Edition 2021 - 22

Engineering Mathematics

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Computer Science Engineering
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Engineering Mathematics

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GATE Syllabus

Linear Algebra : Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.

Calculus : Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.

Probability and Statistics : Random variables. Uniform, normal, exponential, poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem.

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Linear Algebra

1

"Try not to become a man of success but rather to become a man of value."

Albert Einstein

C Classroom Practice Questions

Q.1 If $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ then which of the following is a factor of Δ. **[IIT Delhi, 1998]**

(A) $a+b$ (B) $a-b$

(C) abc (D) $a+b+c$

Q.2 The necessary condition to diagonalize a matrix is that **[IIT Kanpur, 2001]**

(A) Its all eigen values should be distinct.

(B) Its eigen vectors should be independent.

(C) Its eigen values should be real.

(D) The matrix is non-singular.

Q.3 Let A, B, C, D be $n \times n$ matrices, each with non-zero determinant, If ABCD = I, then B^{-1} is **[IIT Delhi, 2004]**

(A) $D^{-1}C^{-1}A^{-1}$

(B) CDA

(C) ADC

(D) Does not necessarily exist

Q.4 Given an orthogonal matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ then the value of $[AA^T]^{-1}$ is **[IIT Bombay, 2005]**

(A) $\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$

Q.5 For the matrix $P = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, one of the eigen values is equal to – 2. Which of the following is an eigen vector?

[IIT Bombay, 2005]

(A) $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ (B) $\begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ (D) $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

Q.6 Consider the matrices $X_{(4\times3)}$, $Y_{(4\times3)}$ and $P_{(2\times3)}$. The order of $[P(X^TY)^{-1}P^T]^T$ will be

[IIT Bombay, 2005]

(A) (2×2) (B) (3×3)

(C) (4×3) (D) (3×4)

Statement For Linked Answer Q.7 & Q.8

Cayley-Hamilton Theorem states that a square matrix satisfies its own characteristic equation. Consider a matrix

$$A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$$

Q.7 A satisfies the relation **[IIT Kanpur, 2007]**

(A) $A+3I+2A^{-1}=0$

(B) $A^2+2A+2I=0$

(C) $(A+I)(A+2I)=0$

(D) $\exp(A)=0$

Q.8 A^9 equals **[IIT Kanpur, 2007]**

(A) $511\ A+510\ I$ (B) $309\ A+104\ I$

(C) $154\ A+155\ I$ (D) $\exp(9A)$

Q.9 Let x and y be two vectors in a 3 dimensional space and $<x,y>$ denote their dot product. Then the determinant

$$\det\begin{bmatrix} <x,x> & <x,y> \\ <y,x> & <y,y> \end{bmatrix}$$

[IIT Kanpur, 2007]

(A) Is zero when x and y are linearly independent.

(B) Is positive when x and y are linearly independent.

(C) Is non-zero for all non-zero x and y.

(D) Is zero only when either x or y is zero.

Q.10 The determinant $\begin{vmatrix} 1+b & b & 1 \\ b & 1+b & 1 \\ 1 & 2b & 1 \end{vmatrix}$ evaluates to **[IIT Kanpur, 2007]**

(A) 0 (B) $2b(b-1)$

(C) $2(1-b)(1+2b)$ (D) $3b(1+b)$

Q.11 The system of linear equations $\begin{bmatrix} 2 & 1 & 3 \\ 3 & 0 & 1 \\ 1 & 2 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}=\begin{bmatrix} 5 \\ -4 \\ 14 \end{bmatrix}$ have

[IIT Kharagpur, 2014 (Set-02)]

(A) a unique solution.

(B) infinitely many solutions.

(C) no solution.

(D) exactly two solutions.

Q.12 The value $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$ is

(A) $a^2b^2c^2$ (B) $-a^2b^2c^2$

(C) $4a^2b^2c^2$ (D) $-4a^2b^2c^2$

Q.13 The value of determinant

$$\Delta=\begin{vmatrix} x & a & a & a \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{vmatrix}$$

(A) $(x+3a)(x-a)^3$ (B) $(x+3a)(x-a)^2$

(C) $(x-3a)(x+a)^3$ (D) $(x-3a)(x+a)^2$

Q.14 Let $A=[a_{ij}],\ 1\le i,\ j\le n$ with $n\ge 3$ and $a_{ij}=i.j$. The rank of A is

[IIT Kanpur, 2015 (Set-02)]

(A) 0 (B) 1

(C) $n-1$ (D) n

Q.15 The matrix $A=\begin{bmatrix} \frac{3}{2} & 0 & \frac{1}{2} \\ 0 & -1 & 0 \\ \frac{1}{2} & 0 & \frac{3}{2} \end{bmatrix}$ has three distinct Eigen values and one of its Eigen vectors is $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Which one of the following can be another Eigen vector of A? **[IIT Roorkee, 2017 (Set-01)]**

(A) $\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$ (B) $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

Q.16 Given that $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Evaluate

$A^3-6A^3+11A-10I$

(A) Null matrix

(B) Identity matrix

(C) $-4I$

(D) None of the above

Q.17 Which of the following is true

(A) The product of the eigen values of a matrix is equal to the trace of the matrix

(B) The eigen values of a skew-symmetric matrix is real

(C) A is a nonzero column matrix and B is a nonzero row matrix, then rank of AB is one

(D) A system of linear non-homogeneous equations is consistent if and only if the rank of the coefficient matrix is less than or equal to the rank of the augmented matrix.

Q.18 The product of Eigen values of the matrix

$$A=\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 1 & \frac{1}{2} & 0 & 0 & \cdots & 0 \\ 1 & \frac{1}{2} & \frac{1}{3} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \cdots & \frac{1}{n} \end{bmatrix}$$

(A) n^2+n+1 (B) $\frac{n(n+1)}{2}$

(C) $\frac{1}{n!}$ (D) $\frac{1}{n}$

Q.19 If a square matrix of order 100 has exactly 15 distinct eigenvalues, then the degree of the minimum polynomial is

(A) at least 15 (B) at most 15

(C) always 15 (D) exactly 100

Q.20 The sum of all elements of square matrix

$A_{nxn}=\{a_{ij}\}_{10\times 10}$ which is defined

as follows

$\{a_{ij}\}_{10\times 10}=\begin{cases} i\ ; & i=j \\ 0\ ; & i\neq j \end{cases}$ is ________.

Q.21 The sum of all elements of square matrix

$A=\{a_{ij}\}_{15\times 15}$ which is fined as

$\{a_{ij}\}_{n\times n}=\begin{cases} i+j; & i=j \\ 0\ \ ; & i\neq j \end{cases}$ is ________.

Q.22 The value $\begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$ is ________.

Q.23 The eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ are written in the form $\begin{bmatrix} 1 \\ a \end{bmatrix}$ and $\begin{bmatrix} 1 \\ b \end{bmatrix}$. What is $a+b$? **[IISc Bangalore, 2008]**

Q.24 For a matrix $[M]=\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$, the transpose of the matrix is equal to the inverse of the matrix $[M]^T=[M]^{-1}$ The value of x is given by **[IIT Roorkee, 2009]**

Q.25 The value of q for which the following set of linear algebraic equations

$$2x+3y=0$$
$$6x+qy=0$$

can have non-trivial solution is

[IIT Guwahati, 2010]

Q.26 The rank of matrix $\begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 14 & -14 & 0 & -10 \end{bmatrix}$ is________.

[IIT Kharagpur, 2014 (Set - 02)]

Q.27 If the matrix A is such that

$$A=\begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix}\begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$$

then the determinant of A is equal to ________.

[IIT Kharagpur, 2014 (Set - 02)]

Q.28 The value of p such that the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is an eigen vector of the matrix $\begin{bmatrix} 4 & 1 & 2 \\ p & 2 & 1 \\ 14 & -4 & 10 \end{bmatrix}$ is

[IIT Kanpur, 2015 (Set-01)]

Q.29 For $A=\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, the determinant of $A^T A^{-1}$ is

[IIT Kanpur, 2015 (Set-03)]

Q.30 The rank of $A=\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ ______.

Q.31 The constant term of the characteristic polynomial of the matrix

$$\begin{bmatrix} 1 & 2 & 6 & 5 \\ -1 & 3 & 2 & -5 \\ 2 & 4 & 12 & 10 \\ 3 & -2 & 1 & -4 \end{bmatrix} \text{ is } \_\_\_\_\_\_.$$

Q.32 If $A=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$ then $\det(A^{-1})$ is ______ (correct to two decimal places).

[IIT Guwahati, 2018]

Q.33 The Nipotency index of matrix A

$$A=\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} \text{ is } \_\_\_\_\_\_.$$

Q.34 $A_r = \begin{vmatrix} r & r-1 \\ r-1 & r \end{vmatrix}$

$|A_1|+|A_2|+.....+|A_{18}|=$ ______.

Q.35 In LU decomposition of the matrix $\begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$ if the diagonal element of U are both 1, lower diagonal entry l_{22} of L is ______.

Q.36 Matrix $M=\begin{bmatrix} 8 & 4 \\ 2 & 3 \end{bmatrix}$, diagonal elements of L are 1. Product of main diagonal elements of matrix U is ______.

Q.37 Given the system of linear equation

$$2y+5z=6$$
$$5x+3z=9$$
$$4x+7y=15$$

What can be said about L and U = ?

(A) L matrix does not exits.

(B) U matrix does not exits.

(C) Both matrix does not exits.

(D) None of these.

S Self-Practice Questions

Q.1 The eigen vectors of the matrix $\begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \alpha \neq 0$ is (are)

(A) $(0,0,\alpha)$ (B) $(\alpha,0,0)$

(C) $(0,0,1)$ (D) $(0,\alpha,0)$

Q.2 Solve the following system of equations

$$x_1+x_2+x_3=3$$
$$x_1-x_2=0$$
$$x_1-x_2+x_3=1$$

(A) Unique solution

(B) No solution

(C) Infinite number of solution

(D) Only one solution

Q.3 Given matrix $\begin{bmatrix} -4 & 2 \\ 4 & 3 \end{bmatrix}$, the eigen vector is

(A) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (B) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

(C) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

Q.4 The eigen values and the corresponding eigen vectors of a 2×2 matrix are given by

Eigen value	Eigen vector
$\lambda_1 = 8$	$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$\lambda_2 = 4$	$v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

The matrix is

(A) $\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 6 \\ 6 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & 8 \\ 8 & 4 \end{bmatrix}$

Q.5 $X=[x_1 \;\; x_2 \;\; ..... \;\; x_n]^T$ is an n-tuple nonzero vector. The $n\times n$ matrix $V=XX^T$

(A) Has rank zero. (B) Has rank 1.

(C) Is orthogonal. (D) Has rank n.

Q.6 Let A be a 4×4 matrix with eigen values $-5, -2, 1, 4$. Which of the following is an eigenvalue of $\begin{bmatrix} A & I \\ I & A \end{bmatrix}$, where I is the 4×4 identity matrix ?

(A) – 5   (B) – 7

(C) 2   (D) 1

Q.7 If the rank of a (5×6) matrix Q is 4, then which one of the following statements is correct?

(A) Q will have four linearly independent rows and four linearly independent columns.

(B) Q will have four linearly independent rows and five linearly independent columns.

(C) QQ^T will be invertible.

(D) Q^TQ will be invertible.

Q.8 The system of equations

$$x+y+z=6$$
$$x+4y+6z=20$$
$$x+4y+\lambda z=\mu$$

has NO solution for values of λ and μ given by

(A) $\lambda=6, \mu=20$ (B) $\lambda=6, \mu\neq20$

(C) $\lambda\neq6, \mu=20$ (D) $\lambda\neq6, \mu\neq20$

Q.9 The matrix has $M=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ eigen values $-3,-3,5$. An eigen vector corresponding to the eigenvalue 5 is $[1 \quad 2 \quad -1]^T$. One of the eigen vectors of the matrix M^3 is

(A) $[1 \quad 8 \quad -1]^T$ (B) $[1 \quad 2 \quad -1]^T$

(C) $\left[1 \quad \sqrt[3]{2} \quad -1\right]^T$ (D) $[1 \quad 1 \quad -1]^T$

Q.10 For the matrix $A=\begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$, ONE of the normalized Eigen vectors is given as

(A) $\begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$ (B) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

(C) $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} \end{pmatrix}$ (D) $\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$

Q.11 Let A be the 2×2 matrix with elements $a_{11}=a_{12}=a_{21}=+1$ and $a_{22}=-1$. Then the Eigen values of the matrix A^{19} are

(A) 1024 and –1024

(B) $1024\sqrt{2}$ and $-1024\sqrt{2}$

(C) $4\sqrt{2}$ and $-4\sqrt{2}$

(D) $512\sqrt{2}$ and $-512\sqrt{2}$

Q.12 The following system of equations

$$x_1+x_2+2x_3=1$$
$$x_1+2x_2+3x_3=2$$
$$x_1+4x_2+ax_3=4$$

has a unique solution. The only possible value(s) for a is/are

(A) 0

(B) either 0 or 1

(C) one of 0, 1 or –1

(D) any real number other than 5

Q.13 Consider the following linear system

$$x+2y-3z=a$$
$$2x+3y+3z=b$$
$$5x+9y-6z=c$$

This system is consistent if a, b and c satisfy the equation

(A) $7a-b-c=0$ (B) $3a+b-c=0$

(C) $3a-b+c=0$ (D) $7a-b+c=0$

Q.14 Consider the matrix $P=\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$.

Which one of the following statements about P is INCORRECT?

(A) Determinant of P is equal to 1.

(B) P is orthogonal.

(C) Inverse of P is equal to its transpose.

(D) All eigen values of P are real numbers.

Q.15 The value of the determinant

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} \text{ is}$$

(A) 0 (B) 1

(C) $(a+b+c)$ (D) 3

Q.16 If every minor of order 'r' of a matrix 'A' is zero, then rank of 'A' is

(A) greater than 'r'

(B) equal to 'r'

(C) less than or equal to 'r'

(D) less than 'r'

Q.17 The adjoint of the matrix

$$A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

(A) A (B) $3A$

(C) $3A^T$ (D) A^T

Q.18 Let $M^4 = I$, (where I denotes the identity matrix) and $M \neq I$, $M^2 \neq I$ and $M^3 \neq I$. Then, for any natural number k, M^{-1} equals,

(A) M^{4k+1} (B) M^{4k+2}

(C) M^{4k+3} (D) M^{4k}

Q.19 Let M be a real 4×4 matrix. Consider the following statements :

S1 : M has 4 linearly independent eigenvectors.

S2 : M has 4 distinct eigenvalues.

S3 : M is non-singular (invertible) matrix.

Which one among the following is TRUE?

(A) S1 implies S2 (B) S1 implies S3

(C) S2 implies S1 (D) S3 implies S2

Q.20 Let N be a 3×3 matrix with real number entries. The matrix is such that $N^2 = 0$. The Eigen values of N are

(A) 0, 0, 0 (B) 0, 0, 1

(C) 0, 1, 1 (D) 1, 1, 1

Q.21 Let $M = \begin{bmatrix} 1 & 1+i & 2i & 9 \\ 1-i & 3 & 4 & 7-i \\ -2i & 4 & 5 & i \\ 9 & 7+i & -i & 7 \end{bmatrix}$ then

(A) M has only real Eigen values

(B) M has only imaginary Eigen values

(C) All Eigen values of M are zero

(D) None of the above

Q.22 If $A = (a_{ij})m \times n$ is defined as $a_{ij} = i + j \forall i, \; j$ then the sum of all elements of matrix $A =$

(A) $\frac{mn}{2}(m+n+1)$ (B) $\frac{mn}{2}(m+n+2)$

(C) $\frac{m}{2}\left(\frac{n(n+1)}{2}\right)$ (D) $\frac{n}{2}\left(\frac{n(m+1)}{2}\right)$

Q.23 Find Eigen values and Eigen vectors for

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}_{3\times 3}$$

Q.24 If $A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 1 \\ 0 & 5 & -2 \end{bmatrix}$ and

$$\text{adj}(A) = \begin{bmatrix} -11 & -9 & 1 \\ 4 & -2 & -3 \\ 10 & k & 7 \end{bmatrix}$$ then $k =$ _____.

Q.25 Let $A = \begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} \frac{1}{2} & a \\ 0 & b \end{bmatrix}$

Then $(a + b)$ is

Q.26 The determinant of matrix $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{bmatrix}$ is________.

Q.27 If the system

$$2x - y + 3z = 2$$

$$x + y + 2z = 2$$

$$5x - y + az = b$$

has infinitely many solutions, then the $a + b$ is ________.

Q.28 The number of linear independent Eigen vectors of A corresponding to '5'

$$A=\begin{bmatrix}5 & 0 & 0 & 1\\0 & 5 & 0 & 0\\0 & 0 & 2 & 1\\0 & 0 & 3 & 1\end{bmatrix} \text{ is } \_\_\_\_\_\_.$$

A Answer Keys

Classroom Practice Questions									
1.	B	2.	A, B	3.	B	4.	A	5.	D
6.	A	7.	A, C	8.	A	9.	B	10.	A
11.	B	12.	C	13.	A	14.	B	15.	C
16.	C	17.	C	18.	C	19.	C	20.	55
21.	240	22.	–27	23.	+1/2	24.	– 4/5	25.	9
26.	2	27.	0	28.	17	29.	–1	30.	3
31.	0	32.	0.25	33.	3	34.	324	35.	5
36.	16	37.	C						
Self-Practice Questions									
1.	B, D	2.	A	3.	C	4.	A	5.	B
6.	C	7.	A	8.	B	9.	B	10.	B
11.	D	12.	D	13.	B	14.	D	15.	A
16.	D	17.	C	18.	C	19.	C	20.	A
21.	A	22.	B	23.	8, 2, 2	24.	– 5	25.	7/50
26.	88	27.	14	28.	2				

❖❖❖❖

2

Integral Calculus

"Success is simple. Do what's right, the way, at the right time"

Arnold H. Glasgow

C Classroom Practice Questions

Q.1 If $f(x)$ is an even function and 'a' is a positive real number, then $\int_{-a}^{a} f(x)\,dx$ equals **[IIT Madras, 2011]**

(A) 0 (B) a

(C) $2a$ (D) $2\int_{0}^{a} f(x)\,dx$

Q.2 What is the value of the definite integral, $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$? **[IIT Madras, 2011]**

(A) 0 (B) $\frac{a}{2}$

(C) a (D) $2a$

Q.3 Given $i=\sqrt{-1}$, what will be the evaluation of the definite integral $\int_0^{\frac{\pi}{2}} \frac{\cos x + i\sin x}{\cos x - i \sin x}dx$ **[IIT Madras, 2011]**

(A) 0 (B) 2

(C) $-i$ (D) i

Q.4 The value of the improper integral $\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)}$ is, **[IIT Madras, 2011]**

(A) -2π (B) 0

(C) π (D) 2π

Q.5 The value of the integral $I=\frac{1}{\sqrt{2\pi}}\int_0^{\infty}\exp\left(\frac{-x^2}{8}\right)dx$ is **[IIT Bombay, 2005]**

(A) 1 (B) π

(C) 2 (D) 2π

Q.6 The integral $\int_0^{\pi}\sin^3\theta\, d\theta$ is given by **[IIT Kharagpur, 2006]**

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$

(C) $\frac{4}{3}$ (D) $\frac{8}{3}$

Q.7 Assuming $i=\sqrt{-1}$ and t is a real number, $\int_0^{\frac{\pi}{3}} e^{it}dt$ is **[IIT Kharagpur, 2006]**

(A) $\frac{\sqrt{3}}{2}+i\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}-i\frac{1}{2}$

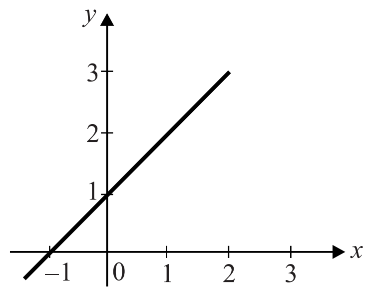
(C) $\frac{1}{2}+i\left(\frac{\sqrt{3}}{2}\right)$ (D) $\frac{1}{2}+i\left(1-\frac{\sqrt{3}}{2}\right)$

Q.8 The following definite integral evaluates to $\int_{-\infty}^{0} e^{-\left(\frac{x^2}{20}\right)}dx$ **[IIT Kharagpur, 2006]**

(A) $\frac{1}{2}$ (B) $\sqrt{5\pi}$

(C) $\sqrt{10}$ (D) π

Q.9 The following plot shows a function y which varies linearly with x. The value of the integral $I=\int_1^2 y\,dx$ is **[IIT Kanpur, 2007]**



(A) 1.0 (B) 2.5

(C) 4.0 (D) 5.0

Q.10 The value of the integral $\int_0^\infty \int_0^\infty e^{-x^2} e^{-y^2} dxdy$ is **[IIT Kanpur, 2007]**

(A) $\sqrt{\frac{\pi}{2}}$ (B) $\sqrt{\pi}$

(C) π (D) $\frac{\pi}{4}$

Q.11 If $\int_0^{2\pi} |x \sin x| dx = K\pi$, then the value of K is equal to ________.

[IIT Kharagpur, 2014 (Set - 03)]

S Self-Practice Questions

Q.1 If a is a constant, then the value of the integral $a^2 \int_0^\infty x e^{-ax} dx$ is,

(A) $\frac{1}{a}$ (B) a

(C) 1 (D) 0

Q.2 The value of the integral $\int_0^2 \frac{(x-1)^2 \sin(x-1)}{(x-1)^2 + \cos(x-1)} dx$ is,

(A) 3 (B) 0

(C) – 1 (D) – 2

Q.3 To evaluate the double integral $\int_0^8 \left(\int_{y/2}^{(y/2)+1} \left(\frac{2x-y}{2} \right) dx \right) dy$, we make the substitution $u = \left(\frac{2x-y}{2} \right)$ and $v = \frac{y}{2}$. The integral will reduce to

(A) $\int_0^4 \left(\int_0^2 2u\, du \right) dv$ (B) $\int_0^4 \left(\int_0^1 2u\, du \right) dv$

(C) $\int_0^4 \left(\int_0^1 u\, du \right) dv$ (D) $\int_0^4 \left(\int_0^2 u\, du \right) dv$

Q.4 Consider the following definite integral :

$$I = \int_0^1 \frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}} dx$$

The value of the integral is,

(A) $\frac{\pi^3}{24}$ (B) $\frac{\pi^3}{12}$

(C) $\frac{\pi^3}{48}$ (D) $\frac{\pi^3}{64}$

Q.5 $\int_0^{\frac{\pi}{4}} \left(\frac{1-\tan x}{1+\tan x} \right) dx$ evaluates to

(A) 0 (B) 1

(C) $\ln 2$ (D) $\frac{1}{2} \ln 2$

Q.6 The value of the integral $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ is

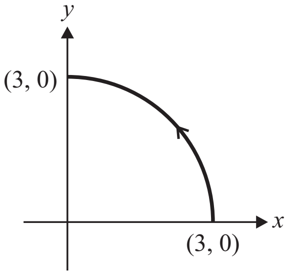
(A) $-\pi$ (B) $-\pi/2$

(C) $\pi/2$ (D) π

Q.7 As shown in the figure, C is the arc from the point (3, 0) to the point (0, 3) on the circle $x^2 + y^2 = 9$. The value of the integral

$$\int_C (y^2 + 2xy)\,dx + (x^2 + 2xy)\,dy$$

is______ (up to 2 decimal places).



Q.8 Consider the following integral $\lim_{a\to\infty} \int_1^a x^{-4}\,dx$ ______________.

(A) Diverges

(B) Converges to $\frac{1}{3}$

(C) Converges to $-\frac{1}{a^3}$

(D) Converges to 0

Q.9 The value of $\int_0^\infty e^{-y^3} y^{\frac{1}{2}}\,dy$ is________.

Q.10 Given $y = \int_1^{x^2} \cos t\,dt$ then $\frac{dy}{dx}$ is __________.

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A Answer Keys

Classroom Practice Questions									
1.	**D**	**2.**	**B**	**3.**	**D**	**4.**	**C**	**5.**	**A**
6.	**C**	**7.**	**A**	**8.**	**B**	**9.**	**B**	**10.**	**D**
11.	**4**								

Self-Practice Questions									
1.	**C**	**2.**	**B**	**3.**	**B**	**4.**	**A**	**5.**	**D**
6.	**D**	**7.**	**0**	**8.**	**B**	**9.**	$\frac{1}{3}\sqrt{\pi}$	**10.**	$2x\cos x^2$

Maxima & Minima

3

"The greatest barrier to success is the fear of failure."

Sven Goran Eriksson

C Classroom Practice Questions

Q.1 The function $f(x, y) = 2x^2 + 2xy - y^3$ has

[IISc Bangalore 2002]

(A) only one stationary point at (0, 0).

(B) two stationary points at (0, 0) and $\left(\frac{1}{6}, -\frac{1}{3}\right)$.

(C) two stationary points at (0, 0) and (1, – 1).

(D) no stationary point.

Q.2 The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval [1, 6] is

[IIT Delhi 2012]

(A) 21 (B) 25

(C) 41 (D) 46

Q.3 What is the maximum value of the function $f(x) = 2x^2 - 2x + 6$ in the interval [0, 2] **[IIT Madras 1997]**

(A) 6 (B) 10

(C) 12 (D) 5.5

Q.4 For real x, the maximum value of $\frac{e^{\sin(x)}}{e^{\cos(x)}}$ is

[IISc Bangalore 2008]

(A) 1 (B) e

(C) $e^{\sqrt{2}}$ (D) $e^{\frac{1}{\sqrt{2}}}$

Q.5 A scalar valued function is defined as $f(x) = x^T Ax + b^T x + c$, where A is a symmetric positive definite matrix with dimension $n \times n$; b and x are vectors of dimension $n \times 1$. The minimum value of $f(x)$ will occur when x equals

[IIT Kharagpur 2014]

(A) $(A^T A)^{-1} b$ (B) $-(A^T A)^{-1} b$

(C) $-\left(\frac{A^{-1}b}{2}\right)$ (D) $\frac{A^{-1}b}{2}$

Q.6 The function

$f(x, y) = x^2 y - 3xy + 2y + x$ has

[IIT Bombay 1993]

(A) no local extremum.

(B) one local maximum but no local minimum.

(C) one local minimum but no local maximum.

(D) one local minimum and one local maximum.

Q.7 Given a function, $f(x, y) = 4x^2 + 6y^2 - 8x - 4y + 8$. The optimal value of $f(x, y)$

[IIT Guwahati 2010]

(A) is a minimum equal to $\frac{10}{3}$

(B) is a maximum equal to $\frac{10}{3}$

(C) is a minimum equal to $\frac{8}{3}$

(D) is a maximum equal to $\frac{8}{3}$

Q.8 If $e^y = x^{1/x}$, then y has a

[IIT Guwahati 2010]

(A) maximum at $x = e$.

(B) minimum at $x = e$.

(C) maximum at $x = e^{-1}$.

(D) minimum at $x = e^{-1}$.

Q.9 The range of values of k for which the function $f(x) = (k^2 - 4)x^2 + 6x^3 + 8x^4$ has a local maxima at point $x = 0$ is

[IISc Bangalore 2016]

(A) $k < -2$ or $k > 2$ (B) $k \le -2$ or $k \ge 2$

(C) $-2 < k < 2$ (D) $-2 \le k \le 2$

S Self-Practice Questions

Q.1 For real values of x, the minimum value of the function $f(x)=\exp(x)+\exp(-x)$ is

(A) 2 (B) 1

(C) 0.5 (D) 0

Q.2 The maximum value of $f(x)=2x^3-9x^2+12x-3$ in the interval $0\le x\le 3$ is _______

Q.3 The maximum area (in square unit) of a rectangle whose vertices lies on the ellipse $x^2+4y^2=1$ is _______.

Q.4 For the function $f(x)=x^2e^{-x}$, the maximum occurs when x is equal to

(A) 2 (B) 1

(C) 0 (D) – 1

Q.5 The function $f(x)=\int_0^x (t-2)^2(t-1)\,dt$ has

(A) a maxima at $x=1$

(B) a minima at $x=1$

(C) a maxima at $x=2$

(D) a minima at $x=2$

Q.6 The maximum value of $f=\tan^{-1}\left(\frac{1-x}{1+x}\right)$ in $[0, 1]$ is ________.

Q.7 If α,β are the roots of $x^2-(a-2)x-(a+1)=0$, where 'a' is a variable then the min value of $\alpha^2+\beta^2$ is ________.

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A Answer Keys

Classroom Practice Questions									
1.	B	2.	C	3.	B	4.	C	5.	C
6.	A	7.	A	8.	A	9.	C		
Self-Practice Questions									
1.	A	2.	6	3.	1	4.	A	5.	B
6.	$\pi/4$	7.	5						

4 Basics of Differential Calculus & Mean Value Theorem

"Failure is success if we learn from it."

Malcolm S. Forbes

C Classroom Practice Questions

Q.1 If $y=|x|$ for $x<0$ and $y=x$ for $x\geq 0$ then **[IIT Madras, 1997]**

(A) $\frac{dy}{dx}$ is discontinuous at $x=0$.

(B) y is discontinuous at $x=0$.

(C) y is not defined at $x=0$.

(D) both y and $\frac{dy}{dx}$ are discontinuous at $x=0$.

Q.2 The function $y=|2-3x|$ **[IIT Guwahati, 2010]**

(A) Is continuously $\forall\ x\in R$ and differentiable $\forall\ x\in R$.

(B) Is continuously $\forall\ x\in R$ and differentiable $\forall\ x\in R$ except at $x=\frac{3}{2}$.

(C) Is continuously $\forall\ x\in R$ and differentiable $\forall\ x\in R$ except at $x=\frac{2}{3}$.

(D) Is continuously $\forall\ x\in R$ except $x=3$ and differentiable $\forall\ x\in R$.

Q.3 The function $f(x)=|x+1|$ on the interval $[-2, 0]$ is **[IIT Kanpur, 1995]**

(A) Continuous and differentiable.

(B) Continuous on the interval but not differentiable at all points.

(C) Neither continuous nor differentiable.

(D) Differentiable but not continuous.

Q.4 What should be the value of λ such that the function defined below is continuous at $x=\frac{\pi}{2}$? **[IIT Madras, 2011]**

$$f(x)=\begin{cases}\dfrac{\lambda\cos x}{\left(\dfrac{\pi}{2}-x\right)} & \text{if } x\neq\dfrac{\pi}{2}\\ 1 & \text{if } x=\dfrac{\pi}{2}\end{cases}$$

(A) 0 (B) $\frac{2}{\pi}$

(C) 1 (D) $\frac{\pi}{2}$

Q.5 For the function :

$$f(x)=\begin{cases}0 & ; & x=0\\ \frac{1}{2}-x & ; & 0<x<\frac{1}{2}\\ \frac{1}{2} & ; & x=\frac{1}{2}\\ \frac{3}{2}-x & ; & \frac{1}{2}<x<1\\ 1 & ; & x\geq 1\end{cases}$$

Which statement is true?

(A) $f(x)$ is right continuous at $x=0$

(B) $f(x)$ is discontinuous $x=\frac{1}{2}$

(C) $f(x)$ is continuous at $x=1$

(D) All are true

Q.6 By applying Lagranges mean value for the function $f(x)=(1+x)\log(1+x)$ on [0, 1] the value of $c\in(0,1)$ is

(A) $\frac{4}{e}$ (B) $\frac{1}{e}$

(C) $\frac{4-e}{e}$ (D) $\frac{1-e}{e}$

Q.7 Consider the function $f(x)=|x|^3$, where x is real. Then the function $f(x)$ at $x=0$ is

[IIT Kanpur, 2007]

(A) Continuous but not differential

(B) Once differential but not twice

(C) Twice differential but not thrice

(D) Thrice differential

Q.8 According to the Mean value theorem, for a continuous function $f(x)$ in the interval $[a, b]$, there exists a value ξ in this interval such that $\int_a^b f(x)\,dx$ is

[IIT Guwahati, 2018]

(A) $f(\xi)(b-a)$ (B) $f(b)(\xi-a)$

(C) $f(a)(b-\xi)$ (D) 0

Q.9 It $f(x)=(1+x)^{\cot x}$ is continuous at $x=0$ then $f(0)$ is _____.

Q.10 The mean value $C\in[1, e]$ for $f(x)=\log(x)$ is ________.

Q.11 If $f(x)=(x-1)^2(x-2)^3 x\in(1, 2)$ mean value is ________.

S Self-Practice Questions

Q.1 Which of the following function is continuous at $x=3$

(A) $f(x)=\begin{cases} 2, & x=3 \\ x-1, & x>3 \\ \dfrac{x+3}{3}, & x<3 \end{cases}$

(B) $f(x)=\begin{cases} 4, & x=3 \\ 8-x, & x\neq 3 \end{cases}$

(C) $f(x)=\begin{cases} x+3, & x\leq 3 \\ x-4, & x>3 \end{cases}$

(D) $f(x)\dfrac{1}{x^3-27}, x\neq 3$

Q.2 A function $f(x)=1-x^2+x^3$ is defined in the closed interval $[-1, 1]$. The value of x, in the open interval $(-1,1)$ for which the mean value theorem is satisfied, is

(A) $-\dfrac{1}{2}$ (B) $-\dfrac{1}{3}$

(C) 1/3 (D) 1/2

Q.3 $f(x)=\begin{cases} 3+x & x\geq 0 \\ 3-x & x<0 \end{cases}$ then $f(x)$ is

(A) Continuous and differential at $x=0$.

(B) Continuous but not differential at $x=0$

(C) Differentiable but not continuous at $x=0$.

(D) Neither continuous nor differentiable.

Q.4 The function $f(x)=\begin{cases} |x-3| & x\geq 1 \\ \dfrac{x^2}{4}-\dfrac{3}{2}x+\dfrac{13}{4} & x<1 \end{cases}$ is

(A) Continuous at $x=1$

(B) Differentiable at $x=1$

(C) Both (A) and (B).

(D) None of these.

Q.5 A real-valued function y of real variable x is such that $y=5|x|$. At $x=0$, the function is

(A) discontinuous but differentiable.

(B) both continuous and differentiable.

(C) discontinuous and not differentiable.

(D) continuous but not differentiable.

Q.6 Consider the function $f(x)=|x|$ in the interval $-1\le x\le 1$. At the point $x=0$, $f(x)$ is

(A) continuous and differentiable.

(B) non-continuous and differentiable.

(C) continuous and non-differentiable.

(D) neither continuous nor differentiable.

Q.7 If

$$f(x)=\begin{cases}\dfrac{\sqrt{1+Px}-\sqrt{1-Px}}{x}; & -1\le x\le 0\\[2mm] \dfrac{2x+1}{x-2} & 0\le x\le 1\end{cases}$$

is continuous in $[-1,\ 1]$ then P is ______.

Q.8 If $f(x)=\begin{cases}ax^2+1; & x\le 1\\ x^2+ax+b; & x>1\end{cases}$

is differential at $x = 1$

Then a and b are

(A) 2, 0　　(B) 0, 2

(C) 1, 0　　(D) 0, 1

A Answer Keys

Classroom Practice Questions									
1.	**A**	**2.**	**C**	**3.**	**B**	**4.**	**C**	**5.**	**B**
6.	**C**	**7.**	**C**	**8.**	**A**	**9.**	e	**10.**	$e-1$
11.	**1.4**								
Self-Practice Questions									
1.	**A**	**2.**	**B**	**3.**	**B**	**4.**	**A**	**5.**	**D**
6.	**C**	**7.**	**– 0.5**	**8.**	**A**				

5

Limits

"Success is the ability to go from failure to failure without losing your enthusiasm."

Winston Churchill

C Classroom Practice Questions

Q.1 $\lim_{x \to \frac{\pi}{2}} \left[\sec x - \frac{1}{1-\sin x} \right] =$

(A) 0 (B) ∞

(C) $-\infty$ (D) 1

Q.2 Limit of the function $\lim_{n \to \infty} \frac{n}{\sqrt{n^2+n}}$ is

[IIT Bombay, 1999]

(A) $\frac{1}{2}$ (B) 0

(C) ∞ (D) 1

Q.3 What is the value of $\lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{x - \frac{\pi}{4}}$?

[IIT Kanpur, 2007]

(A) $\sqrt{2}$

(B) 0

(C) $-\sqrt{2}$

(D) Limit does not exits

Q.4 The value of the expression $\lim_{x \to 0} \left[\frac{\sin(x)}{e^x x} \right]$ is **[IISc Bangalore, 2008]**

(A) 0 (B) $\frac{1}{2}$

(C) 1 (D) $\frac{1}{1+e}$

Q.5 $\lim_{x \to 0} \left(\frac{1-\cos x}{x^2} \right)$ is **[IIT Delhi, 2012]**

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) 1 (D) 2

Q.6 If $\lim_{x \to 0} \frac{\sin 2x + a\sin x}{x^3} = b$ where 'a and b' are finite then $a+b$ is ________.

Q.7 For $\lim_{x \to 0} \frac{a\sin^2 x + b\log(\cos x)}{x^4} = \frac{1}{2}$, the values of $a + b$ is ________.

Q.8 $\lim_{n \to \infty} \left(\sqrt{n^2+n} - \sqrt{n^2+1} \right)$ is ________.

[IISc Bangalore, 2016]

Q.9 $\lim_{x \to \infty} \sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x}$ is

(A) 0 (B) 1

(C) $\frac{1}{2}$ (D) ∞

S Self-Practice Questions

Q.1 The value of $\lim_{x\to 0}\frac{1-\cos(x^2)}{2x^4}$ is

(A) 0 (B) $\frac{1}{2}$

(C) $\frac{1}{4}$ (D) Undefined

Q.2 $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x}$ is equal to

(A) e^{-2} (B) e

(C) 1 (D) e^2

Q.3 $\lim_{x\to\infty}\sqrt{x^2+x-1}-x$ is

(A) 0 (B) ∞

(C) $\frac{1}{2}$ (D) $-\infty$

Q.4 $\lim_{x\to 0}\left(\frac{e^{5x}-1}{x}\right)^2$ is equal to ________.

Q.5 What is the value of $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n}$?

(A) 0 (B) e^{-2}

(C) $e^{\frac{-1}{2}}$ (D) 1

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A Answer Keys

Classroom Practice Questions									
1.	C	2.	D	3.	C	4.	C	5.	B
6.	– 3	7.	– 3	8.	0.5	9.	C		
Self-Practice Questions									
1.	C	2.	D	3.	C	4.	25	5.	B

Probability & Statistics

6

"Success is the maximum utilization of the ability that you have."

Zig Ziglar

C Classroom Practice Questions

Q.1 Consider two events E_1 and E_2 such that $P(E_1)=\frac{1}{2}, P(E_2)=\frac{1}{3}$ and $P(E_1 \cap E_2)=\frac{1}{5}$.

Which of the following statement is true?

[IIT Bombay, 1999]

(A) $P(E_1 \cup E_2)=\frac{2}{3}$

(B) E_1 and E_2 are independent

(C) E_1 and E_2 are not independent

(D) $P\left(\frac{E_1}{E_2}\right)=\frac{4}{5}$

Q.2 If P and Q are two random events, then which of the following is true?

[IIT Bombay, 2005]

(A) Independence of P and Q implies that Probability $(P \cap Q)=0$

(B) Probability $(P \cap Q) \geq$ Probability (P) + Probability (Q)

(C) If P and Q are mutually exclusive then they must be independent

(D) Probability $(P \cap Q) \leq$ Probability (P)

Q.3 A box contains 4 white balls and 3 red balls. In succession, two balls are randomly selected and removed from the box. Given that the first removed ball is white, the probability that the second removed ball is red is

[IIT Guwahati, 2010]

(A) $\frac{1}{3}$ (B) $\frac{3}{7}$

(C) $\frac{1}{2}$ (D) $\frac{4}{7}$

Q.4 Consider a company that assembles computers. The probability of a faulty assembly of any computer is p. The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q. What is the probability of a computer being declared faulty? **[IIT Guwahati, 2010]**

(A) $pq+(1-p)(1-q)$

(B) $(1-q)p$

(C) $(1-p)q$

(D) pq

Q.5 A deck of 5 cards (each carrying a distinct number from 1 to 5) is shuffled thoroughly. Two cards are then removed one at a time from the deck. What is the probability that the two cards are selected with the number on the first card being one higher than the number on the second card? **[IIT Madras, 2011]**

(A) $\frac{1}{5}$ (B) $\frac{4}{25}$

(C) $\frac{1}{4}$ (D) $\frac{2}{5}$

Q.6 Suppose a fair six-sided die is rolled once. If the value on the die is 1, 2, or 3, the die is rolled a second time. What is the probability that the sum total of values that turn up is at least 6? **[IIT Delhi, 2012]**

(A) $\frac{10}{21}$ (B) $\frac{5}{12}$

(C) $\frac{2}{3}$ (D) $\frac{1}{6}$

Common Data Question 7(a) and 7(b)

Suppose we have two bags bag1 contain 2 Red and 5 green marbles bag2 contain 2 Red and 6 green marble. A person tossess a coin and if it heads goes to bag1 otherwise bag2 and draws a marble. In this situation

Q.7(a) What is probability the marble drawn is red

(A) $\frac{2}{7}$ (B) $\frac{2}{14}$

(C) $\frac{1}{8}$ (D) $\frac{15}{56}$

Q.7(b) Given that the marble drawn is Red then what is probability if comes from bag1

(A) $\frac{1}{7}$ (B) $\frac{15}{56}$

(C) $\frac{8}{15}$ (D) $\frac{7}{15}$

Q.8 A box contains 4 red balls and 6 black balls. Three balls are selected randomly from the box one after another, without replacement. The probability that the selected set contains one red ball and two black balls is **[IIT Delhi, 2012]**

(A) $\frac{1}{20}$ (B) $\frac{1}{12}$

(C) $\frac{3}{10}$ (D) $\frac{1}{2}$

Q.9 A and B stand in a ring along with 10 other persons. If the arrangement is at random.

The probability that there are exactly 3 person between A and B is

(A) $\frac{1}{11}$ (B) $\frac{2}{11}$

(C) $\frac{3}{11}$ (D) $\frac{4}{11}$

Q.10 A determinant is selected from the set of all determinants of order 2 with elements 0 and (or) 1. Find the probability that the selected determinant is non zero :

(A) $\frac{13}{16}$ (B) $\frac{10}{16}$

(C) $\frac{6}{16}$ (D) 0

Q.11 A fair coin is lossed independently four times. The probability of event " the number of time head shown up is more that the number of time tails showns up" is

(A) $\frac{1}{16}$ (B) $\frac{1}{8}$

(C) $\frac{1}{4}$ (D) $\frac{5}{16}$

Q.12 A bag contains 10 white balls and 15 black balls. Two balls are drawn in succession. The probability that one of them is black and the other is white is

[IIT Kanpur, 1995]

(A) $\frac{2}{3}$ (B) $\frac{4}{5}$

(C) $\frac{1}{2}$ (D) $\frac{2}{1}$

Q.13 The probability of a defective piece being produced in a manufacturing process is 0.01. The probability that out of 5 successive pieces, only one is defective, is

[IISc Bangalore, 1996]

(A) $(0.99)^4(0.01)$

(B) $(0.99)(0.01)^4$

(C) $5\times(0.99)(0.01)^4$

(D) $5\times(0.99)^4(0.01)$

Q.14 A class of first year B. Tech. students is composed of four batches A, B, C and D, each consisting of 30 students. It is found that the sessional marks of students in Engineering Drawing in batch C have a mean of 6.6 and standard deviation of 2.3. The mean and standard deviation of the marks for the entire class are 5.5 and 4.2, respectively. It is decided by the course instructor to normalize the marks of the students of all batches to have the same mean and standard deviation as that of the entire class. Due to this, the marks of a student in batch C are changed from 8.5 to

[IIT Kharagpur, 2006]

(A) 6.0 (B) 7.0

(C) 8.0 (D) 9.0

Q.15 Consider the continuous random variable with probability density function

$$f(t) = 1 + t \text{ for } -1 \le t \le 0$$
$$= 1 - t \text{ for } 0 \le t \le 1$$

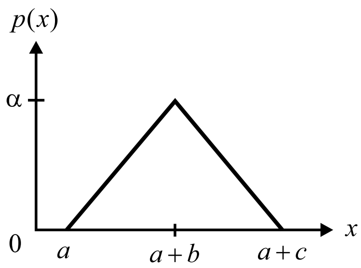
The standard deviation of the random variable is **[IIT Kharagpur, 2006]**

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{\sqrt{6}}$

(C) $\frac{1}{3}$ (D) $\frac{1}{6}$

Q.16 Probability density function differential function $p(x)$ of a random variable x is as shown below. The value of α is

[IIT Kharagpur, 2006]



(A) $\frac{2}{c}$ (B) $\frac{1}{c}$

(C) $\frac{2}{(b+c)}$ (D) $\frac{1}{(b+c)}$

Q.17 Assume that the duration in minutes of a telephone conversation follows the exponential distribution $f(x) = \frac{1}{5}e^{\frac{-x}{5}}, x \ge 0$. The probability that the conversation will exceed five minutes is

[IIT Kanpur, 2007]

(A) $\frac{1}{e}$ (B) $1 - \frac{1}{e}$

(C) $\frac{1}{e^2}$ (D) $1 - \frac{1}{e^2}$

Q.18 An examination consists of two papers, paper 1 and paper 2. The probability of failing in paper 1 is 0.3 and that in paper 2 is 0.2. Given that a student has failed in paper 2, the probability of failing in paper 1 is 0.6. The probability of a student failing in both papers is

[IIT Kanpur, 2007]

(A) 0.5 (B) 0.18

(C) 0.12 (D) 0.06

Q.19 If the standard deviation of the spot speed of vehicles in a highway is 8.8 kmph and the mean speed of the vehicles is 33 kmph, the coefficient of variation in speed is

[IIT Kanpur, 2007]

(A) 0.1517 (B) 0.1867

(C) 0.2666 (D) 0.3646

Q.20 Two independent random variables X and Y are uniformly distributed in the interval [–1, 1]. The probability that max [X, Y] is less than $\frac{1}{2}$ is **[IIT Delhi, 2012]**

(A) $\frac{3}{4}$ (B) $\frac{9}{16}$

(C) $\frac{1}{4}$ (D) $\frac{2}{3}$

Q.21 Consider a random variable to which a Poisson distribution is best fitted. It happens that $P_{(X=1)} = \frac{2}{3}P_{(X=2)}$ on this distribution plot. The variance of this distribution will be **[ESE 2017]**

(A) 3 (B) 2

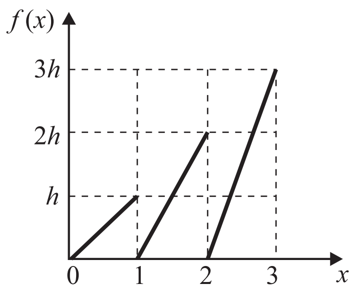
(C) 1 (D) $\frac{2}{3}$

Q.22 Dialing a phone number, A man forgot the last two digits and remembering only that they are different dialed them at random the probability the number being dialled correctly.

(A) $\frac{1}{2}$ (B) $\frac{1}{45}$

(C) $\frac{1}{72}$ (D) $\frac{1}{90}$

Q.23 The graph of a function $f(x)$ is shown in the figure



For $f(x)$ to be a valid probability density function, the value of h is

[IIT Guwahati, 2018]

(A) 1/3　　(B) 2/3

(C) 1　　(D) 3

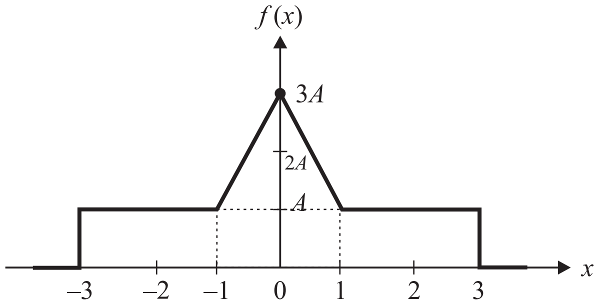
Q.24 A lot has 10% defective items 10 items are chosen randomly. The probability that exactly two item are defective is ________.

Q.25 The mean and variance of binomial distribution are 4 and 3 respectively. Then probability of getting exactly 6 success in this distribution is ________.

Q.26 In binomial distribution $B\left(n, p=\frac{1}{4}\right)$ if the probability of atleast 1success is greater than equal to $9/10$. Then n is approximately equal ________.

Q.27 The function shown in figure can represent a probability density function for A _____.

[IIT Bombay, 1993]



Q.28 If X is a continuous random variable whose probability density function is given by

$$f(x)=\begin{cases} K(5x-2x^2), & 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

Then $P(X>1)$ is ________.

[IIT Kanpur, 2007]

Q.29 If probability density function of a random variable x is :

$$f(x)=\begin{cases} x^2, & -1 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Then, the percentage probability $P\left(\frac{-1}{3} \le x \le \frac{1}{3}\right)$ is ________.

[IISc Bangalore, 2008]

Q.30 An unbiased coin is tossed an infinite number of times. The probability that the fourth head appears at the tenth toss is ________.

[IIT Kharagpur, 2014 (Set - 03)]

Q.31 The number of accidents occurring in a plant in a month follows Poisson distribution with mean as 5.2. The probability of occurrence of less than 2 accidents in the plant during a randomly selected month is ________.

[IIT Kharagpur, 2014 (Set - 4)]

Q.32 For Poisson distribution the probability that ace of spade will drawn from pack of well shuffled cards atleast once in 104 trials

(A) 0.864　　(B) 0.684

(C) 0.486　　(D) 0.846

Q.33 A dice is rolled 180 times using normal distribution find the probability that face 4 will turn up atleast 35 times. Given that area $(0<z<1)=0.3413$

(A) 0.3413　　(B) 0.5

(C) 0.15　　(D) 0.513

Q.34 Let X and Y are two random variables given that $E(X)=10$, $Var(X)=25$. Then find value of a and b such that both are greater than zero for the line $Y=aX-b$ with the help of $E(Y)=0$ and $Var(Y)=1$

(A) 1, 10　　(B) $\frac{1}{5}, 2$

(C) $\frac{1}{3}, \frac{10}{3}$　　(D) 1, 1

Q.35 Mean, standard deviation and percentage COV for following data will be

X	6	7	8	9	10	11	12
f	3	6	9	13	8	5	4

(A) 9, 1.61, 17.77　　(B) 9, 16.1, 1.777

(C) 9, 161, 1777　　(D) 0.9, 16.1, 17.77

Q.36 The mean mode and median for following observation is respectively

Class	0-6	6-12	12-18	18-24	24-30	30-36	36-42
Frequency	6	11	25	35	18	12	6

(A) 20.73, 20.8, 20.88
(B) 20.37, 20.84, 20.22
(C) 20.73, 20.4, 20.22
(D) 20.13, 20.44, 20.12

Q.37 Find the correlation coefficient between X and Y where the lines of regression are $2x-9y+6=0$ and $x-2y+1=0$

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$
(C) $\frac{3}{2}$ (D) 3

Q.38 A regression line has a slope of 8 and intercept of 3. The mean of independent variable is 8. Then the mean of dependent variable is

(A) 19 (B) 11
(C) 76 (D) 67

S Self-Practice Questions

Q.1 Events A and B are mutually exclusive and have nonzero probability. Which of the following statement (s) are true?

(A) $P(A\cup B)=P(A)+P(B)$
(B) $P(B^C)>P(A)$
(C) $P(A\cap B)=P(A)P(B)$
(D) $P(B^C)<P(A)$

Q.2 Throwing two dice the expected value and variance for sum of the numbers on the dice are respectively

(A) $7, \frac{35}{6}$ (B) $\frac{7}{2}, \frac{35}{12}$
(C) $7, \frac{35}{12}$ (D) $\frac{7}{2}, \frac{35}{6}$

Q.3 E_1 and E_2 are events in a probability space satisfying the following constraints

1. $P(E_1)=P(E_2)$
2. $P(E_1\cup E_2)=1$
3. E_1 and E_2 are independent.

The value of $P(E_1)$, the probability of the event E_1, is

(A) 0 (B) 1/4
(C) 1/2 (D) 1

Q.4 Let P(E) denote the probability of an event E. Given P(A) = 1, $P(B)=\frac{1}{2}$ the values of $P\left(\frac{A}{B}\right)$ and $P\left(\frac{B}{A}\right)$ respectively are

(A) $\frac{1}{4},\frac{1}{2}$ (B) $\frac{1}{2},\frac{1}{4}$
(C) $\frac{1}{2},1$ (D) $1,\frac{1}{2}$

Q.5 A box contains 5 black and 5 red balls. Two balls are randomly picked one after another from the box, without replacement. The probability for both balls being red is

(A) $\frac{1}{90}$ (B) $\frac{1}{2}$
(C) $\frac{19}{90}$ (D) $\frac{2}{9}$

Q.6 An examination paper has 150 multiple-choice questions of one mark each, with each question having four choices. Each incorrect answer fetches – 0.25 mark. Suppose 1000 students choose all their answers randomly with uniform probability. The sum total of the expected marks obtained by all these students is

(A) 0 (B) 2550
(C) 7525 (D) 9375

Q.7 Which one of the following statements is NOT true?

(A) The measure of skewness is dependent upon the amount of dispersion.
(B) In a symmetric distribution, the values of mean, mode and median are the same.
(C) In a positively skewed distribution : mean > median > mode.
(D) In a negatively skewed distribution : mode > mean > median.

Q.8 If E denotes expectation, the variance of a random variable X is given by

(A) $E[X^2]-E^2[X]$ (B) $E[X^2]+E^2[X]$
(C) $E[X^2]$ (D) $E^2[X]$

Q.9 The random variable X takes on the values 1, 2, or 3 with probabilities $\frac{(2+5P)}{5}$, $\frac{(1+3P)}{5}$, and $\frac{(1.5+2P)}{5}$, respectively. The values of P and $E[X]$ are respectively

(A) 0.05, 1.87 (B) 1.90, 5.87
(C) 0.05, 1.10 (D) 0.25, 1.40

Q.10 $P_x(x)=Me^{-2|x|}+Ne^{-3|x|}$ is the probability density function for the real random variable X over the entire x axis. M and N are both positive real numbers. The equation relating M and N is

(A) $M+\frac{2}{3}N=1$ (B) $2M+\frac{1}{3}N=1$
(C) $M+N=1$ (D) $M+N=3$

Q.11 A fair coin is tossed 10 times. What is the probability that only the first two tosses will yield heads?

(A) $\left(\frac{1}{2}\right)^2$ (B) ${}^{10}C_2\left(\frac{1}{2}\right)^2$
(C) $\left(\frac{1}{2}\right)^{10}$ (D) ${}^{10}C_2\left(\frac{1}{2}\right)^{10}$

Q.12 A fair coin is tossed independently four times. The probability of the event "the number of time heads shown up is more than the number of times tails shown up" is

(A) $\frac{1}{16}$ (B) $\frac{1}{8}$
(C) $\frac{1}{4}$ (D) $\frac{5}{16}$

Q.13 The annual precipitation data of a city is normally distributed with mean and standard deviation as 1000 mm and 200 mm, respectively. The probability that the annual precipitation will be more than 1200 mm is

(A) < 50 % (B) 50 %
(C) 75 % (D) 100 %

Q.14 A continuous random variable X has a probability density function $f(x)=e^{-x}$, $0<x<\infty$. Then $P\{X>1\}$ is

(A) 0.368 (B) 0.5
(C) 0.632 (D) 1.0

Q.15 A random variable X has the density function $f(x)=\frac{k}{1+x^2}$ where, $-\infty<x<\infty$. Then the value of k is

(A) π (B) $\frac{1}{\pi}$
(C) 2π (D) $\frac{1}{2\pi}$

Q.16 In a sample of 100 students, the mean of the marks (only integers) obtained by them in a test is 14 with its standard deviation of 2.5 (marks obtained can be fitted with a normal distribution). The percentage of students scoring 16 marks is

(A) 36 (B) 23
(C) 12 (D) 10

(Area under standard normal curve between $z=0$ and $z=0.6$ is 0.2257 and between $z=0$ and $z=1.0$ is 0.3413)

Q.17 A box contains 5 black and 5 red balls. Two balls are randomly picked one after another from the box, without replacement. The probability for both balls being red is

(A) $\frac{1}{90}$ (B) $\frac{1}{5}$

(C) $\frac{19}{90}$ (D) $\frac{2}{9}$

Q.18 There are ten coins, of these nine are unbiased and one is a biased coin with two heads. A coin is drawn at random and tossed two times, it appears head on both the times. Then the probability that the head is happened in biased coin is

(A) $\frac{13}{40}$ (B) $\frac{4}{13}$

(C) $\frac{9}{13}$ (D) $\frac{5}{13}$

Q.19 A six-faced fair dice is rolled five times. Then probability (in %) of obtaining 'ONE' at least four times is

(A) 33.3 (B) 3.33

(C) 0.33 (D) 0.0033

Q.20 Two dice are rolled simultaneously. The probability that the sum of digits on the top surface of two dice is even _________.

Q.21 For two rolls of a fair die, the probability of getting a 4 in the first roll and a number less than 4 in the second roll, up to 3 digits after the decimal point, is ________.

Q.22 Let X be a random variable with probability density function

$$f(x)=\begin{cases}0.2, & \text{for } |x|\le 1\\ 0.1, & \text{for } 1<|x|\le 4\\ 0, & \text{otherwise}\end{cases}$$

The probability $P(0.5<X<5)$ is _______.

Q.23 Consider an unbiased cubic dice with opposite faces coloured identically and each face coloured red, blue or green such that each colour appears only two times on the dice. If the dice is thrown thrice, the probability of obtaining red colour on top face of the dice at least twice is________.

Q.24 A random variable X has probability density function $f(x)$ as given below :

$$f(x)=\begin{cases}a+bx & \text{for } 0<x<1\\ 0 & \text{otherwise}\end{cases}$$

If the expected value $E[X]=\frac{2}{3}$, then $P[X<0.5]$ is ______.

Q.25 Assume that in a traffic junction, the cycle of the traffic signal lights is 2 minutes of green (vehicle does not stop) and 3 minutes of red (vehicle stops). Consider that the arrival time of vehicles at the junction is uniformly distributed over 5 minute cycle. The expected waiting time (in minutes) for the vehicle at the junction is ______.

Q.26 If two regression line equation are

$$x=19.13-0.87y$$
$$y=11.64-0.50x$$

Then the value of $E(x), E(y)$ and r respectively,

(A) 15.9, 3.69, – 0.65

(B) 0.87, 0.5, – 0.65

(C) 19.13, 11.64, – 0.5

(D) 15, 3, – 0.45

❖❖❖❖

A Answer Keys

Classroom Practice Questions									
1.	C	2.	D	3.	C	4.	A	5.	A
6.	A	7.	D, C	8.	D	9.	B	10.	C
11.	D	12.	C	13.	D	14.	D	15.	B
16.	A	17.	B	18.	C	19.	C	20.	B
21.	A	22.	D	23.	A	24.	0.1937	25.	0.11
26.	8	27.	0.125	28.	17/28	29.	2.47	30.	0.082
31.	0.034	32.	A	33.	C	34.	B	35.	A
36.	C	37.	B	38.	D				
Self-Practice Questions									
1.	A	2.	A	3.	D	4.	D	5.	D
6.	D	7.	D	8.	A	9.	A	10.	A
11.	C	12.	D	13.	A	14.	A	15.	B
16.	C	17.	D	18.	B	19.	C	20.	0.5
21.	0.083	22.	0.4	23.	0.259	24.	0.25	25.	0.9
26.	A								

Engineering Mathematics

Multiple Subject Questions (MSQs)

Q.1 Out of 300 students in a school, 95 play cricket only, 120 play football only, 80 play volleyball only and 5 play no games. If one student is chosen at random. Which of the option/ s is are correct?

(A) The probability of getting a player who plays volleyball $=\dfrac{5}{15}$

(B) The probability of getting a player who plays either cricket or volleyball $=\dfrac{7}{12}$

(C) The probability of getting a player who plays neither football nor volleyball $=\dfrac{1}{3}$

(D) None of these.

Q.2 Let A be an $n\times n$ invertible matrix with real entries whose column sums are all equal to 1. Which of the following statements is /are FALSE.

(A) Every column in the matrix A_2 sums to 2

(B) Every column in the matrix A_3 sums to 3

(C) Every column in the matrix A^{-1} sums to 1

(D) None of these

Q.3 Which of the following is /are TRUE for the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$:

(A) Sum of Eigen values $=6$

(B) Determinant of the matrix is 8

(C) Eigen values $=1,1,4$

(D) Eigen values $=0,2,4$

Q.4 For two given events A and B, $P(A\cap B)$

(A) Not less than $P(A)+P(B)-1$

(B) Not greater than $P(A)+P(B)$

(C) Equal to $P(A)+P(B)-P(A\cup B)$

(D) Equal to $P(A)+P(B)+P(A\cup B)$

Q.5 If $\bar{E}$ and $\bar{F}$ are the complementary events of E and F respectively and if $0<P(F)<1$, then

(A) $P\left(\dfrac{E}{F}\right)+P\left(\dfrac{\bar{E}}{F}\right)=1$

(B) $P\left(\dfrac{E}{F}\right)+P\left(\dfrac{E}{\bar{F}}\right)=1$

(C) $P\left(\dfrac{\bar{E}}{F}\right)+P\left(\dfrac{E}{\bar{F}}\right)=1$

(D) $P\left(\dfrac{E}{\bar{F}}\right)+P\left(\dfrac{\bar{E}}{\bar{F}}\right)=1$

Q.6 If the system of equations $x=cy+bz$, $y=az+cx$ and $z=bx+ay$ has a non trivial solution, then value of $a^2+b^2+c^2+2abc$ is

(A) 1

(B) 0

(C) -1

(D) Independent of a, b, c

Q.7 If $AB = A$ and $BA = B$, then

(A) $A^2 = A$ (B) $B^2 = B$

(C) $A = I$ (D) $B = I$

Q.8 Let $L = \lim_{x \to 0} \frac{a - \sqrt{(a^2 - x^2)} - (x^2/4)}{x^4}$, $a > 0$. If L is finite, then

(A) $a = 2$ (B) $a = 1$

(C) $L = \frac{1}{64}$ (D) $L = \frac{1}{32}$

Q.9 Which of the following statements are correct for any square matrix A?

(A) $A + A^T$ is symmetric matrix (B) $A - A^T$ is skew-symmetric matrix

(C) AA^T is orthogonal matrix (D) AA^H is unitary matrix

Q.10 Let a fair dice with opposite face having red, blue, yellow color respectively is rolled infinite times then the probability of getting

(A) 3rd time red face on 5th roll is $\frac{16}{81}$ (B) red face on 5th roll is $\frac{1}{6}$

(C) 3 red face in 5 roll is $\frac{40}{243}$ (D) exactly 1 red and 2 blue face on 5 rolls is $\frac{10}{81}$

Q.11 Determinant of matrix $D = \begin{vmatrix} 2 & 3 & 4 & 5 \\ 1 & 4 & 4 & 5 \\ 1 & 3 & 5 & 5 \\ 1 & 3 & 4 & 6 \end{vmatrix}$ is

(A) 20 (B) 14

(C) 10 (D) 8

Q.12 There are 2 fair coins and 1 false coin with tails on both sides. A coin is chosen at random and tossed 4 times. What is the probability that the false coin has been chosen for tossing if ‘tails’ occurs in all 4 times?

(A) 0.44 (B) 0.54

(C) 0.84 (D) 0.48

Q.13 If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then for every positive integer A^n is equal to

(A) $\begin{bmatrix} 1+2n & 4n \\ n & 1+2n \end{bmatrix}$ (B) $\begin{bmatrix} 3^n & (-4)^n \\ n & (-1)^n \end{bmatrix}$

(C) $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ (D) Undefined

Q.14 If $P(X) = \frac{1}{2}$, $P(Y) = \frac{1}{3}$ and $P\left(\frac{X}{Y}\right) = \frac{1}{3}$ then $P(X \cap Y)$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{9}$

(C) $\frac{1}{12}$ (D) $\frac{1}{18}$

Q.15 The value of integral given below is $\int_0^{\pi} x^2 \cos x dx$

(A) -2π (B) π

(C) $-\pi$ (D) 2π

Q.16 Consider the following set of equations $x+2y=5$, $4x+8y=12$, $3x+6y+3z=15$. This set

(A) Has a unique solution (B) Has no solution

(C) Has infinite number of solutions (D) Has 3 solutions

Q.17 $f(x)=ax^2+bx+c$ have maxima at some x, when

(A) $a=2, b=-3$ (B) $a=-2, b=5$

(C) $a=-4, b=-7$ (D) $a=1, b=-5$

Q.18 Let $f(x)=\begin{cases} a+bx, & x<1 \\ 4, & x=1 \\ b-ax, & x>1 \end{cases}$

$f(x)$ will be continuous when

(A) $a=0, b=5$ (B) $a=2, b=-2$

(C) $a=1, b=3$ (D) $a=0, b=4$

Q.19 An open topped box is to be constructed by removing equal square for each corners of 3 m by 8 m rectangular sheet of aluminum and folding up to sides then the volume of such a box will be

(A) Maximum when side of squares cut, is 0.66 m

(B) Maximum when side of square cut is 2 m

(C) Minimum when side of square cut, is 0.66 m

(D) Minimum when side of square cut, is 2 m

A Answer Keys

1.	**B, C**	**2.**	**A, B, C**	**3.**	**A, C**	**4.**	**A, B, C**	**5.**	**A, D**
6.	**A, D**	**7.**	**A, B**	**8.**	**A, C**	**9.**	**A, B**	**10.**	**A, C, D**
11.	**B**	**12.**	**C**	**13.**	**C**	**14.**	**B**	**15.**	**A**
16.	**B**	**17.**	**B, C**	**18.**	**D**	**19.**	**A**		