(Please write your Exam Roll No.) Exam Roll No. 00515611921

END TERM EXAMINATION

FOURTH SEMESTER [B.TECH] JULY 2023

Paper Code: AIDS/AIML/IOT-212	*Subject: Computational Methods*
Time: 3 Hours	*Maximum Marks: 75*

Note: Attempt five questions in all including Q. No.1 which is compulsory. Select one question from each unit.

Q1 Attempt all questions:-

(a) Explain two types of complexities of an algorithm. **(2.5)**

(b) Define rate of convergence of an iterative method. Find the rate of convergence of Newton- Raphson's method. **(2.5)**

(c) Use Lagrange's Interpolation formula to find the value of y when x=10. The following values of x and y are given. **(2.5)**

x	5	6	9	11
y	12	13	14	16

(d) The following table of values are given for a function f(x). **(2.5)**

	x	0.1	0.2	0.3
y				
0.1		2.0200	2.0351	2.0403
0.2		2.0351	2.0801	2.1153
0.3		2.0403	2.1153	2.1803

Determine the value of $\frac{\partial f}{\partial x}$ *and* $\frac{\partial f}{\partial y}$ at (0.2, 0.2) using central difference formula.

(e) Evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ using three-point Gaussian Quadrature formula. **(2.5)**

(f) Classify the PDE $\frac{\partial^2 u}{\partial x^2} - 2x\frac{\partial^2 u}{\partial x \partial y} + x^2\frac{\partial^2 u}{\partial y^2} - 2\frac{\partial u}{\partial x} = 0$. 8/9 5/9 **(2.5)**

UNIT-I

Q2 (a) Find the real root of equation $x^3 - 9x + 1 = 0$ correct to three places of decimal using simple fixed point iteration method. **(7.5)**

(b) Find the 4th root of 32 correct to three places of decimal using Secant method. **(7.5)**

Q3 (a) Using Regula Falsi method find the real root of $x\log_{10}x = 1.2$ correct to three places of decimal. **(7.5)**

(b) Using Bisection method find the root of the equation $x^3 - 5x + 1 = 0$ correct to two places of decimal. **(7.5)**

UNIT-II

Q4 (a) Solve the following system of equations using Gauss Elimination method with partial pivoting: 5, 1, -6 **(7.5)**

$$2x_1 + 2x_2 + x_3 = 6$$
$$4x_1 + 2x_2 + 3x_3 = 4$$
$$x_1 + x_2 + x_3 = 0$$

(b) Find the largest eigen values and corresponding eigen vector of the following matrix: **(7.5)**

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

4 - 4
4 - 2
2 - 3
12 - 4

4 - 4
4 - 2
4 - 3
0 - 4

2 - 2
2 - 2
1 - 2
6 - 0

4 - 4
2 - 4
3 - 4
4 - 0

P.T.O.

Q5 (a) Solve the following system of equations by Cholesky method: **(7.5)**

$$x + 2y + 3z = 5$$
$$2x + 8y + 22z = 6$$
$$3x + 22y + 82z = -10$$

(b) Calculate cubic spline for the given data: **(7.5)**

x	1	2	3	4
y	1	5	11	8

Also find y(1.5) and y'(2)

UNIT-III

Q6 (a) Evaluate $\int_0^1 \frac{dx}{1+x}$ using Simpson's 3/8 rule taking h=1/6. Hence evaluate the approximate value of π. **(7.5)**

(b) Use Romberg's method to find $\int_0^1 \frac{dx}{1+x^2}$ correct to four places of decimals. **(7.5)**

Q7 (a) Find the first derivative of $f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$ at x=0.5 using forward, backward and central differences formulae taking step size h=0.25. **(7.5)**

(b) Find the values of f'(0) and f'(8) from the following data using approximate initial values based on finite differences and Richardson's extrapolation method. **(7.5)**

X	0	1	2	3	4	5	6	7	8
f(x)	-5	-2	7	34	91	190	343	562	859

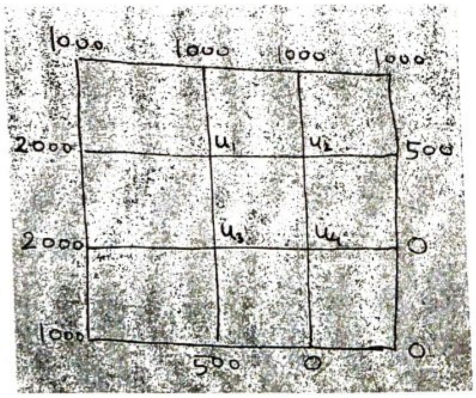
UNIT-IV

Q8 (a) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with initial condition y=1 at x=0. Find y for x=0.1 by Euler's method. **(7.5)**

(b) Using the finite differences method find y(0.25),y(0.5) and y(0.75) satisfying the differential equation $y'' + y = x$ subject to the boundary condition y(0)=0 , Y(1)=2. **(7.5)**

Q9 (a) Using Runge Kutta method solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ with initial condition y(0)=1 at x=0.2 and 0.4. **(7.5)**

(b) Given the value of u(x,y) on boundary of the square in the figure. Evaluate the function u(x,y) satisfying the Laplace equation $\nabla^2 u = 0$ at the pivotal points of the figure by Gauss-Seidal formula. **(7.5)**



4th Sem
End term

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P-2/2
AIDS/AIML/IOT-212

Student Name: Kasak

Enrollment No: 02715611922

Mid-Term Examination- April 2024

Programme: B.Tech. (AI & DS) Semester: Fourth Semester

Paper Code: AIDS212 Paper Name: Computational Method

Time: 1½ Hrs. Maximum Marks: 30

Note:

- Question No. 1 is compulsory.
- Attempt any two questions from the remaining questions.
- All questions carry equal marks
- Only scientific calculator is allowed.

Q. No.	Question 1	Marks	CO
1.a	If the estimated value of $e^{0.5}$ is 1.5 while true value is 1.648721then find a true percent relative error and a percent approximate estimate of the error.	2	CO1
1.b	The equation $2x = \log_{10} x + 7$ has a root between 3 and 4. Find this root, correct to three decimal places, by regula-falsi method.	3	CO1
1.c	Define 'partial pivoting' in the process to find the solution of LSE. Explain by giving an example	2.5	CO2
1.d	To which form the coefficient matrix is transformed when AX=B is solved by Gauss elimination method. Solve x+y= 2 and 2x+3y=5 by Gauss elimination method	2.5	CO2
	Question 2		
2.a	Use the method of fixed point iteration to find a positive root of the equation $xe^x = 1$, given that a root lies between 0 and 1	5	CO1
2.b	Describe the concept applied in the bracketing methods used for solving non-linear equation. Write computational algorithm used in bisection method. Also write the value of error bound at nth iteration in bisection.	1+3+1	CO1
	Question 3		

3.a	Solve the system of non-linear equations: $x^2 + y = 11, y^2 + x = 7.$	5	CO2
3.b	Using Thomas algorithm solve the tridiagonal system of equations: $3x - y = -1$ $-x + 3y - z = 7$ $-y + 3z = z$	5	CO2
	Question 4		
4.a	Explain the limitation of using Newton Raphson method. Find the root of the equation $f(x) = x^2 - 3x + 2$ in the vicinity of x=0, using Newton-Raphson method	5	CO1
4.b	Explain the eigen value and eigen vector of a matrix. Determine the largest eigen value and corresponding eigen vector of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$	1+4	CO2

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Enrolment No. 02715611922

CLASS TEST – May 2024

Semester: Fourth Semester
Subject : Computational Methods
Maximum Marks: 30

Programme: B.Tech (AIML/AI&DS)
Paper Code: AIML/AI&DS-212
Time: One and Half Hours

Note:
Question no. 1 is compulsory.
Attempt any two questions from the remaining questions.
All questions carry equal marks.
Only scientific calculator is allowed.

	Question 1		
1 (a)	Estimate $f'(x_2)$ using forward,backward and central difference formulae for given data $(x_0, y_0) = (1,2), (x_1, y_1) = (2,4), (x_2, y_2) = (3,8), (x_3, y_3) = (4,16), (x_4, y_4) = (5,32)$.	2.5	CO3
1 (b)	Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using three-point Gaussian quadrature formula.	2.5	CO3
1 (c)	Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with initial condition y=1 at x=0.Find the value of y for x=0.1 by Euler's method using step size 0.02.	3	CO4
1 (d)	Classify the PDE $2\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 3\frac{\partial^2 u}{\partial y^2} = 2$	2	CO4

	Question 2		
2 (a)	Find an approximate value of $\int_0^{\pi/2} \sqrt{cos\theta}\, d\theta$ using Simpson's 1/3 rule by dividing the interval into six equal parts.	5	CO3
2 (b)	Find the value of $f'(3)$ with h=4,2,1 using the formula $f'(x) = \frac{f(x_2)-f(x_0)}{2h}$ for the given data	5	CO3

x	-1	1	2	3	4	5	7
y	1	1	16	81	256	625	2401

	Question 3		
3 (a)	Using modified Euler's method, find an approximate value of y when x=0.3. given that $\frac{dy}{dx} = x + y$, y (0) =1 and h=0.1	5	CO4
3 (b)	Using Runge-Kutta method of 4th order solve $\frac{dy}{dx} = 3x^2 + y^2$ with y (1) = 1.2 at x=1.1	5	CO4

	Question 4		
4 (a)	Evaluate $\int_0^1 \frac{dx}{1+x^2}$ correct to four places of decimal by Romberg's method.	5	CO3
4 (b)	Using the finite difference method find y(0.25),y(0.5)and y(0.75) satisfying the differential equation $y'' + y = x$ subject to boundary condition y(0)=0 & y(1)=2.	5	CO4

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SUPPLEMENTARY MID TERM EXAMINATION
May 2024

Programme: B.Tech (AIML/AI&DS)
Paper Code: AIML/AI&DS-212
Time: One and Half Hours

Semester: Fourth Semester
Subject : Computational Methods
Maximum Marks: 30

Note:
Question no. 1 is compulsory.
Attempt any two questions from the remaining questions.
All questions carry equal marks.
Only scientific calculator is allowed.

	Question 1		
1 (a)	Evaluate $\sqrt{5}$ using the equation $x^2 - 5 = 0$ by applying fixed point iteration method.	2.5	CO1
1 (b)	Explain two types of complexities of an algorithim.	2.5	CO1
1 (c)	Define tridiagonal system with an example.	2.5	CO2
1 (d)	Find the value of y when x=10 using Lagrange's interpolation formula for given data	2.5	CO2

x	5	6	9	11
y	12	13	14	16

	Question 2		
2 (a)	Find a root of equation $x^3 - 5x + 1 = 0$ using bisection method correct to two places of decimals.	5	CO1
2 (b)	Develop a recurrence formula for finding $\sqrt{N}$,using Newton -Raphson method and hence evaluate $\sqrt{28}$ correct to three places of decimals.	5	CO1

	Question 3		
3 (a)	Solve the following equations by Cholesky's method. X+2y+3z=5 2x+8y+22z=6 3x+22y+82z=-10	5	CO2
3 (b)	Determine the largest eigen value and the corresponding eigen vector of the matrix A = $\begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}$.	5	CO2

P. T. O.

	Question 4		
4 (a)	Find the root of the equation $xe^x = cos x$ using Secant method correct to three places of decimals.	5	CO1
4 (b)	Solve the following equation by Gauss Seidel method 2x+y+6z=9 8x+3y+2z=13 X+5y+z=7	5	CO2

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END TERM EXAMINATION

FOURTH SEMESTER [B.TECH] JUNE-2024

Paper Code: AIDS/AIML/IOT-212 **Subject: Computational Methods**

Time: 3 Hours **Maximum Marks:75**

Note: Attempt five questions in all including. Q. no. 1 which is compulsory. Select one question from each unit.

Q1 Attempt **any Five** of the following questions: **(5×5=25)**

(a) (i) What is a transcendental equation. Give example.
(ii) What is meant by simple and multiple roots of an equation. Give examples of each.

(b) Taking initial approximation $x_0 = 2$, perform two iterations of Newton-Raphson method to obtain the approximate value of $(18)^{\frac{1}{3}}$ (cube root of 18).

(c) Using LU decomposition with L as the lower triangular matrix and U as the upper triangular matrix to solve the following system of equations. Take U with diagonal entries 1

$$x + y + z = 1, 4x + 3y - z = 6, 3x + 5y + 3z = 4$$

(d) Obtain piecewise linear interpolating polynomials for $f(x)$ using the following data:

x	1	2	4	8
$f(x)$	3	7	21	73

(e) Find the approximate value of α such that the value of $\int_0^1 \frac{\alpha}{1+x}dx$ using trapezoidal rule with step length $h = 0.2$ is 3.

(f) Obtain approximate value of $f'(-3)$ with step size $h = 1$, and $h = \frac{1}{2}$ using the method

$f'(x_0) = \frac{-3f(x_0)+4f(x_1)-f(x_2)}{2h}$ for the following data:

x	-3	-2.5	-2	-1
$f(x)$	-25	-14.125	-7	-1

(g) Use Euler method to find $y(0.9)$ for the problem $3\frac{dy}{dx} + 5y^2 = \sin x, y(0.3) = 5$ with step length $h = 0.3$.

(h) Consider the boundary value problem $\frac{d^2y}{dx^2} = 4(y - x), 0 \le x \le 1, y(0) = 0, y(1) = 2$. Use finite difference method to approximate the solution of the ODE with step size $h = 1/2$.

Q2 (a) Perform three iterations of bisection method to obtain a root of $f(x) = \cos(x) - xe^x = 0$. **(6)**

(b) Use fixed point iteration method to find a positive root, between 0 and 1, of equation $xe^x = 1$. Start with initial iteration $x_0 = 1$. Perform 3 iterations. **(6.5)**

P-1/3 **P.T.O.**

AIDS/AIML/IOT-212

UNIT-I

Q3 (a) Given that a real root of $f(x) = x^3 - 5x + 1 = 0$ lies in the interval $(0,1)$, perform 3 iterations of the following method to obtain this root **(12.5)**

(i) Secant method
(ii) Regula-Falsi method
(iii) Newton-Raphson method with initial approximation $x_0 = 0.5$

UNIT-II

Q4 (a) Use Lagrange interpolation to find a polynomial that passes through the following data points: **(6)**

x	0	1	3	4
$f(x)$	-20	-12	-20	-24

(b) Solve the following system of equation using Gaussian elimination method: **(6.5)**

$$x + 3y + 5z = 2, 2x + y + z = 7, 3x + 2y + 4z = 7$$

Q5 (a) Apply two iterations of Gauss-Siedal method with initial approximation $(x^0, y^0, z^0) = (1,0,1)$ for the following system of equation **(4)**

$$12x + 3y - 5z = 1, x + 5y + 3z = 28, 3x + 7y + 13z = 76.$$

(b) Find inverse of the following matrix using Cholesky decomposition: **(8.5)**

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{pmatrix}$$

UNIT-III

Q6 (a) Compute $\int_0^4 \sqrt{1+5x^2}\, dx$ using Gaussian quadrature rule with $n = 2$, where n is the number of nodal points. **(4)**

(b) Find approximate value of $\int_0^2 x^2 e^{-x^2}$ by taking six subintervals using
(i) Trapezoidal rule (ii) Simpson's $1/3$ rule. **(3.5+5)**

Q7 (a) Find the Jacobian matrix for the system of equations: **(8)**

$$f_1(x,y) = x^2 + y^2 - x = 0,$$
$$f_2(x,y) = x^2 - y^2 - y = 0$$

at the point (1,1) using the methods:

$\frac{\partial f}{\partial x} = \frac{f_{i+1,j} - f_{i-1,j}}{2h}, \frac{\partial f}{\partial y} = \frac{f_{i,j+1} - f_{i,j-1}}{2k}$ at the point (x_i, y_i) with $h = k = 1$.

(b) Obtain approximate value of $f''(-1)$ using the method $f''(x_0) = \frac{f(x_0) - 2f(x_1) + f(x_2)}{h^2}$ with $h = 1$ for the following data: **(4.5)**

x	-1	-0.5	0	1
$f(x)$	2.7183	1.6487	1	0.3679

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UNIT-IV

Q8 (a) Consider $\frac{dy}{dx} = y(1+x^2), y(0) = 1$. Using the method $y_{i+1} = y_i + hf(x_{i+1}, y_{i+1})$. Find $y(0.2)$ and $y(0.4)$ with step length $h = 0.2$. **(5)**

(b) Compute $y(0.1)$ by fourth order Runge Kutta method with step size $h = 0.1$ for the ODE $\frac{dy}{dx} = 3x + \frac{y}{2}, y(0) = 1$. (7.5)

Q9 Explain the finite difference method. Solve the boundary problem $\frac{d^2u}{dx^2} = u + x, 0 \le x \le 1, u(0) = 0, u(1) = 0$, with $h = 1/4$ using finite difference method.

(4+8.5=12.5)

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$x_0 = 0 \quad = 0$

$x_1 = \frac{1}{4}$

$x_2 = \frac{2}{4}$

$x_3 = \frac{3}{4}$

$x_4 = 1 \quad = 0$

$y'' = y + x$

$\frac{1}{h^2}[y_{i+1} - 2y_i + y_{i-1}] =$

$- y_i = x_i$

$16y_{i+1} - 2y_i + y_{i-1}$

$16y_{i+1} - 3y_i + y_{i-1} = x_i$

Put $x = 1$

$16y_2 - 3y_1 + y_0 = x_i \quad \frac{1}{4}$

$16y_2 - 3y_1$

Put $x = 2$

$16y_3 - 3y_2 + y_1 = x_2$

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P-3/3

AIDS/AIML/IOT-212

Ques 2 Order of convergence of Bisection method.

Let if root lies between x_0 & x_1

then $x_2 = \frac{x_0 + x_1}{2}$, ... $x_{i+1} = \frac{x_{i-1} + x_i}{2}$ → (1)

as we know, $\boxed{e_i = x_i - \alpha}$

So, $\boxed{x_i = e_i + \alpha}$

$\boxed{x_{i+1} = e_{i+1} + \alpha}$

$\boxed{x_{i-1} = e_{i-1} + \alpha}$

Put in Eqn (1)

$$e_{i+1} + \alpha = \frac{e_{i-1} + \alpha + e_i + \alpha}{2}$$

$$e_{i+1} + \cancel{\alpha} = \frac{e_{i-1} + e_i}{2} + \cancel{\alpha}$$

$$e_{i+1} = \frac{e_i}{2}\left[\underbrace{\frac{e_{i-1}}{e_i}}_{\text{Neglected}} + 1\right]$$

$$e_{i+1} = \frac{e_i}{2}[1]$$

$$\left[\frac{e_{i+1}}{e_i}\right] \leq \frac{1}{2}$$

compare it with

$$\left|\frac{e_{i+1}}{e_i^{p}}\right| \leq K$$

Here $K = 1/2$ & $p = 1$

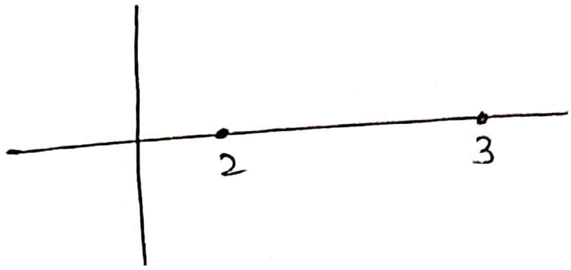
Hence order of convergence is 1.

derivation

Q3 $x^3 - 2x - 5 = 0$

$f(x) = x^3 - 2x - 5$

$$x_2 = \frac{f(x_1)\,x_0 - f(x_0)\,x_1}{f(x_1) - f(x_0)}$$



$f(1) = -6$

$f(2) = -1$ (-ve)

$f(3) = 16$ (+ve)

here $x_0 = 2$ $\quad x_1 = 3$

$f(x_0) = -1$ $\quad f(x_1) = 16$.

$$x_2 = \frac{16(2) - 3(-1)}{16 - \cancel{3}(-1)} = 2.058823$$

$\boxed{f(x_2) = -0.390805}$ (-ve)

Now, Root lies between 2.05882 & 3.

$$x_3 = \frac{16 \times 2.058823 - 3(-0.390805)}{16 - (-0.390805)} = \frac{34.113535}{16.390805}.$$

$$\boxed{\begin{aligned} x_3 &= 2.081260 \\ f(x_3) &= -0.1472378 \text{ (-ve)} \end{aligned}}$$

Root lies between 2.081260 & 3.

$$x_4 = \frac{16 \times 2.081260 - 3(-0.147237)}{16 - (-0.147237)} = \frac{33.741871}{16.147237}.$$

$\boxed{x_4 = 2.089637}$

two places. decimal Digit Repeated.

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Ques 4 ① $\boxed{5x} + 2y + z = 12$
② $x + \boxed{4y} + 2z = 15$
③ $x + 2y + \boxed{5z} = 20$

already arranged hai, no need to arrange them

from Eqn $x = \frac{1}{5}(12 - z - 2y)$

$y = \frac{1}{4}(15 - 2z - x)$

$z = \frac{1}{5}(20 - 2y - x)$

$x_1 = \frac{1}{5}(12 - 0 - 2(0)) = 2.4$ (x_1 ke lie y & z = 0 hoenge)

$y_1 = \frac{1}{4}(15 - 2(0) - 2.4) = 3.15$ (y_1 ke lie x ki pichle vali or z 0 hoga)

$z_1 = \frac{1}{5}(20 - 2(3.15) - 2.4) = 2.26$ (z_1 ke lie y & x ki pichle wali)

$x_2 = \frac{1}{5}(12 - 2.26 - 2(3.15)) = 0.688$ (y_1, z_1)

$y_2 = \frac{1}{4}(15 - 2(2.26) - 0.688) = 2.448$ (z_1, x_2)

$z_2 = \frac{1}{5}(20 - 2(2.448) - 0.688) = 2.8832$ (x_2, y_2)

$x_3 = \frac{1}{5}(12 - 2.8832 - 2(2.448)) = 0.84416$ (y_2, z_2)

$y_3 = \frac{1}{4}(15 - 2(2.8832) - 0.84416) = 2.09736$ (z_2, x_3)

$z_3 = \frac{1}{5}(20 - 2(2.09736) - 0.84416) = 2.992224$ (y_3, x_3)

$x_4 = \frac{1}{5}(12 - 2.9922 - 2(2.09736)) = 0.962616$ (y_3, z_3)

$y_4 = \frac{1}{4}(15 - 0.962616 - 2(2.9922)) = 2.013246$ (x_4, z_3)

$z_4 = \frac{1}{5}(20 - 2(2.01324) - 0.962616) = 3.00218$ (

$x_5 = \frac{1}{5}(12 - 3.00218 - 2(2.013246)) = 0.9942656$

$y_5 = \frac{1}{4}(15 - 2(3.00218) - 0.9942656) = 2.00034375$

$z_5 = \frac{1}{5}(20 - 2(2.00034375) - 0.9942656) = 3.00100953$

Hence, $\boxed{x \simeq 1, \quad y \simeq 2, \quad z \simeq 3}$

$Q5$ $f(x) = x^3$

x	0	1	2
y	0	1	8

$$\underbrace{0 \quad 1}_{S_1} \underbrace{\quad 8}_{S_2}$$

$$S_1(x) = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$ {linear spline formula}

$$= 0 + \frac{(1-0)}{(1-0)}(x-0)$$

$$\boxed{S_1(x) = x}$$

$$S_2(x) = y_2 + \frac{y_3 - y_2}{x_3 - x_2}(x - x_2)$$

$$= 1 + \frac{(8-1)}{(2-1)}(x-1)$$

$$= 1 + 7(x-1)$$

$$= 7x - 6.$$

Q6. actual value = $\frac{1}{3}$ approximate = 0.3333

Absolute Error = |Actual - Absolute|

$$= \frac{1}{3} - 0.333$$

$$= 0.000333$$

Relative = $\frac{\text{Absolute Error}}{\text{Actual value}} = \frac{0.000333}{1/3}$

$$= 0.000999$$

% Error = Relative x 100 %

$$= 0.00099 \times 100\ \%$$

$$= 0.0999\ \%$$

Q1

algorithm Complexities

space

time

Process of defining a formula for prediction of how much memory space is Required for ~~see~~ Successful Execution of algorithm

Process of determining a formula for total time Required towards Execution of that algorithm

Q7. $\int_0^6 \frac{dx}{1+x^2}$

x	0	1	2	3	4	5	6
$f(x)$	1	0.5	0.2	0.1	0.05882	0.03846	0.02702

(i) Trapezoidal

$$I = \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + \dots y_{n})\right]$$

$$= \frac{1}{2}\left[1 + 0.02702 + 2(0.5 + 0.2 + 0.1 + 0.05882 + 0.03846)\right]$$

$$= 1.41079$$

(ii) Simpson $\frac{1}{3}$ Rule.

$$I = \frac{h}{3}\left[y_0 + y_n + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$I = \frac{1}{3}\left[1 + 0.027 + 4(0.5 + 0.1 + 0.03846) + 2(0.2 + 0.05882)\right]$$

$$= 1.366166$$

(iii) Simpson $3/8$ Rule.

$$I = \frac{3h}{8}\left[y_0 + y_n + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)\right]$$

$$= 3 \times \frac{1}{8}\left[1 + 0.02702 + 3(0.5 + 0.1 + 0.05882 + 0.03846) + 2(0.1)\right]$$

$$= \quad 1.3571$$

Q10 $$3\frac{\partial^2 u}{\partial x^2} + \frac{5\partial^2 u}{\partial x \partial y} + \frac{4\partial^2 u}{\partial y^2} = 7$$

$B^2 - 4AC$

$(5)^2 - 4 \times 4 \times 3$

$-23 < 0$ So, Eq^n is elliptic

Ques 8 $\quad I = \int_0^1 \frac{dx}{1+x}$ $\quad$ {limits (-1, 1) nhi hai}

{so, change karenge}

$f(x) = \frac{1}{1+x}$

$x = \frac{1}{2}(b-a)t + \frac{1}{2}(b+a)$ $\quad$ {aaj ke formula mei ½ lagana bhul gyi thi}

$x = \frac{1}{2}(1-0)t + \frac{1}{2}(1+0) = \frac{1}{2}t + \frac{1}{2} = \boxed{\frac{1}{2}(t+1)}$

Now, $\boxed{x = \frac{1}{2}(t+1)}$

$dx = \frac{1}{2}dt$ $\quad$ {constant ko ~~Integration~~ Differentiation ....? aree? zero nhi hota kya.

isko simplify karke btati

$x = \frac{1}{2}(t+1)$

$x = \frac{1}{2}t + \boxed{\frac{1}{2}}$ constant

$dx = \frac{1}{2}dt + 0$

$\boxed{dx = \frac{dt}{2}}$

(smj aaya)

Put all value in (I)

$$I = \int_{-1}^{1} \frac{dt}{2\left[1+\frac{1}{2}(t+1)\right]}$$

$$= \int_{-1}^{1} \frac{dt}{\cancel{2}\left[\frac{2+(t+1)}{\cancel{2}}\right]}$$

$$= \int_{-1}^{1} \frac{dt}{2+(t+1)} = \int_{-1}^{1} \frac{dt}{t+3}$$

Now use formula (tho bhul gye let me tell you)
three pt formula

$$I = \frac{8}{9}f(0) + \frac{5}{9}\left[f\left(-\sqrt{\frac{3}{5}}\right) + f\left(\frac{\sqrt{3}}{\sqrt{5}}\right)\right]$$

$$= \frac{8}{9} \times 0.333 + \frac{5}{9}\left[0.449357 + 0.264929\right]$$

$$= 0.296 + 0.396825$$

$$= 0.692825$$

$f(0) =$ Put $t = 0$

$\frac{1}{3} = 0.33$

$f\left(-\frac{\sqrt{3}}{\sqrt{5}}\right) = \frac{1}{\left(-\frac{\sqrt{3}}{\sqrt{5}}\right)+3}$

$= \frac{1}{2.2254}$

$= 0.44935$

$f\left(\frac{\sqrt{3}}{\sqrt{5}}\right) = \frac{1}{3.77459}$

$= 0.26492$

(Answer miss match hoe to btana)

Ques9 RK method,

Solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ $y(0) = 1$ at $\boxed{y(0.2) = ?}$

$h = 0.2$

$x = 0$
$y = 1$

→ Initial condition (x_0, y_0)

$x = 0.2$
$y = ?$

Solution

$f(x_0, y_0)$

$f(0,1) = \frac{(1)^2 - (0)^2}{(1)^2 + (0)^2} = 1$

$\therefore k_1 = hf(x_0, y_0) = 0.2 \times 1 = 0.2$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}) = 0.2 \times f(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2})$

$= 0.2 f(0.1, 1.1)$

$= 0.2 \times 0.9836$

$= 0.19672$

$f(0.1, 1.1)$

$= \frac{(1.1)^2 - (0.1)^2}{(1.1)^2 + (0.1)^2}$

$= \frac{1.2}{1.22} = 0.9836$

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$

$= 0.2 f(0 + \frac{0.2}{2}, 1 + \frac{0.19672}{2})$

$= 0.2 f(0.1, 1.09836)$

$= 0.1967122$

$f(0.1, 1.09836)$

$= \frac{(1.09836)^2 - (0.1)^2}{(1.09836)^2 + (0.1)^2}$

$= \frac{1.196394}{1.21639}$

$= 0.98356$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.2 f(0 + 0.2, 1 + 0.1967122)$

$= 0.2 f(0.2, 1.1967122)$

$= 0.2 \times 0.94565$

$= 0.18913132$

$f(0.2, 1.1967122)$

$= \frac{(1.1967122)^2 - (0.2)^2}{(1.1967122)^2 + (0.2)^2}$

$= \frac{1.39212}{1.47212} = 0.94565$

$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 0.195999 \simeq 0.1960$

$y = y_0 + k$

$= 1 + 0.1960 = 1.1960$ Ans//

So, at $x = 0.2$
$y = 1.1960$