Computational Methods

unit-2

→ solving linear Equations -

(I) Guass Elimination

Let the Equations be,

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{23}y + a_{33}z = b_3$$

From above Equations -

$\boxed{\text{Step I}}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$\boxed{\text{Step II}}$ write $[A \mid B]$

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right]$$

$\boxed{\text{Step III}}$

Make $\boxed{\text{upper } \Delta^r \text{ matrix}}$

↳ matlab matrix ke main diagnol ke neeche ka part zero kra

Now, the matrix (Argumental) be

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & A_{22} & A_{23} & B_2 \\ 0 & 0 & A_{33} & B_3 \end{array}\right]$$

Now Make Equations & solve them

$A_{33}z = B_3$ (z ki value niklegi)

$A_{22}y + A_{23}z = B_2$ (y ki value niklegi)

$a_{11}x + a_{12}y + a_{13}z = B_1$ (x ki value niklegi)

(II) Guass Jordan

(1) first write Eq^n in Argumental matrix Form. (Consider the Eq^ns of Guass Elimination)

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right]$$

(2) Sirf main diagnol ko chod ke, baki sb Elements ko zero kro

(3) Equations bnao

$$\left[\begin{array}{ccc|c} A_{11} & 0 & 0 & B_1 \\ 0 & A_{22} & 0 & B_2 \\ 0 & 0 & A_{33} & B_3 \end{array}\right]$$

(4) x, y & z ki value nikalo.

$\boxed{\text{Note}}$ Sbse Pehle Matrix ko upper matrix bnana hai, matlab 1st Row ki help se 2nd & 3rd ke Elements ko zero Karna hai. then 2nd Row ki help se 1st & 2nd ko zero Karenge.

(III) Pivoting

1. partial pivoting (By Guass Elimination)

Let us consider,

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{23}y + a_{33}z = b_3$$

(1) write in the form of an argumental matrix. $[A|B]$

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right]$$

(2) ab sbse pehle a_{11}, a_{21}, a_{31} mein se jiski largest value hogi, usko 1st Row Rakhenge (matlab Row interchange Karenge)

(3) 1st Row ke help se ~~a~~ 2nd & 3rd Row ke 1st Element ko 0 Karenge.

~~Ex~~.

$$\begin{bmatrix} ae_1 & ae_2 & ae_3 & | & be_4 \\ 0 & ae_6 & ae_7 & | & be_8 \\ 0 & ae_{10} & ae_{11} & | & be_{12} \end{bmatrix}$$

any Element (jo sbse Bda hoga)

~~isko zero karenge~~

~~(4) ae10 ko~~

(4) ab 2nd coloumn mein, ae_2, ae_6, ae_{10} mein se jo sbse bada Element hoga usko hum 2nd Row mein Rakhenge (Row transformation apply karke.

{For Exp - If ae_{10} is largest then, apply

$R_3 \longleftrightarrow R_2$ }

Or then 2nd Row ke 2nd Element (ae_{10}) ke help se 3rd Row ke 2nd Element ko zero karenge (Exp ke hissab se dekhe to)

After interchanging, the matrix will be

$$\left[\begin{array}{ccc|c} ae_1 & ae_2 & ae_3 & be_4 \\ 0 & ae_{10} & ae_{11} & be_{12} \\ 0 & ae_6 & ae_7 & be_8 \end{array}\right]$$

Make it zero with help of ae_{10}

Now, the Matrix will be,

$$\left[\begin{array}{ccc|c} ae_1 & ae_2 & ae_3 & be_4 \\ 0 & ae_{10} & ae_{11} & be_{12} \\ 0 & 0 & ce_7 & ce_8 \end{array}\right]$$

(5) Now make Equations,

$ce_7 z = ce_8$ (z ki value nikalo

$ae_{10} y + ae_{11} z = be_{12}$ (y ki value nikalo

$ae_1 x + ae_2 y + ae_3 z = be_4$ (x ki value nikalo

(IV) LU Factorisation (Crout's method)

Let the Equations be,

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{32}y + a_{33}z = b_3$$

(1) Write A & B matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

(2) Break down the matrix A in the form of LU, Lower & upper Matrix

$$A = LU$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

(3) Now, let $AX = B$ & $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

write A as LU

$$L\underbrace{UX}_{\substack{\text{Let} \\ UX = V}} = B \qquad V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$LV = B$$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

(4) Make Equations & find the value of v_1, v_2, & v_3.

(5) $UX = V$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}}_{\text{found in above step}}$$

Make Eqn's & find value of x & y & z.

(V) **Cholesky's method** - (LU ache se krna hai iske lie)

$$A = LL^T$$

$$L = \begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \quad L^T = \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$$

Baaki same LU ki trh solve hoega.

$$A = LL^T$$

Let $AX = B$ $\quad$ & $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$LL^T X = B$$

$$LY = B \quad \& \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

(y ki value find karo)

$$L^T X = Y$$

↳ x is value find karo !!

Khtm baat

(Easy Peasy)

(VI) Guass Seidal method -

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

Seedhe $\boxed{\text{formula}}$ lagana hota hai

(complicated hai thoda)

The formula is

$$x = \underbrace{\frac{1}{a_1}}_{x \text{ ka cofficient}} (\underbrace{d_1}_{\text{Eq}^n \text{ ke is Equal to mein jo hai}} - \overbrace{b_1 y - c_1 z}^{-\text{ve kro baki bache 2 ko}})$$

$$y = \frac{1}{b_2}(d_2 - a_2 x - c_2 z)$$

$$z = \frac{1}{c_3}(d_3 - a_3 x - b_3 z)$$

$\boxed{\text{Ruko}}$, abhi Kaha ja rhe ho.

Picture abhi baaki hai dost !!

yeh jo upr Eqn hai inko aise arrange Karna ki

1st Eqn mein x ka cofficient greater ho.

2nd Eqn mein y ka,

3rd Eqn mein z ka.

Let's take an Exp.

Greatest $20x + 13y + 14z = 19 \rightarrow$ ①

Greatest

$12x + 15y + 22z = 16 \rightarrow$ ③

$5x + 19y + 21z = 15 \rightarrow$ ②

Greatest

$20 > 12 > 5$

$19 > 15 > 13$

$22 > 21 > 14$

abhi bhi twist bacha hai !!

Formula mein jab x_0 ki value nikalenge to y_0 & z_0 ko 0 Rakhenge or jab y_1 ki value nikalenge to z_0 ko 0 Rakhenge & x ko x_1 se Replace Karenge & when x_2 ki value nikalenge to y_1 & z_1 ki value Rakhenge 0 nhi Rakhenge

{1 min Ruko, Ques dekhte hue Padho smj ayega}

x, y, z ki value tab tk find Karenge jab tk decimal ke Baad wali values Repeat nhi hoengi.

(VII) Eigen values & Eigen vectors {Power method}

$$AX = \lambda X \quad \text{(Recursive)}$$

largest = $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Dominant = $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

Step 1 Sbse pehle initial Eigen vector ko let karenge acc to order of Matrix & ~~if~~ (dominant & largest) acc to type.

Step 2 Recursively AX^0, AX^1, AX^2 find karenge

$$AX^0 = \underbrace{[\text{Matrix}][\text{jo Eigen vector let kia hai}]}_{\text{multiply karenge}}$$

$= [\quad]$ (Numerically jo value greater hogi common lenge

$= \lambda_1 [\quad]$

↳ usko λ_1 bolenge (jo common lenge)

or aise hi λ ki value ko find karenge jb tk Decimal ke Baad Repeat nhi hoti.

Matrix Inversion

(I) Guass Elimination method -

1. Given Matrix ko argumental matrix ki form mein likho $[\underbrace{A}_{\text{Given Matrix}} | I]$
2. A matrix ko upper Δ'r matrix bnao {matlab jo ~~too~~ main diagnol hai Matrix ka uske neeche ke elements ko 0 karo
3. I ki jagah jo matrix ayegi operations lagane ke Baad vo matrix ko inverse hogi

(II) Guass Jordan

~~Diagnol~~

1. argumental form mein likho $[A|I]$
2. Diagnol Elements ko 1 karo, and Baaki ko zero.
3. I ki jagah jo matrix hogi usko A^{-1} bolenge.

(Easy Peasy)

(III) LU factorisation Method -

(1) Given matrix ko LU mei decompose karenge

$$A = LU$$
$$A^{-1} = (LU)^{-1}$$
$$A^{-1} = U^{-1}L^{-1}$$

(2) Let $\boxed{L^{-1} = X}$

$$LL^{-1} = I$$
$$\boxed{LX = I}$$

{L put karke X matrix found karenge}

Let $U^{-1} = Y$

$$UU^{-1} = I$$
$$\boxed{UY = I}$$

{U put karke Y ko matrix find karenge}

$$\boxed{A^{-1} = \cancel{Y}X}$$

IV Iterative method

$$AB = E + I$$
$$(AB)^{-1} = (E+I)^{-1}$$

↓ Binomial lagao.

$$B^{-1}A^{-1} = \cancel{B}(1 - E + E^2 + \underbrace{\cdots\cdots}_{})$$

$$\boxed{A^{-1} = B(1 - E + E^2)}$$

error chota hota hai islie neglect karenge

A bhi Given hoga.
B bhi Given hoga.
E nikalenge
($AB = E+I$) formula use krke.

Unit-3 (Formula's)

Numerical Differentiation

First order

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i} \quad \text{[Forward]}$$

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}} \quad \text{[Backward]}$$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{x_{i+1} - x_{i-1}} \quad \text{[central]}$$

Second order

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2} \quad \text{[forward]}$$

$$f''(x_i) = \frac{f(x_i) - 2f(x_{i-1}) + \cancel{2}f(x_{i-2})}{h^2} \quad \text{[Backward]}$$

$$f''(x_i) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2} \quad \text{[central]}$$

High accuracy formula (first order)

$$f'(x_i) = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h} \quad \text{[Forward]}$$

$$f'(x_i) = \frac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{2h} \quad \text{[Backward]}$$

$$f'(x_i) = \frac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})}{2h} \quad \text{[Central]}$$

Derivative of unequal spaced Data :-

$$f'(x) = \frac{2x - x_i - x_{i+1}}{(x_{i-1} - x_i)(x_{i-1} - x_{i+1})} f(x_{i-1}) + \frac{2x - x_{i-1} - x_{i+1}}{(x_i - x_{i-1})(x_i - x_{i+1})} f(x_i) + \frac{2x - x_{i-1} - x_i}{(x_{i+1} - x_{i-1})(x_{i+1} - x_i)} f(x_{i+1})$$

Richardson Extrapolation

$$D^K(h) = \frac{4^K D^{K-1}(h/2) - D^{K-1}(h)}{4^K - 1} \quad \text{(for central)}$$

$$D^K(h) = \frac{2^K D^{K-1}(h/2) - D^{K-1}(h)}{2^K - 1} \quad \text{(for Backward \& forward)}$$

Numerical Integration

Trapezoidal Rule -

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})\right]$$

Simpson's 1/3 Rule -

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{3}\left[y_0 + y_n + 4(y_1 + y_3 + y_5 + \cdots) + 2(y_2 + y_4 + \cdots)\right]$$

Simpson's 2/3 Rule -

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{3h}{8}\left[y_0 + y_n + 3(y_1 + y_2 + y_4 + y_5 + \cdots) + 2(y_3 + y_6 + y_9 + \cdots)\right]$$

unequal spaced

$$I = \frac{h_1}{2}(f(x_0) + f(x_1)) + \frac{h_2}{2}(f(x_1) + f(x_2)) + \frac{h_3}{2}(f(x_2) + f(x_3)) + \cdots$$

$h_1 = x_1 - x_0$; $h_2 = x_2 - x_1$; $h_3 = x_3 - x_2$

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Romberg's method

h	I_1		
		$I_1^* = I_2 + \frac{1}{3}(I_2 - I_1)$	
$h/2$	I_2		$I_1^{**} = I_2^* + \frac{1}{3}[I_2^* - I_1^*]$
		$I_2^* = I_3 + \frac{1}{3}(I_3 - I_2)$	
$h/4$	I_3		

Gussian Quadrature

(i) one point formula

$$\int_{-1}^{1} f(x)\,dx = 2f(0)$$

(ii) 2 point formula.

$$\int_{-1}^{1} f(x)\,dx = f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

(iii) 3 point formula

$$\int_{-1}^{1} f(x)\,dx = \frac{8}{9}f(0) + \frac{5}{9}\left[f\left(-\sqrt{\frac{3}{5}}\right) + f\left(\sqrt{\frac{3}{5}}\right)\right]$$

$$\boxed{x = \frac{1}{2}(b-a)t + \frac{1}{2}(b+a)}$$

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unit-4

Runge-Kutta method

$$\frac{dy}{dx} = f(x, y) \qquad y(x_0) = y_0$$

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$\boxed{y_1 = y_0 + k}$$

Finite Difference method

$$y'' + \lambda(x)y' + \mu(x)y = Y(x) \qquad y(x_0) = a \quad y(x_n) = b$$

$$y_i' = \frac{1}{2h}[y_{i+1} - y_{i-1}]$$

$$y_i'' = \frac{1}{h^2}[y_{i+1} - 2y_i + y_{i-1}]$$

$$y_i''' = \frac{1}{2h^3}[y_{i+2} - 2y_{i+} + 2y_{i-1} - y_{i-2}]$$

$$y_i^{iv} = \frac{1}{h^4}[y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}]$$

Numerical solution

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + Fu = G.$$

$\Delta S = B^2 - 4AC < 0$ (Elliptic)

$\Delta S = B^2 - 4AC = 0$ (Parabolic)

$\Delta S = B^2 - 4AC > 0$ (Hyperbolic)

Formula's

unit-2

Interpolation

Linear -

$$S_i(x) = y_i + \frac{y_{i+1} - y_i}{x_{i+1} - x_i}(x - x_i)$$

Quadratic -

$$S_i(x) = \frac{z_{i+1} - z_i}{2(x_{i+1} - x_i)}(x - x_i)^2 + z_i(x - x_i) + y_i$$

$$z_{i+1} = -z_i + 2\left(\frac{y_{i+1} - y_i}{x_{i+1} - x_i}\right)$$

$z_0 = 0$

cubic -

$$S_i(x) = \frac{(x_i - x)^3}{6h}M_{i-1} + \frac{(x - x_{i-1})^3}{6h}M_i + \frac{(x_i - x)}{h}\left(y_{i-1} - \frac{h^2}{6}M_{i-1}\right)$$

$$+ \frac{(x - x_{i-1})}{h}\left(y_i - \frac{h^2}{6}M_i\right)$$

where

$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right]$$

$$\boxed{M_0 = M_n = 0}$$

Inverse finding Method

↳ Guass Elimination.

↳ Guass Jordan.

↳ LU $\{L^{-1} = X\}\ \{U^{-1} = Y\}$

↳ Iterative Method. $\boxed{\begin{array}{c} A^{-1} = B(1 - E + E^2) \\ E = AB - I \end{array}}$

For finding Solution of linear Eq^n.

- Guass Elimination
- Guass Jordan.
- Pivoting
- LU Factorisation. $A = LU$; $LUX = B$; $UX = Y$.
- Cholskey $A = LL^T$; $LL^TX = B$; $L^TX = Y$
- Guass seidal
- Eigen values & Eigen vector.

UNIT - 1

Solution of Algebric & transcedental Equation.

(I) Bisection (Bolzano): $x_n = \frac{a+b}{2}$. $\{f(a)f(b) < 0\}$

order of convergence = 1.

(II) Regula falsi:

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$ $\{f(x_0) \cdot f(x_1) < 0\}$

order of convergence = 1.6.

(III) Newton Raphson:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

Order of convergence = 2.

(IV) Secant Method:

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}.$$

order of Convergence = 1.6.

(V) Fixed point / Iterative

~~f(x) = ϕx~~ $x = \phi(x)$

order of convergence = 1.

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Significant Digits

↳ ER no. ke vo digits jo uski accuracy or precision ko denote karte hai

include -

1 Non-zero Digits (1, 2, 3 . . .)

2. Beech mein zero (102 → 3 significant)

Ex

Number	Significant Digits
123.45	5
0.00456	3
100	1
100.0	4.

Rounding off Rules -

(1) agr rounding ke lie selected digit ke Baad wala digit (5 to 9) hai to selected digit mei 1 increase krdo

(2) agr 0, 1, 2, 3, 4 hai to Same Rehne do.

(3) Exactly 5 aur uske Baad ke sb 0 hai to nearest Even bna do.

Ex → 2.5 = 2 ; 3.5 = 4

Ex

47.856 tenths place tk.

$\boxed{47.9}$ ↳ ~~next~~ digit 5 to ~~or usse next~~ 6 (Rul 1 lagega)

Ex

1234.567 Hundeth.

↳ 6 ke Baad 7 hai Rule 1

1234.57.

Error

	Definition	Formula
Absolute	measured or actual ke beech ka difference mod mei	| Measured - Actual |
Relative	absolute Error ko actual se Divide, Porportion dikhata h	$\frac{\text{Absolute}}{\text{True}}$
% Error	Relative ko 100 ke multiply	Relative × 100
Truncation	jab ER infinite series ya calculation ko finite terms ya digit tk calculate kate hai	|Actual - Truncated value| ~~Measured~~

Algebric - jo sirf polynomials & cofficients ko involve karti hai, yeh Expression ka uske karti hai jisme variables & constants shamile hote hai

Ex → $3x^2 + 2x - 5 = 0$

Transcendental - algebric ke alawa transcendental functions ko involve karti hai, wo hote hai jo algebric se derive hote hai jais→ Exponential, logarithmic, trigonometric

Aspect	algebric	transcendental
Defition	Polynomial & cofficient	functions transcental
function	Polynomial	Exponential, logartmic, trigno
complexity	simple & Easy	Complex & iterative
Can be solved	factorisation, Quadratic formula	fixed point iteration

Numerical algorithms - woh step by step computational procedures hote hai, jo numerical problems ke approximate solution ke sath solve karne ke lie design kie gaye hote hai yeh Mathematical operators ka use karte hai → iteration, interpolation, differentiation, integration. Precise Result achieve karne ki koshish karte hai.

Features

- Approximation (approximate solution)
- Iteration (Repeated step)
- Error analysis (analyze error & try to minimize)
- Efficiency (kam computational Resources)

complexity - computational Resources Quantify karne ke lie jaise ke time & space jo algorithm ko Execute karne mei lagte hai

complexity

Time (complete hone mei lagne wala time measure)

space (Execute hone ke lie Required memory ka measure)

Computer implementation - numerical algorithms ko actual Programming languages me likhna or unhe computer pe run karna.
algo ko code mei translate karna, Data structures ko select karna, aur Performance optimization

Efficiency - algorithm kitna Effectively resources ka use karta hai.
Factors - time, space, algorithm, Data structure

Bracketing methods - woh numerical technique jo roots ke approximation karne ke lie use hoti hai,
yeh Gurantee deta hai ke agar kisi interval mei sign change hota hai, to usme root Exist karta hai

open method - numerical technique jo roots ko approximate karne ke lie single initial Guess ya multiple initial guess ka use karte hai, Bina interval ke

	Bracketing	open
Initial Requirements	Interval with sign change	one or two initial Guess
Converge	Guranteed if interval is valid	Not Guranteed depends on initial Guess
speed	Slower	Faster
Ex	Bisection, False Position	Newton Raphson, Secant

Bisection method-

↳ Ek iterative method hai jo continuous function ke roots ka approximate karne ke lie use hota, hai isme interval ko repeatedly half kia jata hai or subinterval ko select kia jata hai

$$\boxed{x = \frac{a+b}{2}}$$

Newton-Raphson -

↳ Ek iterative method hai jo functions ke roots ko approximate karne ke lie, use hot hai, tangent line use karke root ki taraf converge kiya jata hai.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Secant method -

↳ Newton Raphson ka alternative hai, or isme derivative ki jagah function ke 2 near by points ka gradient use hota hai.

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

Regula Farsi -

interval halving technique ka variation, function ke ek interval mei root ko approximate ke lie istemal hota h

Pivoting - refers to rearranging rows of a matrix for place a better candidate element (called pivot) in a position jo lead karegi more stable & accurate solution ko.

Pivoting

Partial

isme matrix ke rows swap kie jate hai, taki current pivot element ka magnitude highest ho.

Complete

Isme Rows & columns dono ko swap kiya jata hai taaki matrix ke sabhi element ke lie better numerical stability ho.

Interpolation - Ek method hai jo known data points ke beech mein values ko Estimate karne ke lie use hota hai, isme Ek function construct karte hai, jo Given data points pe fit hota hai aur points ke beech mei value predict karta hai

Extrapolation - Ek method jo known data points ke bahar ko values ko Estimate karta hai. Function use karte hai jo Existing Data trend ko follow karke Range ke bahar value predict karega

Interpolation	Extrapolation
Reliable	less Reliable
Produce smooth curves	Trend Based, may not be smooth
within Given Range	outside data points
Datafitting	forecasting predicting future trends

$\boxed{\text{Spline}}$ - Smooth curves ko Repersent karne ke lie use hota hai, Piecewise polynomials hote hai jo Data points ko smoothly connect karte hai

↳ multiple polynomial segments within specific interval

smoothly → transition

Type

- Linear (simplest)
- Quadratic
- Cubic — Provid Good balance

FORMULAS

unit - 1

(I) Bisection method [OOC = 1]

$$x = \frac{a+b}{2} \qquad f(a)f(b) < 0 \ [-ve]$$

(II) Regula falsi / False position [OOC = 1.618]

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} \qquad f(x_1)f(x_0) < 0$$

(III) Newton Raphson's [OOC = 2]

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

(IV) Secant method [OOC = 1.618]

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

(V) Fixed point iteration [OOC = 1]

$$x = \phi(x)$$

$$\boxed{x_{n+1} = \phi(x_n)}$$

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unit - II

Solving Linear Equation → Guass Elimination
→ Guass Jordan
→ Pivoting
→ LU
→ Cholskey
→ Eigen values & vectors
→ Guass Seidial.

Solution of Non linear Equation

Finding Inverse → Guass Elimination, Jordan
→ LU
→ Iterative Method

Interpolation → Lagrange
→ Spline / piecewise — Linear, Quadratic, cubic

Lagrange Interpolation -

$$f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} + \cdots + \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x-x_{n-1})}$$

Spline Interpolation -

$\boxed{\text{Linear}}$ -

$$S_i(x) = y_i + \frac{y_{i+1}-y_i}{x_{i+1}-x_i}(x-x_i)$$

$\boxed{\text{Quadratic}}$ -

$$S_i(x) = \frac{(z_{i+1}-z_i)(x-x_i)^2}{2(x_{i+1}-x_i)} + z_i(x-x_i) + y_i$$

where $z_{i+1} = -z_i + 2\left(\frac{y_{i+1}-y_i}{x_{i+1}-x_i}\right)$ $\boxed{z_0 = 0}$

$\boxed{\text{Cubic}}$ -

$$S_i(x) = \frac{(x_i - x)^3 M_{i-1}}{6h} + \frac{(x-x_{i-1})^3 M_i}{6h} + \frac{(x_i-x)}{h}\left(y_{i-1} - \frac{h^2 M_{i-1}}{6}\right) + \frac{(x-x_{i-1})}{h}\left(y_i - \frac{h^2}{6}M_i\right)$$

where, $M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}[y_{i-1} - 2y_i + y_{i+1}]$ $\boxed{M_0 = M_n = 0}$

Newton Interpolation Formula -

Forward -

$$f(a+hu) = f(a) + u\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \dots + \frac{u(u-1)(u-2)\dots(u-n+1)}{n!}\Delta^n f(a)$$

Backward -

$$f(a+hu) = f(a) + u\nabla f(a) + \frac{u(u+1)}{2!}\nabla^2 f(a) + \dots + \frac{u(u+1)(u+2)\dots(u-n+1)}{n!}\nabla^n f(a)$$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
$x-2h$	y_{-2}				
$x-h$	y_{-1}	Δy_{-2}	$\Delta^2 y_{-2}$		
x	y_0	Δy_{-1}	$\Delta^2 y_{-1}$	$\Delta^3 y_{-2}$	Backward
$x+h$	y_1	Δy_0		$\Delta^3 y_{-1}$	$\Delta^4 y_{-2}$
$x+2h$	y_2	Δy_1	$\Delta^2 y_0$		Forward

Guass Forward -

$$f(a+hu) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_{-1} + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_{-1} + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_{-2} + \dots$$

Guass Backward -

$$f(a+hu) = y_0 + \frac{u}{1!}\Delta y_{-1} + \frac{u(u+1)}{2!}\Delta^2 y_{-1} + \frac{u(u+1)(u+2)}{3!}\Delta^3 y_{-2} + \frac{u(u+1)(u+2)(u+3)}{4!}\Delta^4 y_{-2} + \dots$$

Stirling formula (Forward + Backward)

$$f(a+hu) = y_0 + \frac{u}{1!}\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{u^2}{2!}\Delta^2 y_{-1} + \frac{u(u^2-1)}{3!}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) + \dots$$

unit - 3

For I^{st} order derivative

1) forward $f'(x_i) = \dfrac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$

2) Backward $f'(x_i) = \dfrac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$

3) central $f'(x_i) = \dfrac{f(x_{i+1}) - f(x_{i-1})}{x_{i+1} - x_{i-1}}$

For 2^{nd} order Derivative

1) forward $f''(x_i) = \dfrac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2}$

2) Backward $f''(x_i) = \dfrac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^2}$

3) central $f''(x_i) = \dfrac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$

High accuracy or 3 point - for first order

1) Forward = $f'(x_i) = \dfrac{-f(x_{i+2}) + 4f(x_{i-1}) - 3f(x_i)}{2h}$

2) Backward = $f'(x_i) = \dfrac{+3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{2h}$

3) Central = $f'(x_i) = \dfrac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})}{12h}$

unequal Derivate

$$f'(x) = \frac{(2x - x_i - x_{i+1})f(x_{i-1})}{(x_{i-1} - x_i)(x_{i-1} - x_{i+1})} + \frac{(2x - x_{i-1} - x_{i+1})f(x_i)}{(x_i - x_{i-1})(x_i - x_{i+1})}$$

$$+ \frac{(2x - x_{i-1} - x_i)}{(x_{i+1} - x_{i-1})(x_{i+1} - x_i)} \times f(x_{i+1})$$

**

Richardson Extrapolation

$$D^K(h) = \frac{4^K D^{K-1}(h/2) - D^{K-1}(h)}{4^K - 1} \quad \text{(for central)}$$

$$D^K(h) = \frac{2^K D^{K-1}(h/2) - D^{K-1}(h)}{2^K - 1} \quad \text{(for Backward \& Forward)}$$

Derivative formula

$$f'(x) = \frac{f(x+h) - f(x)}{h} \quad \text{(forward)}$$

$$f'(x) = \frac{f(x) - f(x-h)}{h} \quad \text{(Backward)}$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} \quad \text{(central}$$

Numerical Integration

Trapezoidal

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{2}\left[y_0 + y_n + 2[y_1 + y_2 + \cdots y_{n-1}]\right]$$

Simpson 1/3 Rule

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{3}\left[y_0 + y_n + \underbrace{4(y_1+y_3+y_5+\cdots)}_{\text{odd terms}} + \underbrace{2(y_2+y_4+\cdots)}_{\text{Even}}\right]$$

* only applicable if the interval is in Even number.

Simpson 3/8 Rule

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{3h}{8}\left[y_0 + y_n + \underbrace{3(y_1+y_2+y_5+y_4+\cdots)}_{\text{non three multiple}} + \underbrace{2(y_3+y_6+\cdots)}_{\text{3 multiple terms}}\right]$$

* Only applicable if there are interval in multiple of 3.

...equal segment

$$I = \frac{h_1}{2}(f(x_0)+f(x_1)) + \frac{h_2}{2}(f(x_1)+f(x_2)) + \dots + \frac{h_n}{2}(f(x_{n-1})+f(x_n))$$

$h_1 = x_1 - x_0$; $h_2 = x_2 - x_1$ $h_n = x_n - x_{n-1}$

Romberg's

* First use Trapezoidal
* For table use : $I_1^* = I_2 + \frac{1}{3}[I_2^* - I_1]$

$*$ → Shows the no. of / degree of Integration.

Gussian Quadrature Formula -

* only applicable for $\int_{-1}^{1} f(x)dx$ *

$\boxed{\text{1-point}}$ -

$$\int_{-1}^{1} f(x)dx = 2f(0)$$

$\boxed{\text{2-point}}$ -

$$\int_{-1}^{1} f(x)dx = f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

$\boxed{\text{3-point}}$ -

$$\int_{-1}^{1} f(x)dx = \frac{8}{9}f(0) + \frac{5}{9}\left[f\left(-\frac{\sqrt{3}}{\sqrt{5}}\right) + f\left(\frac{\sqrt{3}}{\sqrt{5}}\right)\right]$$

If Limit will no be $(-1,1)$ then use $\boxed{x = \frac{1}{2}(b-a)t + \frac{1}{2}(b+a)}$

$\boxed{\text{Boole's Law}}$ -

$$\int_{x_0}^{x_n} f(x)dx = \frac{2h}{45}[7(y_0) + 32(y_1+y_3+y_5+\dots) + 12(y_2+y_6+\dots) + 14(y_4+y_8+\dots)]$$

Intervals → no. of 4 (multiple)

$\boxed{\text{Weddle's Rule}}$ -

$$\int_{x_0}^{x_n} f(x)dx = \frac{3h}{10}[y_0 + 5(y_1+y_5+y_{11}+\dots) + y_2+y_4+y_8+\dots + 6(y_3+y_9+\dots) + 2(y_6+y_{12}+\dots)]$$

unit - 4

RK (Range Kutta) Method -

$$\frac{dy}{dx} = f(x, y) \qquad \text{Intial condition} \quad y(x_0) = y_0$$

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf\left(x_0 + h/_2,\ y_0 + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_0 + h/_2,\ y_0 + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_0 + h,\ y_0 + k_3)$$

$$k = \frac{1}{6}(k_1 + 2k_2 + k_3 + k_4)$$

update y values as $= y_0 + k$.

Finite Differences

$$y_i' = \frac{1}{2h}[y_{i+1} - y_{i-1}]$$

$$y_i'' = \frac{1}{h^2}[y_{i+1} - 2y_i + y_{i-1}]$$

$$y_i''' = \frac{1}{2h^3}[y_{i+2} - 2y_{i+1} + 2y_{i-1} - y_{i-2}]$$

$$y_i^{iv} = \frac{1}{h^4}[y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i+2}]$$

PDE - Partial Differential equation -

$$A\frac{\partial^2 U}{\partial x^2} + \frac{B\partial^2 U}{\partial x \partial y} + C\frac{\partial^2 U}{\partial y^2} + D\frac{\partial U}{\partial x} + E\frac{\partial U}{\partial y} + FU = G.$$

the $B^2 - 4AC > 0$ (Hyperbolic)

$B^2 - 4AC = 0$ (Parabolic)

$B^2 - 4AC < 0$ (Elliptic)

Formulas

Partial Differentiation

$x_i = x_0 + ih \qquad y_i = y_0 + jk$

$$\left(\frac{\partial f}{\partial x}\right)_{(x_i, y_j)} = \frac{f_{(i+h, j)} - f_{(i,j)}}{h} \quad \text{(Forward)}$$

$$= \frac{f_{(i,j)} - f_{(i-h,j)}}{h} \quad \text{(Backward)}$$

$$= \frac{f_{(i+h,j)} - f_{(i-h,j)}}{2h} \quad \text{(central)}$$

$$\left(\frac{\partial f}{\partial y}\right)_{(x_i, y_i)} = \frac{f_{(i, j+k)} - f_{(i,j)}}{h} \quad \text{(forward)}$$

$$= \frac{f_{(i,j)} - f_{(i,j-k)}}{h} \quad \text{(Backward)}$$

$$= \frac{f_{(i,j+k)} - f_{(i,j-k)}}{2h} \quad \text{(central)}$$

$$\left(\frac{\partial^2 f}{\partial x^2}\right)_{(x_i, y_i)} = \frac{1}{h^2}\left[f_{(i-h,j)} - 2f_{(i,j)} + f_{(i+h,j)}\right]$$

$$\left(\frac{\partial^2 f}{\partial y^2}\right)_{(x_i, y_i)} = \frac{1}{k^2}\left[f_{(i,j-k)} - 2f_{(i,j)} + f_{(i,j+k)}\right]$$

$$\left(\frac{\partial^2 f}{\partial x \partial y}\right)_{(x_i, y_i)} = \frac{f_{(i+h, j+k)} - f_{(i-h, j+k)} - f_{(i+h, j-k)} + f_{(i-h, j-k)}}{2hk}$$