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Phone : 0788 - 4034176, 0788 - 3224176

Help Desk No. - +91-97131-13156

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GATE Syllabus

Regular expressions and finite automata. Context-free grammars and push-down automata. Regular and contex-free languages, pumping lemma. Turing machines and undecidability.

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1 DFA, NFA, Minimization Technique & Product Automation

Classroom Practice Questions

Q.1 The number of substrings (of all lengths inclusive) that can be formed from a character string of length n is

(A) n (B) n^2

(C) $\frac{n(n-1)}{2}$ (D) $\frac{n(n+1)}{2}+1$

[GATE 1989 : IIT Kanpur]

Q.2 State True or False with one line explanation :

A FSM (Finite State Machine) can be designed to add two integers of any arbitrary length (arbitrary number of digits).

[GATE 1994 : IIT Kharagpur]

Q.3 Let $L \subseteq \Sigma^*$ where $\Sigma = \{a,b\}$ which of the following is true?

(A) $L = \{x \mid x$ has an equal number of a's and b's$\}$ is regular

(B) $L = \{a^n b^n \mid n \geq 1\}$ is regular

(C) $L = \{x \mid x$ has more a's than b's$\}$ is regular

(D) $L = \{a^m b^n \mid m \geq 1, n \geq 1\}$ is regular

[GATE 1997 : IIT Madras]

Q.4 Given $\Sigma = \{a,b\}$, which one of the following sets is not countable.

(A) Set of all strings over Σ

(B) Set of all languages over Σ

(C) Set of all regular language over Σ

(D) Set of all languages over Σ accepted by Turing machines.

[GATE 1997 : IIT Madras]

Q.5 Which of the following sets can be recognized by a deterministic finite-state automaton?

(A) The numbers 1, 2, 4, 8,.... 2^n,..... written in binary.

(B) The numbers 1, 2, 4, ... 2^n, ... written in unary.

(C) The set of binary strings in which the number of zeros is the same as the number of ones.

(D) The set {1, 101, 11011, 1110111, ...}

[GATE 1998 : IIT Delhi]

Q.6 How many substrings of different lengths (non-zero) can be formed from a character string of length n?

(A) n (B) n^2

(C) 2^n (D) n(n+1)/2

[GATE 1998 : IIT Delhi]

Q.7 Let L be the set of all binary strings whose last two symbols are the same. The number of states in the minimum state deterministic finite-state automaton accepting L is

(A) 2 (B) 5

(C) 8 (D) 3

[GATE 1998 : IIT Delhi]

Q.8 Consider the regular expression (0+1), (0+1)....n times. The minimum state finite automaton that recognizes the language represented by this regular expression contains :

(A) n states

(B) n +1 states

(C) n + 2 states

(D) None of the above

[GATE 1999 : IIT Bombay]

Q.9 What can be said about a regular language L over {a} whose minimal finite state automaton has two states?

(A) L must be $\{a^n \mid n \text{ is odd}\}$

(B) L must be $\{a^n \mid n \text{ is even}\}$

(C) L must be $\{a^n \mid n \geq 0\}$

(D) Either L must be $\{a^n \mid n \text{ is odd}\}$ or L must be $\{a^n \mid n \text{ is even}\}$

[GATE 2000 : IIT Kharagpur]

Q.10 Consider the following two statements :

$S_1 : \{0^{2n} \mid n \geq 1\}$ is a regular language

$S_2 : \{0^m 1^n 0^{m+n} \mid m \geq 1 \text{ and } n \geq 1\}$ is a regular language

Which of the following statements is correct?

(A) Only S_1 is correct

(B) Only S_2 is correct

(C) Both S_1 and S_2 are correct

(D) None of S_1 and S_2 is correct

[GATE 2001 : IIT Kanpur]

Q.11 Given an arbitrary non-deterministic finite automaton (NFA) with N states, the maximum number of states in an equivalent minimized DFA is at least

(A) N^2 (B) 2^N

(C) 2N (D) N!

[GATE 2001 : IIT Kanpur]

Q.12 Consider a DFA over $\Sigma = \{a, b\}$ accepting all strings which have number of a's divisible by 6 and number of b's divisible by 8. What is the minimum number of states that the DFA will have?

(A) 8 (B) 14

(C) 15 (D) 48

[GATE 2001 : IIT Kanpur]

Q.13 Consider the following languages :

$L_1 = \{ww \mid w \in \{a,b\}^*\}$

$L_2 = \{ww^R \mid w \in \{a,b\}^*, w^R \text{ is the reverse of } w\}$

$L_3 = \{0^{2^i} \mid i \text{ is an integer}\}$

$L_4 = \{0^{i^2} \mid i \text{ is an integer}\}$

Which of the languages are regular?

(A) Only L_1 and L_2

(B) Only L_2, L_3 and L_4

(C) Only L_3 and L_4

(D) Only L_3

[GATE 2001 : IIT Kanpur]

Q.14 Consider the set Σ^* of all strings over the alphabet $\Sigma = \{0,1\} \cdot \Sigma^*$ with the concatenation operator for strings

(A) Does not form a group

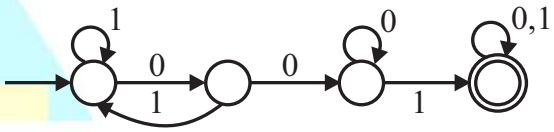
(B) Forms a non-commutative group

(C) Does not have a right identity element

(D) Forms a group if the empty string is removed from Σ^*.

[GATE 2003 : IIT Madras]

Q.15 Consider the following deterministic finite state automation M.



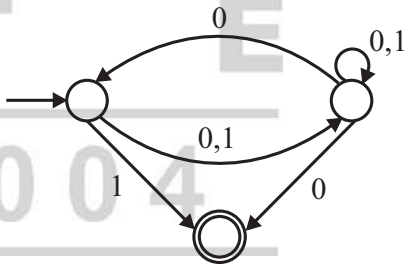
Let S denote the set of seven bit binary strings in which the first, the fourth, and the last bits are 1. The number of strings in S that are accepted by M is

(A) 1 (B) 5

(C) 7 (D) 8

[GATE 2003 : IIT Madras]

Q.16 Consider the NFA M shown below.



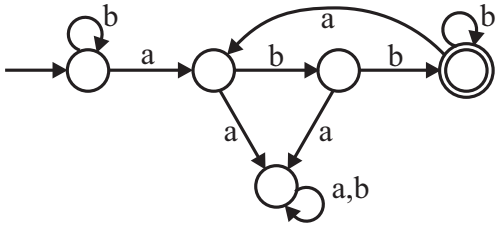
Let the language accepted by M be L. Let L_1 be the language accepted by the NFA, M_1 obtained by changing the accepting state of M to a non-accepting state and by changing the non-accepting state of M to accepting states. Which of the following statements is true?

(A) $L_1 = \{0,1\}^* - L$ (B) $L_1 = \{0,1\}^*$

(C) $L_1 \subseteq L$ (D) $L_1 = L$

[GATE 2003 : IIT Madras]

Q.17 Consider the machine M :



The language recognized by M is :

(A) $\{w \in \{a,b\}^* \mid \text{every } a \text{ in } w \text{ is followed by exactly two } b\text{'s}\}$

(B) $\{w \in \{a,b\}^* \mid \text{every } a \text{ in } w \text{ is followed by at least two } b\text{'s}\}$

(C) $\{w \in \{a,b\}^* \mid w \text{ contains the substring } 'abb'\}$

(D) $\{w \in \{a,b\}^* \mid w \text{ does not contain } 'aa' \text{ as a substring}\}$

[GATE 2005 : IIT Bombay]

Q.18 If s is a string over (0+1)* then let $n_0(s)$ denote the number of 0's in s and $n_1(s)$ the number of 1's in s. Which one of the following languages is not regular?

(A) $L = \{s \in (0+1)^* \mid n_0(s) \text{ is a 3-digit prime}\}$

(B) $L = \{s \in (0+1)^* \mid \text{for every prefix } s' \text{ of } s, |n_0(s') - n_1(s')| \le 2\}$

(C) $L = \{s \in (0+1)^* \mid n_0(s') - n_1(s) | \le 4\}$

(D) $L = \{s \in (0+1)^* \mid n_0(s) \bmod 7 = n_1(s) \bmod 5 = 0\}$

[GATE 2006 : IIT Kharagpur]

Q.19 Which of the following is TRUE?

(A) Every subset of a regular set is regular

(B) Every finite subset of a non-regular set is regular

(C) The union of two non-regular sets is not regular

(D) Infinite union of finite sets is regular

[GATE 2007 : IIT Kanpur]

Q.20 A minimum state deterministic finite automaton accepting the language

$L = \{w \mid w \in \{0,1\}^*, \text{number of } 0\text{'s and } 1\text{'s in } w \text{ are divisible by 3 and 5, respectively}\}$

has

(A) 15 states (B) 11 states

(C) 10 states (D) 9 states

Q.21 Which of the following languages is regular?

(A) $\{ww^R \mid w \in \{0,1\}^+\}$

(B) $\{ww^R x \mid x, w \in \{0,1\}^+\}$

(C) $\{wxw^R \mid x, w \in \{0,1\}^+\}$

(D) $\{xww^R \mid x, w \in \{0,1\}^+\}$

[GATE 2007 : IIT Kanpur]

Q.22 Given below are two finite state automata ($\rightarrow$ indicates the start state and F indicates a final state)

Y :

	a	b
$\rightarrow$1	1	2
2(F)	2	1

Z:

	a	b
$\rightarrow$1	1	2
2(F)	2	1

Which of the following represents the product automaton ZXY?

(A)

	a	b
$\rightarrow$P	S	R
Q	R	S
R(F)	Q	P
S	Q	P

(B)

	a	b
$\rightarrow$P	S	Q
Q	R	S
R(F)	Q	P
S	P	Q

(C)

	a	b
$\rightarrow$P	Q	S
Q	R	S
R(F)	Q	P
S	Q	P

(D) None

[GATE 2008 : IISc Bangalore]

Q.23 Which of the following are regular sets?

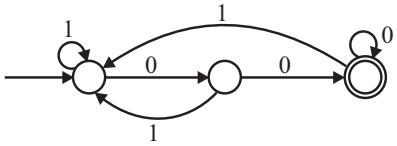
1. $\{a^n b^{2m} \mid n \ge 0, m \ge 0\}$
2. $\{a^n b^{2m} \mid n = 2m\}$
3. $\{a^n b^{2m} \mid n \ne m\}$

4. $\{xcy \mid x, y \in \{a,b\}^*\}$

(A) 1 and 4 only (B) 1 and 3 only
(C) 1 only (D) 4 only

[GATE 2008 : IISc Bangalore]

Q.24 The below DFA accepts the set of all strings over {0, 1} that



(A) Begins either with 0 or 1
(B) End with 0
(C) End with 00
(D) Contains the substring 00.

[GATE 2009 : IIT Roorkee]

Q.25 Let w be any string of length n in {0,1}*. Let L be the set of all substrings of w. What is the minimum number of states in a non-deterministic finite automation that accepts L?

(A) $n-1$ (B) n
(C) n+1 (D) 2^{n-1}

[GATE 2010 : IIT Guwahati]

Q.26 Definition of the language L with alphabet {a} is given as following.

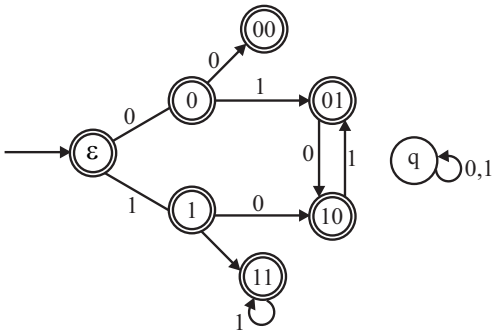
$L = \{a^{nk} \mid k > 0$, and n is a positive integer constant$\}$

What is the minimum number of states needed in a DFA recognize L?

(A) k +1 (B) n + 1
(C) 2^{n+1} (D) 2^{k+1}

[GATE 2011 : IIT Madras]

Q.27 Consider the set of strings on {0, 1} in which, every substring of 3 symbols has at most two zeros. For example, 001110 and 011001 are in the language, but 100010 is not. All strings of length less than 3 are also in the language. A partially completed DFA that accepts this language is shown below.



The missing arcs in the DFA are

(A)

	00	01	10	11	q
00	1	0			
01				1	
10	0				
11			0		

(B)

	00	01	10	11	q
00		0			1
01		1			
10				0	
11		0			

(C)

	00	01	10	11	q
00		1			0
01		1			
10			0		
11		0			

(D)

	00	01	10	11	q
00		1			0
01				1	
10	0				
11			0		

[GATE 2012 : IIT Delhi]

Q.28 Which one of the following is TRUE?

(A) The language $L = \{a^n b^n \mid n \geq 0\}$ is regular.

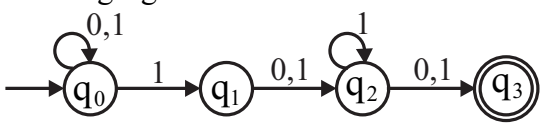
(B) The language $L = \{a^n \mid n$ is prime$\}$ is regular.

(C) The language $L = \left\{ w \mid \begin{array}{l} w \text{ has } 3k+1 \text{ b's for} \\ \text{some } k \in N \text{ with} \\ \Sigma = \{a,b\} \end{array} \right\}$ is regular.

(D) The language $L = \{ww \mid w \in \Sigma^* \text{ with } \Sigma = \{0,1\}\}$ is regular.

[GATE 2014 : IIT Kharagpur]

Q.29 Consider the finite automaton in the following figure.



What is the set of reachable states for the input string 0011?

(A) $\{q_0, q_1, q_2\}$ (B) $\{q_0, q_1\}$
(C) $\{q_0, q_1, q_2, q_3\}$ (D) $\{q_3\}$

[GATE 2014 : IIT Kharagpur]

Q.30 If $L_1 = \{a^n \mid n \geq 0\}$ and $L_2 = \{b^n \mid n \geq 0\}$, consider

(i) $L_1 \cdot L_2$ is a regular language

(ii) $L_1 \cdot L_2 = \{a^n b^n \mid n \geq 0\}$

Which one of the following is CORRECT?

(A) Only (i)

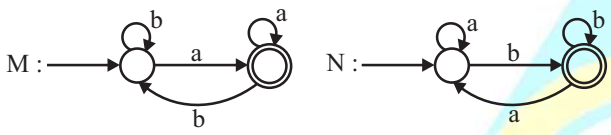
(B) Only (ii)

(C) Both (i) and (ii)

(D) Neither (i) nor (ii)

[GATE 2014 : IIT Kharagpur]

Q.31 Consider the DFAs M and N given below. The number of states in a minimal DFA that accepts the languages $L(M) \cap L(N)$ is _____.



[GATE 2015 : IIT Kanpur]

Q.32 The minimum possible number of states of a deterministic finite automaton that accepts the regular language $L = \{w_1 a w_2 \mid w_1, w_2 \in \{a,b\}^*, |w_1| = 2, |w_2| \geq 3\}$ is _____.

[GATE 2017 : IIT Roorkee]

Q.33 Let δ denote the transition function and $\hat{\delta}$ denote the extended transition function of the $\varepsilon - NFA$ whose transition table is give below :

δ	ε	**a**	**b**
$\rightarrow q_0$	$\{q_2\}$	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_0\}$	ϕ	ϕ
q_3	ϕ	ϕ	$\{q_2\}$

Them $\hat{\delta}(q_2, aba)$ is

(A) ϕ (B) $\{q_0, q_1, q_3\}$

(C) $\{q_0, q_1, q_2\}$ (D) $\{q_0, q_2, q_3\}$

[GATE 2017 : IIT Roorkee]

Q.34 Let N be an NFA with n states. Let k be the number of states of a minimal DFA which is equivalent to N. Which one of the following is necessarily true?

(A) $k \geq 2^n$ (B) $k \geq n$

(C) $k \leq n^2$ (D) $k \leq 2^n$

[GATE 2018 : IIT Guwahati]

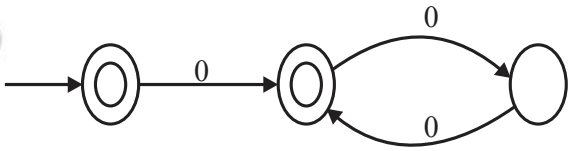
Q.35 Given a language L, define L^i as follows :

$L^0 = \{\varepsilon\}$

$L^i = L^{i-1} \cdot L$ for all $i > 0$

The order of a language L is defined as the smallest k such that $L^k = L^{k+1}$.

Consider the language L_1 (over alphabet 0) accepted by the following automaton.



The order of L_1 is _____.

[GATE 2018 : IIT Guwahati]

Q.36 If L is a regular language over $\Sigma = \{a,b\}$, which one of the following languages is NOT regular?

(A) $L.L^R = \{xy \mid x \in L, y^R \in L\}$

(B) $\{ww^R \mid w \in L\}$

(C) $\text{Prefix}(L) = \left\{x \in \Sigma^* \middle| \exists y \Sigma^* \text{such that } xy \in L\right\}$

(D) $\text{Suffix}(L) = \left\{y \in \Sigma^* \middle| \exists y \in \Sigma^* \text{such that } xy \in L\right\}$

[GATE 2019 : IIT Madras]

Q.37 Let Σ be the set of all bijections from {1, ..., 5} to {1, ..., 5}, where id denotes the identity function, i.e. $id(j) = j$, $\forall j$. Let $\circ$ denote composition on functions. For a string $x = x_1 \; x_2 \ldots . x_n \in \Sigma^n$, $n \geq 0$, let $\pi(x) = x_1 \circ x_2 \circ \ldots . \circ x_n$.

Consider the language $L = \{x \in \Sigma^* \mid \pi(x) = id\}$. The minimum number of states in any DFA accepting L is _____.

[GATE 2019 : IIT Madras]

Q.38 Which of the following statement is false?

(A) Every finite subset of a non-regular set is regular

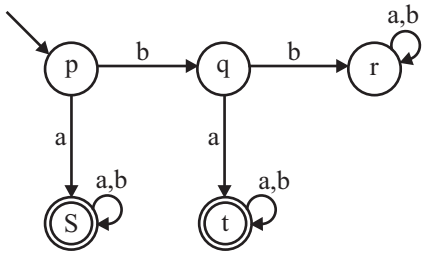
(B) Every subset of a regular set is regular

(C) Every finite subset of a regular set is regular

(D) The intersection of two regular sets is regular

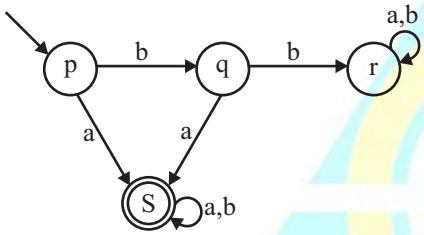
[GATE 1998 : IIT Delhi]

Q.39 A deterministic finite automation (DFA) D with alphabet $\sum = \{a, b\}$ is given below

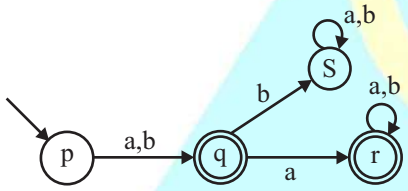


Which of the following finite state machine is a valid minimal DFA which accepts the same language as D?

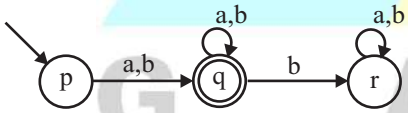
(A)



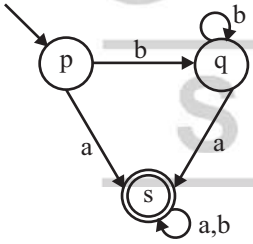
(B)



(C)



(D)



[GATE 2019 : IIT Madras]

Q.40 Consider the following language.

$$L = \left\{ \begin{array}{l} x \in \{a,b\}^* \mid \text{number of} \\ a\text{'sin } x \text{ is} \\ \text{divisible by 2 but not} \\ \text{divisible by 3} \end{array} \right\}$$

The minimum number of states in a DFA that accepts L is _____.

[GATE 2020 : IIT Delhi]

Q.41 Let R_1 and R_2 be regular sets defined over the alphabet $\sum$ then :

(A) $R_1 \cap R_2$ is not regular

(B) $R_1 \cup R_2$ is regular

(C) $\sum^* - R_1$ is regular

(D) R_1^* is not regular

[GATE 1990 : IISc Bangalore]

Q.42 The smallest finite automation which accepts the language

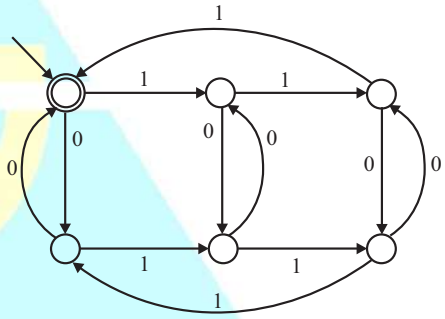
$L = \{x \mid \textit{length of } x \textit{ is divisible by } 3\}$ has

(A) 2 states (B) 3 states

(C) 4 states (D) 5 states

[GATE 2002 : IISc Bangalore]

Q.43 The following finite state machine accepts all those binary strings in which the number of 1's and 0's are respectively



(A) Divisible by 3 and 2

(B) Odd and even

(C) Even and odd

(D) Divisible by 2 and 3

[GATE 2004 : IIT Delhi]

Self - Practice Questions

Q.1 L = {Set of all strings over {a, b} ending with fixed length symbol n}

Find minimum no. of states in DFA that accept L?

(A) 1 (B) n

(C) $n-1$ (D) $n+1$

Q.2 $L = \{a^n \mid n \geq 0 \text{ and } n \neq 2,5,7\}$ which of the following options is/are TRUE about L?

(A) The minimum state DFA will contain 8-states.

(B) No. of non-final state in minimum DFA will be 3.

(C) No. of final states in minimum DFA will be 6.

(D) The language 'L' can be computed by PDA.

Q.3 Find minimum no. of states in a DFA that accept language.

$$L=\{a^n \mid n\geq 0\}\cup\{b^n \mid n\geq 1\}$$ _____?

Q.4 $L=\{W\in\{0,1\}^* \mid |W| \bmod n<k\}$ where k is positive integer constant and $k<n$.

What is the no. of final states in minimum DFA?

(A) $n-1$ (B) k

(B) $k-1$ (D) $k+1$

Q.5 $L=\{W\in\{0,1\}^* \mid |W| \bmod n<k\}$ where k is positive integer constant and $k<n$.

Which of the following options is/are TRUE about minimize DFA that accept L?

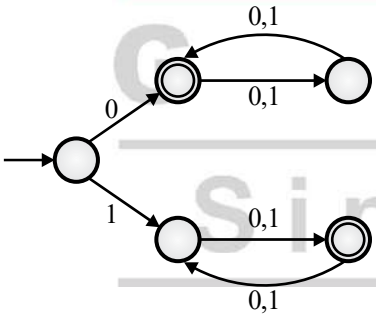
(A) No. of states in DFA is $(n-1)$.

(B) No. of non-final states in DFA is $(n-k)$.

(C) No. of final states is $(k-1)$.

(D) The strings 1011101 is accepted by DFA for $n=5$ and $k=3$.

Q.6 Find minimum no. of states for above DFA _____?



Q.7 $L=\{W\in\{a,b\}^n \mid$ n is positive integer constant$\}$

Find minimum number of states in NFA that accept L?

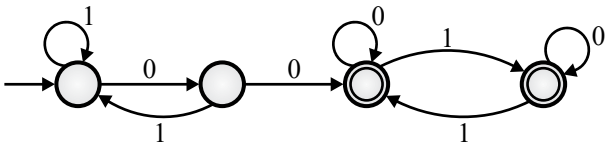
(A) $n-1$ (B) n

(C) $n+1$ (D) $n+2$

Q.8 $L=\{WxW^R \mid W,x\in\{0,1\}^+\}$

Find minimum number of states in DFA that accept L?

Q.9 Which of the following options is/are true about above DFA.



(A) The language accepted by above DFA is $\{W00W \mid W\in\{0,1\}^*\}$

(B) The minimum state DFA for M consists of 3-states.

(C) The regular expression for $L(M)$ is $(0+1)^*00(0+1)^*$

(D) The language accepted y above DFA is $\{W_1 00 W_2 \mid W_1 \text{ and } W_2\in\{0,1\}^+\}$

Q.10 $L=\{W\in\{a,b\}^* \mid \#_a(W) \bmod 3\leq 1$ or $\#_b(W) \bmod 4=1\}$

Find no. of final sates in minimum DFA that accept L?

Q.11 $L=\{W\in\{a,b\}^* \mid \#_a(W) \bmod 4\leq 2$ and $\#_b(W) \bmod 5\leq 1\}$

Find no. of non-final sates in minimum DFA?

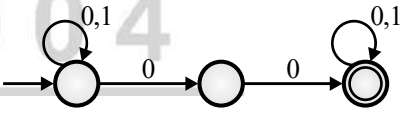
(A) The total no. of states in minimum DFA will be 20.

(B) The no. of final states in minimum DFA will be 8.

(C) The no. of non-final states in minimum DFA is 14.

(D) The total no. of states in minimum DFA is 16.

Q.12 **NFA :**



Which of the following options is/are true?

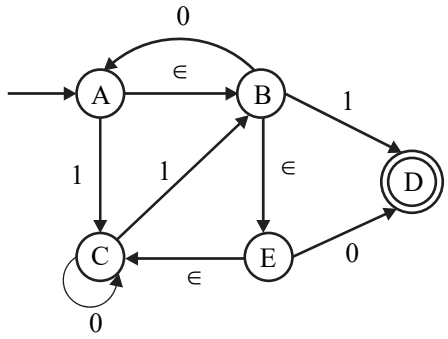
(A) $L(\text{NFA})=\{W_1 00 W_2 \mid W_1 \text{ and } W_2\in\{0,1\}^*\}$

(B) The complement of language accepted by above NFA is $\{W0 \mid W\in\{0,1\}^*\}$.

(C) Conversion of above NFA into DFA will contain 4-states.

(D) The minimized DFA of above NFA will contain 3-states.

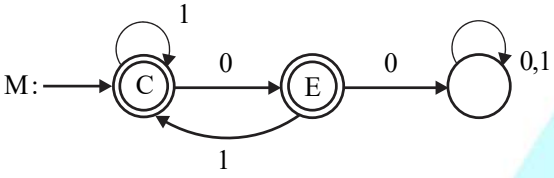
Q.13 What will be $\delta(A,01)$ for the following automation?



(A) $\{D\}$ (B) $\{B, D\}$

(C) $\{B, C, D\}$ (D) $\{B, C, D, E\}$

Q.14 Consider following DFA:-



Which of the following options is /are TRUE?

(A) It accepts all the strings over {0, 1} ending with 0.

(B) The regular expression for $L(M)$ is $(1+01)^*$

(C) It accepts set of all string over {0, 1} having no two consecutive 0' s.

(D) $L(M) = \{w \in \{0,1\}^* \mid \neq (w) \text{ is odd}\}$

Q.15 Let $M = (K, \Sigma, \delta, s, F)$ be a finite state automaton, where

$K = \{A, B\}, \Sigma = \{a, b\}, s = A, F = \{B\}$,

$\delta(A, a) = A, \delta(A, b) = B, \delta(B, a)$

$= B$ and $\delta(B, b) = A$

A grammar to generate the language accepted by M can be specified as

$G = (V, \Sigma, R, S)$, where $V = K \cup \Sigma$, and $S = A$

Which one of the following set of rules will make $L(G) = L(M)$?

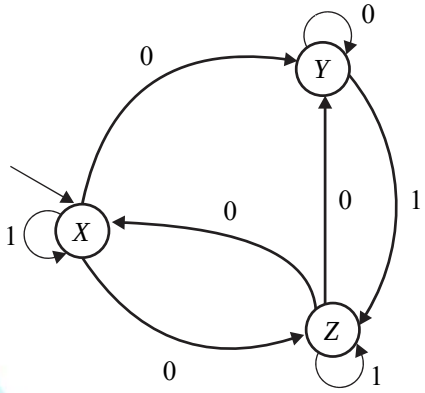
(A) $\begin{Bmatrix} A \to aB, A \to bA, B \to bA \\ B \to aA, B \to \varepsilon \end{Bmatrix}$

(B) $\begin{Bmatrix} A \to aA, A \to bB, B \to aB \\ B \to bA, B \to \varepsilon \end{Bmatrix}$

(C) $\begin{Bmatrix} A \to bB, A \to aB, B \to aA \\ B \to bA, B \to \varepsilon \end{Bmatrix}$

(D) $\begin{Bmatrix} A \to aA, A \to bA, B \to aB \\ B \to bA, A \to \varepsilon \end{Bmatrix}$

Q.16 Consider the non- deterministic finite automation (NFA) shown in the figure. State X is the starting state of the automation. Let the language accepted by the NFA with Y as the only accepting state be L1. Similarly, let the language accepted by the NFA with Z as the only accepting state be L2. Which of the following statements about L1 and L2 is TRUE?



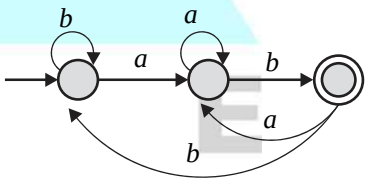
(A) $L1 = L2$

(B) $L1 \subset L2$

(C) $L2 \subset L1$

(D) None of the above

Q.17 If the final states and non-final states in the DGA below are interchanged, then which of the following languages over the alphabet {a, b} will be accepted by the new DFA?



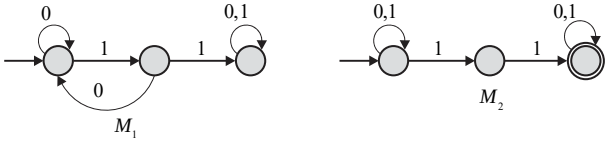
(A) Set of all strings that do not end with ab

(B) Set of all strings that begin with either an a or a b

(C) Set of all strings that do not contain the substring ab

(D) The set described by the regular expression $b*aa*(ba)*b*$

Q.18 Consider the following two finite automata. M_1 accepts L_1 and M_2 accepts L_2 which one of the following is true?



(A) $L_1 = L_2$ (B) $L_1 \subset L_2$

(C) $L_1 \cap \overline{L}_2 = \phi$ (D) $L_1 \cup L_2 \neq L_1$

Q.19 Choose the correct statement –

(A) $A = \{a^n b^n \mid n = 1, 2, 3....\}$ is regular language

(B) The set B, consisting of all strings made up of only a's and b's having equal number of a's and b's defines a regular language

(C) $L(A^* B) \cap B$ gives the set A

(D) None of the above

Answer Keys

Classroom Practice Questions									
1.	D	2.	TRUE	3.	D	4.	B	5.	A
6.	D	7.	B	8.	C	9.	D	10.	A
11.	B	12.	D	13.	D	14.	A	15.	C
16.	B	17.	B	18.	B	19.	B	20.	A
21.	C	22.	D	23.	A	24.	C	25.	C
26.	B	27.	D	28.	C	29.	A	30.	A
31.	1	32.	8	33.	C	34.	D	35.	2
36.	B	37.	120	38.	B	39.	A	40.	6
41.	B, C	42.	3	43.	A				
Self - Practice Questions									
1.	D	2.	B ,C, D	3.	4	4.	C	5.	B, D
6.	3	7.	C	8.	7	9.	B, C, D	10.	9
11.	A, C	12.	A, C, D	13.	D	14.	B, C	15.	B
16.	B	17.	D	18.	A	19.	D		

❖❖❖❖

2 Regular Expression

Classroom Practice Questions

Q.1 Let $r = 1(1+0)^*$, $s = 11^*0$ and $t = 1^*0$ be three regular expressions. Which one of the following is true?

(A) $L(s) \subseteq L(r)$ *and* $L(s) \subseteq L(t)$

(B) $L(r) \subseteq L(s)$ *and* $L(s) \subseteq L(t)$

(C) $L(s) \subseteq L(t)$ *and* $L(s) \subseteq L(r)$

(D) $L(t) \subseteq L(s)$ *and* $L(s) \subseteq L(r)$

[GATE 1991 : IIT Madras]

Q.2 Which Two of the following four regular expressions are equivalent?

(i) $(00)^*(\varepsilon+0)$

(ii) $(00)^*$

(iii) 0^*

(iv) $0\ (00)^*$

(A) (i) and (ii) (B) (ii) and (iii)

(C) (i) and (iii) (D) (iii) and (iv)

[GATE 1996 : IISc Bangalore]

Q.3 Which one of the following regular expressions over {0, 1} denotes the set of all strings not containing 100 as a substring?

(A) 0* (1+0)* (B) 0*1010*

(C) 0*1*01* (D) 0*(10+1)*

[GATE 1997 : IIT Madras]

Q.4 If the regular set A is represented by $A = (01+1)^*$ and the regular set 'B' is represented by $B = ((01)^*1^*)^*$, which of the following is true?

(A) A $\subset$ B

(B) B $\subset$ A

(C) A and B are incomparable

(D) A = B

[GATE 1998 : IIT Delhi]

Q.5 The string 1101 does not belong to the set represented by

(A) 110* (0+1)

(B) 1(0+1)* 101

(C) (10)*(01)* (00 +11)*

(D) (00 + (11)* 0)*

[GATE 1998 : IIT Delhi]

Q.6 Let S and T be languages over $\sum = \{a, b\}$ represented by the regular expressions $(a + b^*)^*$ and (a + b)*, respectively. Which of the following is true?

(A) S $\subset$ T

(B) T $\subset$ S

(C) S = T

(D) $S \cap T = \phi$

[GATE 2002 : IISc Bangalore]

Q.7 The regular expression 0*(10*)* denotes the same set as

(A) (1*0)*1*

(B) 0+ (0+10)*

(C) (0+1)* 10 (0+1)*

(D) None of the above

[GATE 2003 : IIT Madras]

Q.8 Consider the regular language $L = (111+11111)^*$. The minimum number of states in any DFA accepting this language is

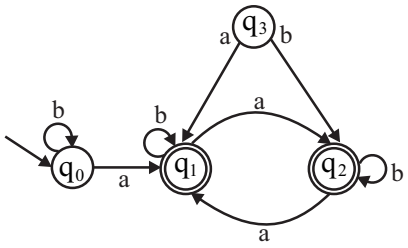
(A) 3 (B) 5

(C) 8 (D) 9

[GATE 2006 : IIT Kharagpur]

Common Data for Q.9 & Q.10 Questions

Consider the following finite state automaton



Q.9 The language accepted by this automaton is given by the regular expression

(A) $b^*ab^*ab^*ab^*$ (B) $(a+b)^*$

(C) $b^*a(a+b)^*$ (D) $b^*ab^*ab^*$

[GATE 2007 : IIT Kanpur]

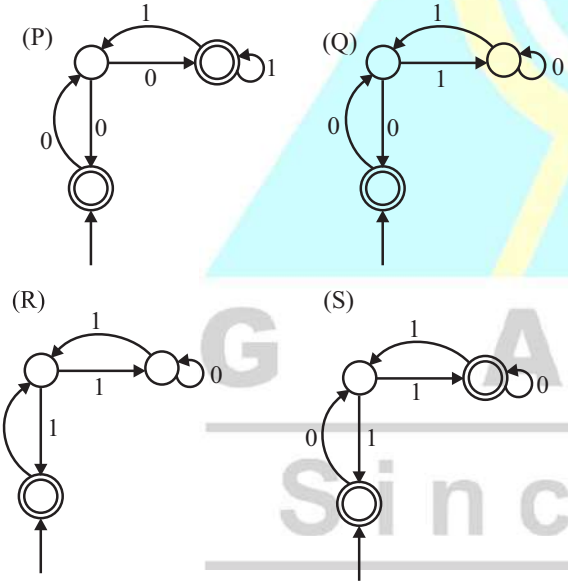
Q.10 The minimum state automation equivalent to the above FSA has the following number of states

(A) 1 (B) 2

(C) 3 (D) 4

[GATE 2007 : IIT Kanpur]

Q.11 Match the following NFA's with the regular expression they correspond to



1. $\varepsilon+0(01^*1+00)^*01^*$
2. $\varepsilon+0(10^*1+00)^*0$
3. $\varepsilon+0(10^*1+10)^*1$
4. $\varepsilon+0(10^*1+10)^*10^*$

(A) $P-2, Q-1, R-3, S-4$

(B) $P-1, Q-3, R-2, S-4$

(C) $P-1, Q-2, R-3, S-4$

(D) $P-3, Q-2, R-1, S-4$

[GATE 2008 : IISc Bangalore]

Q. 12 Let $L=\{w\in(0+1)^* | w$ has even number of 1's$\}$, i.e. L is the set of all bit strings with even number of 1's. Which one of the regular expressions below represents L?

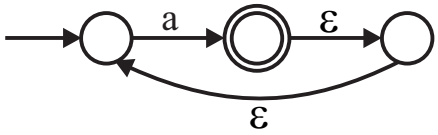
(A) $(0^*10^*1)^*$ (B) $0^*(10^*\ 10^*)^*$

(C) $0^*(10^*1)^*0^*$ (D) $0^*1(10^*1)^*10^*$

[GATE 2010 : IIT Guwahati]

Q.13 What is the complement of the language accepted by the NFA shown below?

Assume $\sum=\{a\}$ and ε is the empty string.



(A) ϕ (B) $\{\varepsilon\}$

(C) a^* (D) $\{a,\varepsilon\}$

[GATE 2012 : IIT Delhi]

Q.14 Consider the languages $L_1=\phi$ and $L_2=\{a\}$. Which one of the following represents $L_1L_2^*\cup L_1^*$?

(A) $\{\varepsilon\}$ (B) ϕ

(C) a^* (D) $\{\varepsilon,a\}$

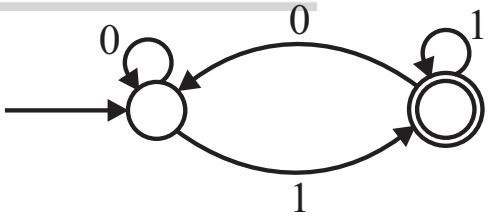
[GATE 2013 : IIT Bombay]

Q. 15 The length of the shortest string NOT in the language (over $\sum=\{a,b\}$) of the following regular expression is _____.

a*b*(ba)*a*

[GATE 2014 : IIT Kharagpur]

Q. 16 Which of the regular expression given below represent the following DFA?



1. 0*1 (1+00*1*)*
2. 0*1*1+11*0*1
3. (0+1)*1

(A) 1 and 2 only (B) 1 and 3 only

(C) 2 and 3 only (D) 1, 2, and 3

[GATE 2014 : IIT Kharagpur]

Q. 17 Let $L_1 = \{w \in \{0,1\}^* | w$ has at least as many occurrences of (110)'s as (011)'s$\}$.
Let $L_2 = \{w \in \{0,1\}^* | w$ has at least as many occurrences of (000)'s as (111)'s$\}$. Which one of the following is TRUE?

(A) L_1 is regular but not L_2

(B) L_2 is regular but not L_1

(C) Both L_1 and L_2 are regular.

(D) Neither L_1 nor L_2 are regular

[GATE 2014 : IIT Kharagpur]

Q.18 Let L be the language represented by the regular expression $\Sigma^*0011\Sigma^*$ where $\Sigma = \{0,1\}$. What is the minimum number of states in a DFA that recognizes $\bar{L}$ (complement of L)?

(A) 4 (B) 5

(C) 6 (D) 8

[GATE 2015 : IIT Kanpur]

Q.19 The number of states in the minimal deterministic finite automaton corresponding to the regular expression (0 +1)* (10) is _____.

[GATE 2015 : IIT Kanpur]

Q.20 Consider the alphabet $\Sigma = \{0,1\}$, the null/empty string λ and the set of strings X_0, X_1, and X_2 generated by the corresponding non-terminals of a regular grammar X_0, X_1, and X_2 are related as follows.

$$X_0 = 1X_1$$
$$X_1 = 0X_1 + 1X_2$$
$$X_2 = 0X_1 + \{\lambda\}$$

Which one of the following choices precisely represents the strings in X_0?

(A) 10(0* + (10)*)1

(B) 10(0*+(10*))*1

(C) 1(0+10)*1

(D) 10(0+10)*1+110(0+10)*1

[GATE 2015 : IIT Kanpur]

Q.21 Which one of the following regular expressions represents the language : the set of all binary strings having two consecutive 0's and two consecutive 1's?

(A) $(0+1)^*0011(0+1)^* + (0+1)^*1100(0+1)^*$

(B) $(0+1)^*(00(0+1)^*11 + 11(0+1)^*00)(1+1)^*$

(C) $(0+1)^*00(0+1)^* + (0+1)^*11(0+1)^*$

(D) $00(0+1)^*11 + 11(0+1)^*00$

[GATE 2016 : IISc Bangalore]

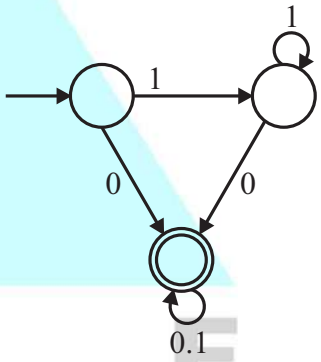
Q.22 The number of states in the minimum sized DFA that accepts the language defined by the regular expression $(0+1)^*(0+1)(0+1)^*$ is _____.

[GATE 2016 : IISc Bangalore]

Q.23 Consider the language L given by the regular expression (a+b)*b(a+b) over the alphabet {a, b}. The smallest number of states needed in a deterministic finite-state automaton (DFA) accepting L is _____.

[GATE 2017 : IIT Roorkee]

Q. 24 Consider the DFA A given below.



Which of the following are FALSE?

1. Complement of L(A) is context – free
2. L(A) = L((11*0+0)(0+1)*0*1*)
3. For the language accepted by A, A is the minimal DFA.
4. A accepts all strings over {0, 1} of length at least 2.

(A) 1 and 3 only (B) 2 and 4 only

(C) 2 and 3 only (D) 3 and 4 only

[GATE 2013 : IIT Bombay]

Q. 25 Consider the following two statements :

I. If all states of an NFA are accepting states then the language accepted by the NFA is Σ^*.

II. There exists a regular language A such that for all languages B, $A \cap B$ is regular.

Which one of the following is CORRECT?

(A) Only I is true
(B) Only II is true
(C) Both I and II are true
(D) Both I and II are false

[GATE 2016 : IISc Bangalore]

Q. 26 For $\Sigma = \{a, b\}$, let us consider the regular language $L = \{x \mid x = a^{2+3k} \text{ or } x = b^{10+12k}, k \geq 0\}$. Which one of the following can be a pumping length (the constant guaranteed by the pumping lemma) for L?

(A) 3 (B) 5
(C) 9 (D) 24

[GATE 2019 : IIT Madras]

Q. 27 Consider the following statements.

I. If $L_1 \cup L_2$ is regular, then both L_1 and L_2 must be regular.

II. The class of regular languages is closed under infinite union.

Which of the above statements is/are TRUE?

(A) I only
(B) II only
(C) Both I and II
(D) Neither I nor II

[GATE 2020 : IIT Delhi]

Q.28 Which one of the following regular expressions represents the set of all binary strings with an odd number of 1's?

(A) $((0+1)*1(0+1)*1)*10*$
(B) $(0*10*10*)*0*1$
(C) $10*(0*10*10*)*$
(D) $(0*10*10*)*10*$

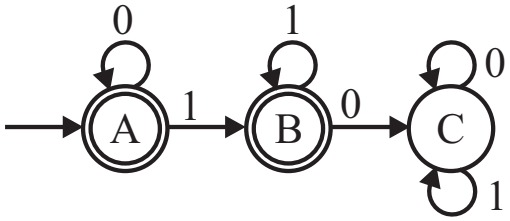
[GATE 2020 : IIT Delhi]

Q.29 Which of the following regular expression identities are true?

(A) r (*) = r*
(B) (r*s*)* = (r + s)*
(C) $(r+s)^* = r*+s*$
(D) $r*s* = r*+s*$

[GATE 1992 : IIT Delhi]

Q.30 The regular expression for the language recognized by the finite state automation of the below figure is _____.



[GATE 2002 : IISc Bangalore]

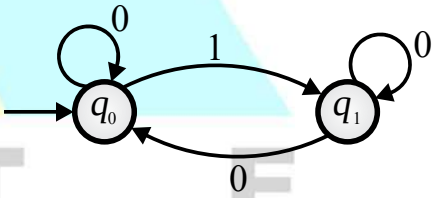
Q.31 Which one of the following languages over the alphabet {0, 1} is described by the regular expression (0 +1)* 0(0+1)* 0(0+1)*

(A) The set of all strings containing the substring 00
(B) The set of all strings containing at most two 0's
(C) The set of all strings containing at least two 0's
(D) The set of all strings that begin and end with either 0 or 1

[GATE 2009 : IIT Roorkee]

Self - Practice Questions

Q.1 Which of the following options represents regular expression accepted by above DFA '*M*'?



(A) (0+1)*1
(B) 0*1(0+00*1)*
(C) (0+10*0)*10*
(D) 1(0+1)*

Q.2 Which of the following language is/are NOT regular?

(A) $L = \{WxW^R \mid W, x \in \{0,1\}^*\}$
(B) $L = \{WxW \mid W, x \in \{0,1\}^*\}$
(C) $L = \{WxW \mid W, x \in \{0,1\}^+\}$
(D) $L = \{WW \mid W \in \{0,1\}^*\}$

Q.3 R.E. = (111+1111)*

Which of the following regular language represents above regular expression?

(A) $L=\{1^n \mid n \geq 0\}$

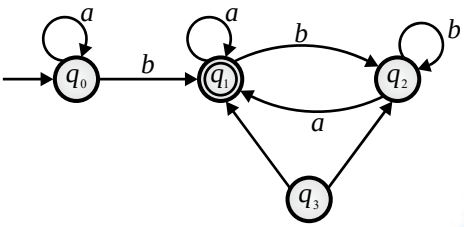
(B) $L=\{1^n \mid n \geq 0 \text{ and } n \neq 1,2\}$

(C) $L=\{1^n \mid n \geq 0 \text{ and } n \neq 1,2,5,7\}$

(D) $L=\{1^n \mid n \geq 0 \text{ and } n \neq 3,4\}$

Q.4 Regular expression $=(0+1)^*(0+1)(0+1)^*$ what is the minimum no. of states in DFA that accept above R.E. _____?

Q.5 The language accepted by above automaton is given by regular expression



(A) $a^*b(a+b)^*$

(B) $a^*ba^*bb^*aa^*$

(C) $a^*b(a+bb^*a)^*$

(D) $a^*b(bb+aa)^*$

Q.6 Which of the following two regular expression is/are equivalent?

(i) $(00)^*(0+\in)$ (ii) $(0+1)^*1$

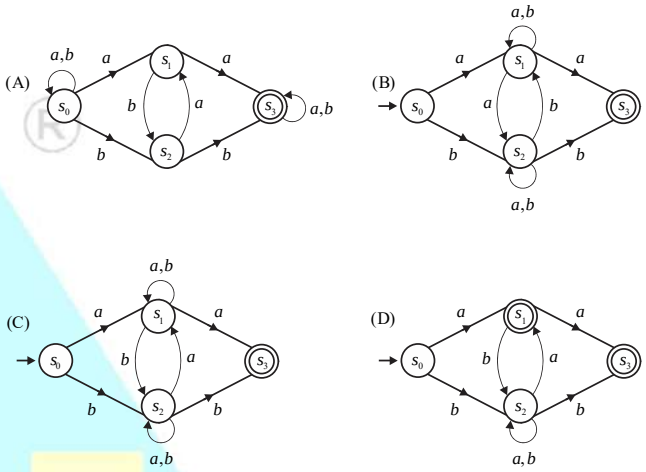
(ii) $(0^*1^*)^*$ (iv) $(0+1)^*$

(v) $0^*1(1+00^*1)^*$ (vi) $(0^*0^*)^*$

(A) (iii) and (iv) (B) (i) and (vi)

(C) (ii) and (v) (D) None

Q.7 Consider the regular expression. R=(a+b)*(aa+bb)(a+b)*. Which of the following non- deterministic finite automata recognizes the language defined by the regular expression R? Edges labeled λ denote transitions on the empty string.



Answer Keys

Classroom Practice Questions									
1.	A	2.	C	3.	D	4.	D	5.	C,D
6.	C	7.	A	8.	D	9.	C	10.	B
11.	C	12.	B	13.	B	14.	C	15.	3
16.	B	17.	B	18.	B	19.	3	20.	C
21.	B	22.	2	23.	4	24.	D	25.	C
26.	D	27.	D	28.	D	29.	B	30.	0*1*
31.	C								
Self - Practice Questions									
1.	A, B	2.	C, D	3.	B	4.	2	5.	C
6.	A, B, C	7.	A						

3 Grammar and Language and it's Application

Classroom Practice Questions

Q.1 If G is a context-free grammar and w is a string of length n in L(G), how long is derivation of w in G, if G is Chomsky normal form?

(A) 2n (B) 2n + 1

(C) 2n – 1 (D) n

[GATE 1992 : IIT Delhi]

Q.2 Which of the following definitions below generates the same language as L where $L = \{x^n y^n \mid n \geq 1\}$

(i) $E \rightarrow xEy \mid xy$

(ii) $xy \mid (x^+ xyy^+)$

(iii) $x^+ y^+$

(A) (i) only (B) (i) and (ii)

(C) (ii) and (iii) (D) (ii) only

[GATE 1995 : IIT Kanpur]

Q.3 Let L denote the language generated by the grammar $S \rightarrow 0S0 \mid 00$. Which of the following is true?

(A) L = 0^+

(B) L is regular but not 0^+

(C) L is context free but not regular

(D) L is not context free

[GATE 2000 : IIT Kharagpur]

Q.4 Let $L = L_1 \cap L_2$, where L_1 and L_1 are languages defined as follows,

$L_1 = \{a^m b^m c\, a^n b^n \mid m, n \geq 0\}$

$L_2 = \{a^i b^j c^k \mid i, j, k \geq 0\}$

Then L is

(A) Not recursive

(B) Regular

(C) Context free but not regular

(D) Recursively enumerable but not context free

[GATE 2009 : IIT Roorkee]

Q.5 Consider the following languages

$L_1 = \{0^p 1^q 0^r \mid p, q, r \geq 0\}$

$L_2 = \{0^p 1^q 0^r \mid p, q, r \geq 0, p \neq r\}$

Which one of the following statements is **FALSE**?

(A) L_2 is context-free

(B) $L_1 \cap L_2$ is context-free

(C) Complement of L_2 is recursive

(D) Complement of L_1 is context-free but not regular

[GATE 2013 : IIT Bombay]

Q.6 Which of the following languages is /are regular?

$L_1 : \{wxw^R \mid w, x \in \{a,b\}^* |w|, |x| > 0, w^R$ is the reverse of string w$\}$

$L_2 : \{a^n b^m \mid m \neq n \text{ and } m, n \geq 0\}$

$L_3 : \{a^p b^q c^r \mid p, q, r \geq 0\}$

(A) L_1 and L_3 only (B) L_2 only

(B) L_2 and L_3 only (D) L_3 only

[GATE 2015 : IIT Kanpur]

Q.7 Which of the following languages is generated by the given grammar?

$S \to aS \mid bS \mid \varepsilon$

(A) $\{a^n b^m \mid n, m \geq 0\}$

(B) $\{w \in \{a,b\}^* \mid w$ has equal number of a's and b's$\}$

(C) $\{a^n \mid n \geq 0\} \cup \{b^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$

(D) $\{a,b\}^*$

[GATE 2016 : IISc Bangalore]

Q.8 Language L_1 is defined by the grammar :

$S_1 \to aS_1 b \mid \varepsilon$

Language L_2 is defined by the grammar :

$S_2 \to abS_2 \mid \varepsilon$

Consider the following statements :

$P : L_1$ is regular

$Q : L_2$ is regular

Which one of the following is TRUE?

(A) Both P and Q are true

(B) P is true and Q is false

(C) P is false and Q is true

(D) Both P and Q are false

[GATE 2016 : IISc Bangalore]

Q.9 A context-free grammar is ambiguous if :

(A) The grammar contains useless non-terminals.

(B) It produces more than one parse tree for same sentence.

(C) Some production has two non-terminals side by side on the right-hand side.

(D) None of the above.

[GATE 1987 : IIT Bombay]

Q.10 FORTRAN is a :

(A) Regular language.

(B) Context free language.

(C) Context sensitive language.

(D) None of the above.

[GATE 1987 : IIT Bombay]

Q.11 Consider the following grammar G :

$S \to bS \mid aA \mid b$

$A \to bA \mid aB$

$B \to bB \mid aS \mid a$

Let $N_a(w)$ and $N_b(w)$ denote the number of a's and b's in a string w respectively. The language $L(G) \subseteq \{a,b\}^+$ generated by G is

(A) $\{w \mid N_a(w) > 3N_b(w)\}$

(B) $\{w \mid N_b(w) > 3N_a(w)\}$

(C) $\{w \mid N_a(w) = 3k, k \in \{0,1,2,...\}\}$

(D) $\{w \mid N_b(w) = 3k, k \in \{0,1,2,...\}\}$

[GATE 2004 : IIT Delhi]

Q. 12 Which of the following statements are true?

1. Every left-recursive grammar can be converted to a right-recursive grammar and vice-versa.
2. All ε-productions can be removed from any context-free grammar by suitable transformations.
3. The language generated by a context-free grammar all of whose productions are of the form $X \to w$ or $X \to wY$ (where, w is a string of terminals and Y is a non-terminal), is always regular.
4. The derivation trees of strings generated by a context-free grammar in Chomsky Normal form are always binary trees.

(A) 1, 2, 3 and 4

(B) 2, 3 and 4 only

(C) 1, 3 and 4 only

(D) 1, 2 and 4 only

[GATE 2008 : IISc Bangalore]

Q.13 Match the following **List-I** with **List-II**

List-I

(E) Checking that identifiers are declared before their use

(F) Number of formal parameters in the declaration of a function agrees with the number of actual parameters in a use of that function

(G) Arithmetic expressions with matched pairs of parentheses

(H) Palindromes

List-II

(P) $L = \{a^n b^m c^n d^m \mid n \geq 1, m \geq 1\}$

(Q) $X \rightarrow XbX \mid XcX \mid dXf \mid g$

(R) $L = \{wcw \mid w \in (a|b)^*\}$ (S)

$X \rightarrow bXb \mid cXc \mid \varepsilon$

Codes :

(A) $E-P, F-R, G-Q, H-S$

(B) $E-R, F-P, G-S, H-Q$

(C) $E-R, F-P, G-Q, H-S$

(D) $E-P, F-R, G-S, H-Q$

[GATE 2008 : IISc Bangalore]

Q. 14 $S \rightarrow aSa \mid bSb \mid a \mid b$

The language generated by the above grammar over the alphabets {a, b} is the set of

(A) All palindromes

(B) All odd length palindromes

(C) Strings that begin and end with the same symbol

(D) All even length palindromes

[GATE 2009 : IIT Roorkee]

Q.15 Consider the languages

$L_1 = \{0^i 1^j \mid i \neq j\}, L_2 = \{0^i 1^j \mid i = j\},$

$L_3 = \{0^i 1^j \mid i = 2j+1\},$

$L_4 = \{0^i 1^j \mid i \neq 2j\},$

Which one of the following statement is true?

(A) Only L_2 is context free

(B) Only L_2 and L_3 are context free

(C) Only L_1 and L_2 are context free

(D) All are context free

[GATE 2010 : IIT Guwahati]

Q.16 Which of the following languages are context-free?

$L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}$

$L_2 = \{a^m b^n a^m b^n \mid m, n \geq 1\}$

$L_3 = \{a^m b^n \mid m = 2n+1\}$

(A) L_1 and L_2 only (B) L_1 and L_3 only

(C) L_2 and L_3 only (D) L_3 only

[GATE 2015 : IIT Kanpur]

Q.17 Consider the following context-free grammars :

$G_1 : S \rightarrow aS \mid B, B \rightarrow b \mid bB$

$G_2 : S \rightarrow aA \mid bB, A \rightarrow aA \mid B \mid \varepsilon, B \rightarrow bB \mid \varepsilon$

Which one of the following pairs of languages is generated by G_1 and G_2, respectively?

(A) $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$

(B) $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ or } n \geq 0\}$

(C) $\{a^m b^n \mid m \geq 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$

(D) $\{a^m b^n \mid m \geq 0 \text{ and } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$

[GATE 2016 : IISc Bangalore]

Q.18 Consider the following context-free grammar over the alphabet $\sum = \{a, b, c\}$ with S as the start symbol :

$S \rightarrow abScT \mid abcT$

$T \rightarrow bT \mid b$

Which one of the following represents the language generated by the above grammar?

(A) $\{(ab)^n (cb)^n \mid n \geq 1\}$

(B) $\{(ab)^n cb^{m_1} cb^{m_2} \ldots cb^{m_n} \mid n, m_1, m_2, \ldots, m_n \geq 1\}$

(C) $\{(ab)^n (cb^m)^n \mid m, n \geq 1\}$

(D) $\{(ab)^n (cb^n)^m \mid m, n \geq 1\}$

[GATE 2017 : IIT Roorkee]

Q.19 Identify the language generated by the following grammar, where S is the start variable.

$S \to XY$

$X \to aX \mid a$

$Y \to aYb \mid \varepsilon$

(A) $\{a^m b^n \mid m \geq n, n > 0\}$

(B) $\{a^m b^n \mid m \geq n, n \geq 0\}$

(C) $\{a^m b^n \mid m > n, n \geq 0\}$

(D) $\{a^m b^n \mid m > n, n > 0\}$

[GATE 2017 : IIT Roorkee]

Q. 20 If G is a grammar with productions

$S \to SaS \mid aSb \mid bSa \mid SS \mid \varepsilon$

Where S is the start variable, then which one of the following string is not generated by G?

(A) abab (B) aaab

(C) abbaa (D) babba

[GATE 2017 : IIT Roorkee]

Q.21 Consider the context-free grammar over the alphabet {a, b, c} given below. S and T are non-terminals.

$G_1 : S \to aSb \mid T, T \to cT \mid \varepsilon$

$G_2 : S \to bSa \mid T, T \to cT \mid \varepsilon$

The language $L(G_1) \cap L(G_2)$ is

(A) Finite

(B) Not finite but regular

(C) Context free but not regular

(D) Recursive but not context free

[GATE 2017 : IIT Roorkee]

Q.22 Which one of following kinds of derivation is used by LR parsers?

(A) Leftmost

(B) Leftmost in reverse

(C) Rightmost

(D) Rightmost in reverse

[GATE 2019 : IIT Madras]

Q.23 Consider the grammar given below:

S → Aa

A → BD

B → b | ε

D → d | ε

Let a,b,d, and $ be indexed as follows:

a	b	d	$
3	2	1	0

Compute the FOLLOW set of the non-terminal B and write the index values for the symbols in the FOLLOW set in the descending order. (For example, if the FOLLOW set is {a, b, d,$}, then the answer should be 3210)

[GATE 2019 : IIT Madras]

Q.24 Consider the augmented grammar given below :

$S' \to S$

$S \to \langle L \rangle \mid \text{id}$

$L \to L, S \mid S$

Let $I_0 = \text{CLOSURE}(\{[S' \to \bullet S]\})$.

The number of items in the set GOTO $(I_0, \langle\ )$ is : _____.

[GATE 2019 : IIT Madras]

Q.25 The C language is

(A) A context free language

(B) A context sensitive language

(C) A regular language

(D) Parsable fully only by a Turing machine

[GATE 2002 : IISc Bangalore]

Q.26 Consider the language $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and the following statements.

I. L is deterministic context-free.

II. L is context-free but not deterministic context-free.

III. L is not LL(k) for any k.

Which of the above statements is/are TRUE?

(A) I only (B) II only

(C) I and III only (D) III only

[GATE 2002 : IISc Bangalore]

Q.27 Which of the following features cannot be captured by context-free grammar?

(A) Syntax of if-then-else statements

(B) Syntax of recursive procedures

(C) Whether a variable has been declared before its use

(D) Variable names of arbitrary length.

[GATE 2019 : IIT Madras]

Q.28 The lexical analysis for a modern computer language such as java needs the power of which one of the following machine models in a necessary and sufficient sense?

(A) Finite state automata
(B) Deterministic pushdown automata
(C) Non-deterministic pushdown automata
(D) Turing machine

[GATE 2011 : IIT Madras]

Self - Practice Questions

Q.1 $G: S \to aS|bA$

$A \to aA|bC|\in$

$C \to aC|bS$

Language accepted by above grammar is

(A) $L(G) = (a+b)^*$

(B) $L(G) = \{W \in \{a,b\}^* | \text{Number of } b\text{'s are odd}\}$

(C) $L(G) = \{W \in \{a,b\}^* | \#_b(W) \bmod 3 = 0\}$

(D) $L(G) = \{W \in \{a,b\}^* | \#_b(W) \bmod 3 = 1\}$

Q.2 $G: S \to AB|BC$

$B \to aAb|\in$

$A \to aA|a$

$C \to bC|a$

Language accepted by above grammar is

(A) $L(G) = \{a^n b^n | n \geq 1\}$

(B) $L(G) = \{a^n b^m | n > m\}$

(C) $L(G) = \{a^n b^m | m > n\}$

(D) $L(G) = \{a^n b^m | n \neq m\}$

Q.3 Which of the following CFLS is represented by following grammar?

$S \to AB / CD$
$C \to 0C / \in$
$A \to 0A1 / \in$
$D \to 1D2 / \in$
$B \to 0B / \in$

(A) $\{0^i 1^j 2^k | i = j < k \text{ OR } i > j = k\}$

(B) $\{0^i 1^j 2^k | i = j \text{ OR } j = k\}$

(C) $\{0^i 1^j 2^k | i = j = k \text{ OR } i > j = k\}$

(D) None.

Q.4 Consider the following grammar.

$G_1: S \to bA|aB$
$A \to aS|bAA|a$
$B \to bS|bBB|b$

$G_2: S \to aSbS|bSaS|\in$

Which one is /are TRUE?

(A) $L(G_1) = L(G_2)$

(B) $L(G_1) \subseteq L(G_2)$

(C) $L(G_1) \cap L(G_2) = \phi$

(D) $L(G_2) \cup \{\in\} = L(G_1)$

Q.5 Consider following grammar:-

$G: \{ S \to aA,$
$A \to aA|bA$
$B \to bB|cC|c$
$C \to cC|c \quad \}$

The $L(G_1)$?

(A) $a^*b^*c^*$ (B) $aa^*b^*c^*$

(C) $aa^*bb^*cc^*$ (D) $(abc)^*$

Q.6 Let L be a regular language and M be a context free language, both over the alphabet Σ. Let L^c and M^c denote the complements of L and M respectively. Which of the following statements about the language $L^c \cup M^c$ is TRUE?

(A) It is necessarily regular but not necessarily context free
(B) It is necessarily context free
(C) It is necessarily non- regular
(D) None of the above

Common Data for Q.7 & Q.8 Questions

Consider the context- free grammar

$E \to E+E$
$E \to (E*E)$
$E \to id$

Where E is the starting symbol, the set of terminals is $\{id,(,+,),*\}$, and the set of non-terminals is {E}.

Q.7 Which of the following terminal strings has more than one parse tree when parsed according to the above grammar?

(A) $id+id+id+id$
(B) $id+(id*(id*id))$
(C) $(id*(id*id))+id$
(D) $((id*id+id)*id)$

Q.8 For the terminal string with more than one parse tree obtained as solution to Q. 8A. How many parse trees are possible?

(A) 5 (B) 4

(C) 3 (D) 2

Q.9 Let L be a context-free language and M a regular language. Then the language $L \cap M$ is

(A) Always regular

(B) Never regular

(C) Always a deterministic context-free language

(D) Always a context-free language

Q.10 Consider an ambiguous grammar G and its disambiguated version D. Let the language recognized by the two grammars be denoted by $L(G)$ and $L(D)$ respectively.

(A) $L(D) \subset L(G)$ (B) $L(D) \supset L(G)$

(C) $L(D) = L(G)$ (D) $L(D)$ is empty

Q.11 The two grammars given below generate a language over the alphabet $\{x, y, z\}$

$G1: S \to x \mid z \mid xS \mid zS \mid yB$

$B \to y \mid z \mid yB \mid zB$

$G1: S \to y \mid z \mid yS \mid zS \mid xB$

$B \to y \mid yS$

Which of the following choices describes the properties satisfied by the strings in these languages?

(A) G1: No y appears before any x
G2: Every x is followed by at least one y

(B) G1: No y appears before any x
G2: No x appears before any y

(C) G1: No y appears after any x
G2: Every x is followed by at least one y

(D) G1: No y appears after any x
G2: Every y is followed by at least one x

Q.12 Consider the grammar

$S \to ABCc \mid bc$

$BA \to AB$

$Bb \to bb$

$Ab \to ab$

$Aa \to aa$

Which of the following sentence can be derived by this grammar?

(A) abc (B) aab

(C) abcc (D) abbc

Answer Keys

Classroom Practice Questions									
1.	C	2.	A	3.	B	4.	C	5.	D
6.	A	7.	D	8.	C	9.	B	10.	B
11.	C	12.	C	13.	C	14.	B	15.	D
16.	B	17.	D	18.	B	19.	C	20.	C
21.	B	22.	D	23.	31	24.	5	25.	B
26.	C	27.	C	28.	A				
Self - Practice Questions									
1.	D	2.	D	3.	A	4.	B	5.	C
6.	A	7.	A	8.	A	9.	D	10.	C
11.	A	12.	A						

4 Push Down Automata

Classroom Practice Questions

Q.1 Context free languages and regular languages are both closed under the operation(s) of :
(A) Union
(B) Intersection
(C) Concatenation
(D) Complementation

[GATE 1989 : IIT Kanpur]

Q.2 Context-free languages are
(A) Closed under union
(B) Closed under complementation
(C) Closed under intersection
(D) Closed under Kleene closure.

[GATE 1992 : IIT Delhi]

Q.3 If L_1 and L_2 are context free languages and R is a regular set, one of the languages below is not necessarily a context free language. Which one?
(A) L_1L_2 (B) $L_1 \cap L_2$
(C) $L_1 \cap R$ (D) $L_1 \cup L_2$

[GATE 1996 : IISc Bangalore]

Q.4 Which of the following languages over {a, b, c} is accepted by a deterministic push down automata?
(A) $\{wcw^R \mid w \in \{a,b\}^*\}$
(B) $\{ww^R \mid w \in \{a,b,c\}^*\}$
(C) $\{a^n b^n c^n \mid n \geq 0\}$
(D) $\{w \mid w$ is a palindrome over {a, b, c}$\}$

[GATE 1997 : IIT Madras]

Q.5 Context free languages are closed under :
(A) Union, intersection
(B) Union, Kleene closure
(C) Intersection, complement
(D) Complement, Kleene Closure

[GATE 1999 : IIT Bombay]

Q.6 Let L_D be the set of all languages accepted by a PDA by final state and L_E the set of all languages accepted by empty stack. Which of the following is true?
(A) $L_D = L_E$
(B) $L_D \supset L_E$
(C) $L_D \subset L_E$
(D) None of the above

[GATE 1999 : IIT Bombay]

Q.7 If L_1 is a context free language and L_2 is a regular language which of the following is/are false?
(A) $L_1 - L_2$ is not context free
(B) $L_1 \cap L_2$ is context free
(C) $\sim L_1$ is context free
(D) $\sim L_2$ is regular

[GATE 1999 : IIT Bombay]

Q.8 Which of the following statement is true?
(A) If a language is context free it can always be accepted by deterministic pushdown automaton
(B) The union of two context free languages is context free
(C) The intersection of two context free languages is context free
(D) The complement of a context free language is context free

[GATE 2001 : IIT Kanpur]

Q.9 The language accepted by a Pushdown Automation in which the stack is limited to 10 items is best described as
(A) Context free
(B) Regular
(C) Deterministic Context Free
(D) Recursive

[GATE 2002 : IISc Bangalore]

Q.10 Let $G = (\{S\}, \{a,b\}, R, S)$ be a context free grammar where the rule set R is

$$S \to aSb \mid S\,S \mid \varepsilon$$

Which of the following statements is true?

(A) G is not ambiguous

(B) There exist $x, y \in L(G)$ such that $xy \notin L(G)$

(C) There is a deterministic pushdown automaton that accepts $L(G)$

(D) We can find a deterministic finite state automaton that accepts $L(G)$

[GATE 2003 : IIT Madras]

Q.11 The language $\{a^m b^n c^{m+n} \mid m, n \geq 1\}$ is

(A) Regular

(B) Context free but not regular

(C) Context sensitive but not context free

(D) Type-0 but not context sensitive

[GATE 2004 : IIT Delhi]

Q.12 Let $M = (K, \sum, F, \Delta, S, F)$ be a pushdown automaton.

Where, K= {s, f}, F = {f},

$\sum = \{a,b\}, F = \{a\}$ and

$$\Delta = \{((s,a,\varepsilon),(s,a)), ((s,b,\varepsilon),(s,a)), ((s,a,\varepsilon), (f,\varepsilon)), ((f,a,a),(f,\varepsilon)), ((f,b,a),(f,\varepsilon))\}.$$

Which one of the following strings is not a member of L(M) ?

(A) aaa (B) aabab

(C) baaba (D) bab

[GATE 2004 : IIT Delhi]

Q.13 Let N_f and N_p denote the classes of languages accepted by non-deterministic finite automata and non-deterministic push-down automata, respectively. Let D_f and D_p denote the classes of languages accepted by deterministic finite automata and deterministic push-down automata, respectively. Which one of the following is TRUE?

(A) $D_f \subset N_f$ *and* $D_p \subset N_P$

(B) $D_f \subset N_f$ *and* $D_p = N_P$

(C) $D_f = N_f$ *and* $D_p = N_P$

(D) $D_f = N_f$ *and* $D_p \subset N_P$

[GATE 2005 : IIT Bombay]

Q.14 Consider the languages :

$L_1 = \{a^n b^n c^m \mid n, m > 0\}$ and

$L_2 = \{a^n b^n c^m \mid n, m > 0\}$

Which one of the following statement is FALSE?

(A) $L_1 \cap L_2$ is a context-free language

(B) $L_1 \cup L_2$ is a context-free language

(C) L_1 and L_2 are context-free languages

(D) $L_1 \cap L_2$ is a context sensitive language

[GATE 2005 : IIT Bombay]

Q.15 Consider the languages :

$L_1 = \{w w^R \mid w \in \{0,1\}^*\}$

$L_2 = \{w \# w^R \mid w \in \{0,1\}^*\}$ where # is a special symbol

$L_3 = \{w w \mid w \in \{0,1\}^*\}$

Which one of the following is TRUE?

(A) L_1 is a deterministic CFL

(B) L_2 is a deterministic CFL

(C) L_3 is a CFL, but not a deterministic CFL

(D) L_3 is a deterministic CFL

[GATE 2005 : IIT Bombay]

Q.16 Let $L_1 = \{0^{n+m} 1^n 0^m \mid n, m \geq 0\}$,

$L_2 = \{0^{n+m} 1^{n+m} 0^m \mid n, m \geq 0\}$, and

$L_3 = \{0^{n+m} 1^{n+m} 0^{n+m} \mid n, m \geq 0\}$

Which of these languages are NOT context free?

(A) L_1 only (B) L_3 only

(C) L_1 *and* L_2 (D) L_2 *and* L_3

[GATE 2006 : IIT Kharagpur]

Q.17 Consider the following statements about the context-free grammar $G = \{S \to SS,\ S \to ab,\ S \to ba,\ S \to \varepsilon\}$

1. G is ambiguous
2. G produces all strings with equal number of a's and b's
3. G can be accepted by a deterministic PDA.

Which combination below expresses all the true statements about G?

(A) 1 only (B) 1 and 3 only

(C) 2 and 3 only (D) 1, 2 and 3

[GATE 2006 : IIT Kharagpur]

Q.18 The language $L = \{0^i 2^i 1^i \mid i \geq 0\}$ over the alphabet $\{0, 1, 2\}$ is

(A) Not recursive.

(B) Is recursive and is a deterministic CFL.

(C) Is a regular language.

(D) None

[GATE 2007 : IIT Kanpur]

Q. 19 Which one of the following is FALSE?

(A) There is a unique minimal DFA for every regular language

(B) Every NFA can be converted to an equivalent PDA

(C) Complement of every context free language is recursive

(D) Every non deterministic PDA can be converted to an equivalent deterministic PDA

[GATE 2009 : IIT Roorkee]

Q.20 Consider the languages L_1, L_2 and L_3 as given below.

$L_1 = \{0^p 1^q \mid p, q \in N\}$

$L_2 = \{0^p 1^q \mid p, q \in N \text{ and } p = q\}$ and

$L_3 = \{0^p 1^q 0^r \mid p, q, r \in N \text{ and } p = q = r\}$

Which of the following statements is not TRUE?

(A) Pushdown automata (PDA) can be used to recognize L_1 and L_2

(B) L_1 is a regular language

(C) All the three languages are context free

(D) Turing machines can be used to recognize all the languages.

[GATE 2011 : IIT Madras]

Q.21 Consider the following languages over the alphabet $\Sigma = \{0, 1, c\}$:

$L_1 = \{0^n 1^n \mid n \geq 0\}$

$L_2 = \{wcw^r \mid w \in \{0,1\}^*\}$

$L_3 = \{ww^r \mid w \in \{0,1\}^*\}$

Here, w^r is the reverse of the string w. Which of these language are deterministic context-free languages?

(A) None of the languages

(B) Only L_1

(C) Only L_1 and L_2

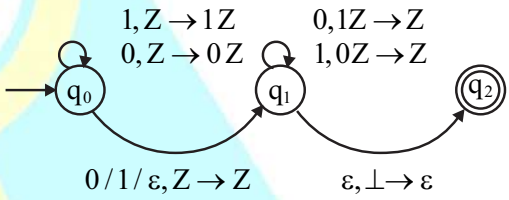
(D) All the three languages.

[GATE 2011 : IIT Madras]

Q.22 Consider the NPDA

$(Q = \{q_0, q_1, q_2\}, \Sigma = \{0,1\},$
$\Gamma = \{0,1,\perp\}, \delta, q_0,$
$\perp, F = \{q_2\}),$

where (as per usual convention) Q is the set of states, Σ is the input alphabet, Γ is the stack alphabet, δ is the state transition function, q_0 is the initial state, $\perp$ is the initial stack symbol, and F is the set of accepting states. The state transition is as follows :



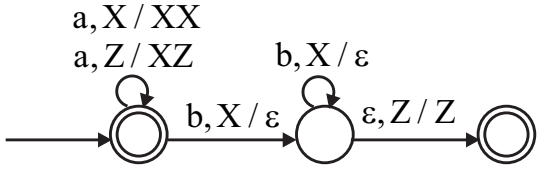
Which one of the following sequences must follow the string 101100 so that the overall string is accepted by the automaton?

(A) 10110 (B) 10010

(C) 01010 (D) 01001

[GATE 2015 : IIT Kanpur]

Q.23 Consider the transition diagram of a PDA given below with input alphabet $\Sigma = \{a, b\}$ and stack alphabet $\Gamma = \{X, Z\}$. Z is the initial stack symbol. Let L denote the language accepted by the PDA.



Which one of the following is TRUE?

(A) $L = \{a^n b^n \mid n \geq 0\}$ and is not accepted by any finite automata

(B) $L=\{a^n \mid n\geq 0\}\cup\{a^n b^n \mid n\geq 0\}$ and is not accepted by any deterministic PDA

(C) L is not accepted by any Turning machine that halts on every input

(D) $L=\{a^n \mid n\geq 0\}\cup\{a^n b^n \mid n\geq 0\}$ and is deterministic context-free

[GATE 2016 : IISc Bangalore]

Q.24 Consider the following languages :

$L_1=\{a^n b^m c^{n+m} \mid m,n\geq 1\}$

$L_2=\{a^n b^n c^{2n} \mid n\geq 1\}$

Which one of the following is TRUE?

(A) Both L_1 and L_2 are context free.

(B) L_1 is context free while L_2 is not context free.

(C) L_2 is context free while L_1 is not context free.

(D) Neither L_1 nor L_2 is context free.

[GATE 2016 : IISc Bangalore]

Q.25 Let L_1, L_2 be any two context-free languages and R be any regular language. Then which of the following is/are CORRECT?

(i) $L_1 \cup L_2$ is context free

(ii) $\overline{L_1}$ is context free

(iii) $L_1 - R$ is context free

(iv) $L_1 \cap L_2$ is context free

(A) (i), (ii) and (iv) only

(B) (i) and (iii) only

(C) (ii) and (iv) only

(D) (i) only

[GATE 2017 : IIT Roorkee]

Q.26 Consider the following language over the alphabet $\Sigma=\{a,b,c\}$.

Let $L_1=\{a^n b^n c^m \mid m,n\geq 0\}$ and

$L_2=\{a^m b^n c^n \mid m,n\geq 0\}$.

Which of the following are context-free languages?

I. $L_1 \cup L_2$

II. $L_1 \cap L_2$

(A) I only

(B) II only

(C) I and II

(D) Neither I nor II

[GATE 2017 : IIT Roorkee]

Q.27 Consider the following languages.

$L_1=\{a^p \mid p \text{ is a prime number}\}$

$L_2=\{a^n b^m c^{2m} \mid n\geq 0, m\geq 0\}$

$L_3=\{a^n b^n c^{2n} \mid n\geq 0\}$

$L_4=\{a^n b^n \mid n\geq 1\}$

Which of the following are CORRECT?

I. L_1 is context free but not regular

II. L_2 is not context –free.

III. L_3 is not context-free but recursive

IV. L_4 is deterministic context-free

(A) I, II and IV only

(B) II and III only

(C) I and IV only

(D) III and IV only

[GATE 2017 : IIT Roorkee]

Q.28 Consider the following languages :

I. $\left\{a^m b^n c^p d^q \mid m+p=n+q, \text{ where } m,n,p,q\geq 0\right\}$

II. $\left\{a^m b^n c^p d^q \mid m=n \text{ and } p=q, \text{ where } m,n,p,q\geq 0\right\}$

III. $\left\{a^m b^n c^p d^q \mid m=n=p \text{ and } p\neq q, \text{ where } m,n,p,q\geq 0\right\}$

IV. $\left\{a^m b^n c^p d^q \mid mn=p+q, \text{ where } m,n,p,q\geq 0\right\}$

Which of the languages above are context-free?

(A) I and IV only (B) I and II only

(C) II and III only (D) II and IV only

[GATE 2018 : IIT Guwahati]

Q.29 Which one of the following languages over $\Sigma=\{a,b\}$ is NOT context-free?

(A) $\{ww^R \mid w\varepsilon\{a,b\}^*\}$

(B) $\{wa^n b^n w^R \mid w\varepsilon\{a,b\}^*, n\geq 0\}$

(C) $\{wa^n w^R b^n \mid w\varepsilon\{a,b\}^*, n\geq 0\}$

(D) $\{a^n b^i \mid i\varepsilon\{n,3n,5n\}^*, n\geq 0\}$

[GATE 2019 : IIT Madras]

Q.30 Consider the following languages.

$L_1 = \{wxyx \mid w, x, y \in (0+1)^+\}$

$L_2 = \{xy \mid x, y \in (a+b)^*, |x| = |y|, x \neq y\}$

Which one of the following is TRUE?

(A) L_1 is regular and L_2 is context-free.

(B) L_1 is context-free but not regular and L_2 is context-free.

(C) Neither L_1 nor L_2 is context-free.

(D) L_1 is context-free but L_2 is not context-free.

[GATE 2020 : IIT Delhi]

Self - Practice Questions

Q.1 Which of the following language can't be solved by deterministic PDA?

(A) $L = \{a^n b^n \mid n \geq 1\}$

(B) $L = \{a^n b^m c^m \mid n \geq 0 \text{ and } m \geq 1\}$

(C) $L = \{a^n b^n \mid n \geq 0\} \cup \{a^n b^{2n} \mid n \geq 0\}$

(D) $L = \{a^n b^m \mid n \neq m\}$

Q.2 Which of the following options OR statement is/ are correct/

$S1: \{a^n b^n c^m \mid n \leq m \leq 2n\}$ is CFL

$S1: \{a^i b^j \mid i = j \text{ OR } i = 2j\}$ is DCFL.

(A) $S1$ only (B) $S2$ only

(C) Both (D) None.

Q.3 Consider the following grammar:-

$G_1 : S \to aSa \mid a$

$G_2 : S \to aSa \mid b$

Which of the options is / are TRUE

(A) $L(G_1)$ and $L(G_2)$ are regular

(B) $L(G_1)$ is regular but $L(G_2)$ is CFL

(C) Both $L(G_1)$ and $L(G_2)$ are CFL

(D) $L(G_2)$ is regular but $L(G_1)$ is CFL

Q.4 Equivalent CFL for

$G = \{S \to aS \mid aSbS \mid \in\}$

(A) $L = \{X \mid X$ is a palindromes$\}$

(B) $L = \{X \mid X = a^n b^n \forall n \geq 0\}$

(C) $L = \{X \mid$ each prefix oF 'X' has atleast as many a' s as b's$\}$

(D) $L = \{X \mid X$ has equal no. of a's and b's$\}$

Q.5 Which of the following options is/are CFL?

(A) $L = \{ww \mid w \in \{a,b\}^*\}$

(B) $L = \{w_1 w_2 \mid w_1 \neq w_2 \text{ and } w_1, w_2 \in \{a,b\}^*\}$

(C) $L = \{a^n b^m c^n d^m \mid n, m \geq 0\}$

(D) $L = \{x\,w\,y\,w \mid a, y, w \in \{a,b\}^+\}$

Q.6 Which of the following options is/are TURE?

(A) $L = \{a^i b^j \mid i \geq 0, j = n^2\}$ is not CFL

(B) For grammar $G = \{S \to aSb \mid SS \mid \in\}$ The $L(G)$ can be solved by DPDA.

(C) $L = \{w\,x^n w^R y^n \mid w \in \{x,y\}^*\}$ is CRL.

(D) For $L = \{ww \mid w \in \{a,b\}^*\}$, $\bar{L}$ is not CFL.

Q.7 Consider the regular grammar below

$S \to bS \mid aA \mid \varepsilon$

$A \to aS \mid bA$

The Myhill- Nerode equivalence classes for the language generated by the grammar are

(A) $\begin{Bmatrix} \{w \varepsilon (a+b)^* \mid \#_a(w) \text{ is even}\} \text{and} \\ \{w \varepsilon (a+b)^* \mid \#_a(w) \text{ is odd}\} \end{Bmatrix}$

(B) $\begin{Bmatrix} \{w \varepsilon (a+b)^* \mid \#_b(w) \text{ is even}\} \text{and} \\ \{w \varepsilon (a+b)^* \mid \#_b(w) \text{ is odd}\} \end{Bmatrix}$

(C) $\begin{Bmatrix} \{w \in (a+b)^* \mid \#_a(w) = \#_a(w)\} \\ \text{and } (w \in (a+b)^* \mid \#_a(w) \text{is even}\} \text{and} \\ \{w \varepsilon (a+b)^* \mid \#_a(w) \neq \#_b(w)\} \end{Bmatrix}$

(D) $\{\varepsilon\}, \begin{Bmatrix} wa \mid w \in (a+b)^* \text{ and} \\ wb \mid w \in (a+b)^* \end{Bmatrix}$

Q.8 Consider the pushdown automaton (PDA) below which runs over the input alphabet (a, b, c). It has the stack alphabet $\{Z_0, X\}$ where Z_0 is the bottom-of-stack market. The set of states of the PDA is {s, t, u, f} where s is the start and f is the final state. The PDA accepts by final state. The transitions of the PDA given below are depicted in a standard manner. For

example, the transition $(s,b,X)\rightarrow(t,XZ_0)$ means that if the PDA is in state a and the symbol on the top of the stack is X, then it can read b from the input and move to state t after popping the top of stack and pushing the symbols Z_0 and X (in that order) on the stack.

$(s,a,Z_0)\rightarrow(s,XXZ_0)$

$(s,\varepsilon,Z_0)\rightarrow(f,\varepsilon)$

$(s,a,X)\rightarrow(s,XXX)$

$(s,b,X)\rightarrow(t,\varepsilon)$

$(t,b,X)\rightarrow(t,\varepsilon)$

$(t,c,X)\rightarrow(u,\varepsilon)$

$(u,c,X)\rightarrow(u,\varepsilon)$

$(u,\varepsilon,Z_0)\rightarrow(f,\varepsilon)$

The language accepted by the PDA is

(A) $\{a^\ell b^m c^n \mid \ell=m=n\}$

(B) $\{a^\ell b^m c^n \mid \ell=m\}$

(C) $\{a^\ell b^m c^n \mid 2\ell=m+n\}$

(D) $\{a^\ell b^m c^n \mid m=n\}$

Q.9 In the context-free grammar below, S is the start symbol, a and b are terminals, and ε denotes the empty string.

$S\rightarrow aSAb\mid\varepsilon$

$A\rightarrow bA\mid\varepsilon$

The grammar generates the language

(A) $\left((a+b)^*b\right)^*$ (B) $\{a^m b^n \mid m\le n\}$

(C) $\{a^m b^n \mid m=n\}$ (D) a^*b^*

Q.10 Which of the following languages is accepted by a non-deterministic pushdown automaton (PDA) but NOT by a deterministic PDA?

(A) $\{a^n b^n c^n \mid n\ge 0\}$

(B) $\{a^\ell b^m c^n \mid \ell\ne m \text{ or } m\ne n\}$

(C) $\{a^n b^n \mid n\ge 0\}$

(D) $\{a^m b^n \mid m,n\ge 0\}$

Q.11 Consider the following languages.

$L_1=\{a^i b^j c^k \mid i=j,k\ge 1\}$

$L_1=\{a^i b^j \mid j=2i,i\ge 0\}$

Which of the following is true?

(A) L_1 is not a CFL but L_2 is

(B) $L_1\cap L_2=\phi$ and L_1 is non-regular

(C) $L_1\cup L_2$ is not a CFL but L_2

(D) There is a 4-state PDA that accepts L_1, but there is no DPDA that accepts L_2

Q.12 Which of the following languages is (are) non-regular?

$L_1=\{0^m1^n \mid 0\le m\le n\le 10000\}$

$L_2=\{w \mid w \text{ reads the same forward and backward}\}$

$L_3=\{w\in\{0,1\}^* \mid w \text{ contains an even number of 0's and an even number of 1's}\}$

(A) L_2 and L_3 only (B) L_1 and L_2 only

(C) L_3 only (D) L_2 only

Answer Keys

Classroom Practice Questions									
1.	A, C	2.	A, D	3.	B	4.	A	5.	B
6.	A	7.	A	8.	B	9.	B	10.	C
11.	B	12.	B	13.	D	14.	A	15.	B
16.	D	17.	B	18.	D	19.	D	20.	D
21.	C	22.	B	23.	D	24.	B	25.	B
26.	A	27.	D	28.	B	29.	C	30.	A
Self - Practice Questions									
1.	C	2.	D	3.	B	4.	C	5.	B, D
6.	A, C	7.	A	8.	C	9.	B	10.	B
11.	B	12.	D						

❖❖❖❖

5 TM and Undecidability

Classroom Practice Questions

Q.1 Let L_1 be a recursive language. Let L_2 and L_3 be languages that are recursively enumerable but not recursive. Which of the following statement is not necessarily true?

(A) $L_2 - L_1$ is recursively enumerable

(B) $L_1 - L_3$ is recursively enumerable.

(C) $L_2 \cap L_3$ is recursively enumerable.

(D) $L_2 \cup L_3$ is recursively enumerable.

[GATE 2010 : IIT Guwahati]

Q.2 Consider the following decision problems :

P_1 : Does a given finite state machine accept a given string.

P_1 : Does a given context free grammar generate an infinite number of strings.

Which of the following statement is true?

(A) Both (P_1) and (P_2) are decidable

(B) Neither (P_1) nor (P_2) are decidable

(C) Only (P_1) is decidable

(D) Only (P_2) is decidable

[GATE 2000 : IIT Kharagpur]

Q. 3 Which of the following statement is false?

(A) Every NFA can be converted to an equivalent DFA.

(B) Every non-deterministic Turing machine can be converted to an equivalent deterministic Turing machine.

(C) Every regular language is also a context free language.

(D) Every subset of a recursively enumerable set is recursive.

[GATE 2007 : IIT Kanpur]

Q.4 Recursive languages are :

(A) A proper superset of context free languages.

(B) Always recognizable by pushdown automata.

(C) Also called type (0) languages.

(D) Recognizable by Turing machines.

[GATE 1990 : IISc Bangalore]

Q.5 In which of the cases stated below is the following statement true?

"For every nondeterministic machine M_1 there exists an equivalent deterministic machine M_2 recognizing the same language".

(A) M_1 is nondeterministic finite automation

(B) M_1 is a nondeterministic PDA

(C) M_1 is a nondeterministic Turing machine

(D) For no machine M_1 use the above statement true

[GATE 1992 : IIT Delhi]

Q.6 Which of the following conversions is not possible (algorithmically)?

(A) Regular grammar to context-free grammar

(B) Non-deterministic FSA to deterministic FSA

(C) Non-deterministic PDA to deterministic PDA

(D) Non-deterministic Turing machine to deterministic Turing machine.

[GATE 1994 : IIT Kharagpur]

Q.7 Which one of the following is not decidable?

(A) Given a Turing machine M, a string s and an integer k, M accepts s within k steps

(B) Equivalence of two given Turing machines

(C) Language accepted by a given finite state machine is non empty

(D) Language generated by a context free grammar is non empty

[GATE 1997 : IIT Madras]

Q.8 Regarding the power of recognition of languages, which of the following statement is false?

(A) The non-deterministic finite-state automata are equivalent to deterministic finite-state automata.

(B) Non- deterministic Push-down automata are equivalent to deterministic Push-down automata.

(C) Non-deterministic Turing machines are equivalent to deterministic Turing machines.

(D) Multi-tape Turing machines are equivalent to single-tape Turing machines.

[GATE 1998 : IIT Delhi]

Q.9 Which of the following is true?

(A) The complement of a recursive language is recursive.

(B) The complement of a recursively enumerable language is recursively enumerable.

(C) The complement of a recursive language is either recursive or recursively enumerable.

(D) The complement of a context-free language is context-free.

[GATE 2002 : IISc Bangalore]

Q.10 If the strings of a language L can be effectively enumerated in lexicographic (i.e. alphabetic) order, which of the following statements is true?

(A) L is necessarily finite

(B) L is regular but not necessarily finite

(C) L is context free but not necessarily regular

(D) L is recursive but not necessarily context free

[GATE 2003 : IIT Madras]

Q.11 A single tape Turing Machine M has two states q_0 and q_1 of which q_0 is the starting state. The tape alphabet of M is {0, 1, B} and its input alphabet is {0,1}. The symbol B is the blank symbol used to indicate end of an input string. The transition function of M is described in the following table.

	0	1	B
q_0	$q_1,1,R$	$q_1 1,R$	Halt
q_1	$q_1,1,R$	$q_1,1,R$	q_0,B,L

The table is interpreted as illustrated below. The entry $(q_1,1,R)$ in row q_0 and column 1 signifies that if M is in state q_0 and reads 1 on the current tape square, then it writes 1 on the same tape square, moves its tape head one position to the right and transitions to state q_1. Which of the following statement is true about M?

(A) M does not halt on any string in $(0+1)^+$

(B) M does not halt on any string in (00 + 1)*

(C) M halts on all string ending in a 0

(D) M halts on all string ending in a 1

[GATE 2003 : IIT Madras]

Q.12 Let L_1 be a recursive language, and let L_2 be a recursive enumerable but not a recursive language. Which one of the following is TRUE?

(A) $\overline{L}_1$ is recursive and $\overline{L}_2$ is recursively enumerable

(B) $\overline{L}_1$ is recursive and $\overline{L}_2$ is not recursively enumerable

(C) $\overline{L}_1$ and $\overline{L}_2$ are recursively enumerable

(D) $\overline{L}_1$ is recursively enumerable and $\overline{L}_2$ is recursive

[GATE 2005 : IIT Bombay]

Q.13 For $s \in (0+1)^*$, let d(s) denote the decimal value of s (e. g. d (101) = 5)

Let $L = \{s \in (0+1)^* | d(s) \bmod 5 = 2$ and $d(s) \bmod 7 \neq 4\}$

Which one of the following statement is true?

(A) L is recursively enumerable, but not recursive

(B) L is recursive, but not context free

(C) L is context free, but not regular

(D) L is regular

[GATE 2006 : IIT Kharagpur]

Q.14 Let L_1 be a regular language, L_2 be a deterministic context-free language and L_3 be a recursively enumerable, but not recursive language. Which one of the following statement is false?

(A) $L_1 \cap L_2$ is a deterministic CFL

(B) $L_3 \cap L_1$ is recursive

(C) $L_1 \cup L_2$ is context free

(D) $L_1 \cap L_2 \cap L_3$ is recursively enumerable

[GATE 2006 : IIT Kharagpur]

Q.15 Which of the following is true for the language $\{a^p | p \text{ is prime}\}$?

(A) It is not accepted by a Turing machine

(B) It is regular but not context-free

(C) It is context-free but not regular

(D) It is neither regular nor context-free, but accepted by a Turing machine

[GATE 2008 : IISc Bangalore]

Q.16 If L and $\bar{L}$ are recursively enumerable then L is

(A) Regular

(B) Context free

(C) Context sensitive

(D) Recursive

[GATE 2008 : IISc Bangalore]

Q.17 Which of the following statement is/are FALSE?

1. For every non-deterministic Turing machine, there exits an equivalent deterministic Turing machine
2. Turing recognizable languages are closed under union and complementation
3. Turing decidable languages are closed under intersection and complementation
4. Turing recognizable languages are closed under union and intersection

(A) 1 and 4 only (B) 1 and 3 only

(C) 2 only (D) 3 only

[GATE 2013 : IIT Bombay]

Q.18 Let L be a language and $\bar{L}$ be its complement. Which one of the following is NOT a viable possibility?

(A) Neither L nor $\bar{L}$ is recursively enumerable (r.e.).

(B) One of L and $\bar{L}$ is r.e. but not recursive; the other is not r.e.

(C) Both L and $\bar{L}$ are r.e. but not recursive.

(D) Both L and $\bar{L}$ are recursive.

[GATE 2014 : IIT Kharagpur]

Q.19 Let $< M >$ be the encoding of a Turing machines as a string over $\Sigma = \{0,1\}$.

Let $L = \{< M > | M$ is a Turing machine that accepts a string of length $\{2014\}$.

Then, L is

(A) Decidable and recursively enumerable

(B) Un-decidable but recursively enumerable

(C) Un-decidable and not recursively enumerable

(D) Decidable but not recursively enumerable

[GATE 2014 : IIT Kharagpur]

Q.20 For any two languages L_1 and L_2 such that L_1 is context-free and L_2 is recursively enumerable but not recursive, which of the following is / are necessarily true?

I. $\overline{L}_1$ (complement of L_1) is recursive

II. $\overline{L}_2$ (complement of L_2) is recursive

III. $\overline{L}_1$ is context-free

IV. $L_1 \cup L_2$ is recursively enumerable

(A) I only
(B) III only
(C) III and IV only
(D) I and IV only

[GATE 2015 : IIT Kanpur]

Q.21 Consider the following types of languages :

L_1 : Regular, L_2 : Context-free,

L_3 : Recursive,

L_4 : Recursively enumerable.

Which of the following is/are TRUE?

I. $\overline{L}_3 \cup L_4$ is recursively enumerable

II. $\overline{L}_2 \cup L_3$ is recursive

III. $L_1^* \cap L_2$ is context-free

IV. $L_1 \cup \overline{L}_2$ is context-free

(A) I only
(B) I and III only
(C) I and IV only
(D) I, II and III only

[GATE 2016 : IISc Bangalore]

Q.22 Consider the following languages.

$L_1 = \{< M > | M$ takes at least 2016 steps on some input$\}$,

$L_2 = \{< M > | M$ takes at least 2016 steps on all inputs$\}$ and

$L_3 = \{<M> | M$ accepts $\varepsilon\}$,

Where for each Turing machine M, <M> denotes a specific encoding of M.

Which one of the following is TRUE?

(A) L_1 is recursive and L_2, L_3 are not recursive

(B) L_2 is recursive and L_1, L_3 are not recursive

(C) L_1, L_2 are recursive and L_3 is not recursive

(D) L_1, L_2, L_3 are recursive

[GATE 2016 : IISc Bangalore]

Q.23 Let L(R) be the language represented by regular expression R. Let L(G) be the language generated by a context free grammar G. Let L(M) be the language accepted by a Turing machine M.

Which of the following decision problems are undecidable?

I. Given a regular expression R and a string w, is $w \in L(R)$?

II. Given a context free grammar G, is $L(G) = \phi$?

III. Given a context free grammar G, is $L(G) = \Sigma^*$ for some alphabet Σ?

IV. Given a Turing machine M and a string w, is $w \in L(M)$?

(A) I and IV only
(B) II and III only
(C) II, III and IV only
(D) III and IV only

[GATE 2017 : IIT Roorkee]

Q.24 The set of all recursively enumerable languages is

(A) closed under complementation.
(B) closed under intersection.
(C) a subset of the set of all recursive languages.
(D) an uncountable set.

[GATE 2018 : IIT Guwahati]

Q.25 Consider the following problems. $L(G)$ denotes the language generated by a grammar G. $L(M)$ denotes the language accepted by a machine M.

(I) For an unrestricted grammar G and a string w, whether $w \in L(G)$

(II) Given a Turning machine M, whether $L(M)$ is regular

(III) Given two grammars G_1 and G_2, whether $L(G_1) = L(G_2)$

(IV) Given an NFA N, whether there is a deterministic PDA P such that N and P accept the same language.

Which one of the following statements is correct?

(A) Only I and II are undecidable

(B) Only III is undecidable
(C) Only II and IV are undecidable
(D) Only I, II and III are undecidable

[GATE 2018 : IIT Guwahati]

Q.26 Which of the following problems are un-decidable?

(A) Membership problem in context-free languages.
(B) Whether a given context-free language is regular.
(C) Whether a finite state automation halts on all inputs.
(D) Membership problem for type 0 languages

[GATE 1989 : IIT Kanpur]

Q.27 It is undecidable whether :

(A) An arbitrary Turing machine halts after 100 steps.
(B) A Turing machine prints a specific letter.
(C) A Turing machine computes the product of two numbers.
(D) None of the above.

[GATE 1990 : IISc Bangalore]

Q.28 Which one of the following is the strongest correct statement about a finite language over some finite alphabet $\sum$?

(A) It could be un-decidable
(B) It is Turing-machine recognizable
(C) It is a regular language
(D) It is a context-sensitive language

[GATE 1991 : IIT Madras]

Q.29 Which of the following statements is false?

(A) The halting problem for Turing machines is un-decidable
(B) Determining whether a context free grammar is un-decidable
(C) Given two arbitrary context free grammars G_1 and G_2 it is undecidable whether $L(G_1) = L(G_2)$
(D) Given two regular grammars G_1 and G_2, it is un-decidable whether $L(G_1) = L(G_2)$

[GATE 1996 : IISc Bangalore]

Q.30 Which of the following are decidable?

1. Whether the intersection of two regular language is infinite
2. Whether a given context-free language is regular
3. Whether two push-down automata accept the same language
4. Whether a given grammar is context-free

(A) 1 and 2 (B) 1 and 4
(C) 2 and 3 (D) 2 and 4

[GATE 2008 : IISc Bangalore]

Q.31 Which of the following problems are decidable?

1. Does a given program ever produce an output?
2. If L is a context-free language, then, is $\overline{L}$ also context-free?
3. If L is a regular language, then, is $\overline{L}$ also regular?
4. If L is a recursive language, then is $\overline{L}$ also recursive?

(A) $1,2,3,4$ (B) $1,2$
(C) $2,3,4$ (D) $3,4$

[GATE 2012 : IIT Delhi]

Q.32 Which of the following is/are undecidable?

1. G is a CFG. Is $L(G) = \Phi$?
2. G is a CFG. Is $L(G) = \sum^*$?
3. M is a Turing machines. Is $L(M)$ regular?
4. A is a DFA and N is an NFA. Is $L(A) = L(N)$?

(A) 3 only (B) 3 and 4 only
(C) 1, 2 and 3 only (D) 2 and 3 only

[GATE 2013 : IIT Bombay]

Q.33 Which one of the following problems is un-decidable?

(A) Deciding if a given context-free grammar is ambiguous.
(B) Deciding if a given string is generated by a given context-free grammar.
(C) Deciding if the language generated by a given context-free grammar is empty.

(D) Deciding if the language generated by a given context-free grammar is finite.

[GATE 2014 : IIT Kharagpur]

Q.34 Which of the following decision problems are undecidable?

I. Given NFAs N_1 and N_2, is $L(N_1) \cap L(N_2) = \phi$?

II. Given a CFG $G = (N, \sum, P, S)$ and a string $x \in \sum^*$, does $x \in L(G)$?

III. Given CFGs G_1 and G_2 is $L(G_1) = L(G_2)$?

IV. Given a TM M, is $L(M) = \phi$?

(A) I and IV only

(B) II and III only

(C) III and IV only

(D) II and IV only

[GATE 2016 : IISc Bangalore]

Q.35 Define languages L_0 and L_1 as follows

$L_0 = \{< M,w,0> | M \text{ halts on } w\}$

$L_1 = \{< M,w,1> | M \text{ does not halts on } w\}$

Here <M, w, i > is a triplet, whose first component, M is an encoding of a Turing machine, second component, w, is a string, and third component, i, is a bit.

Let $L = L_0 \cup L_1$. Which of the following is true?

(A) L is recursively enumerable, but $\overline{L}$ is not

(B) $\overline{L}$ is recursively enumerable, but L is not

(C) Both L and $\overline{L}$ are recursive

(D) Neither L nor $\overline{L}$ is recursively enumerable

[GATE 2003 : IIT Madras]

Q.36 L_1 is recursively enumerable language over $\sum$. An algorithm A effectively enumerates its words as $w_1, w_2, w_3 \ldots$ Define another language L_2 over $\sum \cup \{\#\}$ as $\{w_i \# w_j : w_i, w_j \in L_1, i < j\}$. Here # is a new symbol. Consider the following assertions :

S_1: L_1 is recursive implies L_2 is recursive

S_2 : L_2 is recursive implies L_1 is recursive

Which of the following statements is TRUE?

(A) Both S_1 and S_2 are true

(B) S_1 is true but S_2 is not necessarily true

(C) S_2 is true but S_1 is not necessarily true

(D) Neither is necessarily true

[GATE 2004 : IIT Delhi]

Q.37 Let $A \leq_m B$ denotes that language A is mapping reducible (also known as many-to-one reducible) to language B. Which one of the following is FALSE?

(A) If $A \leq_m B$ and B is recursive then A is recursive.

(B) If $A \leq_m B$ and A is undecidable then B is un-decidable.

(C) If $A \leq_m B$ and B is recursively enumerable then A is recursively enumerable.

(D) If $A \leq_m B$ and B is not recursively enumerable then A is not recursively enumerable.

[GATE 2004 : IIT Delhi]

Q.38 Let X be a recursive language and Y be a recursively enumerable but not recursive language. Let W and Z be two languages such that $\overline{Y}$ reduce to W and Z reduces to $\overline{X}$ (reduction means the standard many-one reduction). Which one of the following statements is TRUE?

(A) W can be recursively enumerable and Z is recursive.

(B) W can be recursive and Z is recursively enumerable.

(C) W is not recursively enumerable and Z is recursive.

(D) W is not recursively enumerable and Z is not recursive.

[GATE 2016 : IISc Bangalore]

Q.39 Let A and B be finite alphabets and let # be a symbol outside both A and B. Let f be a total function from A* to B* . We say f is

computable if there exists a Turing machine M which given an input x in A*, always halts with f(x) on its tape. Let L_f denote the languages $\{x\#f(x) | x \in A^*\}$.

Which of the following statement is TRUE:

(A) f is computable if and only if L_f is recursive

(B) f is computable if and only if L_f is recursively enumerable

(C) If f is computable then if L_f is recursive, but not conversely

(D) If f is computable then if L_f is recursive, but not conversely

[GATE 2017 : IIT Roorkee]

Q.40 Consider the following sets:

S1. Set of all recursively enumerable languages over the alphabet {0, 1}

S2. Set of all synthetically valid C programs

S3. Set of all languages over the alphabet {0, 1}

S4. Set of all non – regular languages over the alphabet {0, 1}

Which of the above sets are uncountable?

(A) S1 and S2 (B) S3 and S4

(C) S2 and S3 (D) S1 and S4

[GATE 2019 : IIT Madras]

Q.41 Consider the following problem X. "Given a Turing machine M over the input alphabet $\sum$, any state q of M and a word $w \in \sum^*$, does the computation of M on w visit the state q"

Which of the following statements about X is correct?

(A) X is decidable

(B) X is un-decidable but partially decidable

(C) X is un-decidable and not even partially decidable

(D) X is not a decision problem

[GATE 2001 : IIT Kanpur]

Q.42 Consider three decision problems P_1, P_2 and P_3. It is known that P_1 is decidable and P_2 is un-decidable. Which of the following is TRUE?

(A) P_3 is decidable if P_1 is reducible to P_3

(B) P_3 is un-decidable if P_3 is reducible to P_2

(C) P_3 is un-decidable if P_2 is reducible to P_3

(D) P_3 is decidable if P_3 is reducible to P_2's complement

[GATE 2005 : IIT Bombay]

Q.43 Let $\sum$ be a finite non-empty alphabet and let 2^{Σ^*} be the power set of $\sum^*$. Which one of the following is TRUE?

(A) Both 2^{Σ^*} and $\sum^*$ are countable

(B) 2^{Σ^*} is countable and $\sum^*$ is uncountable

(C) 2^{Σ^*} in uncountable and $\sum^*$ is countable

(D) Both 2^{Σ^*} and $\sum^*$ are uncountable

[GATE 2014 : IIT Kharagpur]

Q.44 Consider two decision problems Q_1, Q_2 such that Q_1 reduces in polynomial time to 3-SAT and 3-SAT reduces in polynomial time to Q_2. Then which one of the following is consistent with the above statement?

(A) Q_1 is in NP, Q_2 is NP hard.

(B) Q_2 is in NP, Q_1 is NP hard.

(C) Both Q_1 and Q_2 in NP.

(D) Both Q_1 and Q_2 are NP hard.

[GATE 2015 : IIT Kanpur]

Self - Practice Questions

Q.1 Which of the following statements is false?

(A) Every context-sensitive language is recursive

(B) The set of all languages that are not recursively enumerable is countable.

(C) The family of recursively enumerable languages is closed under union.

(D) The families of recursively enumerable and recursive languages are closed under reversal.

Q.2 Which of the following problems is undecidable?
(A) To determine if two finite automata are equivalent
(B) To determine if two finite automata are equivalent
(C) Finiteness problem for finite automata
(D) Ambiguity problem for context free grammar

Q.3 Let S be an NP-complete problem. Q and R are other two problems not known to be NP. Q is polynomial time reducible to S and S is polynomial time reducible to R. Which of the following statements is true?
(A) R is NP-complete
(B) R is NP-hard
(C) Q is NP-complete
(D) Q is NP-hard

Q.4 Let L(R) be the language represented by regular expression R. Let L(G) be the language generated by a context free grammar G. Let L(M) be the language accepted by a Turing machine M. Which of the following decision problems are decidable? I. Whether L(G) is deterministic context free language? II. Whether L(G1)∩L(G2) is a context free language, where G1 and G2 are deterministic grammar? III. Given a context-free grammar G, is $L(G)=\Sigma^*$ for some alphabet Σ? IV. Given a Turing machine M and a string w, is $w \in L(M)$?
(A) III and IV only
(B) II and IV only
(C) I and II only
(D) None of the above

Q.5 Which of the following statement is false?
(A) Checking the ambiguity of CFL is decidable.
(B) Checking whether a given context free language is regular is decidable.
(C) Checking whether a given context free language is empty is decidable.
(D) Both A and B

Q.6 Which of the following statement is false?
(A) Checking the ambiguity of CFL is decidable.
(B) Checking whether a given context free language is regular is decidable.
(C) Checking whether a given context free language is empty is decidable.
(D) Both A and B

Q.7 Given the following two statements: S_1:If L_1 and L_2 are recursively enumerable languages over ∑, then $L_1 \cup L_2$ and $L_2 \cap L_2$ are also recursively enumerable. S_2: The set of recursively enumerable languages is countable. Which of the following is correct ?
(A) S_1 is correct and S_2 is not correct
(B) S_1 is not correct and S_2 is correct
(C) Both S_1 and S_1 are not correct.
(D) Both S_1 and S_1 are correct.

Q.8 A problem whose language is recursive is called?
(A) Unified problem
(B) Boolean function
(C) Recursive problem
(D) Decidable

Q.9 Which of the following is FALSE with respect to possible outcomes of executing a Turing Machine over a given input?
(A) it may halt and accept the input
(B) it may halt by changing the input
(C) it may halt and reject the input
(D) it may never halt

Q.10 Which of the following pairs have different expressive power?
(A) Single-tape-turing machine and multi-dimensional turing machine.
(B) Multi-tape turing machine and multi-dimensional turing machine.
(C) Deterministic push down automata and non-deterministic pushdown automata.
(D) Deterministic finite automata and Non-deterministic finite automata

Q.11 What is the highest type number that can be assigned to the following grammar?

$S \rightarrow Aa$
$A \rightarrow Ba$
$B \rightarrow abc$

(A) Type 0 (B) Type 1
(C) Type 2 (D) Type 3

Q.12 If L and P are two recursively enumerable languages, then they are not closed under
(A) Kleene Star L * of L
(B) Intersection L ∩ P
(C) Union L ∪ P
(D) Set Difference

Q.13 Which of the following statements is not correct?
(A) Every recursive language is recursively enumerable.
(B) L = $\{0^n1^n 0^n \mid n=1, 2, 3, ...\}$ is recursively enumerable.
(C) Recursive languages are closed under intersection.
(D) Recursive languages are not closed under intersection.

Q.14 Let L = $\{a^p \mid p \text{ is a prime}\}$. Then which of the following is true?
(A) It is not accepted by a Turing Machine
(B) It is regular but not context free
(C) It is context free but not regular
(D) It is neither regular nor context free, but accepted by a Turing Machine

Q.15 A language L is called Turing-decidable (or just decidable), if there exists a Turing Machine M such that on input x, M accepts if x ∈ L, and M rejects otherwise. L is called undecidable if it is not decidable. Which of following option is false?
(A) The class of decidable languages is closed under complement.
(B) The class of decidable languages is closed under union
(C) The class of decidable languages is closed under intersection
(D) None of these

Answer Keys

Classroom Practice Questions									
1.	B	2.	C	3.	D	4.	D	5.	A, C
6.	C	7.	B	8.	B	9.	A, C	10.	D
11.	A	12.	B	13.	D	14.	B	15.	D
16.	D	17.	D	18.	B	19.	B	20.	D
21.	D	22.	C	23.	D	24.	B	25.	D
26.	B, D	27.	B	28.	C	29.	D	30.	B
31.	D	32.	D	33.	A	34.	C	35.	D
36.	A	37.	B	38.	C	39.	A	40.	B
41.	B	42.	C	43.	C	44.	A		
Self - Practice Questions									
1.	B	2.	D	3.	B	4.	D	5.	D
6.	D	7.	D	8.	D	9.	B	10.	C
11.	D	12.	D	13.	D	14.	D	15.	D

6 FSM With Output

Classroom Practice Questions

Q.1 A finite state machine with the following state table has a single input X and a single output Z.

Present state	Next state Z	
	X= 1	X = 0
A	D,0	B,0
B	B,1	C,1
C	B,0	D,1
D	B,1	C,0

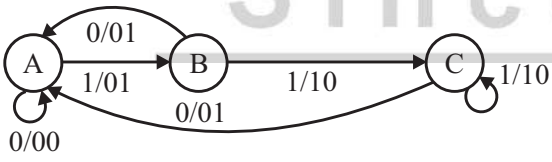
If the initial state is unknown, then the shortest input sequence to reach the final state C is

(A) 01 (B) 10

(C) 101 (D) 110

[GATE 1995 : IIT Kanpur]

Q.2 The finite state machine described by the following state diagram with A as starting state, where an arc label is x/y and x stands for 1-bit input and y stands for 2-bit output



(A) Outputs the sum of the present and the previous bits of the input

(B) Outputs 01 whenever the input sequence contains 11

(C) Outputs 00 whenever the input sequence contains 10

(D) None of the above

[GATE 2002 : IISc Bangalore]

Self - Practice Questions

Q.1 Given the following state table of an FSM with two states A and B, one input and one output :

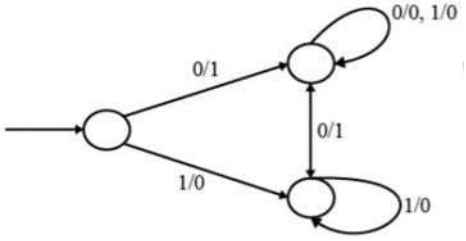
Present state A	Present state B	Input	Next state A	Next state B	Output
0	0	0	0	0	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	1	0	0
0	0	1	0	1	0
0	1	1	0	0	1
1	0	1	0	1	1
1	1	1	0	0	1

If the initial state is $A=0, B=0$, what is the minimum length of an input string which will take the machine to the state $A=0, B=1$ with Output = 1?

(A) 3 (B) 4

(C) 5 (D) 6

Q.2 The FSM (finite state machine) machine pictured in the figure above



(A) Complementsa given bit pattern

(B) Finds 2's complement of a given bit pattern

(C) Increments a given bit pattern by 1

(D) Changes the sign bit

Answer Keys

Classroom Practice Questions									
1.	B	2.	A						
Self - Practice Questions									
1.	A	2.	C						

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