |
| Self-Practice Questions      |   |    |     |    |   |    |   |    |   |
| 1                            | A | 2  | C   | 3  | A | 4  | D | 5  | C |
| 6                            | C | 7  | 5   | 8  | A | 9  | D | 10 | D |
| 11                           | D | 12 | D   | 13 | C |    |   |    |   |

### Classroom Practice Questions

7.  $3x + \log^2 a$  / -



G A T E

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Since 2004

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# 4

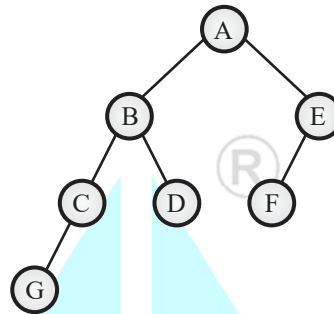
# TREE



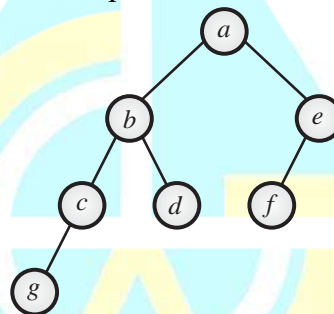
## Classroom Practice Questions :

- Q.1** A complete  $n$ -ary tree is one in which every node has 0 or  $n$  sons. If  $x$  is the number of internal nodes of a complete  $n$ -ary tree, the number of leaves in it is given by  
(A)  $x(n-1)+1$   
(B)  $xn-1$   
(C)  $xn+1$   
(D)  $x(n+1)$  [1998 : 2 Marks]
- Q.2** The number of leaf nodes in a rooted tree of  $n$  nodes, with each node having 0 or 3 children is:  
(A)  $n/2$   
(B)  $(n-1)/3$   
(C)  $(n-1)/2$   
(D)  $\lceil (2n+1)/3 \rceil$  [2002 : 2 Marks]
- Q.3** In a complete  $k$ -ary tree, every internal node has exactly  $k$  children or no child. The number of leaves in such a tree with  $n$  internal nodes is:  
(A)  $nk$   
(B)  $(n-1)k+1$   
(C)  $n(k-1)+1$   
(D)  $n(k-1)$  [2005 : 2 Marks]
- Q.4** A complete  $n$ -ary tree is a tree in which each node has  $n$  children or no children. Let  $I$  be the number of internal nodes and  $L$  be the number of leaves in a complete  $n$ -ary tree. If  $L = 41$ , and  $I = 10$ , what is the value of  $n$ ?  
(A) 3  
(B) 4  
(C) 5  
(D) 6 [2007 : 2 Marks]
- Q.5** In a binary tree, the number of internal nodes of degree 1 is 5, and the number of internal nodes of degree 2 is 10. The number of leaf nodes in the binary tree is  
(A) 10  
(B) 11  
(C) 12  
(D) 15 [2006 : 1 Mark]

- Q.6** State whether the following statements are TRUE or FALSE: it is possible to construct a binary tree uniquely whose pre-order and post-order traversals are given? [1987 : 1 Mark]
- Q.7** Construct a binary tree whose preorder traversal is K L N M P R Q S T and inorder traversal is N L K P R M S Q T [1987 : 1 mark]
- Q.8** If the binary tree in figure is traversed in inorder, then the order in which the nodes will be visited is [1991 : 2 marks]



- Q.9** Which of the following sequences denotes the post order traversal sequence of the given tree?



- (A) f e g c d b a  
 (B) g c b d a f e  
 (C) g c d b f e a  
 (D) f e d g c b a

[1996 : 1 Mark]

- Q.10** Consider the label sequences obtained by the following pairs of traversal on labeled binary tree. Which of these pairs identify a tree uniquely?

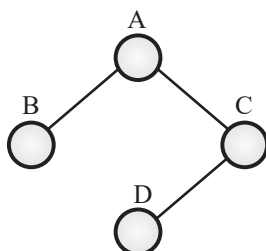
- (i) preorder and postorder  
 (ii) inorder and postorder  
 (iii) preorder and inorder  
 (iv) level order and postorder

- (A) (i) only  
 (B) (ii) and (iii)  
 (C) (iii) only  
 (D) (iv) only

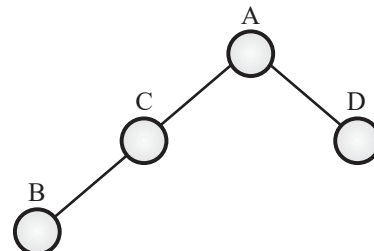
[2004 : 1 Mark]

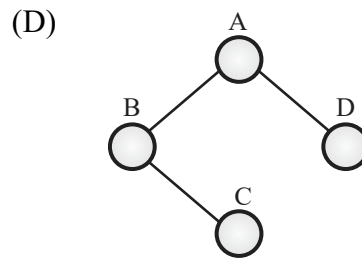
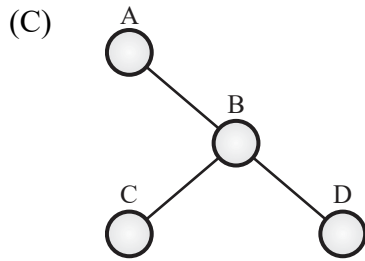
- Q.11** Which one of the following binary trees has its inorder and preorder traversals as BCAD and ABCD, respectively? BCAD ABCD [2004 : 2 Marks]

(A)



(B)





**Q.12** The inorder and preorder traversal of a binary tree are d b e a f c g and a b d e c f g, respectively. The postorder traversal of the binary tree is:

- (A) d e b f g c a
- (B) e d b g f c a
- (C) e d b f g c a
- (D) d e f g b c a

[2007 : 2 Marks]

**Q.13** Let LASTPOST, LASTIN and LASTPRE denote the last vertex visited in a postorder, inorder and preorder traversal, respectively, of a complete binary tree. Which of the following is always true?

- (A) LASTIN = LASTPOST
- (B) LASTIN = LASTPRE
- (C) LASTPRE = LASTPOST
- (D) None of the above

[2000 : 2 Marks]

**Q.14** The following three are known to be the preorder, inorder and postorder sequences of a binary tree. But it is not known which is which.

- I. MBCAFHPYK
- II. KAMCBYPFH
- III. MABCKYFPH

Pick the true statement from the following.

- (A) I and II are preorder and inorder sequences, respectively
- (B) I and III are preorder and postorder sequences, respectively
- (C) II is the inorder sequence, but nothing more can be said about the other two sequences
- (D) II and III are the preorder and inorder sequences, respectively

[2008 : 2 Marks]

**Q.15** Consider the following New-order strategy for traversing a binary tree:

- Visit the root
- Visit the right subtree using New-order
- Visit the left subtree using New-order

The New-order traversal of the expression tree corresponding to the reverse polish expression  $3\ 4\ *\ 5\ -\ 2\ ^\ 6\ 7\ *\ 1\ +\ -$  is given by:

- (A)  $+ - 1\ 6\ 7\ *\ 2\ ^\ 5 - 3\ 4\ *$
- (B)  $- + 1\ *\ 6\ 7\ ^\ 2 - 5\ *\ 3\ 4$
- (C)  $- + 1\ *\ 7\ 6\ ^\ 2 - 5\ *\ 4\ 3$
- (D)  $1\ 7\ 6\ *\ + 2\ 5\ 4\ 3\ *\ -\ ^ -$

[2016 : 2Marks]

- Q.16** A binary search tree contains the numbers 1, 2, 3, 4, 5, 6, 7, 8. When the tree is traversed in pre-order and the values in each node printed out, the sequence of values obtained is 5, 3, 1, 2, 4, 6, 8, 7. If the tree is traversed in post-order, the sequence obtained would be
- (A) 8, 7, 6, 5, 4, 3, 2, 1  
(B) 1, 2, 3, 4, 8, 7, 6, 5  
(C) 2, 1, 4, 3, 6, 7, 8, 5  
(D) 2, 1, 4, 3, 7, 8, 6, 5
- [2005 : 2 Marks]
- Q.17** Postorder traversal of a given binary search tree, T produces the following sequence of keys 10, 9, 23, 22, 27, 25, 15, 50, 95, 60, 40, 29 Which one of the following sequences of keys can be the result of an in-order traversal of the tree T?
- (A) 9, 10, 15, 22, 23, 25, 27, 29, 40, 50, 60, 95  
(B) 9, 10, 15, 22, 40, 50, 60, 95, 23, 25, 27, 29  
(C) 29, 15, 9, 10, 25, 22, 23, 27, 40, 60, 50, 95  
(D) 95, 50, 60, 40, 27, 23, 22, 25, 10, 9, 15, 29
- [2005 : 2 Marks]
- Q.18** The numbers 1, 2, ..., n are inserted in a binary search tree in some order. In the resulting tree, the right subtree of the root contains p nodes. The first number to be inserted in the tree must be
- (A) p  
(B) p + 1  
(C) n - p  
(D) n - p + 1
- [2005 : 1 Mark]
- Q.19** While inserting the elements 71, 65, 84, 69, 67, 83 in an empty binary search tree (BST) in the sequence shown, the element in the lowest level is
- (A) 65  
(B) 67  
(C) 69  
(D) 83
- [2015 : 1 Mark]
- Q.20** The following numbers are inserted into an empty binary search tree in the given order: 10, 1, 3, 5, 15, 12, 16. What is the height of the binary search tree (the height is the maximum distance of a leaf node from the root)?
- (A) 2  
(B) 3  
(C) 4  
(D) 6
- [2004 : 1 Mark]
- Q.21** Suppose that we have numbers between 1 and 100 in a binary search tree and want to search for the number 55. Which of the following sequences CANNOT be the sequence of nodes examined? 2006 2marks
- (A) {10, 75, 64, 43, 60, 57, 55}  
(B) {90, 12, 68, 34, 62, 45, 55}  
(C) {9, 85, 47, 68, 43, 57, 55}  
(D) {79, 14, 72, 56, 16, 53, 55}

**Q.22** What is the worst case time complexity of inserting  $n^2$  elements into an AVL Tree which contains  $n$  elements initially ? [2020 : 1 Mark]

- (A)  $\theta(n^2)$  (B)  $\theta(n^2 \log n)$   
(C)  $\theta(n^4)$  (D)  $\theta(n^3)$

**Q.23** In a balanced BST with  $n$  elements . What is the worst case time complexity of reporting all the elements in range  $[a,b]$ ? Assume that the number of elements reported is  $k$ . [2020 : 1 Mark]

- (A)  $O(n \log k)$  (B)  $O(k \log n)$   
(C)  $O(\log n)$  (D)  $O(\log n + k)$

**Q.24** Which of the following is TRUE? [2008 : 2 Marks]

- (A) The cost of searching an AVL tree is  $\theta(\log n)$  but that of a binary search tree is  $O(n)$   
(B) The cost of searching an AVL tree is  $\theta(\log n)$  but that of a complete binary tree is  $\theta(n \log n)$   
(C) The cost of searching a binary search tree is  $O(\log n)$  but that of an AVL tree is  $\theta(n)$   
(D) The cost of searching an AVL tree is  $\theta(n \log n)$  but that of a binary search tree is  $O(n)$

**Q.25** What is the maximum and minimum number of elements in an AVL search tree of height 4 (Assume height of leaf node is 0)

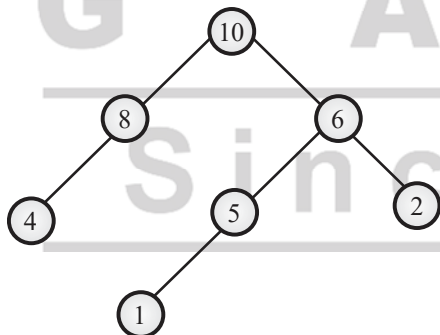
- (A) 31, 12  
(B) 15, 13  
(C) 31, 13  
(D) None of these

**Q.26** What is the maximum height of any AVL-tree with 7 nodes? Assume that the height of a tree with a single node is 0.

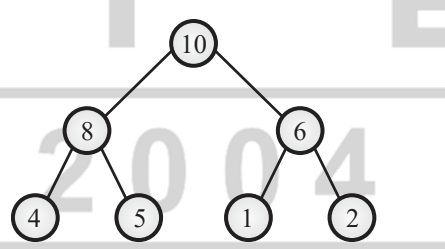
- (A) 2  
(B) 3  
(C) 4  
(D) 5

**Q.27** A max-heap is a heap where the value of each parent is greater than or equal to the values of its children. Which of the following is a max-heap? [2011 : 1 Mark]

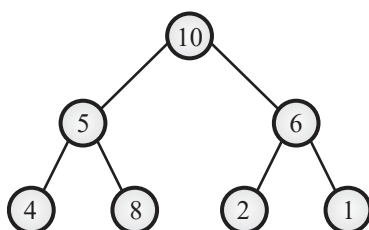
(A)



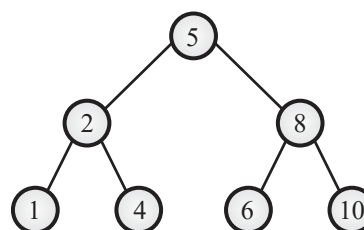
(B)



(C)



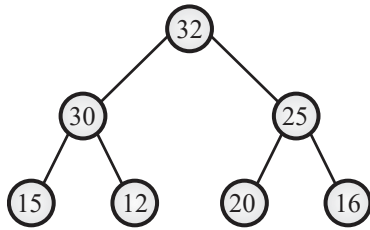
(D)



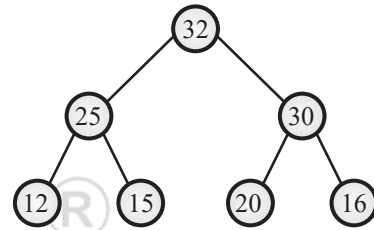


- Q.28** The number of possible min-heaps containing each value from  $\{1,2,3,4,5,6,7\}$  exactly once is [2018 : 2 Marks]
- Q.29** The elements 32, 15, 20, 30, 12, 25, 16 are inserted one by one in the given order into a Max Heap. The resultant Max Heap is. [2004 : 2 Marks]

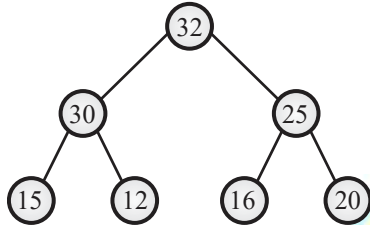
(A)



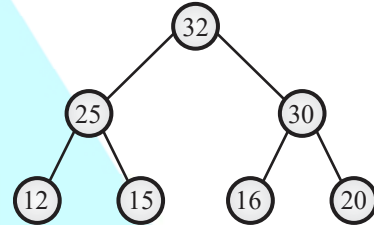
(B)



(C)



(D)



- Q.30** Which of the following sequences of array elements forms a heap?

- (A)  $\{23, 17, 14, 6, 13, 10, 1, 12, 7, 5\}$  (B)  $\{23, 17, 14, 6, 13, 10, 1, 5, 7, 12\}$   
 (C)  $\{23, 17, 14, 7, 13, 10, 1, 5, 6, 12\}$  (D)  $\{23, 17, 14, 7, 13, 10, 1, 12, 5, 7\}$  [2006 : 2 Marks]

#### LINKED QUESTION FOR 31 & 32 :

- Q.31** Consider a binary max-heap implemented using an array. Which one of the following array represents a binary max-heap?

- (A) 25,12,16,13,10,8,14  
 (B) 25,14,13,16,10,8,12  
 (C) 25,14,16,13,10,8,12  
 (D) 25,14,12,13,10,8,16

[2009 : 2 Marks]

- Q.32** What is the content of the array after two delete operations on the correct answer to the previous question?

- (A) 14,13,12,10,8  
 (B) 14,12,13,8,10  
 (C) 14,13,8,12,10  
 (D) 14,13,12,8,10

[2009 : 2 Marks]

- Q.33** Consider the following array of elements.

89, 19, 50, 17, 12, 15, 2, 5, 7, 11, 6, 9, 100

The minimum number of interchanges needed to convert it into a max-heap is

- (A) 4  
 (B) 5  
 (C) 2  
 (D) 3

[2015 : 1 Mark]

**Common data for question 34 and question 35.**

A 3-ary max heap is like a binary max heap, but instead of 2 children, nodes have 3 children. A 3-ary heap can be represented by an array as follows: The root is stored in the first location,  $a[0]$ , nodes in the next level, from left to right, is stored from  $a[1]$  to  $a[3]$ . The nodes from the second level of the tree from left to right are stored from  $a[4]$  location onward. An item  $x$  can be inserted into a 3-ary heap containing  $n$  items by placing  $x$  in the location  $a[n]$  and pushing it up the tree to satisfy the heap property.

**Q.34** Which one of the following is a valid sequence of elements in an array representing 3-ary max heap?

- (A) 1, 3, 5, 6, 8, 9
- (B) 9, 6, 3, 1, 8, 5
- (C) 9, 3, 6, 8, 5, 1
- (D) 9, 5, 6, 8, 3, 1

[2006: 2 Marks]

**Q.35** Suppose the elements 7, 2, 10 and 4 are inserted, in that order, into the valid 3-ary max heap found in the above question, Which one of the following is the sequence of items in the array representing the resultant heap?

- (A) 10, 7, 9, 8, 3, 1, 5, 2, 6, 4
- (B) 10, 9, 8, 7, 6, 5, 4, 3, 2, 1
- (C) 10, 9, 4, 5, 7, 6, 8, 2, 1, 3
- (D) 10, 8, 6, 9, 7, 2, 3, 4, 1, 5

[2006 : 2 Marks]

**Q.36** Consider a max heap, represented by the array:

40, 30, 20, 10, 15, 16, 17, 8, 4.

Now consider that a value 35 is inserted into this heap. After insertion, the new heap is

- (A) 40, 30, 20, 10, 15, 16, 17, 8, 4, 35
- (B) 40, 35, 20, 10, 30, 16, 17, 8, 4, 15
- (C) 40, 30, 20, 10, 35, 16, 17, 8, 4, 15
- (D) 40, 35, 20, 10, 15, 16, 17, 8, 4, 30

[2015 : 2 Marks]

**Q.37** Consider the array representation of a binary min-heap containing 1023 elements. The minimum number of comparisons required to find the maximum in the heap is \_\_\_\_\_. [2020 : 2 Marks]

**Q.38** A complete binary min-heap is made by including each integer in  $[1, 1023]$  exactly once. The depth of a node in the heap is the length of the path from the root of the heap to that node. Thus, the root is at depth 0. The maximum depth at which integer 9 can appear is \_\_\_\_\_. [2016 : 2 Marks]

**Self-Practice Questions :**

**Q.1** State whether the following statements are TRUE or FALSE:

If the number of leaves in a tree is not a power of 2, then the tree is not a binary tree. [1987 : 1 mark]

**Q.2** A binary tree  $T$  has  $n$  leaf nodes. The number of nodes of degree 2 in  $T$  is

- (A)  $\log_2 n$
- (B)  $n-1$
- (C)  $n$
- (D)  $2^n$

[1995 : 1 Mark]



**Q.3** A binary search tree is generated by inserting in order the following integers:

50, 15, 62, 5, 20, 58, 91, 3, 8, 37, 60, 24

The number of nodes in the left subtree and right subtree of the root respectively is [1996 : 2 Marks]

- (A) (4, 7)
- (B) (7, 4)
- (C) (8, 3)
- (D) (3, 8)

**Q.4** A binary search tree is used to locate the number 43. Which one of the following probe sequence is not possible?

- (A) 61, 52, 14, 17, 40, 43
- (B) 10, 65, 31, 48, 37, 43
- (C) 81, 61, 52, 14, 41, 43
- (D) 17, 77, 27, 66, 18, 43

[1996 : 2 Marks]

**Q.5** A binary search tree contains the values 1,2,3,4,5,6,7,8. The tree is traversed in pre-order and the values are printed out. Which of the following sequences is a valid output?

- (A) 5 3 1 2 4 7 8 6
- (B) 5 3 1 2 6 4 8 7
- (C) 5 3 2 4 1 6 7 8
- (D) 5 3 1 2 4 7 6 8

[1997 : 2 Marks]

**Q.6** Which of the following statement is false?

- (A) A tree with  $n$  nodes has  $(n-1)$  edges.
- (B) A labeled rooted binary tree can be uniquely constructed given its postorder and preorder traversal results.
- (C) A complete binary tree with  $n$  internal nodes has  $(n+1)$  leaves.
- (D) The maximum number of nodes in a binary tree of height  $h$  is  $(2^{(h+1)} - 1)$ . [1998 : 1 Mark]

**Q.7** Consider the following nested representation of binary trees: (X Y Z) indicates Y and Z are the left and right sub stress, respectively, of node X. Note that Y and Z may be NULL, or further nested. Which of the following represents a valid binary tree?

- (A) (1 2 (4 5 6 7))
- (B) (1 (2 3 4) 5 6) 7)
- (C) (1 (2 3 4)(5 6 7))
- (D) (1 (2 3 NULL) (4 5))

[2000 : 1 Mark]

**Q.8** Let  $T(n)$  be the number of different binary search trees on  $n$  distinct elements. Then

$$T(n) = \sum_{k=1}^n T(k-1)T(x), \text{ where } x \text{ is}$$

[2003 : 1 Mark]

- (A)  $n-k+1$
- (B)  $n-k$
- (C)  $n-k-1$
- (D)  $n-k-2$

**Q.9** Suppose the numbers 7, 5, 1, 8, 3, 6, 0, 9, 4, 2 are inserted in that order into an initially empty binary search tree. The binary search tree uses the usual ordering on natural numbers. What is the in-order traversal sequence of the resultant tree?

- (A) 7 5 1 0 3 2 4 6 8 9
- (B) 0 2 4 3 1 6 5 9 8 7
- (C) 0 1 2 3 4 5 6 7 8 9
- (D) 9 8 6 4 2 3 0 1 5 7

[2003 : 1 Mark]

**Q.10** Consider the following C program segment

struct CellNode

```
{  
    struct CellNode *leftchild;  
    int element;  
    struct CellNode *rightChild;  
}  
int Dosomething(struct CellNode *ptr)  
{  
    int value = 0;  
    if (ptr != NULL)  
    {  
        if (ptr->leftChild != NULL)  
            value = 1 + Dosomething(ptr->leftChild);  
        if (ptr->rightChild != NULL)  
            value = max(value, 1 + Dosomething(ptr->rightChild));  
    }  
    return (value);  
}
```

The value returned by the function DoSomething when a pointer to the root of a non-empty tree is passed as argument is

- (A) The number of leaf nodes in the tree
- (B) The number of nodes in the tree
- (C) The number of internal nodes in the tree
- (D) The height of the tree

[2005 : 2 Marks]

**Q.11** Consider the label sequences obtained by the following pairs of traversals on a labeled binary tree. Which of these pairs identify a tree uniquely ?

- (i) preorder and postorder
- (ii) inorder and postorder
- (iii) preorder and inorder
- (iv) level order and postorder

- (A) (i) only
- (B) (ii), (iii)
- (C) (iii) only
- (D) (iv) only

[2004 : 2 Marks]

- Q.12** In a binary tree, for every node the difference between the number of nodes in the left and right subtrees is at most 2. If the height of the tree is  $h > 0$ , then the minimum number of nodes in the tree is:
- (A)  $2^{h-1}$   
(B)  $2^{h-1} + 1$   
(C)  $2^h - 1$   
(D)  $2^h$  [2005 : 2 Marks]
- Q.13** How many distinct binary search trees can be created out of 4 distinct keys?
- (A) 5  
(B) 14  
(C) 24  
(D) 35 [2005 : 2 Marks]
- Q.14** The height of a binary tree is the maximum number of edges in any root to leaf path. The maximum number of nodes in a binary tree of height  $h$  is:
- (A)  $2^{h-1}$   
(B)  $2^{h-1} - 1$   
(C)  $2^{h+1} - 1$   
(D)  $2^{h+1}$  [2007 : 1 Mark]
- Q.15** The maximum number of binary trees that can be formed with three unlabeled nodes is:
- (A) 1  
(B) 5  
(C) 4  
(D) 3 [2007 : 2 Mark]
- Q.16** When searching for the key value 60 in a binary search tree, nodes containing the key values 10, 20, 40, 50, 70 80, 90 are traversed, not necessarily in the order given. How many different orders are possible in which these key values can occur on the search path from the root to the node containing the value 60?
- (A) 35  
(B) 64  
(C) 128  
(D) 5040 [2007 : 2 Marks]
- Q.17** Consider the following C program segment where CellNode represents a node in a binary tree:
- ```
struct CellNode
{
    struct CellNode *leftChild;
    int element;
    struct CellNode *rightChild;
};

int GetValue(struct CellNode *ptr)
```

```

{
    int value = 0;
    if (ptr != NULL)
    {
        if ((ptr->leftChild == NULL) && (ptr->rightChild == NULL))
            value = 1;
        else
            value = value + GetValue(ptr->leftChild) + GetValue(ptr->rightChild);
    } return(value);
}

```

The value returned by GetValue() when a pointer to the root of a binary tree is passed as its argument is:

- (A) the number of nodes in the tree
- (B) the number of internal nodes in the tree
- (C) the number of leaf nodes in the tree
- (D) the height of the tree

[2007 : 2 Marks]

### COMMON DATA QUESTION

A Binary Search Tree (BST) stores values in the range 37 to 573. Consider the following sequence of keys.

- I. 81, 537, 102, 439, 285, 376, 305
- II. 52, 97, 121, 195, 242, 381, 472
- III. 142, 248, 520, 386, 345, 270, 307
- IV. 550, 149, 507, 395, 463, 402, 270

**Q.18** Suppose the BST has been unsuccessfully searched for key 273. Which all of the above sequences list nodes in the order in which we could have encountered them in the search? [2008 : 2 Marks]

- (A) II and III only
- (B) I and III only
- (C) III and IV only
- (D) III only

**Q.19** Which of the following statements is TRUE? [2008 : 2 Marks]

- (A) I, II and IV are inorder sequences of three different BSTs
- (B) I is a preorder sequence of some BST with 439 as the root
- (C) II is an inorder sequence of some BST where 121 is the root and 52 is a leaf
- (D) IV is a postorder sequence of some BST with 149 as the root

**Q.20** How many distinct BSTs can be constructed with 3 distinct keys?

- (A) 4
- (B) 5
- (C) 6
- (D) 9

[2008 : 2 Marks]

### COMMON DATA QUESTION

A binary tree with  $n > 1$  nodes has  $n_1$ ,  $n_2$  and  $n_3$  nodes of degree one, two and three respectively. The degree of a node is defined as the number of its neighbors.

- Q.21**  $n_3$  can be expressed as [2008 : 2 Marks]  
 (A)  $n_1 + n_2 - 1$   
 (B)  $n_1 - 2$   
 (C)  $[(n_1 + n_2)/2]$   
 (D)  $n_2 - 1$
- Q.22** Starting with the above tree, while there remains a node  $v$  of degree two in the tree, add an edge between the two neighbors of  $v$  and then remove  $v$  from the tree. How many edges will remain at the end of the process? [2008 : 2 Marks]  
 (A)  $2 * n_1 - 3$   
 (B)  $n_2 + 2 * n_1 - 2$   
 (C)  $n_3 - n_2$   
 (D)  $n_2 + n_1 - 2$
- Q.23** In a binary tree with  $n$  nodes, every node has an odd number of descendants. Every node is considered to be its own descendant. What is the number of nodes in the tree that have exactly one child? [2010 : 1 Mark]  
 (A) 0  
 (B) 1  
 (C)  $(n-1)/2$   
 (D)  $n-1$
- Q.24** We are given a set of  $n$  distinct elements and an unlabeled binary tree with  $n$  nodes. In how many ways can we populate the tree with the given set so that it becomes a binary search tree? [2011 : 2 Marks]  
 (A) 0  
 (B) 1  
 (C)  $n!$   
 (D)  $2n C n / n+1$
- Q.25** The height of a tree is defined as the number of edges on the longest path in the tree. The function shown in the pseudocode below is invoked as `height (root)` to compute the height of a binary tree rooted at the tree pointer `root`. [2012 : 2 Marks]
- ```

int height (treeptr n)
{
    if (n == NULL) return -1;
    if (n → left == NULL)
    if (n → right == NULL) return 0;
    else return [B1]; // Box 1
    else
    {
        hi = height (n → left);
        if (n → right == NULL) return (1
        else
        {
            h2 = height (n → right);
            return [B2]; // Box 2
        }
    }
}

```

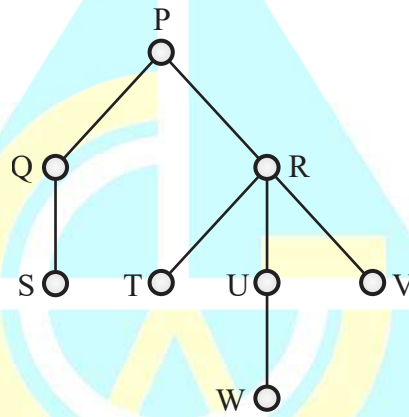
The appropriate expression for the two boxes B1 and B2 are

- (A) B1 : (1 + height(n->right)), B2 : (1 + max(h1,h2))
- (B) B1 : (height(n->right)), B2 : (1 + max(h1,h2))
- (C) B1 : height(n->right), B2 : max(h1,h2)
- (D) B1 : (1 + height(n->right)), B2 : max(h1,h2)

**Q.26** The preorder traversal sequence of a binary search tree is 30, 20, 10, 15, 25, 23, 39, 35, 42. Which one of the following is the postorder traversal sequence of the same tree? **[2013 : 2 Marks]**

- (A) 10, 20, 15, 23, 25, 35, 42, 39, 30
- (B) 15, 10, 25, 23, 20, 42, 35, 39, 30
- (C) 15, 20, 10, 23, 25, 42, 35, 39, 30
- (D) 15, 10, 23, 25, 20, 35, 42, 39, 30

**Q.27** Consider the following rooted tree with the vertex P labeled as root



The order in which the nodes are visited during in-order traversal is

- (A) SQPTRWUV
- (B) SQPTURWV
- (C) SQPTWUVR
- (D) SQPTRUWV

**[2014 : 1 Mark]**

**Q.28** Consider the pseudocode given below. The function DoSomething() takes as argument a pointer to the root of an arbitrary tree represented by the leftMostChild-rightSibling representation. Each node of the tree is of type treeNode. **[2014 : 2 Marks]**

```

typedef struct treeNode* treeptr;
struct treeNode
{
    treeptr leftMostChild, rightSibling;
};
int DoSomething (treeptr tree)
{
    int value=0;
    if (tree != NULL)
    {
        if (tree->leftMostChild == NULL)
            value = 1;
    }
}
  
```



```
else
    value = DoSomething(tree->leftMostChild);
    value = value + DoSomething(tree->rightSibling);
}
return(value);
}
```

When the pointer to the root of a tree is passed as the argument to DoSomething, the value returned by the function corresponds to the

- (A) number of internal nodes in the tree.
- (B) height of the tree.
- (C) number of nodes without a right sibling in the tree.
- (D) number of leaf nodes in the tree.

**Q.29** Which of the following is/are correct inorder traversal sequence(s) of binary search tree(s)?

- 1. 3, 5, 7, 8, 15, 19, 25
- 2. 5, 8, 9, 12, 10, 15, 25
- 3. 2, 7, 10, 8, 14, 16, 20
- 4. 4, 6, 7, 9, 18, 20, 25

- (A) 1 and 4 only
- (B) 2 and 3 only
- (C) 2 and 4 only
- (D) 2 only

[2015 : 1 Mark]

**Q.30** The height of a tree is the length of the longest root-to-leaf path in it. The maximum and minimum number of nodes in a binary tree of height 5 are

- (A) 63 and 6, respectively
- (B) 64 and 5, respectively
- (C) 32 and 6, respectively
- (D) 31 and 5, respectively

[2015 : 1 Mark]

**Q.31** Consider a binary tree T has 200 leaf nodes. Then the number of nodes in T have exactly two children are \_\_\_\_\_

[2015 : 1 Mark]

**Q.32** The number of ways in which the numbers 1, 2, 3, 4, 5, 6, 7 can be inserted in an empty binary search tree, such that the resulting tree has height 6, is \_\_\_\_\_. Note: The height of a tree with a single node is 0.

[2016 : 2Marks]

**Q.33** Let T be a binary search tree with 15 nodes. The minimum and maximum possible heights of T are:

**Note :** The height of a tree with a single node is 0.

- (A) 4 and 15 respectively
- (B) 3 and 14 respectively
- (C) 4 and 14 respectively
- (D) 3 and 15 respectively

[2017 : 1 Mark]



- Q.34** The pre-order traversal of a binary search tree is given by 12, 8, 6, 2, 7, 9, 10, 16, 15, 19, 17, 20. Then the post-order traversal of this tree is: [2017 : 2 Marks]
- (A) 2, 6, 7, 8, 9, 10, 12, 15, 16, 17, 19, 20  
(B) 2, 7, 6, 10, 9, 8, 15, 17, 20, 19, 16, 12  
(C) 7, 2, 6, 8, 9, 10, 20, 17, 19, 15, 16, 12  
(D) 7, 6, 2, 10, 9, 8, 15, 16, 17, 20, 19, 12
- Q.35** The postorder traversal of a binary tree is 8, 9, 6, 7, 4, 5, 2, 3, 1. The inorder traversal of the same tree is 8, 6, 9, 4, 7, 2, 5, 1, 3. The height of a tree is the length of the longest path from the root to any leaf. The height of the binary tree above is \_\_\_\_\_. [2018 : 1 Mark]
- Q.36** Let T be a full binary tree with 8 leaves. (A full binary tree has every level full.) Suppose two leaves a and b of T are chosen uniformly and independently at random. The expected value of the distance between a and b in T (i.e., the number of edges in the unique path between a and b) is (rounded off to 2 decimal places) \_\_\_\_\_. [2019 : 2 Marks]
- Q.37** The preorder traversal of a binary search tree is 15, 10, 12, 11, 20, 18, 16, 19. Which one of the following is the postorder traversal of the tree ? [2020 : 1 Mark]
- (A) 10, 11, 12, 15, 16, 18, 19, 20  
(B) 11, 12, 10, 16, 19, 18, 20, 15  
(C) 20, 19, 18, 16, 15, 12, 11, 10  
(D) 19, 16, 18, 20, 11, 12, 10, 15
- Q.38** A priority queue is implemented as a Max-Heap. Initially, it has 5 elements. The level-order traversal of the heap is: 10, 8, 5, 3, 2.  
Two new elements 1 and 7 are inserted into the heap in that order. The level-order traversal of the heap after the insertion of the elements is:
- (A) 10, 8, 7, 3, 2, 1, 5  
(B) 10, 8, 7, 2, 3, 1, 5  
(C) 10, 8, 7, 1, 2, 3, 5  
(D) 10, 8, 7, 5, 3, 2, 1 [2005 : 2 Marks]
- Q.39** Consider the following array of elements. \_\_\_\_\_  
89, 19, 50, 17, 12, 15, 2, 5, 7, 11, 6, 9, 100  
The minimum number of interchanges needed to convert it into a max-heap is \_\_\_\_\_
- (A) 4  
(B) 5  
(C) 2  
(D) 3 [2015 : 1 Mark]
- Q.40** How many min heaps can be constructed from 5 distinct keys.

## Answers

## Classroom Practice Questions

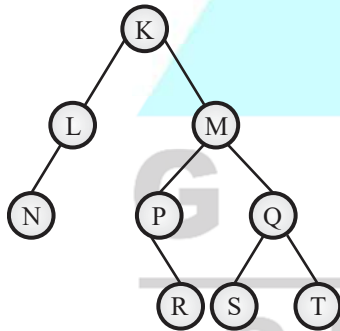
|    |       |    |     |    |    |    |   |    |   |
|----|-------|----|-----|----|----|----|---|----|---|
| 1  | A     | 2  | D   | 3  | C  | 4  | C | 5  | B |
| 6  | False | 7  | *   | 8  | *  | 9  | C | 10 | B |
| 11 | D     | 12 | A   | 13 | D  | 14 | D | 15 | C |
| 16 | D     | 17 | A   | 18 | C  | 19 | B | 20 | A |
| 21 | C     | 22 | B   | 23 | D  | 24 | A | 25 | A |
| 26 | B     | 27 | B   | 28 | 80 | 29 | A | 30 | C |
| 31 | C     | 32 | D   | 33 | D  | 34 | D | 35 | A |
| 36 | B     | 37 | 511 | 38 | 8  |    |   |    |   |

## Self-Practice Questions

|    |       |    |    |    |   |    |   |    |   |
|----|-------|----|----|----|---|----|---|----|---|
| 1  | False | 2  | B  | 3  | B | 4  | D | 5  | D |
| 6  | B, C  | 7  | C  | 8  | B | 9  | C | 10 | D |
| 11 | B     | 12 | B  | 13 | B | 14 | C | 15 | B |
| 16 | A     | 17 | C  | 18 | D | 19 | C | 20 | B |
| 21 | B     | 22 | A  | 23 | A | 24 | B | 25 | A |
| 26 | D     | 27 | A  | 28 | D | 29 | A | 30 | A |
| 31 | 199   | 32 | 64 | 33 | B | 34 | B | 35 | 4 |
| 36 | 4.25  | 37 | B  | 38 | B | 39 | D | 40 | 8 |

## Classroom Practice Questions

7.



8. GCBDAFE

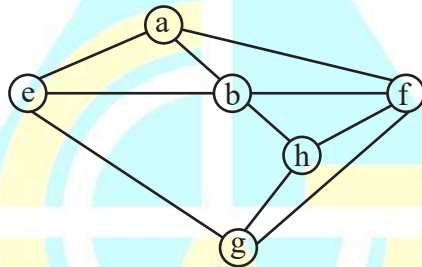


# 5

## GRAPH

### Classroom Practice Questions :

**Q.1** Consider the following graph:



Among the following sequences:

- I. a b e g h f
- II. a b f e h g
- III. a b f h g e
- IV. a f g h b e

Which are depth first traversals of the above graph?

- (A) I, II and IV only
- (B) I and IV only
- (C) II, III and IV only
- (D) I, III and IV only

[2003 : 1Mark]

**Q.2** In a depth-first traversal of a graph  $G$  with  $n$  vertices,  $k$  edges are marked as tree edges. The number of connected components in  $G$  is

- (A)  $k$
- (B)  $k + 1$
- (C)  $n - k - 1$
- (D)  $n - k$

[2005 : 1 Mark]

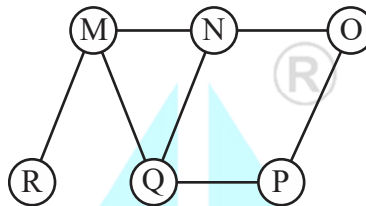
**Q.3** Let  $T$  be a depth first search tree in an undirected graph  $G$ . Vertices  $u$  and  $v$  are leaves of this tree  $T$ . The degrees of both  $u$  and  $v$  in  $G$  are at least 2. which one of the following statements is true?

[2006 : 1 Marks]

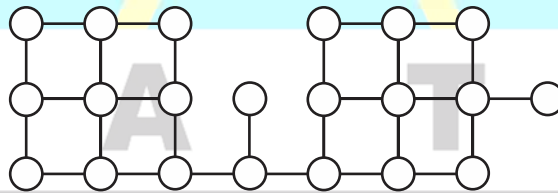
- (A) There must exist a vertex  $w$  adjacent to both  $u$  and  $v$  in  $G$
- (B) There must exist a vertex  $w$  whose removal disconnects  $u$  and  $v$  in  $G$
- (C) There must exist a cycle in  $G$  containing  $u$  and  $v$
- (D) There must exist a cycle in  $G$  containing  $u$  and all its neighbours in  $G$ .

- Q.4** The most efficient algorithm for finding the number of connected components in an undirected graph on  $n$  vertices and  $m$  edges has time complexity
- (A)  $\Theta(n)$   
 (B)  $\Theta(m)$   
 (C)  $\Theta(m+n)$   
 (D)  $\Theta(mn)$
- [2008 : 1 Mark]**

- Q.5** The Breadth First Search algorithm has been implemented using the queue data structure. One possible order of visiting the nodes of the following graph is



- (A) MNOPQR  
 (B) NQMPOR  
 (C) QMNPRO  
 (D) QMNPOR
- [2008 : 1 Mark]**
- Q.6** Let  $G$  be a graph with  $n$  vertices and  $m$  edges. What is the tightest upper bound on the running time of Depth First Search on  $G$ , when  $G$  is represented as an adjacency matrix?
- (A)  $\Theta(n)$   
 (B)  $\Theta(n+m)$   
 (C)  $\Theta(n^2)$   
 (D)  $\Theta(m^2)$
- [2014 (Set-1) : 1 Mark]**
- Q.7** Suppose depth first search is executed on the graph below starting at some unknown vertex. Assume that a recursive call to visit a vertex is made only after first checking that the vertex has not been visited earlier. Then the maximum possible recursion depth (including the initial call) is \_\_\_\_\_.
- [2014 (Set-1) : 1 Mark]**

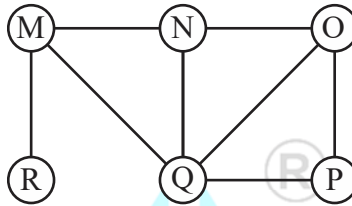


- Q.8** Breadth First Search (BFS) is started on a binary tree beginning from the root vertex. There is a vertex  $t$  at a distance four from the root. If  $t$  is the  $n$ -th vertex in this BFS traversal, then the maximum possible value of  $n$  is \_\_\_\_\_
- [2016 : 1 Mark]**
- Q.9** In an adjacency list representation of an undirected simple graph  $G = (V, E)$ , each edge  $(u, v)$  has two adjacency list entries:  $[v]$  in the adjacency list of  $u$ , and  $[u]$  in the adjacency list of  $v$ . These are called twins of each other. A twin pointer is a pointer from an adjacency list entry to its twin. If  $|E| = m$  and  $|V| = n$ , and the memory size is not a constraint, what is the time complexity of the most efficient algorithm to set the twin pointer in each entry in each adjacency list?
- (A)  $\Theta(n^2)$   
 (B)  $\Theta(m+n)$   
 (C)  $\Theta(m^2)$   
 (D)  $\Theta(n^4)$
- [2016 (Set-2) : 1 Mark]**

**Self-Practice Questions :**

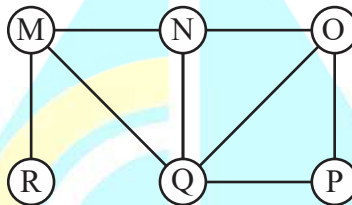
**Q.1** The maximum number of edges possible an undirected graph with 5 nodes ,when Depth First Search (DFS) call is made on any random node in the graph result in stack size 5 i.e. 5 function calls are present in stack simultaneously are \_\_\_\_\_

**Q.2** Consider the following graph :



The number of back edges in the BFS of this graph starting from node M is \_\_\_\_\_

**Q.3** Consider the following graph :



The number of forward edges in the BFS of this graph starting from node M is \_\_\_\_\_

**Q.4** The maximum possible height of BFS tree , if BFS is run on a complete bipartite graph  $K_{m,n}$  is :

- (A) 3
- (B) 2
- (C) 1
- (D) None of these

**Q.5** Which of the following algorithms can be used to most efficiently determine the presence of a cycle in a given graph ?

- (A) Depth First Search
- (B) Breadth First Search
- (C) Prim's Minimum Spanning Tree Algorithm
- (D) Kruskal's Minimum Spanning Tree Algorithm

**Q.6** Traversal of a graph is different from tree because

- (A) There can be a loop in graph so we must maintain a visited flag for every vertex
- (B) DFS of a graph uses stack, but inorder traversal of a tree is recursive
- (C) BFS of a graph uses queue, but a time efficient BFS of a tree is recursive.
- (D) All of the above

**Q.7** Let  $G$  be an undirected graph. Consider a depth-first traversal of  $G$ , and let  $T$  be the resulting depth-first search tree. Let  $u$  be a vertex in  $G$  and let  $v$  be the first new (unvisited) vertex visited after visiting  $u$  in the traversal. Which of the following statements is always true? (GATE CS 2000)

- (A)  $\{u, v\}$  must be an edge in  $G$ , and  $u$  is a descendant of  $v$  in  $T$
- (B)  $\{u, v\}$  must be an edge in  $G$ , and  $v$  is a descendant of  $u$  in  $T$
- (C) If  $\{u, v\}$  is not an edge in  $G$  then  $u$  is a leaf in  $T$
- (D) If  $\{u, v\}$  is not an edge in  $G$  then  $u$  and  $v$  must have the same parent in  $T$

- Q.8** Given two vertices in a graph  $s$  and  $t$ , which of the two traversals (BFS and DFS) can be used to find if there is path from  $s$  to  $t$ ?
- (A) Only BFS  
(B) Only DFS  
(C) Both BFS and DFS  
(D) Neither BFS nor DFS
- Q.9** Which of the following condition is sufficient to detect cycle in a directed graph?
- (A) There is an edge from currently being visited node to an already visited node.  
(B) There is an edge from currently being visited node to an ancestor of currently visited node in DFS forest.  
(C) Every node is seen twice in DFS.  
(D) None of the above
- Q.10** Consider the tree arcs of a BFS traversal from a source node  $W$  in an unweighted, connected, undirected graph. The tree  $T$  formed by the tree arcs is a data structure for computing.
- (A) the shortest path between every pair of vertices.  
(B) the shortest path from  $W$  to every vertex in the graph.  
(C) the shortest paths from  $W$  to only those nodes that are leaves of  $T$ .  
(D) the longest path in the graph

### Answers

#### Classroom Practice Questions

|   |   |   |    |   |    |   |   |   |   |
|---|---|---|----|---|----|---|---|---|---|
| 1 | D | 2 | D  | 3 | D  | 4 | C | 5 | C |
| 6 | B | 7 | 19 | 8 | 31 | 9 | B |   |   |

#### Self-Practice Questions

|   |    |   |   |   |   |   |   |    |   |
|---|----|---|---|---|---|---|---|----|---|
| 1 | 10 | 2 | 0 | 3 | 0 | 4 | B | 5  | A |
| 6 | A  | 7 | C | 8 | C | 9 | B | 10 | B |



Since 2004

# 6

## HASHING

### Classroom Practice Questions :

**Q.1** A hash table with ten buckets with one slot per bucket is shown in the following figure. The symbols S1 to S7 initially entered using a hashing function with linear probing. The maximum number of comparisons needed in searching an item that is not present is

|   |    |
|---|----|
| 0 | S7 |
| 1 | S1 |
| 2 |    |
| 3 | S4 |
| 4 | S2 |
| 5 |    |
| 6 | S5 |
| 7 |    |
| 8 | S6 |
| 9 | S3 |

- (A) 4
- (B) 5
- (C) 6
- (D) 3

[1989 : 2 Marks]

**Q.2** An advantage of chained hash table (external hashing) over the open addressing scheme is

- (A) Worst case complexity of search operations is less
- (B) Space used is less
- (C) Deletion is easier
- (D) None of the above

[1996 : 1 Mark]

**Q.3** Given the following input (4322, 1334, 1471, 9679, 1989, 6171, 6173, 4199) and the hash function  $x \bmod 10$ , which of the following statements are true?

- i. 9679, 1989, 4199 hash to the same value
- ii. 1471, 6171 hash to the same value
- iii. All elements hash to the same value
- iv. Each element hashes to a different value



- (A) i only  
 (B) ii only  
 (C) i and ii only  
 (D) iii or iv

[2004 : 1 Mark]

**Q.4** A hash table contains 10 buckets and uses linear probing to resolve collisions. The key values are integers and the hash function used is  $\text{key} \% 10$ . If the values 43, 165, 62, 123, 142 are inserted in the table, in what location would the key value 142 be inserted?

- (A) 2  
 (B) 3  
 (C) 4  
 (D) 6

[2005 : 1Mark]

**Q.5** Which of the following statement(s) is TRUE?

- I. A hash function takes a message of arbitrary length and generates a fixed length code.  
 II. A hash function takes a message of fixed length and generates a code of variable length.  
 III. A hash function may give the same hash value for distinct messages.

- (A) I only  
 (B) II and III only  
 (C) I and III only  
 (D) II only

[2006: 1Mark]

**Q.6** Consider a hash table of size 11 that uses open addressing with linear probing. Let  $h(k) = k \bmod 11$  be the hash function used. A sequence of records with keys 43 36 92 87 11 4 71 13 14 is inserted into an initially empty hash table, the bins of which are indexed from zero to ten. What is the index of the bin into which the last record is inserted?

- (A) 3  
 (B) 4  
 (C) 6  
 (D) 7

[2008: 2Marks]

**Q.7** The keys 12, 18, 13, 2, 3, 23, 5 and 15 are inserted into an initially empty hash table of length 10 using open addressing with hash function  $h(k) = k \bmod 10$  and linear probing. What is the resultant hash table?

[2009 : 2 Marks]

| (A) | (B) | (C) | (D) |
|-----|-----|-----|-----|
| 0   | 0   | 0   | 0   |
| 1   | 1   | 1   | 1   |
| 2   | 2   | 2   | 2   |
| 3   | 3   | 3   | 3   |
| 4   | 4   | 4   | 4   |
| 5   | 5   | 5   | 5   |
| 6   | 6   | 6   | 6   |
| 7   | 7   | 7   | 7   |
| 8   | 8   | 8   | 8   |
| 9   | 9   | 9   | 9   |

- Q.8** Consider a hash table with 9 slots. The hash function is  $h(k) = k \bmod 9$ . The collisions are resolved by chaining. The following 9 keys are inserted in the order: 5, 28, 19, 15, 20, 33, 12, 17, 10. The maximum, minimum, and average chain lengths in the hash table, respectively, are
- (A) 3, 0, and 1  
(B) 3, 3, and 3  
(C) 4, 0, and 1  
(D) 3, 0, and 2
- [2014 (Set-1): 2 Marks]**
- Q.9** Consider a hash table with 100 slots. Collisions are resolved using chaining. Assuming simple uniform hashing, what is the probability that the first 3 slots are unfilled after the first 3 insertions?
- (A)  $(97 \times 97 \times 97) / 100^3$   
(B)  $(99 \times 98 \times 97) / 100^3$   
(C)  $(97 \times 96 \times 95) / 100^3$   
(D)  $(97 \times 96 \times 95) / (3! \times 100^3)$
- [2014 (Set-3): 2 Marks]**
- Q.10** Given a hash table T with 25 slots that stores 2000 elements, the load factor  $\alpha$  for T is \_\_\_\_\_
- [2015 (Set-3): 1 Mark]**
- Q.11** Consider a double hashing scheme in which the primary hash function is  $h_1(k) = k \bmod 23$ , and the secondary hash function is  $h_2(k) = 1 + (k \bmod 19)$ . Assume that the table size is 23. Then the address returned by probe 1 in the probe sequence (assume that the probe sequence begins at probe 0) for key value  $k = 90$  is \_\_\_\_\_.
- [2020 : 1 Mark]**

### Self-Practice Questions :

- Q.1** Consider a hash table of size seven, with starting index zero, and a hash function  $(3x + 4) \bmod 7$ . Assuming the hash table is initially empty, which of the following is the contents of the table when the sequence 1, 3, 8, 10 is inserted into the table using closed hashing? Note that '\_\_\_' denotes an empty location in the table.
- (A) 8, \_\_, \_\_, \_\_, \_\_, \_\_, 10  
(B) 1, 8, 10, \_\_, \_\_, \_\_, 3  
(C) 1, \_\_, \_\_, \_\_, \_\_, \_\_, 3  
(D) 1, 10, 8, \_\_, \_\_, \_\_, 3
- [2007:2Marks]**
- Q.2** Consider a hash function that distributes keys uniformly. The hash table size is 20. After hashing of how many keys will the probability that any new key hashed collides with an existing one exceed 0.5.
- [2007 : 2 marks]**
- (A) 5  
(B) 6  
(C) 7  
(D) 10

### Linked question Answer for 3 and 4

A hash table of length 10 uses open addressing with hash function  $h(k) = k \bmod 10$ , and linear probing. After inserting 6 values into an empty hash table, the table is as shown below.

|   |    |
|---|----|
| 0 |    |
| 1 |    |
| 2 | 42 |
| 3 | 23 |
| 4 | 34 |
| 5 | 52 |
| 6 | 46 |
| 7 | 33 |
| 8 |    |
| 9 |    |

**Q.3** Which one of the following choices gives a possible order in which the key values could have been inserted in the table?

1. 46, 42, 34, 52, 23, 33
2. 34, 42, 23, 52, 33, 46
3. 46, 34, 42, 23, 52, 33
4. 42, 46, 33, 23, 34, 52

[2010:2Marks]

**Q.4** How many different insertion sequences of the key values using the same hash function and linear probing will result in the hash table shown above?

[2010:2Marks]

- (A) 10
- (B) 20
- (C) 30
- (D) 40

**Q.5** Which one of the following hash functions on integers will distribute keys most uniformly over 10 buckets numbered 0 to 9 for  $i$  ranging from 0 to 2020?

- (A)  $h(i) = i^2 \bmod 10$
- (B)  $h(i) = i^3 \bmod 10$
- (C)  $h(i) = (11 * i^2) \bmod 10$
- (D)  $h(i) = (12 * i) \bmod 10$

[2015 (Set-2): 2 Marks]

**Q.6** Consider an open address hash table with a total of 10000 slots containing 9800 entries. What is the expected number of probes in a successful search?

- (A) 3
- (B) 4
- (C) 5
- (D) 6

**Q.7** A hash function  $h$  defined  $h(\text{key}) = \text{key} \bmod 7$ , with linear probing, is used to insert the keys 44, 45, 79, 55, 91, 18, 63 into a table indexed from 0 to 6. What will be the location of key 18?

- (A) 3
- (B) 4
- (C) 5
- (D) 6

**Q.8** Consider a hash table of size  $m = 10000$ , and the hash function  $h(K) = \text{floor}(m(KA \bmod 1))$  for  $A = (\sqrt{5} - 1)/2$ . The key 123456 is mapped to location \_\_\_\_\_.

- (A) 46
- (B) 41
- (C) 43
- (D) 48

**Q.9** Consider a 13 element hash table for which  $f(\text{key}) = \text{key} \bmod 13$  is used with integer keys. Assuming linear probing is used for collision resolution, at which location would the key 103 be inserted, if the keys 661, 182, 24 and 103 are inserted in that order?

- (A) 0
- (B) 1
- (C) 11
- (D) 12

**Q.10** Consider a hash table of size  $m = 100$  and the hash function  $h(k) = \text{floor}(m(kA \bmod 1))$

$$\text{for } A = \frac{(\sqrt{5}-1)}{2} = 0.618033$$

Compute the location to which the key  $k = 123456$  is placed in hash table.

(A) 77

(B) 82

(C) 88

(D) 89

### Answers

#### Classroom Practice Questions

|    |    |   |   |   |   |   |   |    |    |
|----|----|---|---|---|---|---|---|----|----|
| 1  | B  | 2 | D | 3 | C | 4 | D | 5  | C  |
| 6  | D  | 7 | C | 8 | A | 9 | A | 10 | 80 |
| 11 | 13 |   |   |   |   |   |   |    |    |

#### Self-Practice Questions

|   |   |   |   |   |   |   |   |    |   |
|---|---|---|---|---|---|---|---|----|---|
| 1 | B | 2 | D | 3 | C | 4 | C | 5  | B |
| 6 | B | 7 | C | 8 | B | 9 | B | 10 | C |



**G A T E**  


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**Since 2004**  


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