TOPIC computational methods .... DATE ....

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B.tech AI & DS {Section- S11}

Assignment - 1.

Ques1 What is an algorithm? Explain two types of complexities of an algorithm

Algorithm - is a finite step by step list of well define instructions for solving a particular problem.

Complexities of an algorithm

Time	Space
The time complexities is defined as the process of determining a formula for total time Required towards Execution of that algorithm.	The space complexity is defined as the process of defining for prediction of How much memory space is Required for successful Execution of algorithm.

Ques2 write an algorithm for Bisection Method?

algorithm for Bisection method -

(1) For continous function $f(x)$, take interval $[a, b]$ such that $f(a) f(b) < 0$.

(2) Set $x_0 = a$ $x_1 = b$. & calculate $f(a)$ & $f(b)$

(3) Repeat until accuracy achieved -

(a) calculate midpoint $x_{n+1} = \frac{x_{n-1} + x_n}{2}$

(b) calculat $f(x_{n+1})$.

(c) If $f(x_{n+1}) = 0$ then x_{n+1} is the root.

(d) If $f(x_n) \cdot f(x_{n+1}) < 0$ set $x_{n+1} = x_n$.

then there Exisit a root between them.

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(d) Else if $f(x_{n-1}).f(x_{n+1}) < 0$ there is Root between them.

(4) End

Ques3 Find a Real root of the Equation $x\log_{10}x = 1.2$ by Regula falsi method correct to three places of Decimal.

$$x\log_{10}x = 1.2$$

$$f(x) = x\log_{10}x - 1.2$$

$$f(1) = -1.2 \ (-ve)$$

$$f(2) = -0.59794 \ (-ve)$$

$$f(3) = 0.23136 \ (+ve)$$

So, Real Root lies between 2 & 3.

$$x_0 = 2 \ ; \ x_1 = 3$$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{2(0.23136) - 3(-0.59794)}{0.23136 - (-0.59794)}$$

$$\boxed{\begin{aligned} x_2 &= 2.721011 \\ f(x_2) &= -0.01709 \end{aligned}}$$

$$x_3 = \frac{3(-0.01709) - 2.72101(0.23136)}{-0.01709 - 0.23136}$$

$$\boxed{\begin{aligned} x_3 &= 2.73946 \\ f(x_3) &= -0.001034 \end{aligned}}$$

$$x_4 = \frac{2.72101 \times (-0.001034) - 2.73946(-0.01709)}{-0.001034 + 0.01709}$$

$$\boxed{\begin{aligned} x_4 &= 2.74019 \\ f(x_4) &= -0.0003977 \end{aligned}}$$

$$x_5 = \frac{2.73946(-0.0003977) - (2.74019)(-0.001034)}{0.0006363}$$

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$$\boxed{\begin{aligned} x_5 &= 2.74131 \\ f(x_5) &= 0.0005790 \end{aligned}}$$

$$x_6 = \frac{2.74019 \times 0.0005790 - 2.74131(-0.0003977)}{0.0005790 + 0.0003977}$$

$$\boxed{\begin{aligned} x_6 &= 2.74035 \\ f(x_6) &= -0.0002582 \end{aligned}}$$

$$x_7 = \frac{2.74131 \times (-0.0002582) - 2.74035(0.0005790)}{-0.0002582 - 0.0005790}$$

$$\boxed{x_7 = 2.74066}$$

Hence the Real root of the equation is 2.74066.

Ques 4 Define the order of convergence? Also find the order of convergence of Newton Raphson method.

Order of convergence -

Numerical analysis used to describe the Rate at which an iterative numerical method approaches. It provides the information about how quickly the method's Error decrease with each iteration.

$$e_{i+1} \leq e_i^{\,p} K$$

$$e_i = x_i - \alpha$$

α → Actual.

x_i → Approximated

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Order of convergence for Newton Raphson's method -

Let α be the actual root of Equation $f(x)=0$ & e_{i-1} is the Error

then $x_{i-1}-\alpha = e_{i-1}$

$$x_i - \alpha = e_i \rightarrow (1)$$

$$\boxed{x_{i+1} = e_{i+1}+\alpha} \rightarrow (2)$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \rightarrow (3)$$

Put (1) & (2) in (3)

$$e_{i+1} + \cancel{\alpha} = e_i + \cancel{\alpha} - \frac{f(x_i)}{f'(x_i)}$$

$$e_{i+1} = e_i - \frac{f(e_i+\alpha)}{f'(e_i+\alpha)}$$

$$e_{i+1} = e_i - \frac{[\overset{0}{f(\alpha)} + e_i f'(\alpha) + \cdots]}{[f'(\alpha) + e_i f''(\alpha) + \cdots]}$$

$$e_{i+1} = e_i - \frac{e_i f'(\alpha)}{f'(\alpha)+e_i f''(\alpha)}$$

$$e_{i+1} = \frac{e_i \cancel{f'(\alpha)} + e_i^2 f''(\alpha) - e_i \cancel{f'(\alpha)}}{f'(\alpha) + e_i f''(\alpha)}$$

$$e_{i+1} = e_i^2 \frac{f''(\alpha)}{(f'(\alpha)+e_i f''(\alpha))}$$

$$\boxed{e_{i+1} \doteq e_i^2 K}$$

$$\frac{e_{i+1}}{e_i^2} = K$$

$$\boxed{p=2}$$

Hence the order of convergence is 2.

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Ques5 Find a Real root of the Equation $\cos x = 3x-1$ using fixed point iteration method correct to two places of Decimal

$\cos x = 3x - 1$

$f(x) = \cos x - 3x + 1$

$f(0) = 2 \ (+ve)$

$f(1) = -1.00015 \ (-ve)$

$x_0 = (1+0)/2 = 0.5$

$$\phi(x) = \frac{\cos x + 1}{3}$$

$$\phi'(x) = \frac{-\sin x}{3}$$

$\phi'(0.5) = -0.00290884$ $\quad |(\phi' x)| < 1$

$x_0 = 0.5$

$x_1 = \frac{1}{3}(\cos(0.5)+1) = 0.6259.$

$x_2 = \frac{1}{3}(\cos(0.6259)+1) = 0.6035.$

$x_3 = \frac{1}{3}(\cos(0.6035)+1) = 0.6078$

$x_4 = \frac{1}{3}(\cos(0.6078)+1) = 0.6070$

Hence the Real root of the Equation is 0.6070.

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Ques6 Find the iterative formula for $\sqrt[3]{N}$ using Newton Raphson's Method, and hence evaluate $\sqrt[3]{41}$ correct to three decimal places.

$$x = \sqrt[3]{N}$$

$$x^3 = N$$

$$x^3 - N = 0$$

$$f(x) = x^3 - N$$

Iterative formula for $\sqrt[3]{N}$.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{(x_3^3 - N)}{3x_n^2}$$

$$= \frac{3x_n^3 - x_3^3 + N}{3x_n^2}$$

$$\boxed{x_{n+1} = \frac{2x_n^2 + N}{3x_n^2}} \rightarrow \text{Iterative Formula.}$$

$\boxed{N = 41}$

$f(1) = -40$ $f(3) = -14$

$f(2) = -33$ $f(4) = 23$

So, the Root lies between 3 & 4

So, $x_0 = 3$

$$x_1 = \frac{2(3)^3 + 41}{3(3)^2} = 3.518518$$

$$x_2 = \frac{128.1182869}{37.139906} = 3.449612$$

$$x_3 = \frac{2(3.449612)^3 + 41}{3(3.449612)^2} = 3.44821$$

$$x_4 = \frac{2.(3.44821)^3 + 41}{3(3.44821)^2} = 3.44851$$

So, the Root (Real) is 3.44851

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Q7 Find a root of $x - e^{-x} = 0$ correct to three decimal by Secant method.

Ans7

$$x - e^{-x} = 0$$

$$f(x) = x - e^{-x}$$

$f(0) = -1$ (-ve)

$f(1) = 0.63212$ (+ve)

Hence, the Root lies between 0 to 1

$\boxed{x_0 = 0}$ $\boxed{x_1 = 1}$

$$x_2 = \frac{0 \times 0.63212 - 1(-1)}{0.63212 + 1} = 0.61270004$$

$f(x_2) = 0.0708142$

$$x_3 = \frac{1(0.0708142) - (0.61270)(0.63212)}{(0.0708142 - 0.63212)}$$

$x_3 = 0.563842$

$f(x_3) = -0.0051766.$

$$x_4 = \frac{0.612700 \times (-0.0051766) - (0.563842) \times (0.0708142)}{-0.0051766 - 0.0708142}$$

$x_4 = 0.569356$

$f(x_4) = 0.00346624.$

$$x_5 = \frac{0.563842 \times (0.00346624) - 0.569356 \times (-0.0051766)}{0.00346624 + 0.0051766}$$

$x_5 = 0.567147$

$f(x_5) = 0.0000058135.$

$$x_6 = \frac{0.569356 \times 0.0000058135 - 0.567147 \times 0.00346624}{0.0000058135 - 0.00346624}$$

$x_6 = 0.567145$

so, the Real root of the Eqn is 0.567145.

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Computational methods
Assignment
unit - II.

Q1 Solve the following Equations using Guass Elimination method with partial pivoting?

$$2x_1 + 4x_2 + x_3 = 3$$
$$3x_1 + 2x_2 - 2x_3 = -2$$
$$x_1 - x_2 + x_3 = 6$$

Solution $A = \begin{bmatrix} 2 & 4 & 1 \\ 3 & 2 & -2 \\ 1 & -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}$

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 4 & 1 & 3 \\ 3 & 2 & -2 & -2 \\ 1 & -1 & 1 & 6 \end{array}\right]$$

$R_2 \leftrightarrow R_1$

$$\sim \left[\begin{array}{ccc|c} 3 & 2 & -2 & -2 \\ 2 & 4 & 1 & 3 \\ 1 & -1 & 1 & 6 \end{array}\right]$$

$R_2 \rightarrow R_2 - \frac{2}{3} R_1$, $R_3 \rightarrow R_3 - \frac{R_1}{3}$

$$\sim \left[\begin{array}{ccc|c} 3 & 2 & -2 & -2 \\ 0 & 8/3 & 7/3 & 13/3 \\ 0 & -5/3 & 5/3 & 20/3 \end{array}\right]$$

$R_3 \rightarrow R_3 + \frac{5}{8} R_2$

$$\sim \left[\begin{array}{ccc|c} 3 & 2 & -2 & -2 \\ 0 & 8/3 & 7/3 & 13/3 \\ 0 & 0 & 75/24 & 225/24 \end{array}\right]$$

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$$\frac{75}{24} x_3 = \frac{225}{24}$$

$$\boxed{x_3 = 3}$$

$$\frac{8}{\cancel{3}} x_2 + \frac{7}{\cancel{3}} x_3 = \frac{13}{\cancel{3}}$$

$$8(x_2) + 7(3) = 13$$

$$8x_2 = -8$$

$$\boxed{x_2 = -1}$$

$$3x_1 + 2x_2 - 2x_3 = -2$$

$$3x_1 + 2(-1) - 2(3) = -2$$

$$3x_1 = -2 + 2 + 6$$

$$\boxed{x_1 = 2}$$

So, the solution of Eq^n is :-

$$\boxed{x_1 = 2} \; ; \; \boxed{x_2 = -1} \; ; \; \boxed{x_3 = 3}$$

Ques2 Define tridiagnol system with Example ?

Tridiagnol : A tridiagnol matrix is a band matrix, has non-zero Elements only on the main diagnol, the diagnol upon the main diagnol & the diagnol below the main diagnol.

For Exp :

For 4 order matrix

upon

$$\begin{bmatrix} a & p & 0 & 0 \\ \alpha & b & q & 0 \\ 0 & \beta & c & \gamma \\ 0 & 0 & \gamma & d \end{bmatrix}$$

below main diagnol

For 5 order matrix

$$\begin{bmatrix} a & p & 0 & 0 & 0 \\ x & b & q & 0 & 0 \\ 0 & y & c & \gamma & 0 \\ 0 & 0 & z & d & s \\ 0 & 0 & 0 & w & e \end{bmatrix}$$

Ques3 Solve the following equations by LU factorisation Method.

$$10x - 7y + 3z + 5u = 6$$

$$-6x + 8y - z - 4u = 5$$

$$3x + y + 4z + 11u = 2$$

$$5x - 9y - 2z + 4u = 7$$

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$$A = \begin{bmatrix} 10 & -7 & 3 & 5 \\ -6 & 8 & -1 & -4 \\ 3 & 1 & 4 & 11 \\ 5 & -9 & -2 & 4 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ l_{31} & l_{32} & 1 & 0 \\ l_{41} & l_{42} & l_{43} & 1 \end{bmatrix} \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ 0 & u_{22} & u_{23} & u_{24} \\ 0 & 0 & u_{33} & u_{34} \\ 0 & 0 & 0 & u_{44} \end{bmatrix}$$

$$LU = \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} & l_{21}u_{14}+u_{24} \\ l_{31}u_{11} & l_{31}u_{12}+l_{31}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} & l_{31}u_{14}+l_{32}u_{24}+u_{34} \\ l_{41}u_{11} & l_{41}u_{12}+l_{42}u_{22} & l_{41}u_{13}+l_{42}u_{23}+l_{43}u_{33} & l_{41}l_{14}+u_{24}l_{42}+l_{43}u_{34}+u_{44} \end{bmatrix}$$

$\boxed{u_{11} = 10}$ $\boxed{u_{12} = -7}$ $\boxed{u_{13} = 3}$ $\boxed{u_{14} = 5}$

$l_{21}u_{11} = -6$

$\boxed{l_{21} = \frac{-3}{5}}$

$l_{31}u_{11} = +3$

$\boxed{l_{31} = \frac{3}{10}}$

$l_{41}u_{11} = 5$

$\boxed{l_{41} = \frac{1}{2}}$

$l_{21}u_{12} + u_{22} = 8$

$\frac{-3}{5} \times (-7) + u_{22} = 8$

$\boxed{u_{22} = \frac{19}{5}}$

$l_{31}u_{12} + l_{32}u_{22} = 1$

$\frac{3}{10} \times (-7) + l_{32}\left(\frac{19}{5}\right) = 1$

$\boxed{l_{32} = \frac{31}{38}}$

$l_{21}u_{13} + u_{23} = -1$

$\frac{-3}{5} \times 3 + u_{23} = -1$

$\boxed{u_{23} = \frac{4}{5}}$

$l_{21}u_{14} + u_{24} = -4$

$u_{24} - \frac{3}{5}(5) = -4$

$\boxed{u_{24} = -1}$

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$l_{41}u_{12} + l_{42}u_{22} = -9$

$\frac{1}{2}(-7) + l_{42}\left(\frac{19}{5}\right) = -9$

$l_{42} = -\frac{11}{2} \times \frac{5}{19}$

$\boxed{l_{42} = -\frac{55}{38}}$

$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 4$

$\frac{3}{10} \times (3) + \frac{31}{38} \times \frac{4}{5} + u_{33} = 4$

$u_{33} = 4 - \frac{9}{10} - \frac{62}{95}$

$\boxed{u_{33} = \frac{93}{38}}$.

$l_{41}u_{13} + l_{42}u_{23} + u_{33}l_{43} = -2$

$\frac{1}{2} \times 3 + \left(\frac{-55}{38}\right) \times \frac{4}{5} + \frac{93}{38} \times l_{43} = -2$

$\frac{93}{38} l_{43} = -2 - \frac{3}{3} + \frac{44}{38}$

$\boxed{l_{43} = -\frac{89}{93}}$

$l_{31}u_{14} + l_{32}u_{24} + u_{34} = 11$

$\frac{3}{10} \times 5 + \frac{31}{38}(-1) + u_{34} = 11$

$\boxed{u_{34} = \frac{196}{19}}$

$l_{41}u_{14} + l_{42}u_{24} + l_{43}u_{34} + u_{44} = 4$

$\frac{1}{2} \times 5 + \left(\frac{-55}{38}\right)(-1) + \left(\frac{-89}{93}\right)\frac{196}{19} + u_{44} = 4$

$\boxed{u_{44} = \frac{923}{93}}$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3/5 & 1 & 0 & 0 \\ 3/10 & 31/38 & 1 & 0 \\ 1/2 & -55/38 & -89/93 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 10 & -7 & 3 & 5 \\ 0 & 19/5 & 4/5 & -1 \\ 0 & 0 & 93/38 & 196/19 \\ 0 & 0 & 0 & 923/93 \end{bmatrix}$$

$AX = B$

$LUX = B$

$LV = B$

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$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -3/5 & 1 & 0 & 0 \\ 3/10 & 31/38 & 1 & 0 \\ 1/2 & -55/38 & -89/38 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 2 \\ 7 \end{bmatrix}$$

$\boxed{v_1 = 6}$

$$-\frac{3}{5}v_1 + v_2 = 5$$

$$\boxed{v_2 = 43/5}$$

$$\frac{3}{10}v_1 + \frac{31}{38}v_2 + v_3 = 2$$

$$\frac{3}{10}(6) + \frac{31}{38}\left(\frac{43}{5}\right) + v_3 = 2$$

$$\boxed{v_3 = \frac{-259}{38}}$$

$$\frac{1}{2}v_1 - \frac{55}{38}\left(\frac{43}{5}\right) - \frac{89}{93}v_3 + v_4 = 7$$

$$\frac{1}{2}(6) - \frac{473}{38} - \frac{89}{93}\left(\frac{-259}{38}\right) + v_4 = 7$$

$$\boxed{v_4 = \frac{923}{93}}$$

Now, $UX = V$

$$\begin{bmatrix} 10 & -7 & 3 & 5 \\ 0 & 19/5 & 4/5 & -1 \\ 0 & 0 & 93/38 & 196/19 \\ 0 & 0 & 0 & 923/93 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 6 \\ 43/5 \\ -259/38 \\ 923/93 \end{bmatrix}$$

$$10x - 7y + 3z + 5u = 6$$

$$\frac{19}{5}y + \frac{4}{5}z - u = \frac{43}{5}$$

$$\frac{93}{38}z + \frac{196}{19}u = \frac{-259}{38}$$

$$\frac{923}{93}u = \frac{923}{93}$$

$\boxed{u = 1}$; $\frac{93}{38}z = \frac{-259}{38} - \frac{196}{19}$

$$\boxed{z = -7}$$

$$\frac{19}{5}y + \frac{4}{5}(-7) - 1 = \frac{43}{5}$$

$$\boxed{y = 4}$$

$$10x - 7(4) + 3(-7) + 5 = 6$$

$$10x = 6 + 28 + 21 + 5$$

$$\boxed{x = 5}$$

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So, the solution of linear Equations -

$\boxed{x = 5}$; $\boxed{y = 4}$; $\boxed{z = -7}$; $\boxed{u = 1}$

Ques 5 Solve the following Equation by Guass seidel method ?

$$2x + y + 6z = 9$$
$$8x + 3y + 2z = 13$$
$$x + 5y + z = 7$$

$$x = \frac{1}{8}(13 - 3y - 2z)$$

$$y = \frac{1}{5}(7 - x - z)$$

$$z = \frac{1}{6}(9 - 2x - y)$$

$$x_1 = \frac{1}{8}(13 - 3(0) - (0)) = 1.625$$

$$y_1 = \frac{1}{5}(7 - 1.625) = 1.075$$

$$z_1 = \frac{1}{6}(9 - 2(1.625) - 1.075) = 0.77916$$

$$x_2 = \frac{1}{8}(13 - 3(1.075) - 2(0.77916)) = 1.0270$$

$$y_2 = \frac{1}{5}(7 - 1.0270 - 0.77916) = 1.0387$$

$$z_2 = \frac{1}{6}(9 - 2 \times 1.0270 - 1.0387) = 0.9845$$

$$x_3 = \frac{1}{8}(13 - 3(1.0387) - 2(0.9845)) = 0.9893$$

$$y_3 = \frac{1}{5}(7 - 0.9845 - 0.9893) = 1.00524$$

$$z_3 = \frac{1}{6}(9 - 2(0.9893) - 1.0052) = 1.0027$$

So, Roots (approximated) of the Equations are
$\boxed{x = y = z \simeq 1}$.

Q9 Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ using Guass Jordan Method.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 3 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$; $R_3 \to R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 4 & 8 & -3 & 1 & 0 \\ 0 & -3 & -3 & -2 & 0 & 1 \end{array}\right]$$

$R_1 \to 2R_1 + R_2$; $R_3 \to 4R_3 - 3R_2$

$$\sim \left[\begin{array}{ccc|ccc} 2 & 0 & -2 & -1 & 1 & 0 \\ 0 & -4 & -8 & -3 & 1 & 0 \\ 0 & 0 & 12 & 1 & -3 & 4 \end{array}\right]$$

$R_1 \to R_1/2$ $R_2 \to R_2/-4$ $R_3 \to R_3/12$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1/2 & 1/2 & 0 \\ 0 & 1 & 2 & 3/4 & -1/4 & 0 \\ 0 & 0 & 1 & 1/12 & -3/12 & 4/12 \end{array}\right]$$

$R_1 \to R_1 + R_3$; $R_2 \to R_2 - 2R_3$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5/12 & 3/12 & 4/12 \\ 0 & 1 & 0 & 7/12 & 1/4 & -2/3 \\ 0 & 0 & 1 & 1/12 & -1/4 & 1/3 \end{array}\right]$$

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$$A^{-1} = \begin{bmatrix} -5/12 & 1/4 & 1/3 \\ 7/12 & 1/4 & -2/3 \\ 1/12 & -1/4 & 1/3 \end{bmatrix}$$

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Assignment-3

Ques1 Find $f'(7)$ for the Given data -

x	0	1	5	8
$f(x)$	0	1	8	16.4
	x_0	x_1	x_2	x_3

$$f'(7) = \frac{(2\times7-1-5)\times 0}{(0-1)(0-5)} + \frac{(2\times7-0-5)}{(1-0)(1-5)} + \frac{(2\times7-0-1)\times 8}{(5-0)(5-1)}$$

$$= 0 + (-2.25) + 5.2$$

$$\boxed{f'(7) = 2.95}$$

Ques2 Evaluate $\int_0^1 e^{-x^2}dx$ using Simpson's $1/3^{rd}$ Rule?

	x_0	x_1	x_2	x_3	x_4
x	0	0.25	0.5	0.75	1
$f(x)$	1	0.93941	0.77880	0.56978	0.36787
	y_0	y_1	y_2	y_3	y_4

$$I = \frac{h}{3}\left[y_0 + y_n + 4(y_1+y_3) + 2(y_2)\right]$$

$$= \frac{0.25}{3}\left[1 + 0.36787 + 4(0.93941 + 0.56978) + 0.77880\times2\right]$$

$$\boxed{I = 0.7468525}$$ Ans//

Ques3 Find $f'(4)$ using Richardson's Extrapolation Method for following Data

x	0	1	2	3	4	5	6	7	8
$f(x) = y$	-5	-2	7	34	91	190	343	562	859

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For finding $f'(4)$

use central Difference Formula,

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}.$$

Let us assume that

$\boxed{h=4}$ $$f'(4) = \frac{f(8) - f(0)}{2\times(4)} = \frac{859-(-5)}{2\times 4} = 108$$

$\boxed{h=2}$ $$f'(4) = \frac{f(6) - f(2)}{2\times 2} = \frac{343-7}{2\times 2} = 84$$

$\boxed{h=1}$ $$f'(4) = \frac{f(5) - f(3)}{2\times 1} = \frac{190-34}{2} = 78$$

Table –

h	D	D'	D²
4	108		
		76	
2	84		76
		76	
1	78		

$$D^K(h) = \frac{4^K D^{K-1}(h/2) - D^{K-1}(h)}{4^K - 1}$$

$$D'(h) = \frac{4D(h/2) - D(h)}{4-1} = \frac{4\times 84 - 108}{3} = 76$$

$$D'(h/2) = \frac{4D(h/4) - D(h/2)}{4-1} = 76$$

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$$D^2(h) = \frac{4^2 D'(h/2) - D'(h)}{4^2-1} = \frac{16 \times 76 - 76}{15} = 76$$

Hence the value of $f'(4) = 76$ Ans//

Ques4 The following system of Equation are Given

$f_1(x,y) = x^3 + xy^2 - y^3 = 0$

$f_2(x,y) = xy + 5x + 6y = 0$

find $\frac{\partial f_1}{\partial x}$, $\frac{\partial f_2}{\partial y}$ of the Given system of equation. at $(1,2)$ given $h = k = 1$

$$\left.\frac{\partial f_1}{\partial x}\right|_{(1,2)} = \frac{f_1(2,2) - f(0,2)}{2 \times 1}$$

$$= \frac{(8+8-8) - (0+0-8)}{2}$$

$$= \frac{8+8}{2} = \frac{16}{2} = 8$$

$$\left.\frac{\partial f_2}{\partial y}\right|_{(1,2)} = \frac{f_2(1,3) - f_2(1,1)}{2}$$

$$= \frac{(3+5+18) - (1+5+1)}{2}$$

$$= \frac{26 - 7}{2}$$

$$= 9.5$$

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Q5 Evaluate $\int_0^1 \frac{dx}{1+x}$ using Gaussian Quadrature 3 point formula ?

$f(x) = \frac{1}{1+x}$

Guass Quadrature three point formula,

$$\int_{-1}^{1} f(x)\,dx = \frac{8}{9} f(0) + \frac{5}{9}\left[f\left(-\sqrt{\frac{3}{5}}\right) + f\left(\sqrt{\frac{3}{5}}\right)\right]$$

$$x = \frac{1}{2}(b-a)t + \frac{1}{2}(b+a)$$

$$= \frac{1}{2}(-0+1)t + \frac{1}{2}(1) = \frac{1}{2}(t+1)$$

$$\boxed{x = \frac{1}{2}(t+1)} \qquad f(x) = \frac{2}{3+t}.$$

$$f(0) = \frac{2}{3} = 0.667$$

$$f\left(-\frac{\sqrt{3}}{5}\right) = 1.085953$$

$$f\left(\sqrt{\frac{3}{5}}\right) = 1.734770$$

$$I = \frac{8}{9} \times 0.667 + \frac{5}{9}\left[1.085953 + 1.734770\right]$$

$$\boxed{I = 2.159957}$$

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Assignment-4

Qus1 Using Euler's method, Solve y at $x = 0.1$ from $\frac{dy}{dx} = x + y + xy$, $y(0) = 1$, taking Step Size $h = 0.025$

Sol

x	y	$\frac{dy}{dx} = x + y + xy$	new y = old $y + h\left(\frac{dy}{dx}\right)$
0	1	1	$1 + 0.025(1) = 1.025$
0.025	1.025	1.075625	1.05189
0.50	1.05189	2.077835	1.103835
0.75	1.103835	2.632597	1.10416493
1	1.104164	3.208328	1.2218482

Answer

$y(1) = 1.104164$.

Qus2 Using Runge Kutta method of 4th order Solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$

$$f(x, y) = \frac{y^2 - x^2}{y^2 + x^2}$$

Initial condition are $y_0 = 1$ $x_0 = 0$ $h = 0.2$

To find $y = ?$ at $x = 0.2$

$$f(0, 1) = \frac{1 - 0}{1 + 1} = 1$$

$$k_1 = hf(x_0, y_0) = 0.2 \times 1 = 0.2$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= 0.2\, f(0.1, 1.1)$$

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$$= 0.2\left[\frac{(1.1)^2-(0.1)^2}{(1.1)^2+(0.1)^2}\right] = \frac{1.2}{1.22}\times 0.22$$

$$= 0.196721$$

$$k_3 = hf\left(x_0+\frac{h}{2},\ y_0+\frac{k_2}{2}\right)$$

$$= 0.2\, f(0.1,\ 1.09835) = \frac{0.2\times 1.1962628}{1.216262}$$

$$= 0.196711$$

$$k_4 = hf(x_0+h,\ y_0+k)$$

$$= 0.2\, f(0.2,\ 1.196711)$$

$$= \frac{0.2\times 1.392117}{1.472117} = 0.189131$$

$$k = \frac{1}{6}(k_1+2k_2+2k_3+k_4)$$

$$= \frac{1}{6}\left[0.2+2\times 0.196721+2\times 0.196711+0.189131\right]$$

$$= 0.195991 \simeq 0.1960$$ Ans//

$$y = y_0+k$$

$$= 1+0.1960$$

$$= 1.1960$$

Hence the value of $f(0.2)$ is 1.1960 Ans//

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Ques3 using finite difference method, find $y(0.25)$, $y(0.5)$ & $y(0.75)$ Satisfying the differential equation $y''+y=x$ Subject to the Boundry condition $y(0)=0$, $y(1)=2$

Given Differential Eqn $y''+y=x$

$$y_i''+y_i=x$$

Putting $y_i''=\frac{1}{h^2}[y_{i+1}-2y_i+y_{i-1}]$

$$\frac{1}{h^2}[y_{i+1}-2y_i+y_{i-1}]+y_i=x_i$$

$$16y_{i+1}-31y_i+16y_{i-1}=x_i$$

$i=1$

$$16y_2-31y_1+16y_0=x_1$$

$$16y_2-31y_1=0.25 \rightarrow (1)$$

$i=2$

$$16y_3-31y_2+16y_1=x_2$$

$$16y_3-31y_2+16y_1=0.5 \rightarrow (2)$$

$i=3$

$$16y_4-31y_3+16y_2=x_3$$

$$16\times 2-31y_3+16y_2=0.75$$

$$-31y_3+16y_2=-31.25 \rightarrow (3)$$

Solving (1) & (2) & (3)

$y_1=0.5443$ $(y=0.25)$

$y_2=1.0701$ $(y=0.5)$

$y_3=1.5604$ $(y=0.75)$

Q4 classify the type of PDE -

$$\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x\partial y}+5\frac{\partial^2 u}{\partial y^2}=0$$ wheather it is Parabolic, Elliptic or hyperbolic ?

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$$\frac{\partial^2 u}{\partial x^2} + \frac{2\partial^2 u}{\partial x \partial y} + \frac{5\partial^2 u}{\partial y^2} = 0$$

$B^2 - 4AC$

$(2)^2 - 4 \times 5 \times 1$

$4 - 20$

$-16 < 0$

So, Equation $\frac{\partial^2 u}{\partial x^2} + \frac{2\partial^2 u}{\partial x \partial y} + \frac{5\partial^2 u}{\partial y^2}$ is Elliptic.