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GATE Syllabus

Propositional and first-order logic. Sets, relations, functions, partial orders and lattices. Groups. Graphs: connectivity, matching, coloring. Combinatorics: counting, recurrence relations, generating functions

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1 Sets and Relations

Classroom Practice Questions

Q.1 The cardinality of the set of $\{0,1,2,....,10\}$ is _____.

Q.2 The number of elements in the power set P(S) of the set $S=\{\{\phi\},1,\{2,3\}\}$ is

(A) 2
(B) 4
(C) 8
(D) None of the above

[GATE 1995 : IIT Kanpur]

Q.3 For a set A, the power set of A is denoted by 2^A. If $A=\{5,\{6\},\{7\}\}$, which of the following options are TRUE?

I. $\phi \in 2^A$ II. $\phi \subseteq 2^A$
III. $\{5,\{6\}\} \in 2^A$ IV. $\{5,\{6\}\} \subseteq 2^A$

(A) I and III only (B) II and III only
(C) I, II and III only (D) I, II and IV only

[GATE 2015 : IIT Kanpur]

Q.4 Let ϕ be an empty set, then what is the cardinality of $|P(P(\phi))|$.

(A) 0 (B) 1
(C) 2 (D) 4

Q.5 Check whether the following argument is valid or not?

(1) All my friends are musician
(2) John is my friend.
(3) None of my neighbours are musician
- Therefore John is not my neighbour

Q.6 Consider the following arguments

(1) No practical car is expensive.
(2) Cars with sun-roof are expansive
(3) All wagons are practical.

Then which of the following is not true.

(A) No practical car has sun-roof.
(B) Some wagons are expensive.
(C) Cars with sun- roof are not practical.
(D) No wagon has a sun-roof.

Q.7 Suppose U is the power set of the set $S=\{1,2,3,4,5,6\}$. For any $T \in U$, let $|T|$ denote the number of elements in T and T^1 denote the complement of T. For any T, $R \in U$, let $T \setminus R$ be the set of all elements in T which are not in R. Which one of the following is true?

(A) $\forall X \in U\left(|X|=|X'|\right)$

(B) $\exists X \in U \exists Y \in U \begin{pmatrix} |X|=5,|Y|=5 \\ \text{and } X \cap Y=\phi \end{pmatrix}$

(C) $\forall X \in U \forall Y \in U \begin{pmatrix} |X|=2,|Y|=3 \\ \text{and } X \setminus Y=\phi \end{pmatrix}$

(D) $\forall X \in U \forall Y \in U (X \setminus Y = Y' \setminus X')s$

[GATE 2015 : IIT Kanpur]

Q.8 Let A and B be sets and let A^c and B^c denote the complements of the sets A and B. The set $(A-B)\cup(B-A)\cup(A\cap B)$ is equal to

(A) $A\cup B$ (B) $A^c \cup B^c$
(C) $A\cap B$ (D) $A^c \cap B^c$

[GATE 1996 : IISc Bangalore]

Q.9 Let A, B and C be non-empty sets and let $X=(A-B)-C$ and $Y=(A-C)-(B-C)$

Which one of the following is TRUE?

(A) $X=Y$ (B) $X \subset Y$
(C) $Y \subset X$ (D) None of these

[GATE 2005 : IIT Bombay]

Q.10 Let E, F and G be finite sets.

Let $X=(E\cap F)-(F\cap G)$ and

$Y=(E-(E\cap G))-(E\cap F)$

Which one of the following is true?

(A) $X\subset Y$

(B) $X\supset Y$

(C) $X=Y$

(D) $X-Y\neq\phi$ and $Y-X\neq\phi$

[GATE 2006 : IIT Kharagpur]

Q.11 If P,Q,R are subsets of the universal set U, then

$(P\cap Q\cap R)\cup(P^c\cap Q\cap R)\cup Q^c\cup R^c$ is

(A) $Q^c\cup R^c$ (B) $P\cup Q^c\cup R^c$

(C) $P^c\cup Q^c\cup R^c$ (D) U

[GATE 2008 : IISc Bangalore]

Q.12 Let # be the binary operator defined as X#Y=X′+Y′ where X and Y are Boolean variables.

Consider the following two statements.

- (S1) (P#Q)#R=P#(Q#R)
- (S2) Q#R=(R#Q)

Which are the following is/are true for the Boolean variables P, Q and R?

(A) Only S1 is true

(B) Only S2 is true

(C) Both S1 and S2 are true

(D) Neither S1 nor S2 are true

[GATE 2015 : IIT Kanpur]

Q.13 Let S be an infinite set and $S_1,S_2,...S_n$ be sets such that $S_1\cup S_2\cup.....\cup S_n=S$. Then

(A) At least one of the sets S_i is a finite set.

(B) Not more than one of the sets S_i can be finite.

(C) At least one of sets S_i is infinite

(D) Not more than of one the sets S_i can be infinite.

[GATE 1993 : IIT Bombay]

Q.14 Consider the following statements:

- S1: There exists infinite sets A, B, C such that A∩(B∪C) is finite.
- S2: There exists two irrational numbers x and y such that (x + y) is rational.

Which of the following is true about S1 and S2?

(A) Only S1 is correct

(B) Only S2 is correct

(C) Both S1 and S2 are correct

(D) None of S1 and S2 is correct

[GATE 2001 : IIT Kanpur]

Q.15 Let A be a set with n elements. Let C be a collection of distinct subset of A such that for any two subsets S_1 and S_2 in C, either $S_1\subset S_2$ or $S_2\subset S_1$. What is the maximum cardinality of C?

(A) n (B) $n+1$

(C) $2^{n-1}+1$ (D) $n!$

[GATE 2005 : IIT Bombay]

Q.16 Consider the following relation on subset of the set S integers between 1 and 2014. For two distinct subsets U and V of S, we say $U<V$ if the minimum element in the symmetric difference of the two sets is in U.

Consider the following two statements :

S1 : There is a subset of S that is larger than every other subset.

S2 : There is a subset of S that is smaller than every other subset.

Which one of the following is CORRECT?

(A) Both S1 and S2 are true

(B) S1 is true and S2 is false

(C) S2 is true and S1 is false

(D) Neither S1 nor S2 is true

[GATE 2014 : IIT Kharagpur]

Q.17 In a class of 200 students, 125 students have taken Programming Language course, 85 students have taken Data Structures course, 65 students have taken Computer Organization course ; 50 students have taken both programming Language and Data Structures, 35 students have taken both Programming Language and Computer Organization; 30 students have

taken both Data structures and Computer Organization; 15 students have taken all the three course. How many students have not taken any of the three course?

(A) 15 (B) 20

(C) 25 (D) 30

[GATE 2004 : IIT Delhi]

Q.18 How many multi sets can be constructed from the elements at a finite set with n elements.

(A) n^2 (B) 2^n

(C) $n!$ (D) Unlimited

Q.19 How many multisets of size = 4 are possible with the elements of a finite set of size n, such that atleast one element appears exactly twice.

Q.20 Let A be a finite set of size n. The number of elements in the power set of $A \times A$ is

(A) 2^{2n} (B) 2^{n^2}

(C) $(2^n)^2$ (D) n^2

[GATE 1993 : IIT Bombay]

Q.21 How many binary relations are there on a set A with n elements?

[GATE 1987 : IIT Bombay]

Q.22 The number of binary relations on a set with n elements is :

(A) n^2

(B) 2^n

(C) 2^{n^2}

(D) None of the above

[GATE 1999 : IIT Bombay]

Q.23 What is the possible number of reflexive relations on a set of 5 elements?

(A) 2^{10} (B) 2^{15}

(C) 2^{20} (D) 2^{25}

[GATE 2010 : IIT Guwahati]

Q.24 Let R be the set of all binary relations on the set {1, 2, 3}. Suppose a relation is chosen from R at random. The probability that the chosen relation is reflexive (round off to 3 decimal places) is _____.

[GATE 2020 : IIT Delhi]

Q.25 The binary relation $S = \phi$ (empty set) on set $A = \{1, 2, 3\}$ is

(A) Neither reflexive nor symmetric

(B) Symmetric and reflexive

(C) Transitive and reflexive

(D) Transitive and symmetric

[GATE 2002 : IISc Bangalore]

Q.26 The binary relation

$R = \{(1,1),(2,1),(2,2),(2,3),(2,4),$
$(3,1),(3,2),(3,3),(3,4)\}$

on the set $A = \{1, 2, 3, 4\}$ is

(A) Reflexive, symmetric and transitive

(B) Neither reflexive, nor irreflexive but transitive

(C) Irreflexive, symmetric and transitive

(D) Irreflexive and antisymmetric

[GATE 1997 : IIT Madras]

Q.27 Consider the binary relation $R = \{(x, y), (x, z), (z, x), (z, y)\}$ on the set $\{x, y, z\}$. Which one of the following is TRUE?

(A) R is symmetric but NOT antisymmetric

(B) R is NOT symmetric but antisymmetric

(C) R is both symmetric and antisymmetric.

(D) R is neither symmetric nor antisymmetric.

[GATE 2009 : IIT Roorkee]

Q.28 Let R be a relation on the set of ordered pairs of positive integers such that $((p, q), (r, s)) \in R$ if and only if $p - s = q - r$. Which one of the following is true about R?

(A) Both reflexive and symmetric

(B) Reflexive but not symmetric

(C) Not reflexive but symmetric

(D) Neither reflexive nor symmetric

[GATE 2015 : IIT Kanpur]

Q.29 Let R be the relation on the set of positive integers such that aRb if and only if a and b distinct and have a common divisor other than 1.

Which one of the following statements about R is true?

(A) R is symmetric and reflexive but not transitive

(B) R is reflexive but not symmetric and not transitive

(C) R is transitive but not reflexive and not symmetric

(D) R is symmetric but not reflexive and not transitive

[GATE 2015 : IIT Kanpur]

Q.30 A binary relation R on $N \times N$ is defined as follows : $(a,b)R(c,d)\ if\ a \le c\ \ or\ \ b \le d$. Consider the following propositions :

P : R is reflexive

Q : R is transitive

Which one of the following statements is TRUE?

(A) Both P and Q are true.

(B) P is true and Q is false.

(C) P is false and Q is true.

(D) Both P and Q are false.

[GATE 2016 : IISc Bangalore]

Q.31 Let R_1, R_2, R_3 and R_4 are relations defined on set $A = \{a,b,c\}$ then which of the following is /are true?

(A) $R_1 = \{(a,a)(b,b)\}$ is both symmetric and antisymmetric

(B) $R_2 = \{(a,a)(b,c),(c,b),(a,c)\}$ is neither symmetric nor antisymmetric

(C) $R_3 = \{(a,b)(b,a),(c,c)\}$ is symmetric but not antisymmetric

(D) $R_4 = \{(a,b)(b,c),(c,c)\}$ is anti-symmetric but not transitive

Q.32 Which of the following statement is/ are not true

(A) If R and S are any two reflexive relation on a set A, then $R \cap S$ and $R \cup S$ are also reflexive

(B) If R and S are any two Symmetric relation on a set A, then $R \cap S$ and $R \cup S$ are also Symmetric

(C) If R and S are any two antisymmetric relation on a set A, then $R \cap S$ and $R \cup S$ are also antisymmetric

(D) If R and S are any two transitive relation on a set A, then $R \cap S$ and $R \cup S$ are also transitive

Q.33 Which of the following statement is/are false

(A) If a relation R on a set A is symmetric and transitive then R is reflexive

(B) A relation R on a set is symmetric if $R = R^{-1}$

(C) A relation R on a set A is antisymmetric if $R \cap R^{-1} \subseteq \Delta_A$

(D) A relation R on a set is reflexive if $\bar{R}$ is irreflexive

(E) A relation R on a set is reflexive if R^{-1} is reflexive

(F) If R is antisymmetric relation on set A, then $R \cap S$ is also antisymmetric for any relation S on set A

Q.34 Find reflexive closure of relation $R = \{(1,2),(2,2),(3,3)\}$ on a set $A = \{1,2,3\}$

Q.35 Find symmetric closure of relation $R = \{(1,2),(2,2),(3,3)\}$ can a set $A = \{1,2,3\}$

Q.36 Find transitive closure of relation $R = \{(1,1),(1,3),(2,2)(3,1),(3,2)\}$ on a set $A = \{1,2,3\}$

Q.37 Find the transitive closure of relation $R = \{(a,d),(b,a),(b,c)(c,a),(c,d),(d,c)\}$ on a set $A = \{a,b,c,d\}$

Q.38 The transitive closure of the relation $\{(1,2)(2,3)(3,4)(5,4)\}$ on the set $A = \{1,2,3,4,5\}$ is _____.

[GATE 1989 : IIT Kanpur]

Q.39 Consider the binary relation :

$S = \{(x,y) \mid y = x+1 and\ x,y \in \{0,1,2,...\}\}$

The reflexive transitive closure of S is

(A) $\{(x,y) \mid y > x\ and\ x,y \in \{0,1,2,...\}\}$

(B) $\{(x,y) \mid y \ge x\ and\ x,y \in \{0,1,2,...\}\}$

(C) $\{(x,y) \mid y < x\ and\ x,y \in \{0,1,2,...\}\}$

(D) $\{(x,y) \mid y \le x\ and\ x,y \in \{0,1,2,...\}\}$

[GATE 2004 : IIT Delhi]

Q.40 Suppose A is a finite set with n elements. The number of elements in the largest equivalence relation on A is.

(A) n (B) n^2

(C) 1 (D) $n+1$

Q.41 Let S be a set of n elements. The number of ordered pairs in the largest and the smallest equivalence relations on S are

(A) n and n (B) n^2 and n

(C) n^2 and 0 (D) n and 1

[GATE 2006 : IIT Kharagpur]

Q.42 Let R be a non-empty relation on a collection of sets defined by $A^R B$ if and only if $A \cap B = \phi$. Then, (pick the true statement)

(A) R is reflexive and transitive

(B) R is symmetric and not transitive

(C) R is an equivalence relation

(D) R is not reflexive and not symmetric

[GATE 1996 : IISc Bangalore]

Q.43 Consider the following relations:

- R1 (a, b) iff (a + b) is even over the set of integers
- R2 (a, b) iff (a + b) is odd over the set of integers
- R3 (a, b) iff a.b>0 over the set of non-zero rational numbers
- R4 (a, b) iff $|a-b| \leq 2$ over the set of natural numbers

Which of the following statements is correct?

(A) R1 and R2 are equivalence relations, R3 and R4 are not

(B) R1 and R3 are equivalence relations, R2 and R4 are not

(C) R1 and R4 are equivalence relations, R2 and R3 are not

(D) R1, R2, R3 and R4 all are equivalence relations

[GATE 2001 : IIT Kanpur]

Q.44 Find the number of equivalence relation on a set with 3 element

Q.45 The number of equivalence relations on the set $\{1,2,3,4\}$ is

(A) 15 (B) 16

(C) 24 (D) 4

[GATE 1998 : IIT Madras]

Q.46 Find the number of equivalence relation on a set with 5 elements

Q.47 State whether the following statement are TRUE or FALSE :

(A) The union of two equivalence relations is also an equivalence relation.

(B) The intersection of two equivalence relations is also an equivalence relation.

[GATE 1987 : IIT Bombay]

Q.48 Let R and S be any two equivalence relations on a non-empty set A. Which one of the following statements is TRUE?

(A) $R \cup S$, $R \cap S$ are both equivalence relations

(B) $R \cup S$ is an equivalence relation

(C) $R \cap S$ is an equivalence relation

(D) Neither $R \cup S$ nor $R \cap S$ is an equivalence relation

[GATE 2005 : IIT Bombay]

Q.49 Find the partition of a set $A = \{1,2,3,4,5\}$ corresponding to the equivalence relation

$R = \{(1,1),(2,2),(3,3),(4,4),(5,5),$
$(1,2),(2,1),(3,4),(4,3)\}$

Q.50 Consider the following equivalence relation on the set of all human being. Two persons are related if they are born in the same week. Find the number of distinct equivalence classes

Q.51 A relation R is defined on the set of integers as xRy iff $(x+y)$ is even. Which of the following statements is true?

(A) R is not an equivalence relation

(B) R is an equivalence relation having 1 equivalence class

(C) R is an equivalence relation having 2 equivalence classes

(D) R is an equivalence relation having 3 equivalence classes

[GATE 2000 : IIT Kharagpur]

Q.52 Suppose $A=\{a,b,c,d\}$ and π_1 is the following partition of A

$\pi_1=\{\{a,b,c\},\{d\}\}$

List the ordered pairs of the equivalence relation induced by π_1.

Q.53 Let R be a symmetric and transitive relation on a set A. Then

(A) R is reflexive and hence an equivalence relation.

(B) R is reflexive and hence partial order.

(C) R is not reflexive and hence not an equivalent relation.

(D) None of the above.

[GATE 1995 : IIT Kanpur]

Q.54 A relation R is defined on ordered pairs of integers as follows :

$(x,y)R(u,v)$ *if* $x<u$ and $y>v$. Then R is

(A) Neither a Partial Order nor an Equivalence Relation

(B) A Partial Order but not a Total Order

(C) A Total Order

(D) An Equivalence Relation

[GATE 2006 : IIT Kharagpur]

Q.55 A partial order P is defined on the set of natural numbers as follows. Here x/y denotes integer division.

(i) $(0,0)\in P$.

(ii) $(a,b)\in P$ if and only if

$(a\%10)\le(b\%10)$ and $\left(\frac{a}{10},\frac{b}{10}\right)\in P$.

Consider the following ordered pairs :

(i) (101, 22) (ii) (22,101)

(iii) (145, 265) (iv) (0,153)

Which of these ordered pairs of natural numbers are contained in P?

(A) (i) and (iii)

(B) (ii) and (iv

(C) (i) and (iv)

(D) (iii) and (iv)

[GATE 2006 : IIT Kharagpur]

Q.56 Draw the hasse diagram of the POSET $[\{-1,0,2,3,5.5\};\le]$

Q.57 Draw the hasse diagram of the POSET $[D_6;/]$ Where "/" is relation divides.

Q.58 Draw the hasse diagram of the POSET $[D_{12};/]$

Q.59 Draw the hasse diagram of the POSET $[\{2,3,4,6\};/]$

Q.60 Let $X=\{2,3,6,12,24\}$. Let $\le$ be the partial order defined by $x\le y$ if x divides y. The number of edges in the Hasse diagram of $(x,\le)$ is

(A) 3

(B) 4

(C) 9

(D) None of the above

[GATE 1996 : IISc Bangalore]

Q.61 The POSET $[(1,2,3,4,6,9);/]$ is

(A) Join semi lattice but not a meet semi lattice

(B) Not join semi lattice but a meet semi lattice

(C) A lattice

(D) Not a semi lattice

Q.62 The POSET $[\{2,3,5,30,60,120,180,360\};/]$ is

(A) Join semi lattice but not a meet semi lattice

(B) Not join semi lattice but a meet semi lattice

(C) A lattice

(D) Not a semi lattice

Q.63 The POSET $[\{2,3,4,6,12,18\};/]$ is

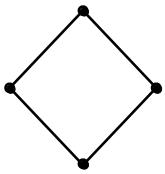
(A) Join semi lattice but not a meet semi lattice

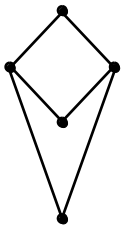
(B) Not join semi lattice but a meet semi lattice

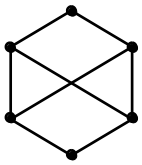
(C) A lattice

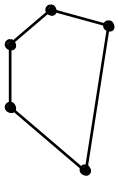
(D) Not a semi lattice

Q.64 Consider the following Hasse diagrams.

(i) 

(ii) 

(iii) 

(iv) 

Which of the above represent a lattice?

(A) (i) and (iv) only

(B) (ii) and (iii) only

(C) (iii) only

(D) (i), (ii) and (iv) only

[GATE 2008 : IISc Bangalore]

Q.65 The inclusion of which of the following sets into

$S = \{\{1,2\},\{1,2,3\},\{1,3,5\},$
$\{1,2,4\},\ \{1,2,3,4,5\}\}$

is necessary and sufficient to make S a complete lattice under the partial order defined by set containment?

[GATE 2004 : IIT Delhi]

(A) $\{1\}$

(B) $\{1\},\{2,3\}$

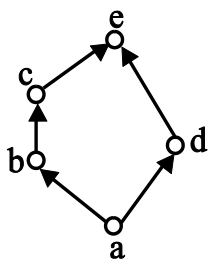
(C) $\{1\},\{1,3\}$

(D) $\{1\},\{1,3\},\{1,2,3,4\},\{1,2,3,5\}$

Q.66 Consider the set $X = \{a,b,c,d,e\}$ under the partial ordering

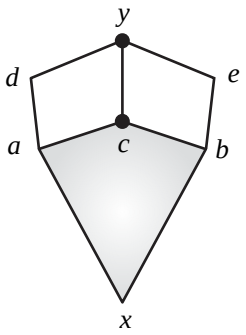
$R = \{(a,a),(a,b),(a,c)$
$(a,d),(a,e),(b,b),(b,c)$
$(b,e),(c,c),(c,e),(d,d)$
$(d,e),(e,e)\}$

The Hasse diagram of the partial order (X,R) is shown below.



The minimum number of ordered pairs that need to be added to R to make (X, R) a lattice is ___. **[GATE 2017 : IIT Roorkee]**

Q.67 For Lattice L shown below



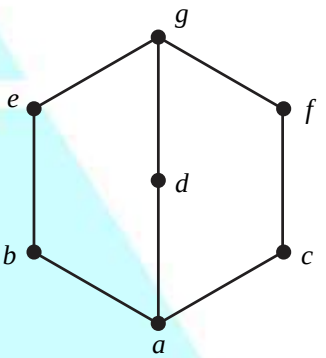
Which of the following subsets is /are sub-lattices of L.

(A) $L_1 = \{x,a,b,y\}$ (B) $L_2 = \{x,a,c,y\}$

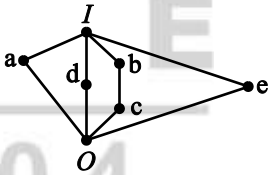
(C) $L_3 = \{x,c,d,y\}$ (D) $L_4 = \{x,a,e,y\}$

Q.68 In the lattice with respect to $[D_{12};/]$ find the complements of the elements of the set D_{12} if exists.

Q.69 For the lattice shown below find the complements of every element.



Q.70 The complement(s) of the element 'a' in the lattice shown in fig. is (are)



[GATE 1988 : IIT Kharagpur]

Q.71 Which of the following statements is/ are not true

(A) If A is any finite set then $[P(A);\subseteq]$ is distributive lattice

(B) Every sub lattice of a distributive lattice is also a distributive lattice

(C) Every totally ordered set is a distributive lattice

(D) Every totally ordered set is bounded

(E) Every distributive lattice is bounded

(F) Every distributive lattice is a complemented lattice

Q.72 Which of the following lattice is /are not distributive

(A) $[D_{125};/]$

(B) $[P(A);\subseteq]$

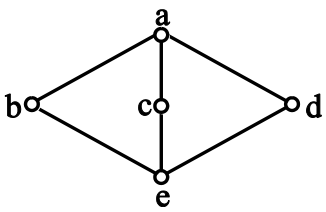
(C) $[D_{12};/]$

(D) $[\{1,2,3,5,30\};/]$

(E) $[\{1,2,3,4,12\};/]$

Q.73 The following is the Hasse diagram of the poset $[\{a,b,c,d,e\},\prec]$.

The poset is :



(A) not a lattice

(B) a lattice but not a distributive lattice

(C) a distributive lattice but not a Boolean algebra

(D) a Boolean algebra

[GATE 2005 : IIT Bombay]

Q.74 Which of the following lattice is /are complemented lattice

(A) $[D_{125};/]$

(B) $[P(A);\subseteq]$

(C) $[D_{12};/]$

(D) $[\{1,2,3,5,30\};/]$

(E) $[\{1,2,3,4,12\};/]$

Q.75 Which of the following lattice is /are Boolean algebra?

(A) $[D_{125};/]$

(B) $[P(A);\subseteq]$

(C) $[D_{12};/]$

(D) $[\{1,2,3,5,30\};/]$

(E) $[\{1,2,3,4,12\};/]$

Q.76 Which of the following lattice is /are Boolean algebra.

(A) $[D_{21};/]$ (B) $[D_{110};/]$

(C) $[D_{24};/]$ (D) $[\{D_{91};/]$

Q.77 What is the complement of element 5 in boolean algebra $[D_{110};/]$

Q.78 Let S be a set consisting of 10 elements. The of tuples of the form (A, B) such that A and B are subsets of S, and $A \subseteq B$ is _____.

[GATE 2021 : IIT Bombay]

Q.79 A relation R is said to be circular if aRb and bRc together imply cRa. Which of the following options is/are correct?

(A) If a relation S is transitive and circular, then S is an equivalence relation.

(B) If a relation S is reflexive and circular, then S is an equivalence relation.

(C) If a relation S is circular and symmetric, then S is an equivalence relation.

(D) If a relation S is reflexive and symmetric, then S is an equivalence relation.

[GATE 2021 : IIT Bombay]

Self Practice Questions

Q.1 The number of partitions of the set $S = \{a,b,c\}$ is

(A) 4 (B) 5

(C) 3 (D) 2

Q.2 If A = {2,4,5,6,7} , B = {3,4,6,7,8} and C = {2,3,4,5,6} ,then $(A \cup B) \cap C =$

(A) {2,4,5,6,7} (B) {1,2,3,4,5}

(C) {3,4,5,6,7} (D) {2,3,4,5,6}

Q.3 If A and B are two sets, then $(A-B) \cap A =$

(A) A (B) B

(C) $A-B$ (D) $A \cup B$

Q.4 The number of elements in the power set $P(S)$ of the set $S = \{\{\phi\},1,\{2,3\}\}$ is

(A) 2 (B) 4

(C) 8 (D) None of these

Q.5 Let $A=\{\phi,\{\phi\},1\{1,\phi\},2\}$ which of the following is FALSE?

(A) $\phi \in A$ (B) $\{\phi\} \in A$

(C) $\{1\} \in A$ (D) $\{2,\phi\} \subset A$

Q.6 If a set has 4 elements, then the number of relations on A is

(A) 46 (B) 28

(C) 416 (D) 216

Q.7 In a class of 125 students 72 can speak Tamil, 60 can speak Malayalam and 45 can speak Kannada, 48 speak Tamil and Malayalam, 37 speak Kannada and Tamil, 36 speak Kannada and Malayalam and 30 speak all the three languages. How many speak none of the languages?

(A) 31 (B) 39

(C) 53 (D) 48

Q.8 If the cardinality of a power set $P(A)$ is 512, then the cardinality of the set A is

(A) 8 (B) 9

(C) 19 (D) 7

Q.9 If $A-B=C-B$ and $A\cap B=C\cap B$, then which of the following is true?

(A) $A\subseteq B$ (B) $A\cap C=B$

(C) $A=C$ (D) $A=B$

Q.10 If $A\subseteq B$ and $C\subseteq A$ then which of the following is necessarily true?

(A) $B\subseteq C$ (B) $B=C$

(C) $A=B\neq C$ (D) $C\subseteq B$

Q.11 If A, B and C are any three sets, then $A-(B\cup C)=$

(A) $(A-B)\cup(A-C)$

(B) $(A-B)\cup(B-C)$

(C) $(A-B)\cap(A-C)$

(D) $(A-B)\cap(B-C)$

Common Data for Questions 12 to 13

In a survey of 250 families in a locality, it was found that 140 families subscribe for newspaper Deccan Chronicle, 110 families subscribe for Times of India and 110 families subscribe for The Hindu, 60 subscribe for both Deccan Chronicle and Times of India, 40 for both Deccan Chronicle and The Hindu, 50 for both Times of India and The Hindu and 20 are not subscribers of any of these newspapers.

Q.12 How many families subscribe for only one newspaper?

(A) 100 (B) 110

(C) 120 (D) 140

Q.13 How many families subscribe at least two newspapers?

(A) 90 (B) 100

(C) 110 (D) 150

Q.14 Which of the following is false?

(A) $A-(B\cap C)=(A-B)\cup(A-C)$

(B) $(A\cup B)^C=A^C\cap B^C$

(C) $(A\cap B)^C=A^C\cap B^C$

(D) $A\cap A^C=\phi$

Q.15 Let A = {1,2,3,4,5} and $S=A\times A$ Let R be a relation is S defined by

$(a_1,b_1)R(a_2,b_2)$ if $a_1b_1=a_2b_2$

Then which of the following is TRUE?

(A) R is a compatibility relation but not an equivalence relation

(B) R is an equivalence relation

(C) R is partial order relation but not a total ordered relation

(D) R is a total ordered relation

Q.16 Which of the following statement is NOT true about relations that are defined from a set to itself?

(A) Every relation that is irreflexive and transitive is also asymmetric

(B) There exists a relation, which is neither reflexive nor irreflexive

(C) Every relation is either symmetric or asymmetric

(D) There exists a relation, which is both symmetric and antisymmetric

Q.17 Let R = {(1,1),(2,2),(3,3)} be a relation in this set A = {1,2,3} , Then R is_____.

(A) Symmetric

(B) Anti-symmetric

(C) Both (A) and (B)

(D) Neither (A) and (B)

Q.18 A relation R, defined as $x+y=10$ in the set of real numbers, satisfies _____ property.

(A) Reflexive (B) Symmetric

(C) Transitive (D) Equivalence

Q.19 R_1 and R_2 are two relations which are equivalence is a set A, Then

(A) $R_1 \cap R_2$ is equivalence on A

(B) $R_1 \cup R_2$ is equivalence on A

(C) $R_1 - R_2$ is equivalence on A

(D) None of these

Q.20 The binary relation $S = \phi$ (empty set) on set $A = \{1, 2, 3\}$ is

(A) Neither reflexive nor symmetric

(B) Symmetric and reflexive

(C) Transitive and reflexive

(D) Transitive and symmetric

Q.21 If A is a finite set with n elements, then number of elements in the largest equivalence relation of A is

(A) 1 (B) n

(C) $n+1$ (D) n^2

Q.22 Consider the following relations:

$R_1(a,b)$ if $(a+b)$ is even over the set of integers

$R_2(a,b)$ if $(a+b)$ is odd over the set of integers

$R_3(a,b)$ if $a \cdot b > 0$ over the set of non-zero rational numbers

$R_4(a,b)$ if $|a-b| \le 2$ over the set of natural numbers

Which of the following statement is correct?

(A) R_1 and R_2 are equivalence relations, R_3 and R_4

(B) R_1 and R_3 are equivalence relations, R_2 and R_4

(C) R_1 and R_4 are equivalence relations, R_2 and R_3 are not

(D) R_1, R_2, R_3 and R_4 are all equivalence relations

Q.23 If $(L, \oplus, *)$ is a lattice, then which of the following properties need not be satisfied by $(L, \oplus, *)$?

(A) Idempotent law

(B) Commutative law

(C) Associative law

(D) Distributive law

Q.24 If S_m denotes the set of all divisiors of m and D is the relation "divides", then the complement of 3 in the lattice (S_{75}, D) is

(A) 5 (B) 15

(C) 25 (D) 75

Q.25 The greatest lower bound and the least upper bound of the set {4,8,12} in the poset $\{Z^+, /\}$ is

(A) 2, 36 (B) 2, 24

(C) 4, 12 (D) 4, 24

Q.26 The maximal and minimal elements of the poset ({2,3,6,12,15,18}, /) is :

(A) {12, 15, 18},{2, 3}

(B) {18}, {2}

(C) {18, 15}, {2}

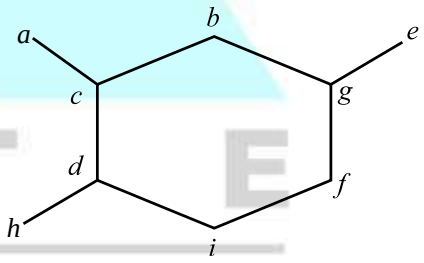
(D) {18}, {2, 3}

Q.27 Which of the following is not a poset? (R-set of real numbers)

(A) $(R, \le)$ (B) $(R, \ge)$

(C) $(R, =)$ (D) $(R, \ne)$

Common Data for Questions 28 to 32



These questions are based on the above Hasse diagrams.

$L = \{a, b, c, d, e, f, g, h, i\}$.

Q.28 Which of the following is the least elements of L?

(A) d (B) f

(C) e (D) Does not exist

Q.29 How many maximal elements does L have?

(A) 1 (B) 2

(C) 3 (D) 4

Q.30 How many minimal elements does L have?

(A) 1 (B) 2

(C) 3 (D) 4

Q.31 Which of the following subset of L, has the least element?

(A) $\{a,b,c,d,h\}$ (B) $\{b,c,d,f\}$

(C) $\{a,b,c,e\}$ (D) $\{c,g\}$

Q.32 Which of the following of elements are not comparable?

(A) $\{a,b\}$ (B) $\{b,c\}$

(C) $\{a,i\}$ (D) $\{e,g\}$

Q.33 If X and Y are two sets, then $X \cap (Y \cap X)^C$ equals

(A) ϕ (B) Y

(C) X (D) $X \cap Y^C$

Q.34 Hasse diagrams are drawn for

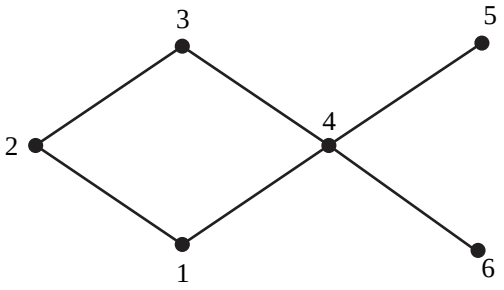
(A) Partially ordered sets

(B) Lattices

(C) Boolean algebra

(D) None of these

Q.35 Maximal and minimal elements of the poset are



(A) Maximal 5, 6; minimal 2

(B) Maximal 5, 6; minimal 1

(C) Maximal 3, 5; minimal 1, 6

(D) None of these

Q.36 Let $D_{30} = \{1,2,3,5,6,10,15,30\}$ and relation / be a partial ordering on D_{30}. The L.U.B. of 10 and 15 is

(A) 6 (B) 10

(C) 15 (D) 30

Answer Keys

Classroom Practice Questions									
1.	2048	2.	C	3.	C	4.	C	5.	Invalid
6.	B	7.	D	8.	A	9.	A	10.	D
11.	D	12.	B	13.	C	14.	C	15.	B
16.	A	17.	C	18.	D	19.	*	20.	B
21.	2^{n^2}	22.	C	23.	C	24.	0.125	25.	D
26.	B	27.	D	28.	C	29.	D	30.	B
31.	A,B,C,D	32.	C,D	33.	A	34.	*	35.	*
36.	*	37.	*	38.	*	39.	B	40.	B
41.	B	42.	B	43.	B	44.	5	45.	A
46.	52	47.	*	48.	C	49.	*	50.	52
51.	C	52.	*	53.	C	54.	A	55.	D
56.	*	57.	*	58.	*	59.	*	60.	B
61.	B	62.	A	63.	D	64.	A	65.	A
66.	"O"	67.	B,D	68.	*	69.	*	70.	d,b,c,e
71.	D,E,F	72.	D,E	73.	B	74.	B,D,E	75.	B
76.	A,B,D	77.	22	78.	*	79.	B		

Self - Practice Questions									
1.	A	2.	D	3.	C	4.	C	5.	C
6.	D	7.	B	8.	B	9.	C	10.	D
11.	C	12.	C	13.	C	14.	C	15.	B
16.	C	17.	C	18.	B	19.	A	20.	D
21.	D	22.	B	23.	D	24.	C	25.	D
26.	A	27.	D	28.	D	29.	C	30.	B
31.	A	32.	A	33.	A	34.	A	35.	C
36.	D								

Classroom Practice Questions

* **19** $\dfrac{n\times(n-1)^2}{2}$

* **33** $\{(1,2),(2,1),(2,2),(3,3)\}$

* **34** $\{(1,1),(1,2),(2,2),(3,3)\}$

* **36** $\{(1,1),(1,2),(1,3),(2,2),(3,1),(3,2),(3,3)\}$

* **37** $\{(a,a),(a,c),(a,d),(b,a),(b,c),(b,d),(c,a),(c,c),(c,d),(d,a),(d,c),(d,d)\}$

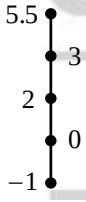
* **38** $\{(1,2),\ (1,3),(1,4),(2,3),(2,4),(3,4),(5,4)\}$

* **47** **(A) False (B) True**

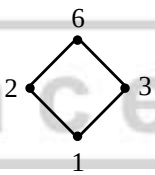
* **49** $\{\{1,2\},\{3\},\{4,5\}\}$

* **52** $\{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c),(d,d)\}$

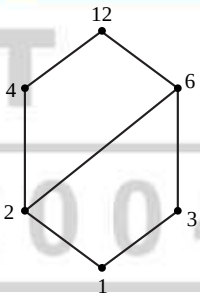
* **56**



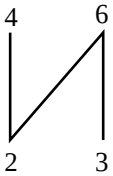
* **57**



* **58**



* **59**



* **68** $\overline{1}=12,\ \overline{2}=$ does not exists, $\overline{3}=4,\ \overline{4}=3,\ \overline{6}=$ does not exists, $\overline{12}=1$

* **69** $\overline{a}=g,\ \overline{b}=d,c,f,\ \overline{c}=b,d,e,\ \overline{d}=b,c,e,f,\ \overline{e}=d,c,f,\ \overline{f}=b,d,e,\ \overline{g}=a$

* **78** $3^{10}=59049$

❖❖❖❖

2 Graph Theory

Classroom Practice Questions

Q.1 Maximum number of edges in a n-node undirected graph without self loops is

(A) n^2 (B) $n(n-1)/2$

(C) $n-1$ (D) $(n+1)(n)/2$

[GATE 2002 : IISc Bangalore]

Q.2 How many undirected graphs (not necessarily connected) can be constructed out of a given set $V=\{v_1,v_2,.....,v_n\}$ of n vertices?

(A) $n(n-1)/2$ (B) 2^n

(C) $n!$ (D) $2^{n(n-1)/2}$

[GATE 2001 : IIT Kanpur]

Q.3 Number of simple graphs possible with 6 vertices and 4 edges is

(A) 15 (B) 360

(C) 1296 (D) 1365

Q.4 How many graphs on n labeled vertices exist which have at least $\left(n^2-3n\right)/2$ edges?

(A) $^{\left(n\hat{}2-n\right)/2}C_{\left(n\hat{}2-3n\right)/2}$

(B) $\sum_{k=0}^{\left(n\hat{}2-3n\right)/2} {}^{\left(n\hat{}2-n\right)}C_k$

(C) $^{\left(n\hat{}2-n\right)/2}C_n$

(D) $\sum_{k=0}^{n} {}^{\left(n\hat{}2-n\right)/2}C_k$

[GATE 2004 : IIT Delhi]

Q.5 A wheel graph on n vertices is regular when n =_____.

Q.6 If a complete Bipartite graph Km, n is regular then

(A) m < n

(B) m > n

(C) m = n

(D) None of the above

Q.7 The number of edges in a k - regular graph with n vertices is

(A) 2nk (B) nk/2

(C) C(n, k) (D) P(n, k)

Q.8 What is the maximum number of edges possible in a bipartite graph with n vertices?

Q.9 What is the maximum number of edges in an acyclic undirected graph with n vertices?

(A) $n-1$ (B) n

(C) $n+1$ (D) $2n-1$

[GATE 2004 : IIT Delhi]

Q.10 What is the number of vertices in an undirected connected graph with 27 edges, 6 vertices of degree 2, 3 vertices of degree 4 and remaining of degree 3?

(A) 10 (B) 11

(C) 18 (D) 19

[GATE 2004 : IIT Delhi]

Q.11 A non-directed graph contains 16 edges and all vertices are of degree 2. Then the number of vertices in G is _____.

Q.12 If a simple non directed graph G contains 24 edges and all vertices are of same degree then which of the following is false?

(A) |V(G)| = 24 (B) |V(G)| = 18

(C) |V(G)| = 16 (D) |V(G)| = 6

Q.13 *G* is undirected graph with n vertices and 25 edges such that each vertex of *G* has degree at least 3. Then the maximum possible value of n is _____.

[GATE 2017 : IIT Roorkee]

Q.14 G is an undirected graph with n vertices and 25 edges such that each vertex of G has degree atmost 4. Then minimum possible value of n is _____.

Q.15 Which of the following statements is/are TRUE for undirected graphs?

P : Number of odd degree vertices is even.

Q : Sum of degree of all vertices is even.

(A) P only (B) Q only

(C) Both P and Q (D) Neither P nor Q

[GATE 2013 : IIT Bombay]

Q.16 If G be a graph with 5 vertices and 7 edges then

(A) $\delta(G) > 2$ (B) $\Delta(G) > 4$

(C) $\delta(G) \le 2$ (D) $\Delta(G) \le 2$

Q.17 Which of the following degree sequences represent a simple non directed graph?

(A) {2, 3, 3, 4, 4, 5}

(B) {2, 3, 4, 4, 5}

(C) {1, 3, 3, 3}

(D) {0, 1, 2….., n-1}

(E) {1, 3, 3, 4, 5, 6, 6}

(F) {2, 3, 3, 3, 3}

Q.18 Which of the following degree sequences represent a simple non directed graph?

S_1 = {6, 6, 6, 6, 4,3,3,0}

S_2 = {6, 5, 5, 4, 3, 3, 2, 2,2}

(A) Only S_1

(B) Only S_2

(C) Both S_1 and S_2

(D) Neither S_1 nor S_2

Q.19 The degree sequences of a simple graph is the sequence of the degree of the nodes in the graph in decreasing order. Which of the following sequences cannot be the degree sequence of any graph?

I. 7, 6, 5, 4, 4, 3, 2, 1

II. 6, 6, 6, 6, 3, 3, 2, 2

III. 7, 6, 6, 4, 4, 3, 2, 2

IV. 8, 7, 7, 6, 4, 2, 1, 1

(A) I and II (B) III and IV

(C) IV only (D) II and IV

[GATE 2010 : IIT Guwahati]

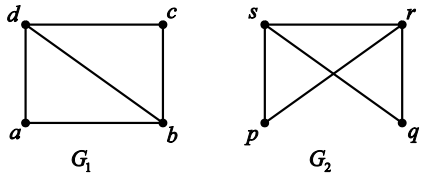
Q.20 Let G is simple graph with 7 vertices and 11 edges. Then find number of edges in complement of G {i.e. $\bar{G}$ }

Q.21 Let G is simple graph such that G has 21 edges and $\bar{G}$ has 24 edges then find number of vertices in graph G.

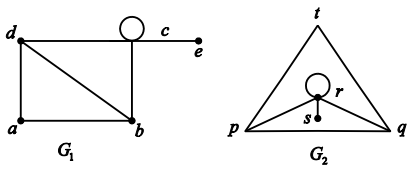
Q.22 If G is a simple graph with degree sequence {5, 2, 2, 2, 2,1} then what is the number of edges in the complement $\bar{G}$?

Q.23 Check whether the following pair of graphs is/are isomorphic?

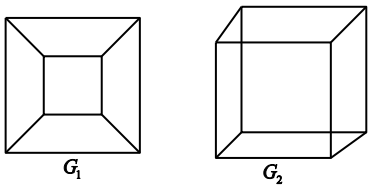
(A)



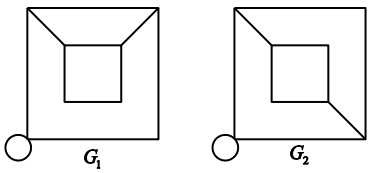
(B)



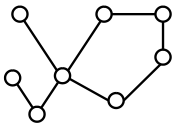
(C)



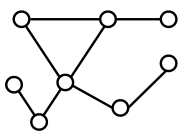
(D)



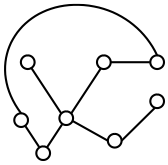
Q.24 Which of the following graphs is isomorphic to



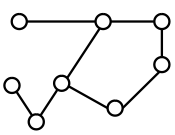
(A)



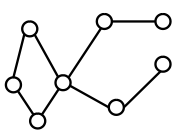
(B)



(C)



(D)



[GATE 2012 : IIT Delhi]

Q.25 How many simple non-isomorphic graphs are possible with 3 vertices?

Q.26 The number of distinct simple graphs with up to three nodes is

(A) 15 (B) 10

(C) 7 (D) 9

[GATE 1994 : IIT Kharagpur]

Q.27 How many simple non-isomorphic graphs are possible with 4 vertices and 2 edges?

Q.28 How many simple non-isomorphic graphs are possible with 4 vertices?

Q.29 A graph is self-complementary if it is isomorphic to its complement. For all self-complementary graphs on n vertices, n is

(A) A multiple of 4

(B) Even

(C) Odd

(D) Congruent to 0 mod 4, 1 mod 4.

[GATE 2015 : IIT Kanpur]

Q.30 If G is isomorphic to $\bar{G}$ which of the following statement is not true.

(A) $|V(G)|=5$ (B) $|V(G)|=8$

(C) $|V(G)|=7$ (D) $|V(G)|=13$

Q.31 A cycle on n vertices is isomorphic to its complement. The value of n is _____.

[GATE 2014 : IIT Kharagpur]

Q.32 A non-planar graph with minimum number of vertices has

(A) 9 edges, 6 vertices

(B) 6 edges, 4 vertices

(C) 10 edges, 5 vertices

(D) 9 edges, 5 vertices

[GATE 1992 : IIT Delhi]

Q.33 Let G be the non-planar graph with minimum possible number of edges. Then G has

(A) 9 edges and 5 vertices

(B) 9 edges and 6 vertices

(C) 10 edges and 5 vertices

(D) 10 edges and 6 vertices

[GATE 2007 : IIT Kanpur]

Q.34 Maximum number of edges in a planar graph with n vertices is _____.

[GATE 1992 : IIT Delhi]

Q.35 Let G be a simple connected planar graph with 25 vertices and 60 edges, then the number of region in the graph G is _____.

Q.36 Let G be a connected planar graph with 20 vertices each of degree 3 then the number of faces in the planar embedding of the graph is _____.

Q.37 Let G be a connected planar graph with 35 regions each of degree 6. Then the number of vertices in graph G is _____.

Q.38 Let G be a connected planar graph with 12 vertices and 30 edges, and degree of each region is k. Then the value of k is _____.

Q.39 Maximum number of edges possible in a simple connected planar graph with 8 vertices is _____.

Q.40 Minimum number of vertices necessary in a simple connected planar graph with 11 edges is _____.

Q.41 Maximum number of regions possible in a simple connected planar graph with 10 vertices is _____.

Q.42 A graph $G=(V,E)$ satisfies $|E|\le 3|V|-6$. The min-degree of G is defined as $\min_{v\in V}\{\text{degree}(v)\}$. Therefore, min-degree of G cannot be

(A) 3 (B) 4

(C) 5 (D) 6

[GATE 2003 : IIT Madras]

Q.43 Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph, then the number of bounded faces in any embedding of G on the plane is equal to

(A) 3 (B) 4

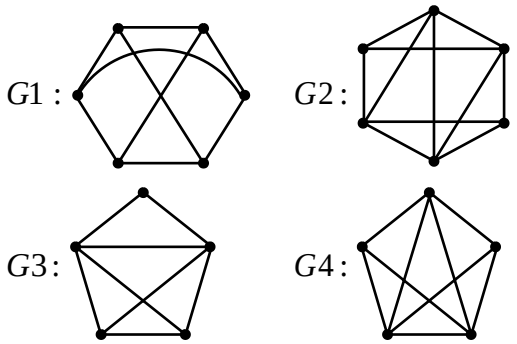
(C) 5 (D) 6

[GATE 2012 : IIT Delhi]

Q.44 Let G be a connected planar graph with 10 vertices. If the number of edges on each face is three, then the number of edges in G is _____.

[GATE 2015 : IIT Kanpur]

Q.45 Which one of the following graphs is NOT planar?

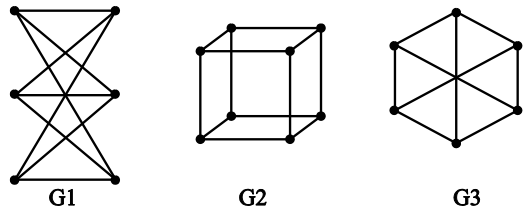


(A) G1 (B) G2

(C) G3 (D) G4

[GATE 2005 : IIT Bombay]

Q.46 Which of the following graphs is/are planar?

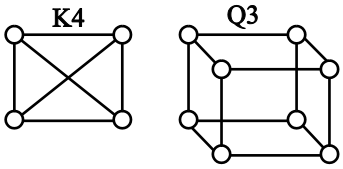


(A) G1 only (B) G1 and G2 only

(C) G2 only (D) G2 and G3 only

[GATE 1989 : IIT Kanpur]

Q.47 K4 and Q3 are graph with the following structures.



Which one of the following statements is TRUE in relation to these graphs?

(A) K4 is planar while Q3 is not

(B) Both K4 and Q3 are planar

(C) Q3 is planar while K3 is not

(D) Neither K4 nor Q3 is planar

[GATE 2010 : IIT Guwahati]

Q.48 Let δ denote the minimum degree of a vertex in a graph. For all planar graphs on n vertices with $\delta \geq 3$, which one of the following is TRUE?

(A) In a planar embedding, the number of faces is at least $\frac{n}{2}+2$

(B) In any planar embedding, the number of faces is less than $\frac{n}{2}+2$

(C) There is a planar embedding in which the number of faces is less than $\frac{n}{2}+2$

(D) There is a planar embedding in which the number of faces is at most $\frac{n}{\delta+1}$

[GATE 2014 : IIT Kharagpur]

Q.49 Consider the following statement

S_1 : A polyhedral graph with exactly 7 edges exists

S_2 : A polyhedral graph with 30 edges and 11 region does not exists.

(A) S_1 is true but S_2 is false

(B) S_1 is false but S_2 is true

(C) Both are true

(D) Both are false

Q.50 A graph is planar if and only if,

(A) it does not contain subgraphs homeomorphic to K_5 and $K_{3,3}$.

(B) it does not contain subgraphs isomorphic to k_5 or $K_{3,3}$.

(C) it does not contain a subgraph isomorphic to k_5 or $K_{3,3}$.

(D) it does not contain a subgraphs homeomorphic to k_5 or $K_{3,3}$.

[GATE 1990 : IISc Bangalore]

Q.51 Find the chromatic number of the following graphs

(i) K_n (ii) C_n

(iii) W_n (iv) $K_{m,n}$

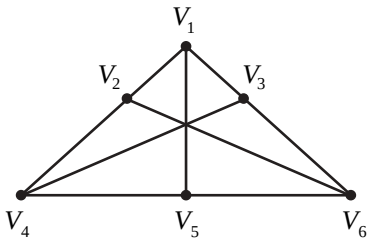
(v) Star graph with n vertices

Q.52 The chromatic number of cycle graph $C_n (n \geq 3)$ is

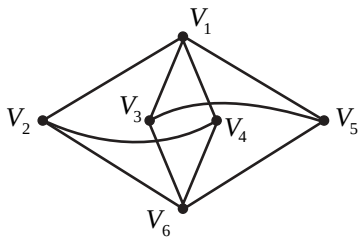
(A) 2 (B) 3

(C) $n-2\left[\frac{n}{2}\right]+2$ (D) $n-2\left[\frac{n}{2}\right]+1$

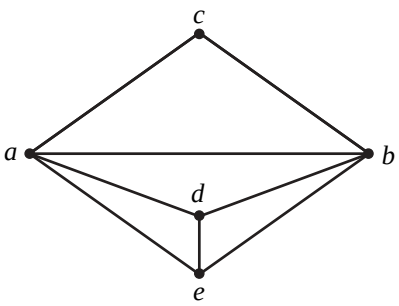
Q.53 The chromatic number of the graph shown below is _____.



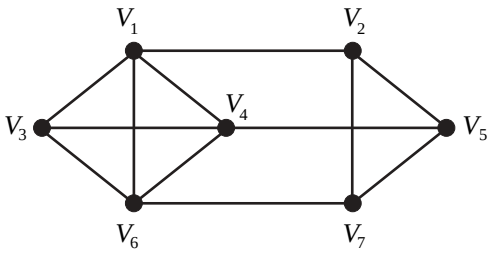
Q.54 For the graph shown below, the chromatic number is _____.



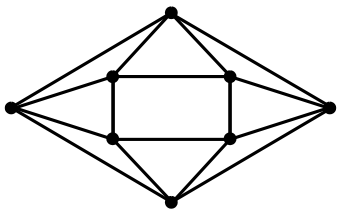
Q.55 For the graph shown below, the chromatic number is _____.



Q.56 For the graph shown below, the chromatic number is _____.



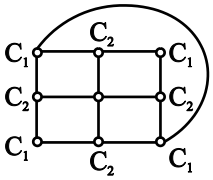
Q.57 The minimum number of colours required to colour the following graph, such that no two adjacent vertices are assigned the same colour, is



(A) 2 (B) 3
(C) 4 (D) 5

[GATE 2004 : IIT Delhi]

Q.58 What is the chromatic number of the following graph?



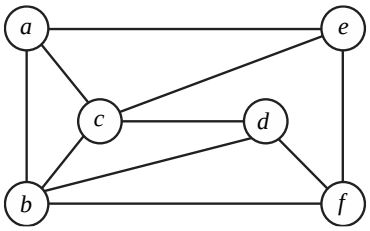
(A) 2 (B) 3
(C) 4 (D) 5

[GATE 2008 : IISc Bangalore]

Q.59 The minimum number of colours that is sufficient to vertex-colour any planar graph is _____.

[GATE 2016 : IISc Bangalore]

Q.60 The chromatic number of the following graph is _____.



[GATE 2018 : IIT Guwahati]

Q.61 Graph G is obtained by adding vertex s to $K_{3,4}$ and making s adjacent to every vertex of $K_{3,4}$. The minimum number of colours required to edge-colour G is _____.

[GATE 2020 : IIT Delhi]

Q.62 What is the chromatic number of an n-vertex simple connected graph, which does not contain any odd length cycle? Assume $n \geq 2$.

(A) 2 (B) 3
(C) $n-1$ (D) n

[GATE 2009 : IIT Roorkee]

Q.63 The number of maximal matching in K_3 is

(A) 1 (B) 2
(C) 3 (D) 4

Q.64 The number of perfect matchings in K_{2n} is

(A) $\frac{(2n)!}{2^n..n!}$ (B) $\frac{(2n-1)!}{2^n.(n-1)!}$

(C) $\frac{2^n.n!}{2n!}$ (D) 0

Q.65 The number of perfect matching in complete by bipartite graph $K_{n,n}$ is

(A) n (B) n^2
(C) 2^n (D) $n!$

Q.66 Which of the following is not true?

(A) Number of perfect matchings in $K_{2n} = \dfrac{2n!}{2^n n!}$

(B) Number of perfect matchings in $K_{n,n} = n!$

(C) Number of perfect matchings in C_n (n is even) = 2

(D) Number of perfect matchings in $W_{2n} = 2n$

Q.67 How many perfect matching's are there in a complete graph of 6 vertices?

(A) 15 (B) 24

(C) 30 (D) 60

[GATE 2003 : IIT Madras]

Q.68 The number of perfect matching in a tree with n vertices $(n \geq 2)$ is

(A) ≤ 1 (B) > 1

(C) $= 1$ (D) ≥ 1

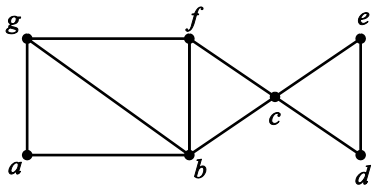
Q.69 Find the matching number of the following graph

(i) K_n (ii) C_n

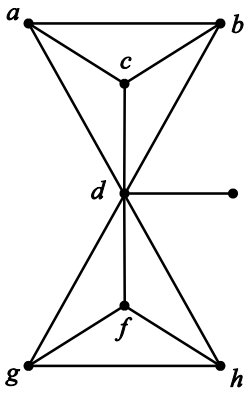
(iii) W_n (iv) $K_{m,n}$

(v) Star graph with n vertices

Q.70 Find the matching number of the following graph



Q.71 Find the matching number of the following graph

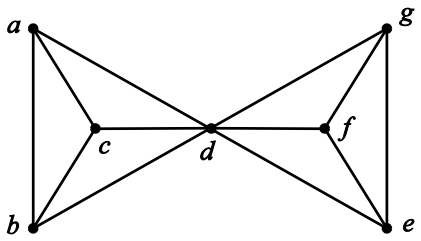


Q.72 The number of edges in a minimal line covering of a graph with n vertices is

(A) $\left\lceil \dfrac{n}{2} \right\rceil$ (B) $\left\lfloor \dfrac{n}{2} \right\rfloor$

(C) $\leq n-1$ (D) $< n-1$

Q.73 Find line covering number (α_1) and line independence number (β_1) for the following graph



Q.74 Find the covering number (α_1) and line independence number (β_1) for the following graph

(i) K_n (ii) C_n

(iii) W_n (iv) $K_{m,n}$

(v) Star graph with n vertices

Q.75 Find the vertex covering number (α_2) and vertex independence number (β_2) for the following graph

(i) K_n (ii) C_n

(iii) W_n (iv) $K_{m,n}$

(v) Star graph with n vertices

Q.76 What is the size of the smallest MIS (Maximal Independent Set) of a chain of nine nodes?

(A) 5 (B) 4

(C) 3 (D) 2

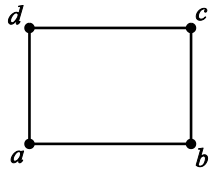
[GATE 2008 : IISc Bangalore]

Q.77 Let G be a simple graph with 20 Vertices and 100 edges. The size of the minimum vertex cover of G is 8. Then, the size of the maximum independent set of G is:

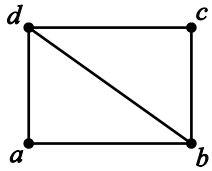
(A) 12 (B) 8

(C) Less than 8 (D) More than 12

Q.78 Find the number of spanning tree of graph shown below

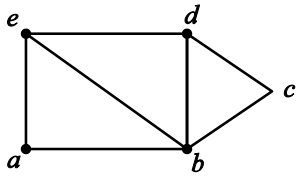


Q.79 Find the number of spanning tree of graph shown below

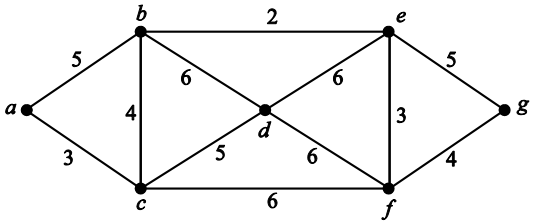


(A) 6 (B) 10
(C) 8 (D) 12

Q.80 Find the number of spanning trees of the following graph



Q.81 Find the minimum spanning trees of the following graph



(i) Using Kruskal's algorithm :-
(ii) Using Prim's algorithm (consider d as initial vertex)

Q.82 Let G be an undirected graph with distinct edge weights. Let e_{max} be the edge with maximum weight and e_{min} the edge with minimum weight. Which of the following statements is false?

(A) Every minimum spanning tree of G must contain e_{min}

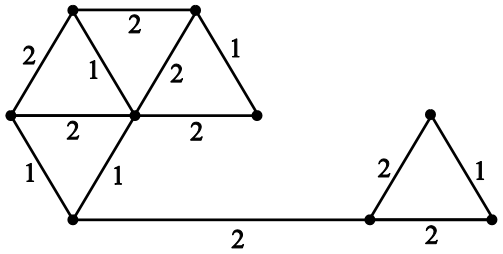
(B) If e_{max} is in minimum spanning tree, then its removal must disconnect G

(C) No minimum spanning tree contain e_{max}

(D) G has a unique minimum spanning tree

[GATE 2000 : IIT Kharagpur]

Q.83 The number of distinct minimum spanning trees for the weighted graph below is _____.

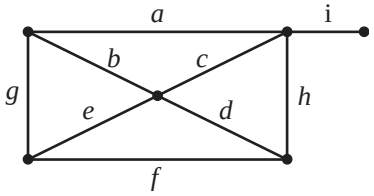


[GATE 2014 : IIT Kharagpur]

Q.84 Let G be connected undirected graph of 100 vertices and 300 edges. The weight of a minimum spanning tree of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes _____.

[GATE 2015 : IIT Kanpur]

Q.85 For the graph shown here,



Which of the following is not a cut set?

(A) {a, b, g} (B) {a, b, e, f}
(C) {a, c, h, d} (D) {a, c, d, f}

Q.86 Find vertex connectivity and edge connectivity of the following graph

(i) K_n (ii) C_n
(iii) W_n (iv) $K_{m,n}$
(v) Star graph with n vertices

Q.87 Let G be an arbitrary graph with n nodes and k components. If a vertex is remove from G, the number of components in the resultant graph must necessarily lie between

(A) k and n (B) $k-1$ and $k+1$
(C) $k-1$ and $n-1$ (D) $k+1$ and $n-k$

[GATE 2003 : IIT Madras]

Q.88 If G is a forest with n vertices and k connected components, how many edges does G have?

(A) $\lfloor n/k \rfloor$ (B) $\lceil n/k \rceil$
(C) $n-k$ (D) $n-k+1$

Q.89 In a connected graph G if we delete an edge then no. of components are

(A) 0 or 1 (B) 1 or 2

(C) 2 (D) ≥ 2

Q.90 In a connected graph G is we delete a vertex then number of components are

(A) ≤ 2

(B) Lies between 1 and $n-1$

(C) 1 or 2

(D) ≥ 2

Q.91 In a connected graph, a bridge is an edge whose removal disconnects a graph. Which one of the following statements is true?

(A) A tree has no bridge

(B) A bridge cannot be part of a simple cycle

(C) Every edge of a clique with size ≥ 3 is a bridge (A clique is any complete subgraph of graph)

(D) A graph with bridge cannot have a cycle

[GATE 2015 : IIT Kanpur]

Q.92 A simple graph with n vertices is necessarily connected if no. of edges are more than E. Then find the value of E.

Q.93 The maximum number of possible edges in an undirected graph with n vertices and k components is _____.

[GATE 1991 : IIT Madras]

Q.94 If all the edge weights of an undirected graph are positive, then any subset of edges that connects all the vertices and has minimum total weight is a

(A) Hamiltonian cycle

(B) Grid

(C) hypercube

(D) tree

[GATE 2006 : IIT Kharagpur]

Q.95 If G is a simple graph on n vertices then which of the following statements is true?

(A) Atleast one of G and its complement is connected.

(B) If G is connected then its complement is also connected.

(C) If G is connected then its complement is disconnected.

(D) If complement G is connected then G is disconnected.

Q.96 Which of the following graphs has an Eulerian circuit?

(A) Any k-regular graph where k is an even number.

(B) A complete graph on 90 vertices.

(C) The complement of a cycle on 25 vertices.

(D) None of the above.

[GATE 2007 : IIT Kanpur]

Q.97 G is a simple undirected graph. Some vertices of G are of odd degree. Add a node v to G and make it adjacent to each odd degree vertex of G. The resultant graph is sure to be

(A) Regular (B) Complete

(C) Hamilton (D) Euler

[GATE 2008 : IISc Bangalore]

Q.98 Let G be an undirected complete graph on n vertices $n > 2$. Then, the number of different Hamiltonian cycles in G is equal to

(A) n! (B) (n-1)!

(C) 1 (D) $\frac{(n-1)!}{2}$

[GATE 2019 : IIT Madras]

Common Data for Questions 99 to 101

The 2^n vertices of graph G correspond to all subsets of a set of size n, for $n \geq 6$. Two vertices of G are adjacent if and only if the corresponding sets intersect in exactly two elements.

Q.99 The number of vertices of degree zero in G is

(A) 1 (B) n

(C) $n+1$ (D) 2^n

[GATE 2006 : IIT Kharagpur]

Q.100 The maximum degree of a vertex in G is

(A) $\binom{n/2}{2}2^{n/2}$ (B) 2^{n-2}

(C) $2^{n-3} \times 3$ (D) 2^{n-1}

[GATE 2006 : IIT Kharagpur]

Q.101 The number of connected components in G is

(A) n (B) $n+2$

(C) $2^{n/2}$ (D) $\frac{2^n}{n}$

[GATE 2006 : IIT Kharagpur]

Q.102 Let G be a complete undirected graph on 6 vertices. If vertices of G are labeled, then the number of distinct cycles of length 4 in G is equal to

(A) 15 (B) 30

(C) 90 (D) 360

[GATE 2012 : IIT Delhi]

Q.103 Consider an undirected random graph of eight vertices. The probability that there is an edge between a pair of vertices is ½. What is the expected number of unordered cycle of length three?

(A) 1/8 (B) 1

(C) 7 (D) 8

[GATE 2013 : IIT Bombay]

Q.104 Consider an unidirectional graph G where self-loops are not allowed. The vertex set of G is $\{(i, j): 1 \le i \le 12, 1 \le j \le 12\}$. There is an edge between (a,b) and (c,d) if $|a-c| \le 1$ and $|b-d| \le 1$. The number of edges in this graph is _____.

[GATE 2014 : IIT Kharagpur]

Q.105 An ordered n-tuple $(d_1, d_2 ..... d_n)$ with $d_1 \ge d_2 \ge ... \ge d_n$ is called graphic if three exists a simple undirected graph with n vertices having degrees $d_1, d_2, ..., d_n$ respectively. Which of the following 6-tuples is NOT graphic?

(A) (1,1,1,1,1,1) (B) (2,2,2,2,2,2)

(C) (3,3,3,1,0,0) (D) (3,2,1,1,1,0)

[GATE 2014 : IIT Kharagpur]

Q.106 Let G be a graph with 100! Vertices, with each vertex labelled by a distinct permutation of the numbers 1,2,…,100. There is an edge between vertices u and v if and only if the label of u can be obtained by swapping two adjacent numbers in the label of v. Let y denote the degree of a vertex in G, and z denote the number of connected components in G.

Then, $y+10z =$ _____.

[GATE 2018 : IIT Guwahati]

Q.107 The maximum number of edges in a bipartite graph on 12 vertices is _____.

[GATE 2014 : IIT Kharagpur]

Q.108 Consider the undirected graph G defined as follows. The vertices of G are bit strings of length n. We have an edge between vertex u and vertex v if and only if u and v differ in exactly one bit position (in other words, v can be obtained from u by flipping a single bit). The ratio of the chromatic number of G to the diameter of G is

(A) $1/2^{n-1}$ (B) $1/n$

(C) $2/n$ (D) $3/n$

[GATE 2006 : IIT Kharagpur]

Q.109 Let T be a tree with 10 vertices. The sum of the degree of all the vertices in T is _____.

[GATE 2017 : IIT Roorkee]

Q.110 Let $G=(V,E)$ be a graph. Define $\xi(G) = \sum_d i_d \times d$, where i_d is the number of vertices of degree d in G. If S and T are two different trees with $\xi(S) = \xi(T)$, then

(A) $|S| = 2|T|$ (B) $|S| = |T| - 1$

(C) $|S| = |T|$ (D) $|S| = |T| + 1$

[GATE 2009 : IIT Roorkee]

Q.111 Consider a weighted complete graph G on the vertex set $\{v_1, v_2, ..., v_n\}$ such that the weight of the edge (v_i, v_j) is $2|i-j|$. The weight of minimum spanning tree of G is

(A) $n-1$ (B) $2n-2$

(C) $\binom{n}{2}$ (D) n^2

[GATE 2006 : IIT Kharagpur]

Q.112 Let $G_1=(V,E_1)$ and $G_2=(V,E_2)$ be connected graphs on the same vertex set V with more than two vertices. If $G_1\cap G_2=(V,E_1\cap E_2)$ is not a connected graph then the graph $G_1\cup G_2=(V,E_1\cup E_2)$

(A) Can not have a cut vertex

(B) must have a cycle

(C) must have a cut-edge (bridge)

(D) has chromatic number strictly greater than those of G_1 and G_2

[GATE 2004 : IIT Delhi]

Common Data for Questions 113 & 114

A binary tree with $n>1$ nodes has n_1, n_2 and n_3 nodes of degree one two and three respectively. The degrees of a node is defined as the number of its neighbours.

Q.113 n_3 can be expressed as :

(A) n_1+n_2-1 (B) n_1-2

(C) $\left[\frac{n_1+n_2}{2}\right]$ (D) n_2-1

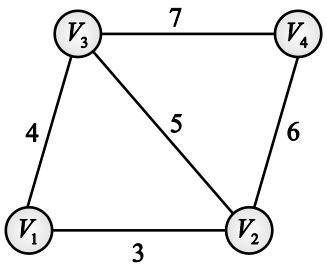
Q.114 Starting with the above tree, while there remains a node v of degree two in the tree, add an edge between the two neighbours of v and then remove v from the tree. How many edges will remain at the end of the process?

(A) $2*n_1-3$ (B) n_2+2*n_1-2

(C) n_3-n_2 (D) n_2+n_1-2

Common Data for Questions 115 & 116

An undirected graph $G(V,E)$ contains $n(n>2)$ nodes named $v_1, v_2, ..., v_n$. Two nodes v_i, v_j are connected if and only if $0<|i-j|\leq 2$. Each edges (v_i, v_j) is assigned a weight $i+j$. A sample graph with $n=4$ is shown below.



Q.115 What will be the cost of minimum spanning tree (MST) of such a graph with n nodes?

(A) $\frac{1}{12}(11n^2-5n)$ (B) n^2-n+1

(C) $6n-11$ (D) $2n+1$

[GATE 2011 : IIT Madras]

Q.116 The length of the path from v_5 to v_6 in the MST of previous question with $n=10$ is

(A) 11 (B) 25

(C) 31 (D) 41

[GATE 2011 : IIT Madras]

Q.117 Consider a set U of 23 different compounds in a Chemistry lab. There is a subset S of U of 9 compounds, each of which reacts with exactly 3 compounds of U.

Consider the following statements :

I. Each compound in U/S reacts with an odd number of compounds.

II. At least one compound in U/S reacts with an odd number of compounds.

III. Each compound in U/S reacts with an even number of compounds.

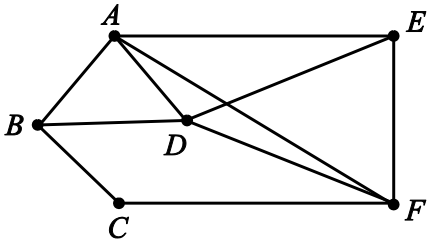
Which one of the above statements is ALWAYS TRUE?

(A) Only I (B) Only II

(C) Only III (D) None

[GATE 2016 : IISc Bangalore]

Q.118 Let G be a connected undirected graph. A cut in G is a set of edges whose removal results in G being broken into two or more components which are not connected with each other. The size of the cut is called its cardinality. A *min−cut* of G is a cut in G of minimum cardinality. Consider the following Graph.



(A) Which of the following sets of edges is a cut?

(i) $\{(A,B),(E,F),(B,D),(A,E),(A,D)\}$

(ii) $\{(B,D),(C,F),(A,B)\}$

(B) What is the cardinality of a $min-cut$ in this graph?

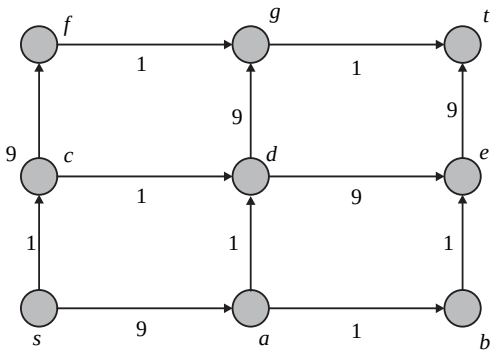
(C) Prove that if a connected undirected graph G with n vertices has a $min-cut$ of cardinality k, then G has at least $\left(\frac{nk}{2}\right)$ edges.

[GATE 1999 : IIT Bombay]

Q.119 Let G be a connected undirected weighted graph consider the following to statements.
S_1. There exists a minimum weight edge in G which is present in every MST of G.
S_2. If every edge in G has distinct weights, then G has a unique MST.
Which of the following is true?
(A) S_1 is true and S_2 is false
(B) S_1 is false and S_2 is true
(C) Both S_1 and II are true
(D) Both S_1 and II are false

[GATE 2021 : IIT Bombay]

Q.120 In a directed acyclic graph with a source vertex s, the quality-score of a directed path is defined to be the product of the weights of the edges on the path. Further, for a vertex v other than s. the quality-score of v is defined to be the maximum among the quality-scores of all the paths from s to v. The quality-score of s is assumed to be 1.

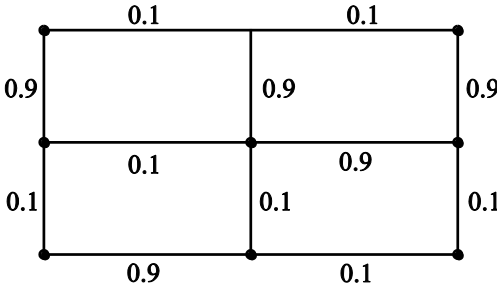


The sum of the quality-score of all the vertices in the graph shown above is _____. **[GATE 2021 : IIT Bombay]**

Q.121 In an undirected connected planar graph G, there are eight vertices and five faces. The number of edges in G is _____.

[GATE 2021 : IIT Bombay]

Q.122 Consider the following undirected graph with edge weights as shown:



The number of minimum-weight spanning trees of the graph is _____.

[GATE 2021 : IIT Bombay]

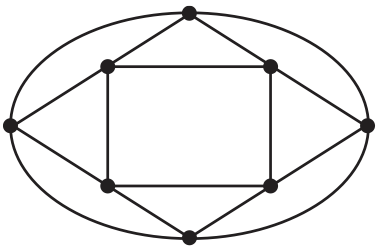
Classroom Practice Questions

Q.1 Which of the following graphs is NOT a regular graph?
(A) A wheel graph of 4 vertices.
(B) A complete graph of 6 vertices.
(C) A cycle graph of 9 vertices.
(D) A path graph of 12 vertices.

Q.2 If G is an undirected graph with 10 vertices and 3 components and every pair of vertices in each component are adjacent, then the number of edges of G is _____.
(A) 28 (B) 45
(C) 30 (D) 20

Q.3 If G is a complete graph of six vertices and T is a spanning tree of G, then the number of edges in the complement graph $\overline{T}$ of T is _____.
(A) 10 (B) 9
(C) 8 (D) 7

Q.4 The vertex connectively of the graph given below is _____.



(A) 3 (B) 4
(C) 5 (D) 6

Q.5 In a simple graph with $(P+1)$ vertices, the maximum degree of any vertex is
(A) $p+1$ (B) p
(C) $p-1$ (D) $p-2$

Q.6 Which of the following is not true about graphs?

(A) Every path is a walk.

(B) Every cycle is a path.

(C) Every trail is a cycle.

(D) Every circuit is a trail.

Q.7 If the number of edges in a simple graph G having 10 vertices is 37, then the graph is definitely

(A) A disconnected graph

(B) A connected graph

(C) A closed graph

(D) All the above

Q.8 The minimum number of edges in a connected graph having 19 vertices is

(A) 19 (B) 20

(C) 17 (D) 18

Q.9 The maximum number of edges in a simple graph with 'm' vertices and 's' components is

(A) $\frac{(m+s)(m+s-1)}{2}$

(B) $\frac{(m-s)(m-s+1)}{2}$

(C) $\frac{(m-s)(m+s-1)}{2}$

(D) None of these

Q.10 Find the minimum number of connected components of a simple graph with 10 vertices and 7 edges.

(A) 3 (B) 4

(C) 10 (D) 7

Q.11 The maximum number of edges possible in a simple bipartite graph with 18 vertices is

(A) 49 (B) 64

(C) 81 (D) None of these

Q.12 The minimum number of vertices for simple 3-regular graph is

(A) 3 (B) 4

(C) 5 (D) 6

Q.13 The number of regions in a graph G with 20 vertices, each of degree 4 is

(A) 22 (B) 21

(C) 20 (D) None of these

Q.14 The chromatic number of a complete graph of five vertices is

(A) 5 (B) 4

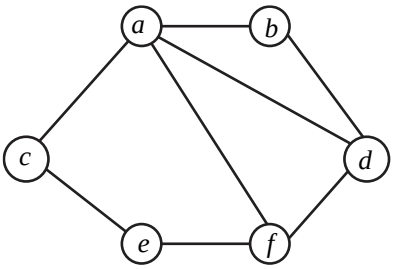
(C) 6 (D) 3

Q.15 A wheel graph with eleven vertices has a chromatic number of

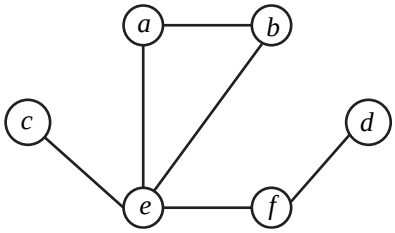
(A) 3 (B) 4

(C) 2 (D) None of these

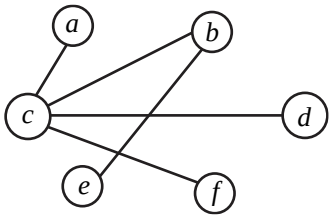
Q.16 Which one of the following option is the complement of following graph?



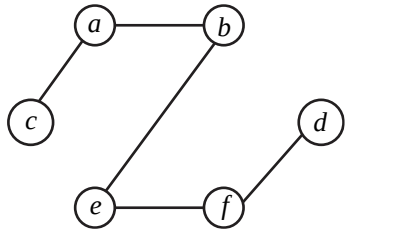
(A)



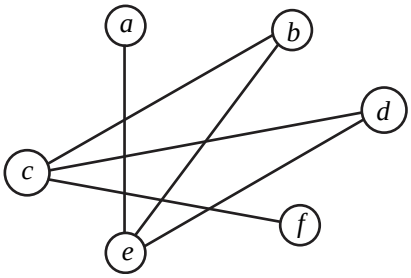
(B)



(C)



(D)



Q.17 The number of distinct simple graphs with up to five nodes is

(A) 15 (B) 10

(C) 31 (D) 9

Q.18 The minimum number of edges in a connected cyclic graph on n vertices is

(A) $n-1$ (B) n

(C) $n+1$ (D) None of these

Q.19 Let G be a graph with 100 vertices numbered 1 to 100. Two vertices i and j are adjacent if $|i-j|=8$ or $|i-j|=12$.

The number of connected components in G is

(A) 8 (B) 4
(C) 12 (D) 25

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Answer Keys

Classroom Practice Questions									
1.	B	2.	D	3.	D	4.	D	5.	4
6.	C	7.	B	8.	*	9.	A	10.	D
11.	16	12.	B	13.	16	14.	13	15.	C
16.	A	17.	F	18.	B	19.	D	20.	10
21.	10	22.	8	23.	A,B,C	24.	B	25.	4
26.	7	27.	2	28.	11	29.	D	30.	C
31.	5	32.	C	33.	B	34.	3n – 6	35.	37
36.	12	37.	72	38.	3	39.	18	40.	6
41.	16	42.	D	43.	D	44.	24	45.	A
46.	B	47.	B	48.	A	49.	B	50.	C
51.	*	52.	C	53.	2	54.	3	55.	4
56.	4	57.	C	58.	B	59.	4	60.	3
61.	7	62.	A	63.	C	64.	A	65.	D
66.	D	67.	A	68.	A	69.	*	70.	3
71.	3	72.	C	73.	*	74.	*	75.	*
76.	C	77.	A	78.	4	79.	C	80.	21
81.	*	82.	C	83.	6	84.	995	85.	C
86.	*	87.	C	88.	C	89.	B	90.	B
91.	B	92.	*	93.	*	94.	D	95.	A
96.	C	97.	D	98.	*	99.	C	100.	C
101.	B	102.	45	103.	C	104.	506	105.	C
106.	109	107.	36	108.	C	109.	18	110.	C
111.	B	112.	B	113.	B	114.	A	115.	B
116.	C	117.	B	118.	*	119.	B	120.	929
121.	11	122.	3						
Self Practice Questions									
1.	D	2.	A	3.	A	4.	A	5.	B
6.	C	7.	B	8.	D	9.	B	10.	A
11.	C	12.	B	13.	A	14.	A	15.	A
16.	D	17.	C	18.	B	19.	B		

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3 Mathematical Logic

Propositional Logic

Classroom Practice Questions

Q.1 The propositional function $(P \cup Q) \cup \sim P$ is

(A) Tautology

(B) Contradiction

(C) Contingency

(D) Satisfiable

Q.2 Simplify the propositional function $\sim (P \cup Q) \cup (\sim P \cap Q) \cup P$

Q.3 The simplest form of propositional function $\{(P \to Q) \leftrightarrow (\sim P \cup Q)\} \cap R$ is ____.

Q.4 Consider the following statements.

$S_1 : P \cup (\sim P \cap Q)$

$S_2 : (P \cap Q) \cap (\sim P \cup \sim Q)$

Which of the following is correct

(A) S_1 is false and S_2 is true.

(B) S_1 is true and S_2 is false

(C) Both are true

(D) None of the above

Q.5 The statement formula

$\{\sim P \cap (\sim Q \cap R)\} \cup (Q \cap R) \cup (P \cap R)$

(A) T (B) F

(C) Q (D) R

Q.6 The simplified form of statement formula

$\left[(P \cup Q) \cap \sim \{\sim P \cap (\sim Q \cup \sim R)\}\right]$

$\cup (\sim P \cap \sim Q) \cup (\sim P \cap \sim R)$

is

(A) P (B) R

(C) T (D) F

Q.7 How many non- equivalent propositional functions are possible with n propositional variables.

(A) 2^n (B) 2^{n^2}

(C) $2^{(2^n)}$ (D) n^2

Q.8 A set of boolean connective is functionally complete if all Boolean functions can be synthesized using those. Which of the following sets of connectives is not functionally complete.

(A) EX-NOR

(B) Implication, negation

(C) OR, negation

(D) NAND

Q.9 A logical binary relation $\odot$, is defined as follows :

A	B	$A \odot B$
True	True	True
True	False	True
False	True	False
False	False	True

Let $\sim$ be the unary negation (NOT) operator, with higher precedence then $\odot$. Which one of the following is equivalent to $A \wedge B$?

(A) $(\sim A \odot B)$ (B) $\sim (A \odot \sim B)$

(C) $\sim (\sim A \odot \sim B)$ (D) $\sim (\sim A \odot B)$

[GATE 2006 : IIT Kharagpur]

Q.10 The binary operation $\Rightarrow$ is defined as follows :

P	Q	$P \Rightarrow Q$
T	T	T
T	F	T
F	T	F
F	F	T

Which one of the following is equivalent to $P \vee Q$?

(A) $\neg Q \Rightarrow \neg P$ (B) $P \Rightarrow \neg Q$

(C) $\neg P \Rightarrow Q$ (D) $\neg P \Rightarrow \neg Q$

[GATE 2009 : IIT Roorkee]

Q.11 Let a, b, c, d be propositions. Assume that the equivalence $a \leftrightarrow (b \vee \neg b)$ and $b \leftrightarrow c$ hold. Then the truth value of the formulae $(a \wedge b) \rightarrow ((a \wedge c) \vee d)$ is always

(A) True

(B) False

(C) Same as truth value of b

(D) Same as truth value of d

[GATE 2000 : IIT Kharagpur]

Q.12 Which of the following statement is/are false

(A) $(P \cap Q) \Rightarrow (P \leftrightarrow Q)$

(B) $(P \leftrightarrow Q) \Rightarrow (P \rightarrow Q)$

(C) $(P \leftrightarrow Q) \Rightarrow (P \rightarrow \sim Q)$

(D) $Q \Rightarrow P \rightarrow Q$

Q.13 Which of the following is /are not tautology

(A) $(P \cap Q) \rightarrow (P \cup Q)$

(B) $\sim P \rightarrow (P \rightarrow Q)$

(C) $P \rightarrow (P \cap Q)$

(D) $P \rightarrow (P \cup Q)$

Q.14 Which of the following is/are tautology?

(A) $(a \vee b) \rightarrow (b \wedge c)$

(B) $(a \wedge b) \rightarrow (b \vee c)$

(C) $(a \vee b) \rightarrow (b \rightarrow c)$

(D) $(a \rightarrow b) \rightarrow (b \rightarrow c)$

[GATE 1992 : IIT Delhi]

Q.15 The proposition $p \wedge (\sim p \vee q)$ is

(A) a tautology

(B) logically equivalent to $(p \wedge q)$

(C) logically equivalent to $(p \vee q)$

(D) a contradiction

[GATE 1993 : IIT Bombay]

Q.16 Let p and q be propositions. Decide whether $(p \leftrightarrow q)$ does not imply $(p \rightarrow \neg q)$ is true or false.

[GATE 1994 : IIT Kharagpur]

Q.17 If the proposition $\neg p \Rightarrow q$ is true, then the truth value of the proposition $\neg p \vee (p \Rightarrow q)$, where '$\neg$' is negation, '$\vee$' is inclusive or and '$\Rightarrow$' is implication, is

(A) True

(B) Multiple-valued

(C) False

(D) Cannot be determined

[GATE 1995 : IIT Kanpur]

Q.18 Which one of the following is false? Read $\wedge$ as AND, $\vee$ as $OR, \sim$ as $NOT, \rightarrow$ as one way implication and $\leftrightarrow$ as two way implication.

(A) $((x \rightarrow y) \wedge x) \rightarrow y$

(B) $((\sim x \rightarrow y) \wedge (\sim x \rightarrow \sim y)) \rightarrow x$

(C) $(x \rightarrow (x \vee y))$

(D) $((x \vee y) \leftrightarrow (\sim x \rightarrow \sim y))$

[GATE 1996 : IISc Bangalore]

Q.19 Let P, Q and R be three atomic prepositional assertions.

Let X denote $(P \vee Q) \rightarrow R$ and Y denote $(P \rightarrow R) \vee (Q \rightarrow R)$. Which one of the following is a tautology?

(A) $X \equiv Y$ (B) $X \rightarrow Y$

(C) $Y \rightarrow X$ (D) $\neg Y \rightarrow X$

[GATE 2005 : IIT Bombay]

Q.20 Which one of the following Boolean expressions is NOT a tautology?

(A) $((a \rightarrow b) \wedge (b \rightarrow c)) \rightarrow (a \rightarrow c)$

(B) $(a \leftrightarrow c) \rightarrow (\sim b \rightarrow (a \wedge c))$

(C) $(a \wedge b \wedge c) \rightarrow (c \vee a)$

(D) $a \rightarrow (b \rightarrow a)$

[GATE 2014 : IIT Kharagpur]

Q.21 Consider the following expressions:

(i) false

(ii) Q

(iii) True

(iv) $P \vee Q$

(v) $\neg Q \vee P$

The number of expressions given above that are logically implied by $P \wedge (P \Rightarrow Q)$ is ———.

[GATE 2016 : IISc Bangalore]

Q.22 Let p, q and r be propositions and the expression $(p \to q) \to r$ be a contraction. Then, the expression $(r \to p) \to q$ is

(A) a tautology

(B) a contradiction

(C) always TRUE when p is FALSE

(D) always TRUE when q is TRUE

[GATE 2017 : IIT Roorkee]

Q.23 The statement $(\neg P) \Rightarrow (\neg q)$ is logically equivalent to which of the statement below?

I. $p \Rightarrow q$

II. $q \Rightarrow p$

III. $(\neg q) \vee p$

IV. $(\neg p) \vee q$

(A) I only

(B) I and IV only

(C) II only

(D) II and III only

[GATE 2017 : IIT Roorkee]

Q.24 Consider the following propositional statements :

$P1: ((A \wedge B) \to C))$

$\equiv ((A \to C) \wedge (B \to C))$

$P2: ((A \vee B) \to C))$

$\equiv ((A \to C) \vee (B \to C))$

Which one of the following is true?

(A) P1 is a tautology, but not P2

(B) P2 is a tautology, but not P1

(C) P1 and P2 are both tautologies

(D) Both P1 and P2 are not tautologies

[GATE 2006 : IIT Kharagpur]

Q.25 Which of the following is false

(A) $a \to (b \cup c) \equiv \{(a \cap \sim b)\} \to c\}$

(B) $(P \to Q) \cap (R \to Q) \equiv (P \cup R) \to Q$

(C) $(P \to Q) \cup (R \to Q) \equiv (P \cap R) \to Q$

(D) None

Q.26 Indicate which of the following well-formed formulae are valid :

(A) $(P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$

(B) $(P \Rightarrow Q) \Rightarrow (\neg P \Rightarrow \neg Q)$

(C) $(P \wedge (\neg P \vee \neg Q)) \Rightarrow Q$

(D) $((P \Rightarrow R) \vee (Q \Rightarrow R)) \Rightarrow ((P \vee Q) \Rightarrow R)$

[GATE 1990 : IISc Bangalore]

Q.27 Consider two well-formed formulas in propositional logic

$F1: P \Rightarrow \neg P$

$F2: (P \Rightarrow \neg P) \vee (\neg P \Rightarrow P)$

Which of the following statements is correct?

(A) F1 is satisfiable, F2 is valid

(B) F1 is unsatisfiable, F2 is satisfiable

(C) F1 is unsatisfiable, F2 is valid

(D) F1 and F2 are both satisfiable

[GATE 2001 : IIT Kanpur]

Q.28 The following propositional statement is

$(P \to (Q \vee R)) \to ((P \wedge Q) \to R)$

(A) Satisfiable but not valid

(B) Valid

(C) A conttradication

(D) None of the above

[GATE 2004 : IIT Delhi]

Q.29 Which of the following arguments is/are valid

(A) The conclusion R follows from the premises $\{P \to (Q \to R), P \cap Q\}$

(B) $\{P \to (Q \to R), P, Q\}$ tautologically implies

(C) $\{P \to (R \to S), \sim R \to \sim P, P\} \to R$

(D) $(P \cap Q)$ Follows from the premises $\{(P \cup Q), \sim P\}$

(E) $\{\sim T \to \sim R, \sim S, T \to W, R \cup S\} \Rightarrow W$

Q.30 Which of the following is /are true.

(A) $\{R \to S, P \to Q, R \cup P\} \Rightarrow S \cup Q$

(B) $\{\sim R \to (S \to \sim T), \sim R \cup W, \sim P \to S, \sim W\} \Rightarrow T \to P$

(C) $\{\sim P \cap Q, Q \to (P \to R)\} \Rightarrow \sim R$

(D) $\{P \to R, P, Q \cup \sim R\} \Rightarrow Q$

Q.31 The following resolution rule is used in logic programming.

Derive clause $(P \vee Q)$ from clauses $(P \vee R), (Q \vee \neg R)$.

Which of the following statements related to this rule is FALSE?

(A) $((P \vee R) \wedge (Q \vee \neg R)) \Rightarrow (P \vee Q)$ is logically valid

(B) $(P \vee Q) \Rightarrow ((P \vee R) \wedge (Q \vee \neg R))$ is logically valid

(C) $(P \vee Q)$ is satisfiable if and only if $(P \vee R) \wedge (Q \vee \neg R)$ is satisfiable

(D) $(P \vee Q) \Rightarrow FALSE$ if and only if both P and Q are unsatisfiable

[GATE 2003 : IIT Madras]

Q.32 Which one of the following propositional logic formulas is TRUE when exactly two of p, q, & r are TRUE ?

(A) $((p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(B) $(\sim (p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(C) $((p \to q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(D) $(\sim (p \leftrightarrow q) \wedge r) \wedge (p \wedge q \wedge \sim r)$

[GATE 2014 : IIT Kharagpur]

Q.33 Which of the following is NOT equivalent to $p \leftrightarrow q$

(A) $(\sim p \vee q) \wedge (p \vee \sim q)$

(B) $(\sim p \vee q) \wedge (q \to p)$

(C) $(\sim p \wedge q) \vee (p \wedge \sim q)$

(D) $(\sim p \wedge \sim q) \vee (p \wedge q)$

[GATE 2015 : IIT Kanpur]

Q.34 Let p, q, r and s be four primitive statements. Consider the following arguments :

$P : [(\neg p \vee q) \wedge (r \to s) \wedge (p \vee r)] \to (\neg s \to q)$

$Q : [(\neg p \wedge q) \wedge [q \to (p \to r)]] \to \neg r$

$R : [[(q \wedge r) \to p] \wedge (\neg q \vee p)] \to r$

$S : [p \wedge (p \to r) \wedge (q \vee \neg r)] \to q$

Which of the above arguments are valid?

(A) P and Q only

(B) P and R only

(C) P and S only

(D) P, Q, R and S

[GATE 2004 : IIT Delhi]

Q.35 Consider the following logical inferences.

I_1 : If it rains then the cricket match will not be played.

The cricket match was played.

Inference : There was no rain.

I_2 : If it rains then the cricket match will, not be played.

It did not rain

Inference : The cricket match was played.

Which of the following is TRUE?

(A) Both I_1 and I_2 are correct inferences

(B) I_1 is correct but I_2 is not a correct inference

(C) I_1 is not correct but I_2 is a correct inference

(D) Both I_1 and I_2 are not correct inferences

[GATE 2012 : IIT Delhi]

Q.36 Consider the following statements :

P : Good mobile phones are not cheap

Q : Cheap mobile phones are not good

L : P implies Q

M : Q implies P

N : P is equivalent to Q

Which of the following about L, M and N is CORRECT?

(A) Only L is TRUE.

(B) Only M is TRUE.

(C) Only N is TRUE.

(D) L, M and N are TRUE.

[GATE 2014 : IIT Kharagpur]

Q.37 Consider the following two statements.

S1 : If a candidate is known to be corrupt, then he will not be elected

S2 : If a candidate is kind, he will be elected

Which one of the following statements follows from S1 and S2 as per sound inference rules of logic?

(A) If a person is known to be corrupt, he is kind

(B) If a person is not known to be corrupt, he is not kind

(C) If a person is kind, he is not known to be corrupt

(D) If a person is not kind, he is not known to be corrupt

[GATE 2015 : IIT Kanpur]

Q.38 Let p,q,r,s represent the following propositions.

$p: x \in \{8,9,10,11,12\}$

$q: x$ is a composite number

$r: x$ is a perfect square

$s: x$ is a prime number

The integer $x \geq 2$ which satisfies $\neg((p \Rightarrow q) \wedge (\neg r \vee \neg s))$ ————.

[GATE 2016 : IISc Bangalore]

Q.39 Let p,q,r denote the statements "It is raining", "It is cold" and "It is pleasant", respectively. Then the statement "It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold" is represented by

(A) $(\neg p \wedge r) \wedge (\neg r \rightarrow (p \wedge q))$

(B) $(\neg p \wedge r) \wedge ((p \wedge q) \rightarrow \neg r)$

(C) $(\neg p \wedge r) \vee ((p \wedge q) \rightarrow \neg r)$

(D) $(\neg p \wedge r) \vee (r \rightarrow (p \wedge q)$

[GATE 2017 : IIT Roorkee]

Q.40 "If X then Y unless Z" is represented by which of the following formulae in proportional logic?

("$\neg$" *is negation* "$\wedge$" *is conjuction, and* "$\rightarrow$" *is implication*)

(A) $(X \wedge \neg Z) \rightarrow Y$

(B) $(X \wedge Y) \rightarrow Y \neg Z$

(C) $X \rightarrow (Y \wedge \neg Z)$

(D) $(X \rightarrow Y) \wedge \neg Z$

[GATE 2002 : IISc Bangalore]

Q.41 What is the converse of the following assertion?

I stay only if you go

(A) I stay if you go

(B) If stay then you go

(C) If you do not go then I do not stay

(D) If I do not stay then you go

[GATE 1998 : IIT Delhi]

Q.42 Consider the following argument is valid or not

(i) If A works hard then B or C will enjoy themselves

(ii) If B enjoys himself then A will not work hard

(iii) If C enjoys himself then D will not enjoy himself

Inference: - If A work hard then D will not enjoy himself

Q.43 Check the validity of the argument using proof by contradiction

$\{P \rightarrow Q, \sim (P \cap Q)\} \Rightarrow \sim P$

Q.44 Check the validity of the argument using conditional proof rule

$\{P \rightarrow (Q \rightarrow S), \sim R \cup P, Q\} \Rightarrow (R \rightarrow S)$

Q.45 Which of the following argument is /are invalid

(A) $\{(x \rightarrow y) \cap x\} \rightarrow y$

(B) $\{(\sim x \rightarrow y) \cap (\sim x \cap \sim y)\} \rightarrow x$

(C) $x \rightarrow (x \cup y)$

(D) $(x \cup y) \leftrightarrow (\sim x \rightarrow \sim y)$

Q.46 Let p and q be two propositions. Consider the following two formulae in propositional logic.

$S_1 : (\neg p \wedge (p \vee q) \rightarrow q$

$S_2 : q \rightarrow (\neg p \wedge (p \vee q))$

(A) Both S_1 and S_2 both are tautologies.

(B) Neither S_1 nor S_2 is a tautology.

(C) S_1 is not a tautology but S_2 is a tautology.

(D) S_1 is a tautology but S_2 is not a tautology.

[GATE 2021 : IIT Bombay]

Q.47 Choose the correct choice(s) regarding the following propositional logic assertion

$S:((P\wedge Q)\rightarrow R)\rightarrow((P\wedge Q)\rightarrow(Q\rightarrow R))$

(A) S is a contradiction

(B) S is a tautology

(C) The antecedent of S is logically equivalent to consequent of S

(D) Neither tautology nor contradiction

[GATE 2021 : IIT Bombay]

First Order Logic

Classroom Practice Questions

Q.1 Negate the statement formula

$\forall_x[P(x)\rightarrow Q(x)]$

Q.2 Consider the following well-formed formulae:

I. $\neg\forall x(P(x))$

II. $\neg\exists x(\neg P(x))$

III. $\neg\exists x(P(x))$

IV. $\exists x(\neg P(x))$

Which of the above are equivalent?

(A) I and II (B) I and IV

(C) II and III (D) II and IV

[GATE 2009 : IIT Roorkee]

Q.3 The statement formula

$\forall_x\{B(x)\cap I(x)\}$ is equivalent to

(A) $\exists_x\{B(x)\rightarrow\sim I(x)\}$

(B) $\sim\{\exists_x\{B(x)\rightarrow\sim I(x)\}\}$

(C) $\exists_x\{\sim B(x)\rightarrow I(x)\}$

(D) $\sim\exists_x\{\sim B(x)\rightarrow I(x)\}$

Q.4 Which one of the following is NOT logically equivalent to $\neg\exists x(\forall y(\alpha)\wedge(\forall z(\beta))$?

(A) $\forall x(\exists z(\neg\beta)\rightarrow\forall y(\alpha))$

(B) $\forall x(\forall z(\beta)\rightarrow\exists y(\neg\alpha))$

(C) $\forall x(\forall y(\alpha)\rightarrow\exists z(\neg\beta))$

(D) $\forall x(\exists y(\neg\alpha)\rightarrow\exists z(\neg\beta))$

[GATE 2013 : IIT Bombay]

Q.5 Which of the following is the negation of $[\forall x,\alpha\rightarrow(\exists y,\beta\rightarrow(\forall u,\exists v,y))]$?

(A) $[\exists x,\alpha\rightarrow(\forall y,\beta\rightarrow(\exists u,\forall v,y))]$

(B) $[\exists x,\alpha\rightarrow(\forall y,\beta\rightarrow(\exists u,\forall v,\neg y))]$

(C) $[\forall x,\neg\alpha\rightarrow(\exists y,\neg\beta\rightarrow(\forall u,\exists v,\neg y))]$

(D) $[\exists x,\alpha\wedge(\forall y,\beta\wedge(\exists u,\forall v,\neg y))]$

[GATE 2002 : IISc Bangalore]

Q.6 Which of the following is a valid first order formula? (Here α and β are first order formulae with x as their only free variable)

(A) ((∀x)[α]⇒(∀x)[β])⇒(∀x)[α⇒β]

(B) (∀x)[α]⇒(∃x)[α∧β]

(C) ((∀x)[α∨β]⇒(∃x)[α])⇒(∀x)[α]

(D) (∀x)[α⇒β]⇒(((∀x)[α])⇒(∀x)[β])

[GATE 2003 : IIT Madras]

Q.7 Which of the following first order formulae is logically valid? Here $\alpha(x)$ is a first order formula with x as a free variable, and β is a first order formula with no free variable.

(A) $[\beta\rightarrow(\exists x,\alpha(x))]\rightarrow[\forall x,\beta\rightarrow\alpha(x)]$

(B) $[\exists x,\beta\rightarrow\alpha(x)]\rightarrow[\beta\rightarrow(\forall x,\alpha(x))]$

(C) $[(\exists x,\alpha(x))\rightarrow\beta]\rightarrow[\forall x,\alpha(x)\rightarrow\beta]$

(D) $[(\forall x,\alpha(x))\rightarrow\beta]\rightarrow[\forall x,\alpha(x)\rightarrow\beta]$

[GATE 2008 : IISc Bangalore]

Q.8 Consider the first-order logic sentence $F:\forall x(\exists yR(x,y))$. Assuming non-empty logical domain, which of the sentences below are implied by F?

I. $\exists y(\exists xR(x,y))$

II. $\exists y(\forall xR(x,y))$

III. $\forall y(\exists xR(x,y))$

IV. $\neg\exists x(\forall y\neg R(x,y))$

(A) IV only

(B) I and IV only

(C) II only

(D) II and III only

[GATE 2017 : IIT Roorkee]

Q.9 Which of the following statement is/are false

(A) $\forall_x \exists_y (P(x,y)) \Rightarrow \exists_x \forall_y (P(x,y))$

(B) $\forall_x \exists_y (P(x,y)) \Rightarrow \forall_x \forall_y (P(x,y))$

(C) $\forall_y \exists_x (P(x,y)) \Rightarrow \exists_x \forall_y (P(x,y))$

(D) $\forall_x \exists_y (P(x,y)) \Rightarrow \exists_y \forall_x (P(x,y))$

Q.10 Let $x \& y$ are any two real numbers then which of the following statement is /are False

(A) $\forall_x \exists_y (x+y=5)$

(B) $\exists_y \forall_x (xy=x)$

(C) $\exists_x \forall_y (x+y=0)$

(D) None of the above

Q.11 Let $a(x,y), b(x,y)$ and $c(x,y)$ be three statements with variables x and y chosen from some universe. Consider the following statement

$(\exists x)(\forall y)\left[(a(x,y) \wedge b(x,y)) \wedge \neg c(x,y)\right]$

Which one of the following is its equivalent?

(A) $(\forall x)(\exists y)\begin{bmatrix} a(x,y) \vee b(x,y)) \\ \rightarrow c(x,y) \end{bmatrix}$

(B) $(\exists x)(\forall y)\begin{bmatrix} (a(x,y) \vee b(x,y)) \\ \wedge \neg c(x,y) \end{bmatrix}$

(C) $\neg(\forall x)(\exists y)\begin{bmatrix} (a(x,y) \wedge b(x,y)) \\ \rightarrow c(x,y) \end{bmatrix}$

(D) $\neg(\forall x)(\exists y)\begin{bmatrix} (a(x,y) \vee b(x,y)) \\ \rightarrow c(x,y) \end{bmatrix}$

[GATE 2004 : IIT Delhi]

Q.12 Which one of these first-order logic formulae is valid?

(A) $\forall x(P(x) \Rightarrow Q(x)) \Rightarrow (\forall x P(x) \Rightarrow \forall x Q(x))$

(B) $\exists x(P(x) \vee Q(x)) \Rightarrow (\exists x P(x) \Rightarrow \exists x Q(x))$

(C) $\exists x(P(x) \wedge Q(x)) \Leftrightarrow (\exists x P(x) \wedge \exists x Q(x))$

(D) $\forall x \exists y P(x,y) \Rightarrow \exists y \forall x P(x,y)$

[GATE 2007 : IIT Kanpur]

Q.13 Which one of the following well-formed formulae is a tautology?

(A) $\forall x \exists y\, R(x,y) \leftrightarrow \exists y \forall x\, R(x,y)$

(B) $(\forall x[\exists y\, R(x,y) \rightarrow S(x,y)]) \rightarrow \forall x \exists y S(x,y)$

(C) $[\forall x \exists y (P(x,y) \rightarrow R(x,y))] \leftrightarrow [\forall x \exists y (\neg P(x,y) \vee R(x,y))]$

(D) $\forall x \exists y P(x,y) \rightarrow \forall x \forall y P(y,x)$

[GATE 2015 : IIT Kanpur]

Q.14 Which one of the following predicate formulae is NOT logically valid?

Note that W is a predicate formula without any free occurrence of x.

(A) $\forall x(p(x) \vee W) \equiv \forall x\, p(x) \vee W$

(B) $\exists x(p(x) \wedge W) \equiv \exists x\, p(x) \wedge W$

(C) $\forall x(p(x) \rightarrow W) \equiv \forall x\, p(x) \rightarrow W$

(D) $\exists x(p(x) \rightarrow W) \equiv \forall x\, p(x) \rightarrow W$

[GATE 2020 : IIT Delhi]

Q.15 Which of the following argument is valid

(A) $\forall_x \{P(x)\}$ Follows from the set of premises $\{\exists_x \{P(x) \cap Q(x)\}$

(B) $\{\exists_x \{P(x) \cap Q(x)\}$ Follows from the set of premises $\{\exists_x \{P(x)\}, \exists_x \{Q(x)\}$

(C) $\exists_z Q(z)$ Follows from the set of premises $\{\forall_x \{P(x) \rightarrow Q(x)\}, \exists_y \{P(y)\}\}$

(D) $\sim P(a,b)$ Follows from the set of premises $\{\forall_x \forall_y \{P(x,y) \rightarrow W(x,y)\}, \sim W(a,b)\}\}$

Q.16 Which of the following predicate calculus statements is/are valid?

(A) $((\forall x)P(x) \vee (\forall x)Q(x)) \rightarrow (\forall x)\{P(x) \vee Q(x)\}$

(B) $\{(\exists x)P(x) \wedge (\exists x)Q(x)\} \rightarrow (\exists x)\{P(x) \wedge Q(x)\}$

(C) $(\forall x)\{P(x) \vee Q(x)\} \rightarrow \{(\forall x)P(x) \vee (\forall x)Q(x)\}$

(D) $(\exists x)\{P(x) \vee Q(x)\} \rightarrow \sim \{(\forall x)P(x)\} \vee \{(\exists x)Q(x)\}$

[GATE 1992 : IIT Delhi]

Q.17 Let $P(x)$ and $Q(x)$ be arbitrary predicates. Which of the following statement is always TRUE?

(A) $(\forall x(P(x) \vee Q(x))) \Rightarrow ((\forall x P(x)) \vee (\forall x Q(x)))$

(B) $(\forall x(P(x) \Rightarrow Q(x))) \Rightarrow ((\forall x P(x)) \Rightarrow (\forall x Q(x)))$

(C) $(\forall x(P(x)) \Rightarrow (\forall x Q(x))) \Rightarrow (\forall x(P(x) \Rightarrow Q(x)))$

(D) $(\forall x(P(x))) \Leftrightarrow (\forall x Q(x)) \Rightarrow (\forall x(P(x) \Leftrightarrow Q(x)))$

[GATE 2005 : IIT Bombay]

Q.18 Which one of the following well-formed formulae in predicate calculus is NOT valid?

(A) $(\forall x p(x) \Rightarrow \forall x q(x)) \Rightarrow (\exists x \neg p(x) \vee \forall x q(x))$

(B) $(\exists x p(x) \vee \exists x q(x)) \Rightarrow \exists x(p(x) \vee q(x))$

(C) $\exists x(p(x) \wedge q(x)) \Rightarrow (\exists x p(x) \wedge \exists x q(x))$

(D) $\forall x(p(x) \vee q(x)) \Rightarrow (\forall x p(x) \vee \forall x q(x))$

[GATE 2016 : IISc Bangalore]

Q.19 What is the logical translation of the following statement?

"None of my friends are perfect".

(A) $\exists x(F(x) \wedge \neg P(x))$

(B) $\exists x(\neg F(x) \wedge P(x))$

(C) $\exists x(\neg F(x) \wedge \neg P(x))$

(D) $\neg \exists x(F(x) \wedge P(x))$

[GATE 2013 : IIT Bombay]

Q.20 The CORRECT formula for the sentence, "not all rainy days are cold" is

(A) $\forall d(Rainy(d) \wedge \sim Cold(d))$

(B) $\forall d(\sim Rainy(d) \rightarrow Cold(d))$

(C) $\exists d(\sim Rainy(d) \rightarrow Cold(d))$

(D) $\exists d(Rainy(d) \wedge \sim Cold(d))$

[GATE 2014 : IIT Kharagpur]

Q.21 Identify the correct translation into logical notation of the followings assertion.

Some boys in the class are taller than all the girls

Note : taller (x, y) is true if x is taller than y.

(A) $(\exists x)(boy(x) \rightarrow (\forall y)(girl(y) \wedge taller(x, y)))$

(B) $(\exists x)(boy(x) \wedge (\forall y)(girl(y) \wedge taller(x, y)))$

(C) $(\exists x)(boy(x) \rightarrow (\forall y)(girl(y) \rightarrow taller(x, y)))$

(D) $(\exists x)(boy(x) \wedge (\forall y)(girl(y) \rightarrow taller(x, y)))$

[GATE 2004 : IIT Delhi]

Q.22 What is the first order predicate calculus statement equivalent to the following? "Every teacher is liked by some student".

(A) $\forall(x)\left[teacher(x) \rightarrow \exists(y)\left[\begin{matrix} student(y) \\ \rightarrow likes(y, x)\end{matrix}\right]\right]$

(B) $\forall(x)\left[teacher(x) \rightarrow \exists(y)\left[\begin{matrix} student(y) \\ \wedge likes(y, x)\end{matrix}\right]\right]$

(C) $\exists(y)\forall(x)\left[teacher(x) \rightarrow \left[\begin{matrix} student(y) \\ \wedge likes(y, x)\end{matrix}\right]\right]$

(D) $\forall(x)\left[teacher(x) \wedge \exists(y)\left[\begin{matrix} student(y) \\ \rightarrow likes(y, x)\end{matrix}\right]\right]$

[GATE 2005 : IIT Bombay]

Q.23 What is the correct translation of the following statement into mathematical logic?

"Some real numbers are rational"

(A) $\exists x(real(x) \vee rational(x))$

(B) $\forall x(real(x) \rightarrow rational(x))$

(C) $\exists x(real(x) \wedge rational(x))$

(D) $\exists x(rational(x) \rightarrow real(x))$

[GATE 2012 : IIT Delhi]

Q.24 Consider the statement

"Not all that glitters is gold"

Predicate glitters (x) is true if x glitters and predicate gold (x) is true if x is gold.

Which one of the following logical formulae represents the above statement?

(A) $\forall x : glitters(x) \Rightarrow \neg gold(x)$

(B) $\forall x : gold(x) \Rightarrow glitters(x)$

(C) $\exists x : gold(x) \wedge \neg glitters(x)$

(D) $\exists x : glitters(x) \wedge \neg gold(x)$

[GATE 2014 : IIT Kharagpur]

Q.25 Which one of the following is the most appropriate logical formula to represent the statement:

"Gold and silver ornaments are precious"

The following notations are used:

$G(x) : x$ is a gold ornament.

$S(x) : x$ is a silver ornament.

$P(x) : x$ is precious.

(A) $\forall x(P(x) \rightarrow (G(x) \wedge S(x)))$

(B) $\forall x((G(x) \wedge S(x)) \rightarrow P(x))$

(C) $\exists x((G(x) \wedge S(x)) \rightarrow P(x))$

(D) $\forall x((G(x) \vee S(x)) \rightarrow P(x))$

[GATE 2009 : IIT Roorkee]

Q.26 Which one of the first order predicate calculus statements given below correctly expresses the following English statement?

Tigers and lions attack if they are hungry or threatened

(A) $\forall x \left[\begin{array}{l} (tiger(x) \wedge lion(x)) \\ \rightarrow \{(hungry(x) \vee threatened \\ (x)) \rightarrow attacks(x) \end{array} \right]$

(B) $\forall x \left[\begin{array}{l} (tiger(x) \wedge lion(x)) \\ \rightarrow \{(hungry(x) \vee threatened \\ (x)) \rightarrow attacks(x) \end{array} \right]$

(C) $\forall x \left[\begin{array}{l} (tiger(x) \vee lion(x)) \\ \rightarrow \{attacks(x) \rightarrow (hungry \\ (x) \vee threatened(x)) \end{array} \right]$

(D) $\forall x \left[\begin{array}{l} (tiger(x) \vee lion(x)) \\ \rightarrow \{(hungry(x) \vee threatened \\ (x)) \rightarrow attacks(x) \end{array} \right]$

[GATE 2006 : IIT Kharagpur]

Q.27 Let fsa and pda be two predicates such that $fsa(x)$ means x is a finite state automation, and $pda(y)$ means that y is a pushdown automation. Let equivalent be another predicate such that equivalent (a,b) means a and b are equivalent. Which of the following first order logic statements represents the following :

Each finite state automaton has an equivalent pushdown automaton.

(A) $(\forall x\ fsa(x)) \Rightarrow (\exists y\ pda(y) \wedge equivalent(x,y))$

(B) $\sim \forall y(\exists x\ fsa(x) \Rightarrow pda(y) \wedge equivalent(x,y))$

(C) $\forall x \exists y(\ fsa(x) \wedge pda(y) \wedge equivalent(x,y))$

(D) $\forall x \exists y(\ fsa(y) \wedge pda(x) \wedge equivalent(x,y))$

[GATE 2008 : IISc Bangalore]

Q.28 Suppose the predicate $F(x,y,t)$ is used to represent the statement that person x can fool person y at time t. Which one of the statements below expresses best the meaning of the formula $\forall x \exists y \exists t(\neg F(x,y,t))$?

(A) Everyone can fool some person at some time.

(B) No one can fool everyone all the time.

(C) Everyone cannot fool some person all the time.

(D) No one can fool some person at some time.

[GATE 2010 : IIT Guwahati]

Q.29 Consider the following formula and its two interpretations I1 and I2.

α: (∀x) [Px ⇔ (∀y) [Qxy ⇔ ¬Qyy]] ⇒ (∀x) [¬Px]

I1 : Domain: the set of natural numbers

Px = 'x is a prime number'

Qxy = 'y divides x'

I2 : same as I1 except that Px = 'x is a composite number'.

Which of the following statements is true?

(A) I1 satisfies α, I2 does not

(B) I2 satisfies α, I1 does not

(C) Neither I1 nor I2 satisfies α

(D) Both I1 and I2 satisfies α

[GATE 2003 : IIT Madras]

Q.30 Consider the following first order logic formula in which R is a binary relation symbol.

$$\forall x \forall y (R(x,y) \Rightarrow R(y,x)) .$$

The formula is

(A) Satisfiable and valid

(B) Satisfiable and so is its negation

(C) Unsatisfiable but its negation is valid

(D) Satisfiable but its negation is unsatisfiable

[GATE 2006 : IIT Kharagpur]

Q.31 Which one of the following options is correct given three positive integers x, y and z, and a predicate

$$P(x) : \neg(x=1) \wedge \forall y((\exists z(x = y * z) \Rightarrow (y = x) \vee (y = 1))$$

(A) $P(x)$ being true means that x is a prime number

(B) $P(x)$ being true means that x is a number other than 1

(C) $P(x)$ is always true irrespective of the value of x

(D) $P(x)$ being true means that x has exactly two factors other than 1 and x

[GATE 2011 : IIT Madras]

Q.32 In a room there are only two types of people, namely Type 1 and Type 2. Type 1 people always tell the truth and Type 2 people always lie. You give a fair coin to a person in that room, without knowing which type he is from and tell him to toss it and hide the result from you till you ask for it. Upon asking the person replies the following "The result of the toss is head if and only if I am telling the truth" Which of the following option is correct?

(A) The result is head

(B) The result is tail

(C) If the person is of Type 2, then the result is tail

(D) If the person is of Type 1, then the result is tail

[GATE 2015 : IIT Kanpur]

Q.33 Consider the first order predicate formula φ:

$$\forall x[(\forall z \ z \mid x \Rightarrow ((z = x) \vee (z = 1))) \Rightarrow \exists w(w > x) \wedge (\forall z \ z \mid w \Rightarrow ((w = z) \vee (z = 1)))]$$

Here '$a \mid b$' denotes that 'a divides b', where a and b are integers. Consider the following sets :

S1. {1, 2, 3, ……, 100}

S2. Set of all positive integers

S3. Set of all integers

Which of the above sets satisfy φ:

(A) S1 and S2

(B) S1 and S3

(C) S2 and S3

(D) S1, S2 and S3

[GATE 2019 : IIT Madras]

Q.34 Consider the first-order logic sentence

$$\varphi \equiv \exists s \exists t \exists u \forall v \forall w \forall x \forall y \ \psi(s,t,u,v,w,x,y)$$

where $\psi(s,t,u,v,w,x,y)$ is a quantifier free first order logic formula using only predicate symbols, and possibly equality, but no function symbols. Suppose φ has a model with a universe containing 7 elements.

Which one of the following statements is necessarily true?

(A) There exists at least one model of φ with universe of size less than or equal to 3.

(B) There exists no model of φ with universe of size less than or equal to 3.

(C) There exists no model of φ with universe of size greater than 7.

(D) Every model of φ has a universe of size equal to 7.

[GATE 2018 : IIT Guwahati]

Self Practice Questions

Q.1 The dual of the statement

"If Sonu plays cricket, then he will go to the gym everyday" is

(A) Sonu plays cricket if and only if he goes to the gym everyday.

(B) Sonu does not play cricket and he goes to the gym everyday.

(C) If Sonu goes to the gym everyday, then he will play cricket.

(D) Sonu does not play cricket or he goes to the gym everyday.

Q.2 The negation of the predicate formula $\exists x[(P(x)\vee Q(x))\to R(x)]$

(A) $\forall x[(P(x)\vee Q(x))\wedge\neg R(x)]$

(B) $\exists x[P(x)\vee Q(x)\wedge\neg R(x)]$

(C) $\forall x[(R(x)\to P(x)\wedge\neg(R(x)\to Q(x))]$

(D) $\exists x[(R(x)\to P(x))\wedge R(x)\to Q(x)]$

Q.3 Let triangle (x) : x is a triangle

interior (x) : all the interior angle of x are equal

sides (x) : all the sides of x are equal.

With the set of all polygons as the universe, the symbolic form of the following statement is

"For any triangle, if all the sides are not equal, then all its interior angles are not equal".

(A) $\forall x[\text{triangle}(x)\wedge\neg(\text{sides}(x)\to\text{interior}(x))]$

(B) $\forall x[\text{triangle}(x)\wedge\to(\neg(\text{sides}(x))\to\neg(\text{interior}(x))]$

(C) $\forall x[\text{triangle}(x)\to(\neg(\text{sides}(x))\to(\text{interior}(x))]$

(D) $\forall x[\text{triangle}(x)\to(\text{sides}(x)\to\neg(\text{interior}(x))]$

Q.4 If A, B and C are any three statements, then $[(A\to B)\wedge(B\to C)]\to(A\to B)$ is a :

(A) Tautology

(B) Contradiction

(C) Contingency

(D) None of these

Q.5 Let $P(x)$ and $Q(x)$ denote the predicate statements given below.

$P(x): x^2-4x-5=0$ and

$Q(x): x^2-3x+2\le 0$

Then which of the following predicate formulae is/ are valid?

$P_1: \forall x(P(x)\to Q(x))$

$P_3: \exists x(P(x)\wedge Q(x))$

(A) P_1 Only

(B) P_2 Only

(C) Both P_1 and P_2

(D) Neither P_1 nor P_2

Q.6 With the set of rational numbers as the universe and $P(x,y)$ denoting $x+2y=3$, which of the following predicate formulae is valid?

(A) $\forall x\,\forall y\,P(x,y)$

(B) $\forall y\,\forall x\,P(x,y)$

(C) $(\exists x)(\exists y)\,P(x,y)$

(D) $(\forall y)(\exists x)\,P(x,y)$

Q.7 The contrapositive of the implication" Ramu is a doctor is sufficient for Madhu is not a lawyer" is:

(A) If Ramu is a doctor, if Madhu is not a lawyer.

(B) Ramu is a doctor, if Madhu is not a lawyer.

(C) Ramu is not a doctor is necessary for Madhu is a lawyer.

(D) Madhu is a lawyer is necessary for Ramu is not a doctor.

Q.8 If $A(x)$ and $B(x)$ are any two predicate variables, then $(\exists x)A(x)\to B(x))$ is equivalent to :

(A) $(\exists x)A(x)\to(\exists x)B(x)$

(B) $(\forall x)A(x) \rightarrow (\forall x)B(x))$

(C) $(\exists x)A(x) \rightarrow (\forall x)B(x))$

(D) $(\forall x)A(x) \rightarrow (\exists x)B(x))$

Q.9 The negation of statement, 'If you are lazy, then you won't get the marks', is

(A) 'You are not lazy or you won't get the marks'.

(B) 'You are lazy or you will get the marks'.

(C) 'You are lazy and you will get the marks'.

(D) 'You are not lazy and you will get the marks'.

Q.10 The negation of the statement 'Book is good if and only if print is good' is

(A) 'Book is not good if and only if print is good'.

(B) 'Book is good if and only if print is not good'.

(C) Either (A) or (B)

(D) None of these

Q.11 Write the contrapositive of $p \Rightarrow (p \wedge q)$.

(A) $(\sim(p) \wedge \sim q) \Rightarrow \sim p$

(B) $((\sim p) \vee q) \Rightarrow p$

(C) $((\sim p) \wedge q) \Rightarrow p$

(D) $(\sim p \vee \sim q) \Rightarrow \sim p$

Common Data for Question 12 and 13

Let p : All sides of the triangle are equal.

q : The triangle is equilateral.

Q.12 The inverse of $p \Rightarrow q$ is

(A) If all sides of the triangle are equal, then it is not equilateral triangle.

(B) If all sides of the triangle are not equal, then it is not equilateral triangle.

(C) If all sides of the triangle are not equal, then it is equilateral triangle.

(D) None of these.

Q.13 The contrapositive of $p \Rightarrow q$ is

(A) If the triangle is equilateral, then all sides are not equal

(B) If the triangle is not equilateral, then all sides of it are equal.

(C) If the triangle is equilateral, then all sides are equal.

(D) If the triangle is not an equilateral triangle, then all sides are not equal.

Common data for question 14 to 16

Let $L(x, y)$ be the statement 'x likes y' where the universe of discourse is set of all people in the world. Choose the appropriate quantifier for the statements given in these questions.

Q.14 Everybody likes Ranbir.

(A) $\exists x\, L$ (x, Ranbir)

(B) $\forall x\, L$ (Ranbir, x)

(C) $\forall x\, L$ (x, Ranbir)

(D) $\exists x\, L$ (Ranbir, x)

Q.15 There is somebody whom everybody likes.

(A) $\exists y \exists x\, L(x, y)$ (B) $\exists y \forall x\, L(x, y)$

(C) $\forall y \forall x\, L(x, y)$ (D) None of these

Q.16 There is somebody whom nobody likes.

(A) $\exists y[(\forall x) \sim L(x, y)]$

(B) $(\forall x)[\sim \exists y L(x, y)]$

(C) $\exists x[\forall \sim L(x, y)]$

(D) None of these

Q.17 $P(x, y)$: x is taller than y

$Q(x, y)$: x is heavier than y

$R(x)$: x is bold

Write the symbolic form of the statement 'If x is taller than y and not heavier than y, then he is bold'.

(A) $\forall x(\forall y P(x, y) \wedge \sim Q(x, y) \Rightarrow R(x))$

(B) $\exists x(\forall y P(x, y) \wedge Q(x, y) \Rightarrow R(x))$

(C) $\forall x(\exists y P(x, y) \wedge \sim Q(x, y) \Rightarrow R(x))$

(D) None of these

Q.18 The notation $\exists! x P(x)$ denotes the proposition 'there exists a unique x such that $P(x)$ is true', if the universe of discourse is the set of integers, identify true statement.

(A) $\exists! x(x > 1)$ (B) $\exists! x(x^2 = 1)$

(C) $\exists! x(2x + 5 = 3x)$ (D) $\exists! x(x^2 = x)$

Q.19 Which of the following states the principle of mathematical induction on the universe of positive integers?

(A) $(P(1) \wedge \forall k[P(k) \Rightarrow P(k+1)]) \Rightarrow \forall x P(x)$

(B) $P(1) \wedge \exists k[P(k) \Rightarrow P(k+1)]) \Rightarrow \forall x P(x)$

(C) $P(1) \wedge \forall k[P(k+1) \Rightarrow P(k)]) \Rightarrow \forall x P(x)$

(D) $P(1) \wedge \exists k[P(k+1) \Rightarrow P(k+1)]) \Rightarrow \forall x P(x)$

Q.20 Which one of the following is a tautology?

(A) $X \rightarrow (X \vee Y)$

(B) $X \wedge \neg Y$

(C) $X \rightarrow (X \wedge Y)$

(D) $(X \vee Y) \rightarrow (X \wedge Y)$

Q.21 Which one of the following is True?

$\alpha = (X \vee \neg Y) \wedge (Y \rightarrow Z) \vee (Z \vee X)$

(A) α is tautology

(B) α is a contradiction

(C) If X is True, Y is True and Z is False, then α is True

(D) If X is True, Y is False and Z is False, then α is True

Answer Keys

Classroom Practice Questions - Propositional Logic									
1.	A	2.	True	3.	R	4.	D	5.	D
6.	C	7.	C	8.	A	9.	D	10.	B
11.	A	12.	C	13.	C	14.	B	15.	B
16.	True	17.	D	18.	D	19.	B	20.	B
21.	3	22.	D	23.	D	24.	D	25.	D
26.	A	27.	A	28.	A	29.	A,B,C,E	30.	A,B,D
31.	B	32.	B	33.	C	34.	C	35.	B
36.	D	37.	C	38.	11	39.	A	40.	A
41.	A	42.	Valid	43.	Valid	44.	Valid	45.	B,D
46.	D	47.	B,C						
Classroom Practice Questions - First Order Logic									
1.	*	2.	B	3.	B	4.	A,D	5.	D
6.	D	7.	C	8.	B	9.	A,B,C,D	10.	C
11.	C	12.	A	13.	C	14.	C	15.	C,D
16.	A	17.	B	18.	D	19.	D	20.	D
21.	D	22.	B	23.	C	24.	D	25.	D
26.	D	27.	None	28.	B	29.	D	30.	B
31.	A	32.	A	33.	None	34.	A		
Self Practice Questions									
1.	B	2.	A	3.	B	4.	A	5.	D
6.	D	7.	C	8.	D	9.	C	10.	C
11.	D	12.	B	13.	D	14.	C	15.	B
16.	A	17.	B	18.	C	19.	A	20.	A
21.	D								

* 1. $\exists x\{P(x) \wedge \sim Q(x)\}$

4 Functions

Classroom Practice Questions

Q.1 Let X, Y, Z be sets of sizes x, y and z respectively. Let $W = X \times Y$ and E be the set of all subsets of W. The number of functions from Z to E is

(A) $2^{2^{xy}}$ (B) 2×2^{xy}

(C) $2^{2^{x+y}}$ (D) 2^{xyz}

[GATE 2006 : IIT Kharagpur]

Q.2 Let S denote the set of all functions $f : \{0,1\}^4 \to \{0,1\}$. Denote by N the number of functions from S to the set $\{0,1\}$. The value of $\log_2 \log_2 N$ is _____.

[GATE 2014 : IIT Kharagpur]

Q.3 How many onto (or surjective) functions are there from an n-element $(n \geq 2)$ set to a 2-element set?

(A) 2^n (B) $2^n - 1$

(C) $2^n - 2$ (D) $2(2^n - 2)$

[GATE 2012 : IIT Delhi]

Q.4 The number of onto functions (surjective functions) from set $X = \{1,2,3,4,\}$ to set $Y = \{a,b,c\}$ is _____.

[GATE 2015 : IIT Kanpur]

Q.5 Let X and Y denote the sets containing 2 and 20 distinct objects respectively and F denote the set of all possible functions defined from X to Y. Let f be randomly chosen from F. The probability of f being one-to-one is_____.

[GATE 2015 : IIT Kanpur]

Q.6 Consider the set of all functions $f : \{0,1,....,2014\} \to \{0,1,....,2014\}$ such that $f(f(i)) = i$, for all $0 \leq i \leq 2014$. Consider the following statements:

P: For each such function, it must be the case that for every i, $f(i) = i$.

Q: For each such function, it must be the case that for some i, $f(i) = i$,

R: Each such function must be onto.

Which one of the following in CORRECT?

(A) P, Q and R are true

(B) Only Q and R are true

(C) Only P and Q are true

(D) Only R is true

[GATE 2014 : IIT Kharagpur]

Q.7 Suppose X and Y are sets and $|X|$ and $|Y|$ are their respective cardinalities. It is given that there are exactly 97 functions from X to Y. From this, one can conclude that

(A) $|X| = 1, \quad |Y| = 97$

(B) $|X| = 97, \quad |Y| = 1$

(C) $|X| = 97, \quad |Y| = 97$

(D) None of the above

[GATE 1996 : IISc Bangalore]

Q.8 Suppose we have 5 employees and 4 projects. In how many ways we can assign. The employee to projects such that each employee is assigned to only 1 project and every project is assign to at least 1 employee

Q.9 For the set N of natural numbers and a binary operation $f: N \times N \to N$, an element $z \in N$ is called an identity for f if $f(a,z) = a = f(z,a)$, for all $a \in N$. Which of the following binary operations have an identity?

(I) $f(x,y) = x + y - 3$

(II) $f(x,y) = \max(x,y)$

(III) $f(x,y) = x^y$

(A) I and II only (B) II and III only

(C) I and III only (D) None of them

[GATE 2006 : IIT Kharagpur]

Q.10 Which of the following statements is /are not True?

(A) A constant function is one-one iff domain of the functions has exactly one element

(B) A constant function is onto iff co-domain of the functions has exactly one element

(C) Every one-one function on a finite set A is bijection

(D) Every onto function on a finite set A is bijection

(E) Every equivalence relation on a finite set A is a function

(F) If $f: A \to B$ is one-one then $|A| = |B|$

Q.11 Let $A = R - \{3\}$

$B = R - \{1\}$

Where R is the set of all real numbers if $f: A \to B$ such that $f(x) = \dfrac{x-2}{x-3}$ then

(A) f is one-one but not onto

(B) f is not one-one but onto

(C) f is bijection

(D) f is neither one-one nor onto

Q.12 Consider the following relation from the set of all bit strings to set of all integers

(i) $f_1(s)$ defines the number of bits in S

(ii) $f_2(s)$ defines the position of zero bits in S

(A) f_1 is a function but f_2 is not a function

(B) f_1 is not a function but f_2 is a function

(C) Both are functions

(D) None are functions

Q.13 Which of the following statements is true

(A) The function $f(x) = x$ and $g(x) = \sqrt{x}$ are identical

(B) The function $f(x) = \log(x^2)$ and $g(x) = 2\log(x)$ are identical

(C) Every function can be represented graphically by a graph

(D) The domain of the functions $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is the set of all negative numbers

Q.14 The domain of the function

$$f(x) = \sin\left\{\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right\} \text{ is}$$

(A) $(-2, 0)$

(B) $(-2, 1)$

(C) $(-2, 2) - 1$

(D) $(0, 2) - 1$

Q.15 The range of function $f(x) = \dfrac{x^2}{x^4 + 1}$ is

(A) $(0,1)$ (B) $\left[0, \dfrac{1}{2}\right]$

(C) $\left[0, \dfrac{1}{4}\right]$ (D) $[0,1]$

Q.16 Which of the following is a function on set of all real number

(A) $f(x) = \dfrac{1}{x}$

(B) $f(x) = \sqrt{x}$

(C) $f(x) = |x|$

(D) $f(x) = \pm\sqrt{x^2 + 1}$

Q.17 For which of the following function inverse is /are not defined in the range $R \to R^+$

(A) $f(x) = x^2$ (B) $f(x) = x^3$

(C) $f(x) = \sin x$ (D) $f(x) = e^x$

Q.18 Let R denote the set of real numbers. Let $f: R\times R \to R\times R$ be a bijective function defined by $f(x,y)=(x+y,x-y)$. The inverse function of f is given by

(A) $f^{-1}(x,y)=\left(\frac{1}{x+y},\frac{1}{x-y}\right)$

(B) $f^{-1}(x,y)=(x-y,x+y)$

(C) $f^{-1}(x,y)=\left(\frac{x+y}{2},\frac{x-y}{2}\right)$

(D) $f^{-1}(x,y)=(2(x-y),2(x+y))$

[GATE 1996 : IISc Bangalore]

Q.19 If $g(x)=1-x$ and $h(x)=\frac{x}{x-1}$, then $\frac{g(h(x))}{h(g(x))}$ is:

(A) $\frac{h(x)}{g(x)}$ (B) $\frac{-1}{x}$

(C) $\frac{g(x)}{h(x)}$ (D) $\frac{x}{(1-x)^2}$

[GATE 2015 : IIT Kanpur]

Q.20 If p, q, r, s are distinct integers such that:

f(p,q,r,s)= max (p,q,r,s)

g(p,q,r,s)= min (p,q,r,s)

h(p,q,r,s)=remainder of $\frac{(p\times q)}{(r\times s)}$ if (p×q)>(r×s)

or

remainder of $\frac{(r\times s)}{(p\times q)}$ if (r×s)>(p×q)

Also a function

fgh(p,q,r,s)=f(p,q,r,s)×g(p,q,r,s)×h(p,q,r,s)

Also the same operations are valid with two variable functions of the form f(p,q)

What is the value of fg(h(2,5,7,3),4,6,8)?

[GATE 2015 : IIT Kanpur]

Q.21 A function $f: N^+ \to N^+$, defined on the set of positive integers N^+, satisfies the following properties:

$f(n)=f(n/2)\ if\ n\ is\ even$

$f(n)=f(n+5)\ if\ n\ is\ odd$

Let $R=\{i \mid \exists j:\ f(j)=i\}$ be the set of distinct values that f takes. The maximum possible size of R is_____.

[GATE 2016 : IISc Bangalore]

Q.22 let $f(x)=\frac{x}{x+1}$ and $g(x)=\frac{x}{1-x}$ then $(fog)^{-1}(x)=?$

Q.23 Let 'f' be a function from a set A to a set B, 'g' be a function from B to C, and 'h' be a function from A to C, such that $h(a)=g(f(a))$ for all $a\in A$. Which of the following statements is always true for all such functions f and g?

(A) g is onto $\Rightarrow$ h is onto

(B) h is onto $\Rightarrow$ f is onto

(C) h is onto $\Rightarrow$ g is onto

(D) h is onto $\Rightarrow$ f and g are onto

[GATE 2005 : IIT Bombay]

Q.24 Let $f: B\to C$ and $g: A\to B$ be two functions and let $h=fog$. Given that h is an onto function, which one of the following is TRUE?

(A) f and g should both be onto functions

(B) f should be onto but g need not be onto

(C) g should be onto but f need not be onto

(D) both f and g need not be onto

[GATE 2005 : IIT Bombay]

Q.25 Let $f: A\to B$ be a function, and let E and F be subsets of A. Consider the following statements about images.

$S1: f(E\cup F)=f(E)\cup f(F)$

$S2: f(E\cap F)=f(E)\cap f(F)$

Which of the following is true about S1 and S2?

(A) Only S1 is correct

(B) Only S2 is correct

(C) Both S1 and S2 are correct

(D) None of S1 and S2 are correct

[GATE 2001 : IIT Kanpur]

Q.26 Let X and Y be finite sets and f be a function. Which one of the following statements TRUE?

(A) For any subsets A and B of X,

$$|f(A\cup B)| = |f(A)| + |f(B)|$$

(B) For any subset A and B of X, $f(A\cap B) = f(A)\cap f(B)$

(C) For any subset A and B of X, $|f(A\cap B)| = \min\{|f(A)|, |f(B)|\}$

(D) For any subsets S and T of Y, $f^{-1}(S\cap T) = f^{-1}(S)\cap f^{-1}(T)$

[GATE 2014 : IIT Kharagpur]

Q.27 Let $f: A\to B$ be an injective (one-to-one) function. Define $g: 2^A \to 2^B$ as : $g(C) = \{f(x) | x\in C\}$, for all subsets C of A. Define $h: 2^B \to 2^A$ as : $h(D) = \{x | x\in A,\ f(x)\in D\}$, for all subsets D of B. Which of the following statements is always true?

(A) $g(h(D))\subseteq D$

(B) $g(h(D))\supseteq D$

(C) $g(h(D))\cap D = \phi$

(D) $g(h(D))\cap (B-D)\neq \phi$

[GATE 2003 : IIT Madras]

Q.28 Let N be the set of natural numbers. Consider the following sets.

P: Set of Rational numbers (positive and negative)

Q: Set of functions from {0, 1} to *N*

R: Set of functions from *N* to{0, 1}

S: Set of finite subsets of *N*.

Which of the sets above are countable?

(A) *Q* and *S* only

(B) *P* and *S* only

(C) *P* and *R* only

(D) *P*, *Q* and *S* only

[GATE 2018 : IIT Guwahati]

Q.29 There are 6 jobs with distinct difficulty levels, and 3 computers with distinct processing speeds. Each job is assigned to a computer such that:

- The fastest computer gets the toughest job and the slowest computer gets the easiest job.
- Every computer gets at least one job.

The number of ways in which this can be done is_____.

[GATE 2021 : IIT Bombay]

Self Practice Questions

Q.1 $f: A\to B$ be a function, and let E and F be subsets of A. Consider the following statements about images.

$$S_1 : f(E\cup F) = f(E)\cup f(F)$$

$$S_2 : f(E\cup F) = f(E)\cup f(F)$$

Which of the following is true about S_1 and S_2

(A) Only S_1 is correct

(B) Only S_2 is correct

(C) Both S_1 and S_2 are correct

(D) None of S_1 and S_2 are correct

Q.2 If $f: R\to R$ and $g: R\to R$ be functions defined by

$$f(x) = x^2 + 1 \text{ and } g(x) = \sin x$$

Then find *fog* and *gof*.

(A) $fog = \sin^2 x + 1,\ gof = \sin x^2 + 1$

(B) $fog = \sin x^2 + 1,\ gof = \sin^2 x + 1$

(C) $fog = \sin(x+1)^2,\ gof = \sin x + 1$

(D) $fog = \sin x + 1,\ gof = \sin(x+1)^2$

Q.3 The number of one-one mappings from, set *A* to set *B*

When *A* = {1,2,3,4} and *B* = {2,3,4,5,6} is

(A) 5 (B) 4^5

(C) 120 (D) 0

Q.4 $f: R\to R$ and $f(x) = \dfrac{2}{1+x^2}$ then f is

(A) One-One but not onto.

❖❖❖❖

(B) Onto, but not one-one
(C) Bijective
(D) None of these

Q.5 If $f(x)=2x+2-x$ and $g(x)=2x-2-x$, then the value of $f(x)g(y)+f(y)g(x)=$
(A) $g(x+y)$ (B) $f(x+y)$
(C) $2g(x+y)$ (D) $2f(x+y)$

Q.6 The function $f:N\to N$ defined by $f(n)=2n+3$ is
(A) Injective and surjective
(B) Injective and not surjective
(C) Not injective and surjective
(D) Not injective and not surjective

Q.7 Let A and B be two sets. $f:A\times B\to B\times A$ defined by $f(a,b)=(b,a)$ is
(A) Bijective
(B) Injective but not surjective
(C) Surjective but not injective
(D) Neither surjective nor injective

Q.8 The domain and range are same for
(A) Constant function
(B) Identity function
(C) Absolute value function
(D) Greatest integer function

Q.9 If the function $f:[1,\infty]\to[1,\infty]$ defined by $f(x)=2x(x-1)$ is invertible, find $f^{-1}(x)$.
(A) $\frac{1+\sqrt{1+2\log_2 x}}{4}$ (B) $\frac{1-\sqrt{1+2\log_2 x}}{4}$
(C) $\frac{1+\sqrt{1+4\log_2 x}}{2}$ (D) $\frac{1-\sqrt{1+4\log_2 x}}{2}$

Q.10 Let R denote the set of real numbers and $f:R\times R\to R\times R$ be a bijective function defined by $f(x,y)=f(x+y,x-y)$. The inverse of f is given by
(A) $f^{-1}(x,y)=\left[\frac{1}{(x+y)},\frac{1}{(x-y)}\right]$
(B) $f^{-1}(x,y)=(x-y,x+y)$
(C) $f^{-1}(x,y)=\left[\frac{x+y}{2},\frac{x-y}{2}\right]$
(D) $f^{-1}(x,y)=[2(x-y),2(x+y)]$

Answer Keys

Classroom Practice Questions									
1.	D	2.	16	3.	C	4.	36	5.	0.95
6.	B	7.	A	8.	240	9.	A	10.	E,F
11.	C	12.	A	13.	D	14.	B	15.	B
16.	C	17.	A,B,C	18.	C	19.	A	20.	8
21.	2	22.	*	23.	C	24.	B	25.	A
26.	D	27.	A	28.	D	29.	65		
Self - Practice Questions									
1.	A	2.	A	3.	C	4.	D	5.	C
6.	B	7.	A	8.	B	9.	C	10.	C

* **22** $(fog)^{-1}(x)$ is an Identity function

❖❖❖❖

5 Group Theory

Classroom Practice Questions

Q.1 Let $A=\{0,\pm2,\pm4,\pm6,.....\}$

$B=\{0,\pm1,\pm3,\pm5,.....\}$

Which of the following is not a semi- group

(A) $(A,+)$ (B) $(A,\bullet)$

(C) $(B,+)$ (D) $(B,\bullet)$

Q.2 Consider the set Σ^* of all strings over the alphabet $\Sigma=\{0,1\}$. Σ^* with the concatenation operator for strings

(A) Does not form a group

(B) Forms a non commutative group

(C) Does not have a right identity element

(D) Forms a group if the empty string is removed from Σ^* .

[GATE 2003 : IIT Madras]

Q.3 Let A be the set of all non-singular matrices over real number and let $*$ be the matrix multiplication operation. Then

(A) A is closed under $*$ but $\langle A,*\rangle$ is not a semigroup

(B) $\langle A,*\rangle$ is a semigroup but not a monoid.

(C) $\langle A,*\rangle$ is a monoid but not a group.

(D) $\langle A,*\rangle$ is a group but not an abelian group.

Q.4 Let S be any finite set, and $F(s)$ is defined as set of all function on set S. Then $F(s)$ with respect to function composition operation (ie.,o) is.

(A) Not a semigroup

(B) Semi group but not a monoid.

(C) Monoid but not a group.

(D) A group

Q.5 Let Z is the set of all integers. The binary operation * is defined as $a*b=\max(a,b)$ then the structure $(Z,*)$ is

(A) Not a semigroup

(B) Semi group but not a monoid.

(C) Monoid but not a group.

(D) A group

Q.6 Let Q^* be the set of all positive rational numbers. The binary operation $*$ is defined as $a*b=\frac{ab}{3}\quad \forall a,b,\in Q^*$ If $(Q^*,*)$ is a group then find

(i) identity element of the group

(ii) inverse of any element a, $\forall a \in$ Group

Q.7 Which of the following statement is/are not true.

(A) $\{0,\pm2k,\pm4k,\pm6k,.....\}$ is a group with respect to addition where any fixed positive integer

(B) $\{x \mid x$ is a real number and $0<x\le1\}$ is a group with respect to multiplication

(C) $\{2^n \mid n$ is an integer$\}$ is a group with respect to multiplication

(D) None of the above

Q.8 Consider the set H of all 3×3 matrices of the type

$$\begin{bmatrix} a & f & e \\ 0 & b & d \\ 0 & 0 & c \end{bmatrix}$$

Where a, b, c, d, e and f are real numbers and $abc\neq0$. Under the matrix multiplication operation, the set H is

(A) A group
(B) A monoid but not group
(C) A semigroup but not a monoid
(D) Neither a group nor a semigroup

Q.9 Which of the following statements is false?
(A) The set of rational numbers is an abelian group under addition
(B) The set of integers in an abelian group under addition
(C) The set of relational numbers form an abelian group under multiplication
(D) The set of real numbers excluding zero in an abelian group under multiplication

Q.10 Which one of the following in NOT necessarily a property of a Group?
(A) Commutativity
(B) Associativity
(C) Existence of inverse for every element
(D) Existence of identity

[GATE 2009 : IIT Roorkee]

Q.11 Some group (G,o) is known to be abelian. Then, which one of the following is true for G?
(A) $g = g^{-1}$ *for every* $g \in G$.
(B) $g = g^2$ *for every* $g \in G$.
(C) $(goh)^2 = g^2 o h^2$ *for every* $g,h \in G$.
(D) G is of finite order.

[GATE 1994 : IIT Kharagpur]

Q.12 A Binary operation $\oplus$ on a set of integers is defined as $x \oplus y = x^2 + y^2$. Which one of the following statements is TRUE about $\oplus$?
(A) Commutative but not a associative
(B) Both commutative and associative
(C) Associative but not Commutative
(D) Neither commutative nor associative

[GATE 2013 : IIT Bombay]

Q.13 Let $(\{p,q\},*)$ be a semi group where $p*p = q$ show that:
(i) $p*q = q*p$
(ii) $q*q = q$

Q.14 The set $\{1,2,4,7,8,11,13,14\}$ is a group under multiplication module 15. The inverse of 4 and 7 are respectively :
(A) 3 and 13 (B) 2 and 11
(C) 4 and 13 (D) 8 and 14

[GATE 2005 : IIT Bombay]

Q.15 The set $\{1,2,3,4,5,6\}$ is a group with respect to $\otimes_7$

Which of the following is /are not true
(A) Inverse of 6 is 6
(B) Inverse of 2 is 4
(C) Inverse of 3 is 5
(D) None of the above.

Q.16 The set $\{0,1,2,3,4,5\}$ is a group with respect to $\oplus_6$, which of the following is not true
(A) $inv\,(1) = 5$
(B) $inv\,(2) = 4$
(C) $inv\,(3) = 5$
(D) $inv\,(0) = 0$

Q.17 Which of the following statement is/are false.
(A) $\{1,2,3,4,5\}$ is a group with respect to $\otimes_6$.
(B) $\{1,2,3,4,5\}$ is a group with respect to $\oplus_6$.
(C) $\{0,1,2,3,4\}$ is a group with respect to $\otimes_5$
(D) $\{1,5\}$ is a group with respect to $\otimes_6$

Q.18 The set $\{1,2,3,5,7,8,9\}$ under multiplication modulo 10 is not a group. Given below are four plausible reasons. Which one of them is false?
(A) It is not closed
(B) 2 does not have an inverse
(C) 3 does not have an inverse
(D) 8 does not have an inverse

[GATE 2006 : IIT Kharagpur]

Q.19 Which one of the following is false?

(A) The set of all bijective functions on a finite set forms a group under function composition.

(B) The set $\{1,2,.....,p-1\}$ forms a group under multiplication mod p where p is a prime number.

(C) The set of all strings over a finite alphabet $\sum$ forms a group under concatenation.

(D) A subset $s \neq \phi$ of G is a subgroup of the group $\langle G,* \rangle$ if and only if for any pair of elements $a,b \in s, a*b^{-1} \in s$.

[GATE 1996 : IISc Bangalore]

Q.20 Consider the set $S=\{1,\omega,\omega^2\}$, where ω and ω^2 are cube roots of unity. If * denotes the multiplication operation, the structure $\{S,*\}$ forms

(A) a group

(B) a ring

(C) an integral domain

(D) a field

[GATE 2010 : IIT Guwahati]

Q.21 There are two elements x, y in a group $(G,*)$ such that every in the group can be written as a product of some number of $x's$ and $y's$ in some order. It is known that

$x*x=y*y=x*y*x*y=y*x*y*x=e$

Where e is the identity element. The maximum number of elements in such a group is ———.

[GATE 2014 : IIT Kharagpur]

Q.22 Which of the following set is /are group with respect to binary operation $\otimes_8$

(A) (0,1) (B) (1,3)

(C) (1,5) (D) (1,7)

Q.23 Let $S=\{0,1,2,3,4,5,6,7\}$ and $\otimes$ denote multiplication modulo 8, that is, $x \otimes y=(xy) \bmod 8$

(a) Prove that $(0,1,\otimes)$ is not a group.

(b) Write 3 distinct groups $(G,\otimes)$ where $G \subset S$ and G has 2 elements.

Q.24 Find the orders of every element of the group $\{0,1,2,3\}$ with respect to $\oplus_4$

Q.25 Find the orders of every element of the group $\{1,2,3,4\}$ with respect to $\otimes_5$

Q.26 Let $(G,*)$ be a subgroup of prime order P. Then how many sub group at group G are possible

Q.27 Which of the following statement is /are false

(A) Every sub-group of an abelian group is also abelian

(B) The union of any two sub-group of a group G is also a subgroup of G

(C) The intersection of any two sub-group of a group G is also a subgroup of G

(D) None of the above

Q.28 Let G_1 and G_2 be subgroups of a group G.

(a) Show that $G_1 \cap G_2$ is also a subgroup of G

(b) Is $G_1 \cup G_2$ always a subgroup of G ?

Q.29 Let G be a group with 15 elements. Let L be a subgroup of G. It is known that $L \neq G$ and that the size of L is at least 4. The size of L is ————.

[GATE 2014 : IIT Kharagpur]

Q.30 Let G be a finite group on 84 elements. The size of a largest possible proper subgroup of G is _____.

[GATE 2018 : IIT Guwahati]

Q.31 Let G be a group of 35 elements. Then the largest possible size of a subgroup of G other than G itself is _____.

[GATE 2020 : IIT Delhi]

Q.32 Check if $\{0,1,2,3,4\}$ is a cyclic group with respect to $\oplus_5$. And if cyclic find all the generators of the group.

Q.33 Let $(G,*)$ be the cyclic group of order 8, then find the number of generators of group G.

Q.34 Let $(G,*)$ be a cyclic group of order 50, then how many generators does the group G has?

Q.35 Let $(G,*)$ be a cyclic group of prime order P, then how many generators does the group G has?

Q.36 The set $\{1,2,3,4,5,6\}$ is a cyclic group with respect to $\otimes_7$. Find the number of generators of the group and what are they.

Q.37 Which of the following statement is /are true.

(A) Every cyclic Group is abelian

(B) Every group of prime order is cyclic

(C) Every group of prime order is abelian

(D) Every sub- group of a cyclic group is cyclic but generators of cyclic subgroup need not be same as generator of original group.

Q.38 Which of the following is/are not true

(A) Any group of order ≤ 5 is abelian

(B) In a group $(G,*)$

If $a*a=a$

Then $a=e$

(C) In a group $(G,*)$

If $a^{-1}=a\ \forall a\in G$

Then $(G,*)$ is abelian group

Q.39 Let G be an arbitrary group. Consider the following relations on G:

$R_1: \forall a,b\in G,\ a\,R_1\,b$ if and only if $\exists g\in G$ such that $a=g^{-1}bg$

$R_2: \forall a,b\in G,\ a\,R_2\,b$ if and only if $a=b^{-1}$

Which of the abive is/are equivalence relation/relatiions?

(A) R_1 only

(B) Neither R_1 nor R_2

(C) R_1 and R_2

(D) R_2 only

Q.40 How many different non-isomorphic Abelian groups of order 4 are there?

(A) 2 (B) 3

(C) 4 (D) 5

[GATE 2006 : IIT Kharagpur]

Cyclic Group

Q.41 For the composition table of a cyclic group shown below

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

Which one of the following choices is correct?

(A) a, b are generators

(B) b, c are generators

(C) c, d are generators

(D) d, a are generators

[GATE 2009 : IIT Roorkee]

Q.42 Consider the set $\{a, b, c\}$ with binary operators + and * defined as follows :

+	a	b	c
a	b	a	c
b	a	b	c
c	a	c	b

*	a	b	c
a	a	b	c
b	b	c	a
c	c	c	b

For example, $a+c=c,\ c+a=a,\ c*b=c$ and $b*c=a$

Given the following set of equations :

* $(a*x)+(a*y)=c$

* $(b*x)+(c*y)=c$

The number of solution (s) (i.e., pair (s) (x, y) that satisfy the equation) is

(A) 0 (B) 1

(C) 2 (D) 3

[GATE 2003 : IIT Madras]

Q.43 The following is the incomplete operation table of a 4-element group.

*	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b				
c				

The last row of the table is

(A) $c\ a\ e\ b$ (B) $c\ b\ a\ e$

(C) $c\ b\ e\ a$ (D) $c\ e\ a\ b$

[GATE 2004 : IIT Delhi]

Q.44 Let G be a group of order 6, and H be a subgroup of G such that $1 < |H| < 6$. Which one of the following options is correct?

(A) Both G and H may not be cyclic.

(B) Both G and H are always cyclic.

(C) G is always cyclic, but H may not be cyclic.

(D) G may not be cyclic, but H is always cyclic.

[GATE 2021 : IIT Bombay]

Self Practice Questions

Q.1 A binary operator * defined on a set of real numbers as $a*b=a^3+b^3$ then * is:

(A) commutative, associative

(B) commutative but not associative

(C) associative but not commutative

(D) neither commutative nor associative

Q.2 The number of generators of the cyclic group G of order 10 is :

(A) 3 (B) 6

(C) 5 (D) 4

Q.3 H is the subgroup of G. The order of G is 21, then the order of H can be

(A) 3 (B) 7

(C) 21 (D) All the above

Q.4 If $G = \{1, 2, 3, 4, 5, 6\}$ $(G, X7)$ is an abelian group, then the order of the element 4 is_____

(A) 3 (B) 4

(C) 2 (D) 5

Q.5 On the set of integers Z, which of the following is not a binary operation?

(A) Addition (B) Subtraction

(C) Multiplication (D) Division

Q.6 Let a binary operation* be defined in R by $a*b=6ab$. Then identify e =

(A) 1/6 (B) 1/4

(C) 1/3 (D) 1/2

Q.7 Which of the following algebraic structure does not form a group?

(A) $(Z,+)$ (integers)

(B) $(R,+)$ (Real numbers)

(C) $(R^+,\times)$ (Positive real numbers)

(D) $(N,\times)$ (natural numbers)

Q.8 The set $G = \{1,2,3,4,5\}$ under multiplication modulo 6 is

(A) An algebraic structure

(B) A non-abelian group

(C) An abelian group

(D) None of the above

Q.9 In a group $G = \{0,1,2,3,4\}$ under addition modulo 5, the inverse of 4 is

(A) 0 (B) 1

(C) 2 (D) 3

Q.10 Let $(Z, *)$ be an algebraic structure, where Z is the set of integers and the operation * is defined by $n*m$ = maximum (n, m). Which of the following statement is true for $(Z, *)$

(A) $(Z, *)$ is a monoid

(B) $(Z, *)$ is an abelian group

(C) $(Z, *)$ is a group

(D) None of these

Q.11 Let A be the set of all non-singular matrices over real numbers and let $*$ be the matrix multiplication operator. Then

(A) A is close under $*$ but $< A,*>$ is not a semigroup.

(B) $< A,*>$ is a semigroup but not a monoid.

(C) $< A,*>$ is a monoid but not a group.

(D) $< A,*>$ is a group but not an abelian group.

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Answer Keys

Classroom Practice Questions									
1.	C	2.	A	3.	D	4.	C	5.	B
6.	*	7.	B	8.	A	9.	C	10.	A
11.	C	12.	A	13.	*	14.	C	15.	D
16.	C	17.	A,B,C	18.	C	19.	C	20.	A
21.	4	22.	B,C,D	23.	*	24.	*	25.	*
26.	2	27.	B	28.	*	29.	5	30.	42
31.	7	32.	*	33.	4	34.	20	35.	(P-1)
36.	*	37.	A,B,C,D	38.	A,B,C	39.	A	40.	A
41.	C	42.	C	43.	D	44.	D		
Self Practice Questions									
1.	B	2.	D	3.	D	4.	A	5.	D
6.	A	7.	D	8.	D	9.	B	10.	D
11.	D								

* **6** (i) 3, (ii) 9/a

* **13** (i) to be proved, (ii) to be proved

* **23** (i) to be proved (ii) $(1,3,\otimes)$, $(1,5,\otimes)$ & $(1,7,\otimes)$

* **24** $O(0)=1,\ O(1)=4,\ O(2)=2,\ O(3)=4$

* **25** $O(1)=1,\ O(2)=4,\ O(3)=4,\ O(4)=2$

* **28** (i) to be proved, (ii) No

* **32** Yes, and generators are = 1,2,3 & 4

* **36** Number of generators = 2 & they are 3 & 5

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6 Combinatorics

Classroom Practice Questions

Q.1 A boy wants to invite 8 friends for his birthday party. In how many ways can he send invitation cards to them, if he has 4 different types cards to send invite?

Q.2 Chintu's TV has only 4 channels, all of them quite boring. Hence its not surprising that he desires to switch channel after every one minute. Then find the number of ways in which he can change the channels so that he is back to his original channel for the first time after 4 min?

Q.3 Find the total number of integer n such that $1<n<2001$ and HCF of n and 36 is 1.

Q.4 Find the number of distinct rational numbers x such that $0<x<1$ and $x = p/q$, where p, q belongs to the set {1,2,3,4,5,6}.

Q.5 Total number of 'n' digits numbers (n>1), having the property, that no two consecutive digits are same.

Q.6 There are ten points in the plane, no three of which are collinear. How many different lines can be drawn through these points?

Q.7 Find number of four-digit numbers in which repetition is not allowed. Also find number of four-digit numbers in which at least one digit is repeated.

Q.8 Find number of four digit even numbers in which repetition is not allowed.

(A) 2240 (B) 2296

(C) 2620 (D) 4536

[GATE 2001 : IIT Kanpur]

Q.9 Find number of three-digit numbers in which repetition is allowed and sum of digits is even.

Q.10 Find the number of non-empty subsets of {1,2, 3…12} having the property that sum of the largest and smallest element is 13.

Q.11 The number of substrings (of all lengths inclusive) that can be formed from a character string of length n is

(A) n (B) n^2

(C) $\frac{n(n-1)}{2}$ (D) $\frac{n(n+1)}{2}$

[GATE 1994 : IIT Kharagpur]

Q.12 There are 5 bags labeled 1 to 5. All the coins in given bag have the same weight. Some bags have coins of weight 10 gm, others have coins of weight 11 gm. I pick 1, 2, 4, 8, 16 coins respectively from bags 1 to 5. Their total weight comes out to 323 gm. Then the products of the labels of the bags having 11 gm coin is ———.

Q.13 A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them. How many balls must she select to be sure of having at least three balls of the same color?

(A) 3 (B) 5

(C) 6 (D) 4

Q.14 A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them. How many balls must she select to be sure of having at least three blue balls?

(A) 13 (B) 9

(C) 12 (D) 14

Q.15 How many numbers must be selected from the set {1, 2, 3, 4, 5, 6} to guarantee that at least one pair of these numbers add up to 7?

(A) 4 (B) 3

(C) 2 (D) 5

Q.16 The rules for the University of Bombay five-a-side cricket competition specify that the members of each team must have birthdays in the same month. What is the minimum number of mathematics students needed to be enrolled in the department to guarantee that they can raise a team of students?

(A) 23 (B) 91

(C) 60 (D) 49

Q.17 What is the minimum number of ordered pairs of non-negative numbers that should be chosen to ensure that there are two pairs (a, (B) and (c, (D) in the chosen set such that, a=c mod 3 and b=d mod 5.

(A) 4 (B) 6

(C) 16 (D) 24

[GATE 2005 : IIT Bombay]

Q.18 A company stores products in a warehouse. Storage bins in this warehouse are specified by their aisle, location in the aisle, and shelf. There are 50 aisles, 85 horizontal locations in each aisle, and 5 shelves throughout the warehouse. What is the least number of products the company can have so that at least two products must be stored in the same bin?

(A) 21251 (B) 1250

(C) 12150 (D) 12151

Q.19 In how many ways 6 different books can be distributed among 10 persons so that no person can take more than one book and maximum number of books are to be distributed?

Q.20 In how many ways 4 boys and 4 girls can sit in a row so that no two girls are sitting side by side?

(A) 720 (B) 2560

(C) 2880 (D) 576

Q.21 In how many ways can ten adults and five children stand in a line so that no two children are next to each other?

Q.22 Suppose that a state's license plates consist of 3 letters followed by 3 digit then total number of different plates can be formed (no repetitions allowe(D) ?

(A) 26^3 X 10^3

(B) 260

(C) P(26,3) X P(10,3)

(D) None of these

Q.23 How many distinguishable permutation can be generated from word "ACADEMY" ?

(A) 2520 (B) 720

(C) 5040 (D) 3620

Q.24 A group contains n men and n women. How many ways are there to arrange these people in a row if then men and women alternate?

Q.25 In how many different ways can 5 ones and 20 twos be permuted so that each one is followed by at least 2 twos?

Q.26 Ten different letters of alphabet are given. Words with six letters formed from these given letters. Find the number of words which can have at least one letter repeated?

(A) 10 C_6 (B) 10^6

(C) $10^6 - 10P_6$ (D) 10 P_6

Q.27 How many 2 letter passwords can be made from {a, b, c}, if a letter can be used any number of times?

Q.28 The number of 4-digit numbers having their digits in non-decreasing order (from left to right) constructed by using the digits belonging to the set $\{1,2,3\}$ is ———.

Q.29 In how many ways can 10 students and 4 teachers be seated at a round table without any restrictions?

(A) $14C_{10}$ (B) 14!

(C) 13! (D) 16!

[GATE 2015 : IIT Kanpur]

Q.30 Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

(A) 25200 (B) 21300

(C) 24400 (D) 210

Common Data Questions 31 to 32

A coin is flipped 10 times where each flip comes up either heads or tails.

Q.31 How many possible outcomes contain at most three tails?

Q.32 How many possible outcomes contain the same number of heads and tails ?

Q.33 There are n kingdoms and $2n$ champions. Each kingdom gets 2 champion. The number of ways in which this can be done is:

(A) $\frac{(2n)!}{2^n}$ (B) $\frac{(2n)!}{n!}$

(C) $\frac{(2n)!}{n!2^n}$ (D) $\frac{n!}{2}$

Q.34 The number of binary strings of n zeros and k ones $(n \geq k-1)$ such that no two ones are adjacent is:

(A) ${}^{n+1}C_k$

(B) ${}^{n}C_k$

(C) ${}^{n}C_{k+1}$

(D) None of the above

[GATE 1999 : IIT Bombay]

Q.35 Dhokla come in 30 different varieties and Raju wants to buy a dozen. How many choices does he have?

Q.36 Let A be a sequence of 8 distinct integers sorted in ascending order. How many distinct pairs of sequences, B and C are there such that

(i) Each is sorted in ascending order.

(ii) B has 5 and C has 3 elements, and

(iii)The result of merging B and C gives A?

(A) 2 (B) 30

(C) 56 (D) 256

[GATE 2003 : IIT Madras]

Q.37 A medical student has to work in a hospital for five days in January. However, he is not allowed to work two consectuive days in the hospital. In how many different ways can be choose the five days he will work in the hospital?

(A) $C(27,5)$ (B) $C(26,5)$

(C) $C(27,4)$ (D) $C(26,4)$

Q.38 Suppose that we draw a card from a deck of 52 cards and replace it before the next draw. In how many ways can 10 cards be drawn so that the tenth card is a repetition of a previous draw.

(A) $52(51)^{19}$

(B) $(52)^{10}-(51)^9 52$

(C) $52^{10}-51^9$

(D) $52^{10}-51^{10}$

Q.39 n couples are invited to a party with the condition that every husband should be accompanied by his wife. However, a wife need not be accompanied by her husband. The number of different gatherings possible at the party is

(A) $\binom{2n}{n} * 2^n$ (B) 3^n

(C) $\frac{(2n)!}{2^n}$ (D) $\binom{2n}{n}$

[GATE 2003 : IIT Madras]

Q.40 Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have more women than men?

Q.41 Suppose that a robot is placed on the Cartesian plane. At each step it is allowed to move either one unit up or one unit right, i.e., if it is at (i, j) then it can move to either $(i+1, j)$ or $(i+j+1)$ Suppose that the robot is not allowed to traverse the line segment from $(4,4)$ to $(5,4)$. With this constraint, how many distinct paths are there for the robot to reach $(10,10)$ starting from $(0,0)$?

(A) 2^9

(B) 2^{19}

(C) $\binom{8}{4} \times \binom{11}{5}$

(D) $\binom{20}{10} - \binom{8}{4} \times \binom{11}{5}$

[GATE 2007 : IIT Kanpur]

Q.42 What is the coefficient of $x^{16}y^{23}$ in the expansion of $(7x-5y)^{39}$?

Q.43 How many subsets with more than two elements does a set with 100 elements have ?

Q.44 Suppose that a cookie shop has four different kinds of cookies. How many different ways can six cookies be chosen?

Q.45 Six distinct symbols are transmitted through a communication channel. A total of 12 blanks are to be inserted between the symbols with atleast 2 blanks between every pair of symbols. The number of ways we can arrange the symbols and blanks is _____.

Q.46 Two girls have picked 10 roses, 15 sunflowers and 14 daffodils. What is the number of ways they can divide the flowers among themselves?

(A) 1638

(B) 2100

(C) 2640

(D) None of the above

[GATE 1999 : IIT Bombay]

Q.47 A professor wishes to give an examination with 10 questions. In how many ways can the test be given a total of 30 marks if each question is to be worth 2 or more marks?

(A) $C(30,10)$ (B) $C(20,10)$

(C) $C(19,10)$ (D) $C(21,10)$

Q.48 m identical balls are to be placed in n distinct bags. You are given that $m \geq kn$, where k is a natural number ≥ 1. In how many ways can the balls be placed in the bags if each bag must contain at least k balls?

(A) $(m-k)$

(B) $\binom{m-kn+n-1}{n-1}$

(C) $\binom{m-1}{n-k}$

(D) $\binom{m-kn+n+k-2}{n-k}$

[GATE 2003 : IIT Madras]

Q.49 In how many ways can b blue balls and r red balls be distributed in n distinct boxes?

(A) $\frac{(n+b-1)!(n+r-1)}{(n-1)!b!(n-1)!r!}$

(B) $\frac{(n+(b+r)-1)!}{(n-1)!(n-1)!(b+r)!}$

(C) $\frac{n!}{b!r!}$

(D) $\frac{(n+(b+r)-1)!}{n!(b+r-1)!}$

[GATE 2008 : IISc Bangalore]

Q.50 There are 6 jobs with distinct difficulty levels, and 3 computers with distinct processing speeds. Each job is assigned to a computer such that:

- The fastest computer gets the toughest job and the slowest computer gets the easiest job.
- Every computer gets at least one job.

The number of ways in which this can be done is _____

[GATE 2021 : IIT Bombay]

Q.51 Let $n = p^2q$, where p and q are distinct prime numbers. How many numbers m satisfy $1 \leq m \leq n$ and $\gcd(m,n) = 1$? Note that $\gcd(m,n)$ is the greatest common divisor of m and n.

(A) $p(q-1)$

(B) pq

(C) $(p^2-1)(q-1)$

(D) $p(p-1)(q-1)$

[GATE 2005 : IIT Bombay]

Q.52 Number of positive integers which are less than 180 and coprime to 180 is _____.

Q.53 Mala has a colouring book in which each English letter is drawn two times. She wants to paint each of these 52 prints with one of k colours, such that the colour-pairs used to colour any two letters are different. Both prints of a letter can also be coloured with the same colour. What is the minimum value of k that satisfies this requirement?

(A) 9 (B) 8

(C) 7 (D) 6

[GATE 2004 : IIT Delhi]

Q.54 The exponent of 11 in the prime factorization of 300! Is

(A) 27 (B) 28

(C) 29 (D) 30

[GATE 2008 : IISc Bangalore]

Q.55 Let, $P = \sum_{\substack{1\le i\le 2k \\ i \text{ odd}}} i$ *and* $Q = \sum_{\substack{1\le i\le 2k \\ i \text{ even}}} i$

Where, k is positive integer. Then

(A) $P = Q - k$ (B) $P = Q + k$

(C) $P = Q$` (D) $P = Q + 2k$

[GATE 2008 : IISc Bangalore]

Q.56 The number of distinct positive integral factors of 2014 is ————.

[GATE 2014 : IIT Kharagpur]

Q.57 $\sum_{x=1}^{99} \frac{1}{x(x+1)} =$ ————.

[GATE 2015 : IIT Kanpur]

Q.58 The number of divisors of 2100 is ———— .

[GATE 2015 : IIT Kanpur]

Q.59 Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the woman choose the chairs amongst the chairs 1 to 4, and then the men select from remaining chairs. The number of possible arrangements are

(A) 1440 (B) 1290

(C) 1160 (D) 900

Common Data Questions 60 to 65

How many solutions are there to the given equations

x1 + x2 + x3 + x4 = 18,

Q.60 All xi belongs to integers

Q.61 All xi belongs to non-negative integers

Q.62 x1 ≥ 1?

Q.63 xi ≥ 2 ?

Q.64 0 ≤ x1 ≤ 10?

Q.65 x1 ≥ -2 ?

Q.66 How many solution for the equation

x1 + x2 + x3 + x4 ≤ 18, all xi belongs to non negative integers

Q.67 Find the sum of all 4-digit numbers which can be formed using the digits 1,2,4 and 5 exactly once

Q.68 A fortune cookie company makes 213 different fortunes. A student eats at a restaurant that uses fortunes from this company and gives each customer one fortune cookie at the end of a meal. What is the largest possible number of times that the student can eat at the restaurant without getting the same fortune four times?

Q.69 There are 12 stations on a rail route. How many ways a special train can stop at 4 of these stations, so that no two stops are consecutive stations?

Q.70 A playoff between two teams consists of at most five games. The first team that wins three games wins the playoff. In how many different ways can the playoff occur?

Common Data Questions 71 to 74

Find the following for the expansion of $(x + y + z)^{10}$

Q.71 Number of terms, if terms with identical sets of exponents are not added?

Q.72 Number of terms, after all terms with identical sets of exponents are added?

Q.73 Sum of all the coefficients of the terms in the expansion*

Q.74 Coefficient of $x^3y^2z^5$

Q.75 In a room containing 28 people, there are 18 people who speak English, 15 people who speak Hindi and 22 people who speak Kannada. 9 persons speak both English and Hindi, 11 persons speak both Hindi and Kannada, whereas 13 persons speak both Kannada and English. How many people speak all three languages?

(A) 9 (B) 8

(C) 7 (D) 6

[GATE 1998 : IIT Delhi]

Q.76 What is the cardinality of the set of integers X defined below?

$X = \{n \mid 1 \le n \le 123, \text{n is not divisible by either } 2,3 \text{ or } 5\}$

(A) 28 (B) 33

(C) 37 (D) 44

[GATE 2006 : IIT Kharagpur]

Q.77 In how many ways can we distribute 5 distinct balls, $B_1, B_2, ..., B_5$ in 5 distinct cells, $C_1, C_2, ..., C_5$ such that Ball B_i is not in cell $C_i, \forall i = 1, 2, ..., 5$ and each cell contains exactly one ball?

(A) 44 (B) 96

(C) 120 (D) 3125

[GATE 2004 : IIT Delhi]

Q.78 In how many ways can you put 7 letters into their respective envelopes such that exactly 3 go into the right envelope?

(A) 240 (B) 320

(C) 315 (D) 450

Q.79 The number of permutations of the characters in LILAC so that no character appears in its original position, if the two L's are indistinguishable, is _____.

[GATE 2020 : IIT Delhi]

Q.80 Let $G(x) = (1-x)^{-2} = \sum_{i=0}^{\infty} g(i)\, x^i$, where $|x| < 1$ What is $g(i)$?

(A) 1 (B) $i+1$

(C) $2i$ (D) $2i$

[GATE 2005 : IIT Bombay]

Q.81 The co-efficient of x^{12} in $(x^3 + x^4 + x^5 + x^6 + ...)^3$ is _____.

[GATE 2016 : IISc Bangalore]

Q.82 If the ordinary generating function of a sequence $\{a_n\}_{n=0}^{\infty}$ is $\frac{1+Z}{(1-Z)^3}$, then $a_3 - a_0$ is equal to _____.

[GATE 2017 : IIT Roorkee]

Q.83 The generating function of the sequence $\{1, -2, 4, -8, 16, ...\infty\}$ is

(A) $(1+2x)^{-1}$ (B) $(1-2x)^{-1}$

(C) $(1+x)^{-2}$ (D) $(1-x)^{-2}$

Q.84 The solution to the recurrence equation $T(2^k) = 3T(2^{k-1}) + 1,\ T(1) = 1 is:$

(A) 2^k (B) $\left(3^{k+1} - 1\right)/2$

(C) $3^{\log_2^k}$ (D) $2^{\log_2^k}$

[GATE 2002 : IISc Bangalore]

Q.85 The recurrence equation

$T(1) = 1$

and $T(n) = 2T(n-1) + n \quad (n \ge 2)$ evaluate to

(A) $2^{n+1} - n - 2$ (B) $2^n - n$

(C) $2^{n+1} - 2n - 2$ (D) $2^n + n$

[GATE 2004 : IIT Delhi]

Q.86 Let x_n denote the number of binary strings of length n that contain no consecutive 0s. Which of the following recurrences does x_n satisfy?

(A) $x_n = 2x_{n-1}$

(B) $x_n = x_{[n/2]} + 1$

(C) $x_n = x_{[n/2]} + n$

(D) $x_n = x_{n-1} + x_{n-2}$

[GATE 2008 : IISc Bangalore]

Q.87 The solution of $a_n - 3a_{n-1} + 2a_{n-2} = 2^n$ is _____.

(A) $c_1 + c_2\, 2^n + n2^{n+1}$

(B) $c_1 - c_2\, 2^n + n2^{n+1}$

(C) $c_1 + c_2\, 2^n + 3n2^n$

(D) $c_1 + c_2\, 2^n - n2^n$

Q.88 A pennant is a sequence of numbers, each number being 1 or 2. An n-pennant is a sequence of numbers with sum equal to n. For example, $(1,1,2)$ is a 4–pennant. The set of possible 1–pennants is $\{(1)\}$, the set of all possible 2–pennants is $\{(2),(1,1)\}$ and the set of all 3–pennants is $\{(2,1),(1,1,1),(1,2)\}$.

Note that the pennant $(1,2)$ is not the same as the pennant $(2,1)$. The number of 10-pennants is _____.

[GATE 2014 : IIT Kharagpur]

Q.89 Let a_n represent the number or bit strings of length n containing two consecutive 1's. What is the recurrence relation for a_n ?

[GATE 2015 : IIT Kanpur]

(A) $a_{n-2}+a_{n-1}+2^{n-2}$

(B) $a_{n-2}+2a_{n-1}+2^{n-2}$

(C) $2a_{n-2}+a_{n-1}+2^{n-2}$

(D) $2a_{n-2}+2a_{n-1}+2^{n-2}$

Q.90 Consider the recurrence relation $a_1=8, a_n=6n^2+2n+a_{n-1}$. Let $a_{99}=K\times10^4$. The value of K is ———.

[GATE 2016 : IISc Bangalore]

Self- Practice Questions

Q.1 The minimum number of cards to be dealt from an arbitrarily shuffled deck of 52 cards to guarantee that three cards are from same suit is

(A) 3 (B) 8

(C) 9 (D) 12

[GATE 2000 : IIT Kharagpur]

Q.2 Suppose that a robot is placed on the Cartesian plane. At each step it is allowed to move either one unit up or one unit right, i.e., if it is at (i, j) then it can move to either $(i+1, j)$ or $(i+j+1)$. How many distinct paths are there for the robot to reach the point (10,10) starting from the initial position (0,0)?

(A) $\binom{20}{10}$

(B) 2^{20}

(C) 2^{10}

(D) None of the above

[GATE 2007 : IIT Kanpur]

Q.3 Let x_n denote the number of binary strings of length n that contain no consecutive 0s. The value of x_5 is

(A) 5 (B) 7

(C) 8 (D) 13

[GATE 2008 : IISc Bangalore]

Q.4 A group of 30 people have been trained as astronauts to go on the first mission to Mars. How many ways are there to select a crew of six people to go on this mission ?

Q.5 Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department?

Q.6 Find the total number of rectangles on the normal chessboard.

Q.7 How many ways are there to pack 5 copies of the same book into 3 identical boxes, where a box can contain as many as 5 books?

Q.8 How many ways are there to divide 5 Gulab Jamun among 3 siblings?

Common Data Questions 9 to 12

How many ways are there to distribute five balls into three boxes if each box must have at least one ball in it, if

Q.9 Both the balls and boxes are labeled?

Q.10 The balls are labeled, but the boxes are unlabeled?

Q.11 The balls are unlabeled, but the boxes are labeled?

Q.12 Both the balls and boxes are unlabeled?

Common Data Questions 13 to 14

A coin is flipped 10 times where each flip comes up either heads or tails.

Q.13 How many possible outcomes are there in total?

Q.14 How many possible outcomes contain exactly two heads?

Q.15 A box contains 12 Red, 7 Blue and x green balls. If the minimum number of balls we have to choose randomly from the box to guarantee that we have 6 balls of same color is 15, then $x=$_____

Q16 In how many ways 10 persons can be divided into 5 teams of 2 each?

Q.17 The solution of $a_n - 2a_{n-1} = 3(2)^n$ is___.

(A) $C_1 2^n + 3n2^n$

(B) $C_1 3^n + 2n2^{n-1}$

(C) $C_1 2^n - 3n2^{n-1}$

(D) $C_1 2^n + 2n3^{n-1}$

Q.18 Suppose 10 persons are in a canteen which offers Lassi, Tea and Mojito. How many ways they can order their drinks as a group, if each person wants one of the 3 drinks?

Q.19 Which one of the following is a closed form expression for the generating function of the sequence $\{a_n\}$, where $a_n = 2n+3$ for all $n = 0,1,2,\ldots$?

(A) $\frac{3}{(1-x)^2}$ (B) $\frac{3x}{(1-x)^2}$

(C) $\frac{2-x}{(1-x)^2}$ (D) $\frac{3-x}{(1-x)^2}$

[GATE 2018 : IIT Guwahati]

Q.20 Four dice are rolled. The number of possible outcomes in which at least one dice shows 2 is

(A) 1296 (B) 625

(C) 671 (D) 584

Q.21 Suppose that there are 100 players entered in a single elimination tennis tournament. How many matches must be conducted to declare the winner?

(A) 101 (B) 100

(C) 99 (D) 98

Q.22 On new year, all the students of a class send greeting cards to one another. If the post man delivers 1640 cards to the students of this class, then the number of students in the class is

(A) 41 (B) 37

(C) 44 (D) 36

Q.23 A veg. salad can be made using at least one of the given veggies Onion, Tomato, Cucumber, carrot and cabbage. How many varieties of salads are possible?

Q.24 If all the words created from the letters of word "SCIENCE", are arranged in a dictionary what will be the rank of word "SCIENCE".

Q.25 What are total number of ways so that we can select at least one eatable from 3 Apples, 5 Mangoes, 4 Bananas and 6 different Chocolates. Only type of fruit matters while selecting one ?

Q.26 Let a_n be the number of n-bit strings that do NOT contain two consecutive 1s. Which one of the following is the recurrence relation for a_n ?

(A) $a_n = a_{n-1} + 2a_{n-2}$

(B) $a_n = a_{n-1} + a_{n-2}$

(C) $a_n = 2a_{n-1} + a_{n-2}$

(D) $a_n = 2a_{n-1} + 2a_{n-2}$

[GATE 2016 : IISc Bangalore]

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Answer Keys

Classroom Practice Questions									
1.	4^8	2.	12	3.	666	4.	11	5.	9^n
6.	45	7.	4536, 4464	8.	B	9.	450	10.	1365
11.	D	12.	12	13.	B	14.	A	15.	A
16.	D	17.	C	18.	A	19.	$^{10}P_6$	20.	C
21.	$10! \times {}^{11}P_5$	22.	C	23.	A	24.	$2\times(n!)^2$	25.	$^{15}C_5$
26.	C	27.	9	28.	15	29.	C	30.	A
31.	176	32.	$^{10}C_5$	33.	A	34.	A	35.	$^{41}C_{12}$
36.	C	37.	A	38.	B	39.	B	40.	*
41.	D	42.	*	43.	*	44.	84	45.	10800
46.	C	47.	C	48.	B	49.	A	50.	65
51.	D	52.	48	53.	D	54.	C	55.	A
56.	8	57.	0.99	58.	36	59.	A	60.	Infinite
61.	$^{21}C_3$	62.	$^{20}C_3$	63.	$^{13}C_3$	64.	$^{21}C_3 - {}^{10}C_3$	65.	$^{23}C_3$
66.	$^{22}C_4$	67.	319968	68.	639	69.	$^{9}C_4$	70.	20
71.	3^{10}	72.	$^{12}C_2$	73.	3^{10}	74.	10!/(3!2!5!)	75.	D
76.	D	77.	A	78.	C	79.	12	80.	B
81.	10	82.	15	83.	A	84.	B	85.	A
86.	D	87.	A	88.	89	89.	A	90.	198
Self - Practice Questions									
1.	C	2.	A	3.	D	4.	$^{30}C_6$	5.	$^{9}C_3 \times {}^{11}C_4$
6.	1296	7.	5	8.	21	9.	150	10.	25
11.	6	12.	2	13.	1024	14.	45	15.	4
16.	$10!/(2!)^5$	17.	A	18.	66	19.	D	20.	C
21.	C	22.	A	23.	31	24.	1260	25.	7679
26.	B								

Classroom Practice Questions

* 40 $C(10,2) \times C(15,4) + C(10,1) \times C(15,5) + C(10,0) \times C(15,6)$

* 42 $^{39}C_{16} \times 7^{16} \times (-5)^{23}$

* 43 $2^{100} - 5051$

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