PSLA →.

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Probability:

Experiment

* Determinstic Experimient
* Random Experiment.

Deterministic - If an Experiment when Repeated under identical conditions produce the same outcome Everytime then this type Of Experiment known as "Deterministic Experiment"

Random - If an Experiment when Repeated under identical condition do not produce the same Result but the outcome in a trial is one of the sevral possible outcome such an experiment is known as Random Experiment

Sample space :- The set of all possible outcomes in a Random Experiment is called sample space (genrally denoted by

* The Element of sample space are called sample Element / sample point or Elementary Elements

* $S = \{H, T\}$ $n(S) = 2$

$S = \{HH, HT, TH, TT\}$ $n(S) = 4$

$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

$n(S) = 8$

Event - A subset of sample space associated with Random experiment is called an Event.
E, A, B, C ... denoted (capital letter)

$S = \{1, 2, 3, 4, 5, 6\}$
A = Even no = $\{2, 4, 6\}$
B = odd no = $\{1, 3, 5\}$
C = Prime no = $\{2, 3, 5\}$

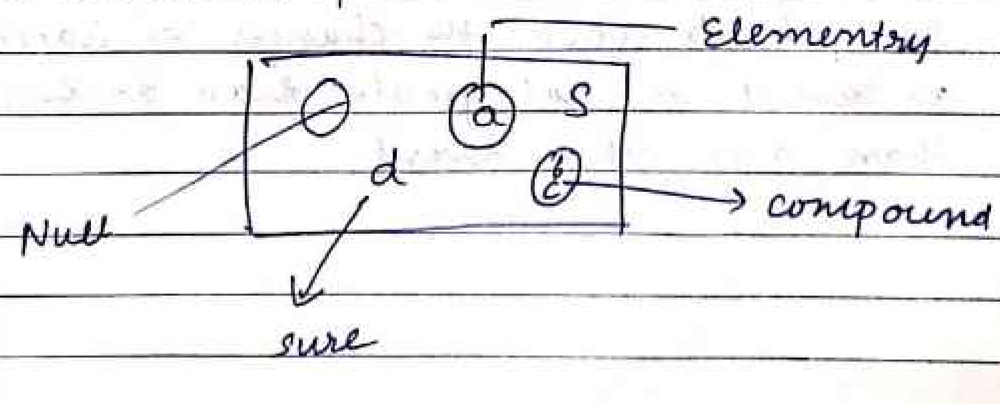
Types of Events -

- **Elementry Event**. If an Random Experiment is performed than an event which contain only one Element is called Elementary Event
For Exp.

- **Impossible Event** : Null subset.
we know that ϕ is subset of Every set & therefore it is an event is called Impossible event.

- **Certain/ Sure Event** : we know that every set is a subset of itself. Hence S is an event & it is known as sure event.

- **compound Event** : More than 1
If an event contain more than one Element is called compound event.



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* Exhaustive Event :-
Event of a Random Experiment are said to be Exhaustive if atleast one of them necessiraly occours.

$$\boxed{A \cup B = S}$$

$S = \{1,2,3,4,5,6\}$
A = Even no = $\{2,4,6\}$
$B = \{1,3,5\}$ = odd no.

$$\boxed{A \cup B = S}$$

* ~~Exa~~ Mutually Exclusive Events
$A \cap B = \phi$
Two Events A & B are said to be mutually Exclusive if the occurence of anyone of them Excludes the occurence of other. i.e., They cannot occur simultaneously.

$A = \{2,4,6\}$
$B = \{1,3,5\}$
$$\boxed{A \cap B = \phi}$$

Equally likely Events :
A set of events are said to be Equally likely Events when the chances of happening an Event is not greater than or less than any other event.

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Favourable Elementry Event -
jiski probab find karni hai

Probability -
If there are n Elementry events associated with a Random Experiment and m of them are favourable to an event A then the probability of occurance of "A" is denoted by P(A) and is defined as

$$P(A) = \frac{\text{favourable number of element}}{\text{Total number of elements}}$$

$$= \frac{m}{n}$$

Results
→ $P(\text{sure event}) = 1$
→ $P(\phi) = 0$
→ $0 \leq P(A) \leq 1$
→ $P(A) + P(\bar{A}) = 1$
→ $P(A+B)$ or $P(A \cup B)$ · {Probablity of happenin of event A or event B}

→ Probability of happening of event A and event B

$P(A \times B)$ or $P(A \cap B)$

Addition theorem on probability
If A & B are two events associated with a Random experiment then

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Note - If A and B are mutually Exclusive events then $\boxed{A \cap B = \phi}$ or $\boxed{P(A \cap B) = \phi}$

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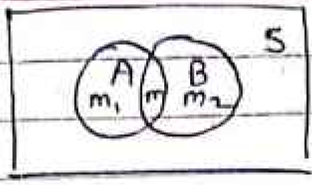
So, addition theorem,

$$\boxed{P(A \cup B) = P(A) + P(B)}$$



$$\cancel{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$
$$\cancel{= m_1 + m_2 - m}$$

Let total no. of Elementry events $= n$
No. of Elementry event fav. to $A = m_1$
No. of Elementry events fav. to $B = m_2$
No. of Elementry events fav. to $A \cap B = m$

No. of Elementry fav. to only $B = m_2 - m$
No. of Elementry fav. to only $A = m_1 - m$

No. of Element elvent fav. to $A \cup B = m_1 + m_2 - m$

$$P(A \cup B) = \frac{m_1 + m_2 - m}{n}$$

$$P(A \cup B) = \frac{m_1}{n} + \frac{m_2}{n} - \frac{m}{n}$$

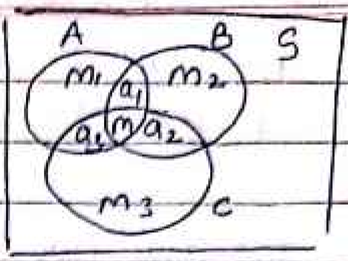
$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Homework

Ques If A, B and C are three events associated with Random Experiment

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$$
$$- P(B \cap C) - P(C \cap A) + P(A \cup B \cup C)$$

$$P(A\cup B\cup C) = P(A)+P(B)+P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$$



Let total no. of Elementry Event $= n$

No. of Events fav. to $A = m_1$

No. of Events fav. to $B = m_2$

No. of Events fav. to $C = m_3$

No. of Events fav. to

only $A = m_1 - a_1 - a_3 + m$.

only $B = m_2 - a_1 - a_2 + m$.

only $C = m_3 - a_2 - a_3 + m$.

No. of Events fav. to $A\cap B = a_1 - m$

No. of Events fav. to $B\cap C = a_2 - m$

No. of Events fav. to $C\cap A = a_3 - m$.

No. of Events fav. to $A\cap B\cap C = m$

Now, Probability $= \dfrac{\text{No. of favourable cases}}{\text{No. of total cases}}$

$$P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$$

$$= \frac{m_1}{n} + \frac{m_2}{n} + \frac{m_3}{n} - \frac{(a_1-m)}{n} - \frac{(a_2-m)}{n} - \frac{(a_3-m)}{m} + \frac{m}{n}$$

~~$P(A\cup B\cup C) =$~~

No. of Element fav. to $A\cup B\cup C$ $= m_1 + m_2 + m_3 - (a_1 - m) - (a_2 - m) - (a_3 - m) + m$

$$= \frac{m_1}{n} + \frac{m_2}{n} + \frac{m_3}{n} - \frac{(a_1-m)}{n} - \frac{(a_2-m)}{n} - \frac{(a_3-n)}{n} + \frac{m}{n}$$

$$= P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$$

Hence proved,,

31/08/23 For three events A, B & C.
For mutually Exclusive Events,
$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

For any two events A and B' Prove that
$P(A \cap B') = P(A) - P(A \cap B)$.

$(A \cap B') \cup (A \cap B) = A$
$P[(A \cap B') \cup (A \cap B)] = P(A)$

Since $A \cap B'$ and $A \cap B$ are mutually Exclusive
therefore,
$P(A \cap B') + P(A \cap B) = P(A)$
$P(A \cap B') = P(A) - P(A \cap B)$

Similarly
$P(A' \cap B) = P(B) - P(A \cap B)$

Ques what is the chences that a leap year ~~will~~ be selected at Random will contain 53 Sunday

$$\begin{array}{r|l} & 52 \\ 7 & 366 \\ & \underline{35} \\ & 16 \\ & 14 \end{array}$$

~~Total~~ outcome = 7

A leap year consists of 366 days, so that there are 52 full weeks and two Extra days
These 2 Extra days are-

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{ (S, M) (M, Tue) (Tue, W) (W, Th), (Th, Fri) (Fri, Sat) (Sat, Sun) }

Favourable = 2 Total = 7

Req. Probab = $\frac{2}{7}$ Ans.

Ques Each cofficient in the Equation $ax^2+bx+c=0$ is determined by throwing an ordinary die. find the possibility that the Eq^n will have Real roots?

ax^2+bx+c

Dis $b^2-4ac \geq 0$ (The Eq^n will have Real Roots)

ac	a	c	4ac	b^2 ($b^2 \geq 4ac$)	No. of cases
1	1	1	4	2, 3, 4, 5, 6	5
2	1 2	2 1	8	3, 4, 5, 6	4 × 2 = 8
3	1 3	3 1	12	4, 5, 6	3 × 2 = 6
4	4 1 2	1 4 2	16	4, 5, 6	3 × 3 = 9
5	5 1	1 5	20	5, 6	2 × 2 = 4
6	1 6 2 3	6 1 3 2	24	5, 6	4 × 2 = 8
7	N	O	T	Possible	×
8	2 4	4 2	32	6	2 × 1 = 2
9	3	3	36	6	1 × 1 = 1

Total = 43.

$n_{C_r} = \frac{\lfloor n}{\lfloor n-r \cdot \lfloor r}$ ~~...~~ $= 43$

Favourable outcome = 43

Total outcom = $6 \times 6 \times 6 = 216$

Probability $= \frac{43}{216}$ Ans//

Ques The probab that atleast one of the events A & B occour is 0.6 If A & B occour simultaneously with probab. 0.2 then find the $P(\bar{A}) + P(\bar{B})$

$P(A \cup B) = 0.6 \qquad P(A \cap B) = 0.2$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.6 = P(A) + P(B) - 0.2$

$0.8 = P(A) + P(B)$

$0.8 = 1 - P(\bar{A}) + 1 - P(\bar{B})$

$P(\bar{A}) + P(\bar{B}) = 2 - 0.8$

$\boxed{P(\bar{A}) + P(\bar{B}) = 1.2}$

Ques. A card is drawn from a deck of 52 cards find the Probab of getting a King or a heart or a Red card.

A → King ; B → Heart ; C → Red card

$P(A) = \frac{4}{52} \qquad P(B) = \frac{13}{52} \qquad P(C) = \frac{26}{52}$

$P(A \cup B \cup C) = ?$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) - P(A \cap B \cap C)$

A B C
6 6 6

13 + 26 + 5 = 44

44

26 + 4 + 14 = 30

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$$= \frac{4}{52} + \cancel{\frac{13}{52}} + \frac{26}{52} - \cancel{\frac{1}{52}} - \cancel{\frac{13}{52}} - \cancel{\frac{2}{52}} + \frac{1}{52}$$

$$= \frac{}{52}$$

$$= \frac{28}{52} \text{ Ans//}$$

1/09/23

Que A Bag contain 8 white & 6 Red Balls. find the probabrality of drawing two balls of same colour.

Ans

8-W
6-R

Total outcome $= {}^{14}C_2$

favourable $= {}^{8}C_2 + {}^{6}C_2$

$$P(\text{of drawing Ball of same clr}) = \frac{{}^8C_2 + {}^6C_2}{{}^{14}C_2}$$

$$= \frac{\frac{\lfloor 8}{\lfloor 2 \lfloor 6} + \frac{\lfloor 6}{\lfloor 2 \lfloor 4}}{\frac{\lfloor 14}{\lfloor 12 \lfloor 2}}$$

$$= \frac{\frac{8\times 7}{2} + \frac{6\times 5}{2}}{\frac{14\times 13}{2}}$$

$$= \frac{8\times 7 + 6\times 5}{14\times 13}$$

$$= \frac{56+30}{182} = \frac{86}{182}$$

unit - 1

Probability spaces :-
is a three tuple (S F, P) in which three components are

(1) Sample space (S) {A Non Empty set $S \neq 0$ is called ~~repAct~~ sample space which Repersents all possible outcomes}

(2) Event space (F) { ~~Sub~~ A collection of subset of S is called Event space (F)}

(3) Probability measure (P) { P assigned each Event in the Event space F a probability

$(P : F \rightarrow R)$

Example Tossing a coin:
$S = \{H, T\}$
$F = \{\{\ \}, \{H, T\} \{H\}, \{T\}\}$
$P\{\ \} = 0$
$P\{H, T\} = 1$
$P\{H\} = \{T\} = 1/2$

Conditional probability
Let A and B be two event associated with a Random Experiment then the probability of occurance of Event "A" under the condition that B has already occured. and probability Of B is not zero is called conditional probability (A/B)

$$\boxed{P(A/B) = \frac{n(A \cap B)}{n(B)}}$$

Probablity of A when B has already occured

Probability of B when A has already occur

$$\boxed{P(B/A) = \frac{n(A \cap B)}{n(A)}}$$

(5,1) (2,4)
(1,5)
(3,3)
(4,2)

Ques A dice is thrown twice and sum of number appearing is observed to be 6. what is probablity that the no. 4 has apperead atleast once.

A : 4 appeared atleast once {(2,4)(4,2)}

B : Sum is 6. {(5,1)(1,5)(2,4)(4,2)(3,3)}

$$P(A/B) = \frac{P(A \cap B)}{P(A)} = \frac{2}{5} \text{ Ans}$$

Q. what is the probability of Rolling a die & is value is less than 4. knowing that the value is an odd number? 1,2,3,4,5,6

A : Number less than 4.

✓ B : value is an odd number

$$P(A/B) = \frac{n(A \cap B)}{n(B)} = \frac{2}{3} \text{ Ans}$$

Multiplication theorem :-

If A and B are two events associated with a Random Experiment. then

$$P(A \cap B) = P(A) \cdot P\left(\frac{B}{A}\right) \quad P(A) \neq 0$$

or

$$P(A \cap B) = P(B) \cdot P\left(\frac{A}{B}\right) \quad P(B) \neq 0$$

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Proof - Homework.

Proof multiplication theorem

as we know that conditional probab is given by-

$$P(A|B) = P(A\cap B)/P(B)$$

$\forall\ P(B) \neq 0$

$$P(A\cap B) = P(B)\cdot P(A|B) \longrightarrow (1)$$

$$P(B|A) = P(B\cap A)/P(A)$$

$\forall\ P(A) \neq 0$

$$P(B\cap A) = P(A)\cdot P(B|A) \longrightarrow (2)$$

~~Eq~~ ~~and~~ ~~1 & 2~~

~~ets~~, since, $P(A\cap B) = P(B\cap A)$

~~P(A A~~ from Eq^n 1 & 2

$$\boxed{P(A\cap B) = P(B)\cdot P\left(\frac{A}{B}\right) = P(A)\cdot P\left(\frac{B}{A}\right)}$$ Ans.

Hence proved,,

04/09/23

Baye's theorem

If E_1, E_2 and .... E_n are mutually Exclusive and Exhaustive Events with $P(E_i) \neq 0$ $(i = 1, 2, 3 \ldots, n)$ of a Random Experiment then for any arbitrary Event A of the Sample Space of the above Experiment with $P(A) > 0$.

we have,

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}$$

Proof Let S be the sample space of the Random Experiment. The events $E_1, E_2 \ldots E_n$ being Exhaustive.

$$S = E_1 \cup E_2 \cup \ldots E_n$$

$$A = A \cap S$$

$$A = A \cap (E_1 \cup E_2 \cup \ldots \cup E_n)$$

$P(E_i) \neq 0 \rightarrow$ Event Impossible nhi hai

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$A = (A \cap E_1) \cup (A \cap E_2) \cup \dots \cup (A \cap E_n)$

(Distributive law)

$P(A) = P(A \cap E_1) + P(A \cap E_2) + \dots + P(A \cap E_n)$

$= P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \dots + P(E_n)P(A/E_n)$

$P(A) = \sum_{i=1}^{n} P(E_i)P(A/E_i)$

$$P(E_i/A) = \frac{P(A \cap E_i)}{P(A)} = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}$$

Hence proved//

Ques Each of the identical Boxes B_1, B_2 & B_3 contains two coins.

B_1 contains Both Gold coins.

B_2 Both silver coins.

B_3 contains one gold & one silver coin.

Picked at Random and if coin is

If a Box is chosen at Random and the coin is gold, what is the probability that the other coin in the box is also gold?

$P(E_1) = 1/3$ $\quad P(A/E_1) = 2/2 = 1$

$P(E_2) = 1/3$ $\quad P(A/E_2) = 0/2 = 0$

$P(E_3) = 1/3$ $\quad P(A/E_3) = 1/2$

$\frac{1}{3}$
$\frac{1}{2}$

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{\sum_{i=1}^{3} P(E_1)P(A/E_1)}$$

$\frac{2+1}{6}$

$\frac{\cancel{3}}{\cancel{6}} \frac{1}{2}$

$$P(E_1/A) = \frac{1/3 \times 1}{\frac{1}{3}(1) + \frac{1}{3}(0) + \frac{1}{3} * \frac{1}{2}} = \frac{1/3}{\frac{1}{3} + \frac{1}{6}}$$

$= \frac{2}{3}$ Ans//

$\frac{0.01}{100}$

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Ques An insurence company insured 2000 Scooter drivers 4000 car drivers & 6000 truck drivers The probability of any accident involving a scooter, car and truck. is 0.01, 0.03 and 0.15 respectively. one of the insured person meets with an accident what is the probability the he is a scooter driver?

$P(E_1) = 2000/12000$ $\quad P(A/E_1) = 0.01$

$P(E_2) = 4000/12000$ $\quad P(A/E_2) = 0.03$

$P(E_3) = 6000/12000$ $\quad P(A/E_3) = 0.15$

$$P(E_1/A) = \frac{\frac{2000}{12000} \times \frac{1}{100}}{\frac{2000}{12000} \times \frac{1}{100} + \frac{4000}{12000} \times \frac{3}{100} + \frac{6000}{12000} \times \frac{15}{100}}$$

90000
12000
2000
104000

12000
9000
2000
23000

$$= \frac{2000}{2000 + 12000 + 90000}$$

$$= \cancel{\frac{2000}{23000}} \quad \frac{2000}{104000} \quad \frac{1}{52} \text{ Ans}$$

$$= \cancel{\frac{2}{23}} \text{ Ans}$$

Ques A university bought 45%, 25% & 30% of computers from HCL, WIPRO & IBM Resp. & 2%, 3% & 1% Of these were found to be defective. Find the prob of a computer Selected at Random is found to be defective. what is the probability that the defective. what is the probability that the defective computer is from HCL?

$P(E_1) = 45/100$ $P(A/E_1) = 2/100$

$P(E_2) = 25/100$ $P(A/E_2) = 3/100$

$P(E_3) = \frac{30}{100}$ $P(A/E_3) = \frac{1}{100}$

$$P(E_1/A) = \frac{\frac{45}{100} \times \frac{2}{\cancel{100}}}{\frac{45}{\cancel{100}} \times \frac{2}{100} + \frac{25}{\cancel{100}} \times \frac{3}{100} + \frac{30}{\cancel{100}} \times \frac{1}{100}}$$

$$= \frac{90}{90+75+30}$$

$$\begin{array}{r} 90 \\ 75 \\ 30 \\ \hline 195 \end{array}$$

$$= \frac{90}{195}$$

$$= 0.46$$

Ques In a bolt factory machine A, B & C manufacture resp 25%, 35% & 40% of the total on their output 5%, 4% & 2% are defective bolts. A bolt is drawn at Random from the product & is found to be defective. what is the probality that it was manufactured by machine B?

Sol $P(E_1) = 25/100$ $P(A/E_1) = 5/100$

$P(E_2) = 35/100$ $P(A/E_2) = 4/100$

$P(E_3) = 40/100$ $P(A/E_3) = 2/100$

$$P(E_2/A) = \frac{\frac{35}{100} \times \frac{4}{\cancel{100}}}{\frac{25}{\cancel{100}} \times \frac{5}{\cancel{100}} + \frac{35}{\cancel{100}} \times \frac{4}{100} + \frac{40}{100} \times \frac{2}{\cancel{100}}}$$

$$\begin{array}{r} {}^{2}35 \\ \times 4 \\ \hline 140 \\ +125 \\ +80 \\ \hline 345 \end{array}$$

$$= \frac{140}{125+140+80}$$

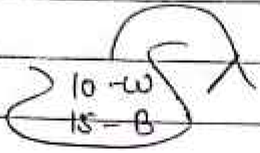
$$= \frac{140}{345}$$

$$= 0.40 \text{ Ans}$$

Ques A Bag contain 10 white 15 Black balls Two Balls are drawn in sucession without Replacement what is the prob that 1^{st} Ball is white & 2^{nd} is Black ?

A : I^{st} white Ball

B : II^{nd} Black Ball



$$P(A \cap B) = P(A)\, P(B/A)$$

$$= \frac{\cancel{10}\,2}{\cancel{25}\,5} \times \frac{\cancel{15}\,3}{\cancel{24}\,12} \quad \frac{1}{4}$$

$$= \frac{1}{4} \text{ Ans}$$

Independent Event -

Events are said to be independent if the ocurence or non occurrnce of one event doesn't effect the probability of the occurence or non occurence of other event.

$$P(B/A) = P(B)$$
$$P(A/B) = P(A)$$

If A & B are Independent Events then

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

If A, B & C are Independent Events

$$P(A \cap B \cap C) = P(A)\, P(B)\, P(C)$$

If $P(A \cap B) \neq P(A)\, P(B)$

then A & B are not Independent.

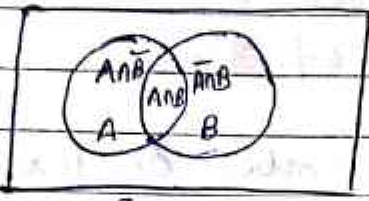
$\overline{A \cup B} = \bar{A} \cap \bar{B}$ Exh $A \cup B = S$

Theorem : If A & B are Independents Events then

- $\bar{A}$ & $\bar{B}$ are also Independent
- A & $\bar{B}$ are also Independent
- $\bar{A}$ & $\bar{B}$ are also Independent

$y = f$

i) **Proof**

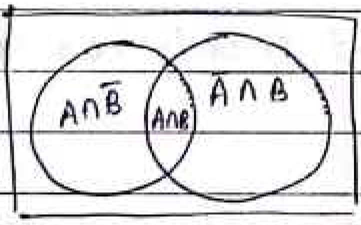


$(A \cap B) \cup (\bar{A} \cap B) = B$

BA

$$P[(A \cap B) \cup (\bar{A} \cap B)] = P(B)$$
$$P(A \cap B) + P(\bar{A} \cap B) = P(B)$$
$$P(A) \cdot P(B) + P(\bar{A} \cap B) = P(B)$$
$$P(\bar{A} \cap B) = P(B) - P(A)P(B)$$
$$P(\bar{A} \cap B) = P(B)[1 - P(A)$$
$$\boxed{P(\bar{A} \cap B) = P(B)P(\bar{A})}$$ Ans//

iii)



$\boxed{\text{To prove}}$ $P(\bar{A} \cap \bar{B}) = P(\bar{B})P(\bar{A})$

proof

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$
$$\cancel{P(\overline{A \cup B}) = P(\bar{A} \cap \bar{B})}$$

$$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$$
$$= 1 - P(A \cup B)$$
~~[illegible]~~
$$= 1 - [P(A) + P(B) - P(A \cap B)]$$
$$= 1 - P(A) - P(B) + P(A \cap B)$$
$$= 1 - P(A) - P(B) + P(A)P(B)$$
$$= \cancel{P(A)[P(B)+1]}$$
~~[illegible]~~
$$= P(\bar{A}) - P(B) + P(A)P(B)$$
$$= P(\bar{A})P(\bar{B})$$

Ques Two dice are thrown find the probab of getting an odd number on the first die ~~first~~ and multiple of 3 on other die?

A = odd number on first die

B = Multiple of 3

(1 1)
(1 2)
(1 3)
(1 4)
(1 5)
(1 6)

$P(A \cap B) = P(A) \cdot P(B)$

$= \frac{1}{2} \cdot \frac{1}{3}$

$= \frac{1}{6}$ Ans

Homework

Ques Proof that two independent Events cannot be mutually Exclusive.

Q In problem in Statistics is given to 3 students A, B & C whose chances of solving it are 1/2, 3/4, 1/4 Respectively. what is the probab that the problem will be solved all of them if all of them try Independently?

$P(A) = 1/2$ $\quad P(B) = 3/4$ $\quad P(C) = \frac{1}{4}$

$P(\bar{A}) = 1/2$ $\quad P(\bar{B}) = \frac{1}{4}$ $\quad P(\bar{C}) = \frac{3}{4}$

The problem will be solved if atleast one of them will solve the problem i.e,

Req. Prob $= P(A \cup B \cup C) = 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$$= 1 - P(\bar{A})P(\bar{B})P(\bar{C})$$

$$= 1 - \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}$$

$$= 1 - \frac{9}{32}$$

$$= \frac{29}{32}$$

$P(A) = \frac{1}{2}$

imp.

Ques. A problem in statistics is given to 3 students A, B & C whose chances of solving it are 0.5, 0.75 & 0.25. What will the probability will not be solved?

Problem will not be solved $= 1 - \frac{29}{32} = \frac{3}{32}$

Homework

Two independent Event cannot be mutually Exclusive? i.e., T.P $\Rightarrow P(A \cap B) \neq 0$

$A \cap B \neq \phi$

Proof Let us consider two Events A & B which are independent.

i.e., $P(A \cap B) = P(A)P(B) \rightarrow (1)$

as we know that for mutually Exclusive Events

$P(A \cap B) = 0 \rightarrow (2)$

From (1) & (2)

$P(A)P(B) = 0$

$\Rightarrow P(A) = 0$ or $P(B) = 0$

and it is not possible because A & B are independent Events and they do not Effect by the occurence of other.

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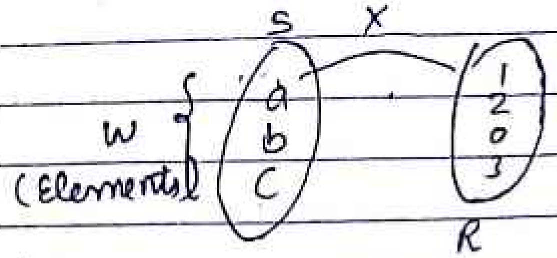
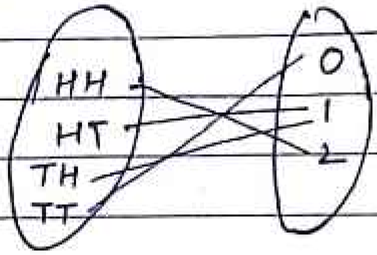
Random variables / varite -

Let S be the sample space associated with a Random Experiment. A function X assigning to every Element $w \in S$ to a real number, $X(w)$ is called a Random variable.





$X = 0, 1, 2$ = No. of head
↳ Random variable.

X = No. of head − No. of tail
$= 0, 2, -2$

Types of Random variable

(1) discreate Random variable (2) continous.

(1) discreate Random : A ~~Random variable x is called~~ discreate Random variable is a variable that can take a finite number of distinct values.

Eg: The number of Employees in a company

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(2) continous Random variable - A Random variable is said to be continous if it can assume any value in some interval or intervals.

for Exp $x \in [0,1]$, weight of a man $\in (40,69)$

$$f(x) = \begin{cases} x & 1 < x < 2 \\ 2 & 2 < x < 3 \end{cases}$$

Theorems on Random variable :-

(1) A function $X(\omega)$ from $S \rightarrow R$ is a Random variable iff for a Real number a $\{\omega : X(\omega) < a\} \in \mathcal{F}$

(2) If X_1 & X_2 are Random variables and C is a constant then CX_1, CX_2, $X_1 + X_2$ and $X_1 X_2$ are also Random variables

Distribution function :-

Let X be a Random variable The function F defined for all real x by $F(x) = P(X \le x)$ where $-\infty < x < \infty$ is called distribution function of Random variable X ($F_X(x)$)

$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

$X \rightarrow$ no. of heads $= 0, 1, 2, 3$

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

$F(x) = P(X \le x)$

$F(0) = P(X \le 0)$

$= 0 + 1/8$

$= \frac{1}{8}$

$F(1) = P(X \le 1) = P(X=0) + P(X=1)$

$= \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$

$F(2) = P(X \le 2)$

$= P(X=0) + P(X=1) + P(X=2)$

$= \frac{7}{8}$

$F(3) = P(X \le 3)$

$= P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 1$

Distribution function

$$F(x) = \begin{cases} 0 & -\infty < x < 0 \\ 1/8 & 0 \le x < 1 \\ 1/2 & 1 \le x < 2 \\ 7/8 & 2 \le x < 3 \\ 1 & x \geq 3 \end{cases}$$

Properties of Distribution function.

1. If F is distribution function of Random variable X and $a < b$ then

$$P(a < x \le b) = F(b) - F(a)$$

2. If $0 \le F(x) \le 1$ and $x < y$ then $F(x) \le F(y)$
i.e., distribution function is increasing function.

$$P(a < x < b) = P(a < X \le b) - P(X = b)$$
$$= F(b) - F(a) - P(X = b)$$

$$P(a \le X < b) = P(X = a) + P(a < x < b)$$
$$= F(b) - F(a) - P(X = b) + P(X = a)$$

$$P(a \le x \le b) = P(X = a) + P(a < X \le b)$$
$$= P(X = a) + F(b) - F(a)$$

Ques A Random variable x assumes the value $-1, 0, 1$ with Probability $P(x) = \frac{1}{3}, \frac{1}{2}, \frac{1}{6}$ Determine the distribution function of x.

x	-1	0	1
$P(x)$	$1/3$	$1/2$	$1/6$

$$F(0) = P(x \le 0) = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

$$F(-1) = P(x \le -1) = \frac{1}{3}$$

$$F(1) = P(x \le 1) = \frac{1}{3} + \frac{1}{2} + \frac{1}{6} = 1$$

$$F(x) = \begin{cases} 0 & -\infty < x < -1 \\ 1/3 & -1 \le x < 0 \\ 5/6 & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

Probability mass function -

If X is a discreate Random variable with distinct values $x_1, x_2, \ldots x_n$ then the function $p(x)$ defined as

$$p(x) = \begin{cases} P(X = x_i) = P_i & x = x_i \\ 0 & x \ne x_i \end{cases}$$

is called the probability mass function of Random variable x

$\{x_i, P_i\} \rightarrow$ Probability distribution of Random variable X.

x	0	1	2
P	1/2	1/3	1/6

$= \{(0, \frac{1}{2}) (1, \frac{1}{3}) (2, \frac{1}{6})\}$

tossing three coin -

S = { HHH, HHT, HTH, THH, TTH, THT, HTT, T

x = Number of heads = 0, 1, 2, 3

x_1, x_2, x_3, x_4

~~$p(x_1) = \frac{1}{8}$~~ ~~$p(x_2) = \frac{3}{8}$~~

$p(x_1) = \{ p(X = x_1) = \frac{1}{8}$

$p(x_2) = 3/8$ $p(x_3) = \frac{3}{8}$ $p(x_4) = \frac{1}{8}$

$$p(x) \begin{cases} 1/8 & x = 0 = x_1 \\ 3/8 & x = 1 = x_2 \\ 3/8 & x = 2 = x_3 \\ 1/8 & x = 3 = x_4 \\ 0 & x = x_i \ (i = 1, \ldots \end{cases}$$

$S = \{(0, \frac{1}{8}), (1, 3/8), (2, \frac{3}{8}) (3, \frac{1}{8})$

The number $p(x_i)$ must satisfy the following condition

(1) $P(x_i) \geq 0 \quad \forall i$

(2) $\sum_{i=1}^{n} p(x_i) = 1$

<u>Note</u> - The set of values which X takes is call the spectrum of Random variable.

A Random variable X has probabilits following.

X	0	1	2	3	4	5	6	7
P(X)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

find the value of K and (i) $P(X \geq 6)$

$P(0<X<5)$

$P(X \leq 6)$

$$0+K+2K+2K+3K+K^2+2K^2+7K^2+K=1$$

$$9K+10K^2-1=0$$

$$10K^2+9K-1=0$$

$$10K^2+10K-K-1=0$$

$$10K(K+1)-1(K+1)$$

$$\boxed{K=\frac{1}{10}}, \cancel{-1}$$

$$P(X \not< 6) = P(X=0)+P(X=1)+P(X=2)+P(X=3)$$

$$+P(X=4)+P(X=5)$$

$$= 0+\frac{1}{10}+\frac{2}{10}+\frac{2}{10}+\frac{3}{10}+\frac{1}{100}$$

$$= \frac{10+20+20+30+1}{100}$$

$$= \frac{81}{100} \qquad \left\{\begin{array}{l} 1-0.19 \\ =0.81 \end{array}\right\}$$

$$=0.81$$

$$P(0<x<5) = P(X=1)+P(X=2)+P(X=3)+P(X=4)$$

$$= \frac{1}{10}+\frac{2}{10}+\frac{2}{10}+\frac{3}{10}$$

$$= \frac{8}{10} = 0.8$$

$P(X \geq 6) = 2K^2 + 7K^2 + K = 9K^2 + K$

$= \frac{19}{100} = 0.1$

Ques If $p(x) = \begin{cases} x/15 & x = 1, 2, 3, 4, 5 \\ 0 & \text{otherwise.} \end{cases}$

find probability -

(i) $P(X = 1 \text{ or } 2)$

(ii) $P\{(\frac{1}{2} < x < \frac{5}{2}) \mid x > 1\}$

(i) $P(X = 1 \text{ or } 2) = P(1 \cup 2)$

$= P(1) + P(2)$

$= \frac{1}{15} + \frac{2}{15} = \frac{3}{15}$

(ii) $P\{\underbrace{(1/2 < X < 5/2)}_{A} \mid \underbrace{x > 1}_{B}\} =$

Multiplication thm,

$\frac{P(A \cap B)}{P(B)} = P(A/B)$

$P(A \cap B) = 2/15 \qquad P(B) = 14/15$

$P(A/B) = \frac{2}{15} \times \frac{15}{14} = \frac{1}{7}$ Ans

Ques Two dice are rolled Let X denote the Ran variable which counts the total number of points on the unturned faces construct A tab giving the non zero values of the prob. mass f Also find distribution function of X ?

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Ans X = total number of ~~faces~~ points on unturned faces.

Soln: $X = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$

X		P(X)
1	NOT Possible	
2	(1,1)	1/36
3	(1,2)(2,1)	2/36
4	(1,3)(3,1)(2,2)	3/36
5	(2,3)(3,2)(4,1)(1,4)	4/36
6	(5,1)(1,5)(3,3)(2,4)(4,2)	5/36
7	(6,1)(1,6)(3,4)(4,3)(2,5)(5,2)	6/36
8	(4,4)(2,6)(6,2)(3,5)(5,3)	5/36
9	(6,3)(3,6)(5,4)(4,5)	4/36
10	(5,5)(6,4)(4,6)	3/36
11	(5,6)(6,5)	2/36
12	(6,6)	1/36

$F(2) = P(X \le 2) = \frac{1}{36}$

$F(3) = P(X \le 3) = \frac{1}{36} + \frac{2}{36} = \frac{3}{36}$

$F(4) = P(X \le 4) = \frac{6}{36}$

$F(5) = P(X \le 5) = \frac{10}{36}$

$F(6) = P(X \le 6) = \frac{15}{36}$

$F(7) = P(X \le 7) = \frac{21}{36}$

$F(8) = P(X \le 8) = \frac{26}{36}$

$F(9) = P(X \le 9) = \frac{30}{36}$

$F(10) = P(X \le 10) = \frac{33}{36}$

$F(11) = P(X \le 11) = \frac{35}{36}$

$F(12) = P(X \le 12) = 1$

discreate → Mass function
continous → density function

$$f(x) = \begin{cases} 0 & x < 2 \\ 1/36 & 2 \leq x < 3 \\ 1/13 & 3 \leq x < 4 \\ 1/6 & 4 \leq x < 5 \\ 5/18 & 5 \leq x < 6 \\ 5/13 & 6 \leq x < 7 \\ 7/13 & 7 \leq x < 8 \\ 13/18 & 8 \leq x < 9 \\ 5/6 & 9 \leq x < 10 \\ 11/13 & 10 \leq x < 11 \\ 35/36 & 11 \leq x < 12 \\ 1 & x \geq 12 \end{cases}$$

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Probability density function -

X → continous Random variable

$$f(x) = \int_{-\infty}^{\infty} f(x)\,dx$$

Properties

$f(x) \geq 0$

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$P(\alpha \leq X \leq \beta) = \int_{\alpha}^{\beta} f(x)\,dx$$

kisi particular point pe probab 0 hogi

$P(\alpha \le X \le \beta) = P(\alpha < X < \beta) = P(\alpha \le X < \beta) = P(\alpha < X \le \beta)$
$= F(b) - F(a)$

In case of continous Random variable it doesn't matter weather we include the end point or not.

$P(2 \le x \le 3) = F(3) - F(2)$

Ques Let X be the continous Random variable with probability density function:

$$f(x) = \begin{cases} ax & 0 \le x \le 1 \\ a & 1 \le x \le 2 \\ -ax+3a & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

Find (i) a (ii) find $P(x \le 1.5)$

(i) $a = ?$

$$1 = \int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (-ax+3a)\,dx$$

$$1 = \left[\frac{ax^2}{2}\right]_0^1 + \left[ax\right]_1^2 + \left[-\frac{ax^2}{2} + 3ax\right]_2^3$$

$$1 = \frac{a}{2} + 2a - a - \frac{9a}{2} + 9a + 2a - 6a$$

$$\cancel{1 = -\frac{8a}{2} + 8a} = \cancel{-4a + 8a}$$

$\cancel{4a = 1}$

$\boxed{a = 1/4}$

$-\frac{8a}{2} + a + 9a - 4a = 1$

$-4a + 6a = 01$

$2a = 1$

$\boxed{a = 1/2}$

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For discreate

(ii) $P(X \leq 1.5) = P(-\infty < x \leq 1.5)$

$= P(-\infty < x \leq 0) + P(0 \leq x \leq 1) + P(1 \leq x \leq 1.5)$

$= 0 + F(1) - F(0) + F(1.5) - F(1)$

$= \frac{1}{2} - 0 + \frac{1}{2} - \frac{1}{2}$

$= \frac{1}{2}$

$$P(-\infty \leq X \leq 1.5) = \int_{-\infty}^{1.5} f(x)\,dx$$

$$= \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} ax\,dx + \int_{1}^{1.5} a\,dx$$

$$= \left[\frac{ax^2}{2}\right]_0^1 + \left[ax\right]_1^{1.5}$$

$$= \frac{a}{2} + 1.5a - a$$

$$= \frac{1}{4} - \frac{1.5}{2} - \frac{1}{2}$$

$$= \frac{1}{2}$$ Ans.

Que The diameter of an Electric cable is assumed to be a continous Random variable X with probability density function $f(x) = 6x(1-x)$ $0 \le x \le 1$

i) check that the above is a probability density function. ~~obtain~~

ii) obtain the Expression for continous distribution function.

iii) compute $P(X \le 1/2 \mid 1/2 \le x \le 2/3)$

(i) $\cancel{\int_0^1 f(x)\,dx = \int_0^1 6x(x-1)\,dx}$

$\cancel{= \int_0^1 (6x^2 - 6x)\,dx}$

$\cancel{= [2x^3 - 3x^2]_0^1}$

$$\int_0^1 f(x)\,dx = \int_0^1 6x(1-x)\,dx$$

$$= \int_0^1 [6x - 6x^2]_0^1$$

$$= [3x^2 - 2x^3]_0^1$$

$$= 3 - 2$$

$$= 1 \quad \text{Ans}$$

Hence the function is Probability density function

ii) $F(x) = P(X \le x)$

$= P(0 \le X \le x)$

$= \int_0^x f(x)\,dx$

$= \int_0^x 6x(1-x)\,dx$

$$\int_0^x (6x - 6x^2)\,dx$$

$$\left[\frac{6x^2}{2} - \frac{6x^3}{3}\right]_0^x$$

$$\left[3x^2 - 2x^3\right]_0^x$$

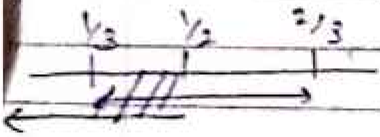
$$= 3x^2 - 2x^3$$

(iii) $P(A \cap B) = P(A) \cdot P\left(\frac{B}{A}\right)$

$B = x \leq \frac{1}{2}$ $\qquad$ $A = \frac{1}{2} \leq x \leq \frac{2}{3}$

$$P\left(\frac{B}{A}\right) = \frac{P\left\{\left(x \leq \frac{1}{2}\right) \cap \left(\frac{1}{3} \leq x \leq \frac{2}{3}\right)\right\}}{P\left(\frac{1}{3} \leq x \leq \frac{2}{3}\right)}$$

$$P\left(\frac{B}{A}\right) = \frac{P\left(\frac{1}{3} \leq x \leq \frac{1}{2}\right)}{P\left(\frac{1}{3} \leq x \leq \frac{2}{3}\right)}$$



$$= \frac{\int_{1/3}^{1/2} 6x(1-x)\,dx}{\int_{1/3}^{2/3} 6x(1-x)\,dx}$$

$$= \frac{\left[3x^2 - 2x^3\right]_{1/3}^{1/2}}{\left[3x^2 - 2x^3\right]_{1/3}^{2/3}}$$

$$= \frac{3}{4} - \frac{1}{4} - \frac{1}{3} + \frac{2}{27} \Big/ \frac{4}{3} - \frac{16}{27} - \frac{1}{3} + \frac{2}{27}$$

$\frac{1}{2} - \frac{11}{27}$ $\frac{27-22}{54}$ $\frac{5}{54}$

$$= \frac{\frac{1}{2} - \frac{1}{3} + \frac{3}{27}}{1 - \frac{14}{27}}$$

-16
$+2$
-14

$27-14$
23

$$= \frac{\frac{27-18+4}{54}}{\frac{13}{27}}$$

$\frac{1}{27}$
$\times 2$
$3)\overline{54}($
18

$$= \frac{\frac{\cancel{13}}{54}}{\frac{\cancel{13}}{27}} = \frac{27}{54} \quad \frac{1}{2}$$

27
-14

$\frac{1}{2}$

<u>Mathematical Expectation of a Random variable</u> -

(I) If X is a Discreate Random variable with which can take the value $x_1, x_2 \dots x_n$ with Respective probabilies $p_1, p_2 \dots p_n$ such that $\sum_{i=1}^{n} p_i = 1$ then the Expectation of X denoted by $E(X)$ is defined as $x_1p_1 + x_2p_2 + x_3p_3 \dots + x_np_n = \sum_{i=1}^{n} x_i p_i(x) = \sum x p(x)$

toss two coin

X	0	1	2
p	1/4	2/4	1/4

$$E(X) = 0 + \frac{2}{4} + \frac{2}{4} = 1$$

II) If X is continous Random variable with prob. density function $f(x)$ then Expectation of X

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$$E(X) = \int_{-\infty}^{\infty} x f(x) dx$$ {limit jo bhi given hogi}

Que If X is continous Random variable with probab density function

$$f(x) = \begin{cases} 2/x^3 & x \geq 1 \\ 0 & x < 1 \end{cases}$$

$E(X) = ?$

$$E(X) = \int_{1}^{\infty} \frac{2}{x^3} \times x \, dx + \int_{-\infty}^{1} 0 \, dx$$

$$\int \frac{1}{x^2} dx \qquad \int x^{-2} dx \qquad \frac{x^{-2+1}}{-2+1}$$

$$= \int_{1}^{\infty} \frac{2}{x^2} = \left[-\frac{2}{x} \right]_{1}^{\infty}$$

$$= 0 + 2$$

$$= 2$$

Note (i) Expected value is also known as Mean.

$$\mu = E(X) = \overline{X}$$

For Random discreate variable

$$E(g(x)) = \Sigma g(x) p(x) dx$$

~~variance~~

$$E(X) = \Sigma x p(x) = u$$
$$E(1) = \Sigma 1 \cdot px \quad (\Sigma px = 1)$$

variance of a discreate Random variable

$X \rightarrow$ discreate Random variable
$f(x) \rightarrow$ Mass function.

$$var(X) = E\{(X - E(X))^2\}$$

$$= E\{(X - u)^2\}$$

$$= E\{X^2 + u^2 - 2Xu)$$

$$= E\{X^2\} + E\{u^2\} - 2E(Xu)$$

$$= E(X^2) + u^2 - 2u E(X)$$
$$= E(X^2) + (E(X))^2 - 2(E(X))^2$$

$$\boxed{var(x) = E(X^2) - (E(X))^2} = \sigma^2$$

$$\boxed{S \cdot D = \sqrt{var \cdot (X)} = \sigma}$$

Tossing two coin

x	0	1	2
$p(x)$	1/4	2/4	1/4
$\cancel{E}\ x^2$	0	1	4

$$E(X^2) = \Sigma x^2 p(x)$$
$$= 0 \times \frac{1}{4} + 1 \times \frac{2}{4} + 1 = \frac{3}{2}$$

$$var(X) = \frac{3}{2} - 1 = \frac{1}{2}$$

$$\boxed{S \cdot D = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}}$$

~~Cheyt~~ Chebyshev's Inequality

If X is the Random variable with mean μ and variance σ^2 then for any +ve number K.

$$P\{|X-\mu| \geq K\sigma\} \leq 1/K^2$$

or

$$P\{|X-\mu| < K\sigma\} \leq 1-\frac{1}{K^2}$$

Chebyshev's Inequality

Proof case I

Let X is a continous Random variable

then $\sigma^2 = E(X-E(X))^2$

$$= E(X-\mu)^2$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

$-\infty$ $\mu-K\sigma$ μ $\mu+K\sigma$ ∞

$$= \int_{-\infty}^{\mu-K\sigma} (x-\mu)^2 f(x)\,dx + \int_{\mu-K\sigma}^{\mu+K\sigma} (x-\mu)^2 f(x)\,dx + \int_{\mu+K\sigma}^{\infty} (x-\mu)^2 f(x)\,dx$$

$$\boxed{\mathrm{I}+\mathrm{II}+\mathrm{III} > \mathrm{I}+\mathrm{III}}$$

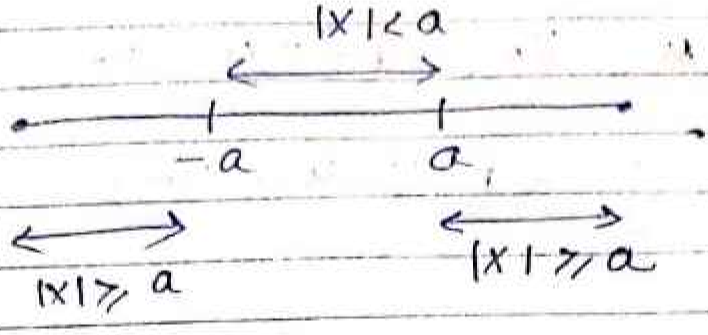
$$\sigma^2 \geq \int_{-\infty}^{\mu-K\sigma} (x-\mu)^2 f(x)\,dx + \int_{\mu+K\sigma}^{\infty} (x-\mu)^2 f(x)\,dx \rightarrow (1)$$

Since, * $x \leq \mu-K\sigma$ and $x \geq \mu+K\sigma$

$|x-\mu| \geq K\sigma$

$\leq x p x$

$\int x f(x)$ Density



$|x-a| \leq \delta$

$x \in (a-\delta, \delta+a)$

$|x-a| \geq \delta$

$x \leq a-\delta \quad ; \quad x \geq a+\delta$

x ———— x

$(x-\mu)^2 = K^2\sigma^2$ { ~~Put the value of $(x-\mu)^2$~~ in Eqn (1) }

$$\sigma^2 \geq \int_{-\infty}^{\mu-K\sigma} K^2\sigma^2 f(x)\,dx + \int_{\mu+K\sigma}^{\infty} K^2\sigma^2 f(x)\,dx$$

$$\sigma^2 \geq K^2\sigma^2 \left[\int_{-\infty}^{\mu-K\sigma} f(x)\,dx + \int_{\mu+K\sigma}^{\infty} f(x)\,dx \right]$$

{ Probability density function }

$$\sigma^2 \geq K^2\sigma^2 \left[P(X \leq \mu - K\sigma) + P(X \geq \mu - K\sigma) \right]$$

$$\sigma^2 \geq K^2\sigma^2 \left[P(|X-\mu|) \geq K\sigma \right]$$

$$\boxed{\frac{1}{K^2} \geq P(|X-\mu| \geq K\sigma)}$$

Hence proved //

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$$P\{|X-\mu| \geq k\sigma\} + P\{|X-\mu| < k\sigma\} = 1$$

$$X \quad \boxed{\begin{aligned} &\frac{1}{k^2} + P\{|X-\mu| < k\sigma\} = 1 \\ &P\{|X-\mu| < k\sigma\} = 1 - \frac{1}{k^2} \end{aligned}}$$

$$P\{|X-\mu| < k\sigma\} = 1 - P\{|X-\mu| \geq k\sigma\}$$

$$\leq 1 - 1/k^2$$

$A + B = 1$
$B = 1 - A$
$B \geq 1 - \frac{1}{k^2}$

$A \leq 1/k^2$
$-A \geq -1/k^2$

$$\boxed{P\{|X-\mu| < k\sigma\} \leq 1 - \frac{1}{k^2}}$$

Hence proved //

<u>Ques</u> - If x is the number ~~Stored~~ Scored in a throw of a fair die. Show that the Chebshev Inequality give.

$$P\{|X-\mu| \geq 2.5\} < 0.47$$

where μ is the mean of X also Show that actual probability is zero

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Solution $x = (1, 2, 3, 4, 5, 6)$

x	1	2	3	4	5	6
$P(x)$	1/6	1/6	1/6	1/6	1/6	1/6
x^2	1	4	9	16	25	36

$$\mu = E(x) = \frac{1}{6} + 2\times\frac{1}{6} + 3\times\frac{1}{6} + 4\times\frac{1}{6} + 5\times\frac{1}{6} + 6\times\frac{1}{6}$$

$$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{5} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6}$$

$$\boxed{\mu = E(x) = \frac{7}{2}}$$

$\sigma \rightarrow$ S.D varienc $= \sigma^2$

$$\sigma^2 = E(x^2) - (E(x))^2$$

$$E(x^2) = \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6}$$

$$= \frac{91}{6}$$

$$\begin{array}{r} 3 \\ 36 \\ 25. \\ 16. \\ 9 \\ 4 \\ +\ 1 \\ \hline 91 \end{array}$$

$$\sigma^2 = \frac{91}{6} - \left(\frac{21}{6}\right)^2$$

$$\sigma^2 = \frac{91}{6} - \frac{441}{36} = \frac{105}{36}$$

$$\sigma = \frac{\sqrt{105}}{36} = \cancel{4.70}\quad 2.91$$

$$P(|x-\mu| \geq K\sigma) \leq \frac{1}{K^2}.$$

$$P(|x-\mu| > c) \leq \frac{\sigma^2}{c^2}$$

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$|x-a| \geq \delta \quad x \leq a-\delta \quad x \geq a+\delta$

$|x-a| < \delta \quad x \in (a-\delta, a+\delta)$

$$P\{|x-\mu| \geq 2.5\} < \frac{2.9167}{(2.5)^2}$$

$$< 0.47$$

$P(|x-3.5| > 2.5) + P(|x-3.5| \leq 2.5) = 1$

$P(|x-3.5| > 2.5) = 1 - P\{|x-3.5| \leq 2.5\}$

$= 1 - P\{1 \leq x \leq 6\}$

$= 1-1$

$\boxed{P(|x-3.5| > 2.5) = 0}$

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UNIT - II

Mean - average value
Median - Middle value
Mode - Repeating. number (most)

Moments -

The ~~rth~~ r^{th} moment of a Random variable X. about any Point A ,

$$\mu_r' = E(X-A)^r$$

Moment is used to study the nature of frequency distribution.

Moment about origin -

$$\mu_r' = E(X-0)^r = E(X)^r$$

$\boxed{\mu_0' = 1}$ $\boxed{\mu_1' = E(X) = \mu}$

→ ~~Since~~ μ_1' is the first moment about origion is mean.

$$\mu_2' = E(X)^2$$

$var(x) = E(x^2) - (E(x))^2$

$Var(x) = \mu_2' - (\mu_1)^2$

Moment about mean -

$$\mu_r = E(X-\overline{X})^r$$
$$= E(X-\mu)^r$$

$$\mu_0 = E(x-\mu)^0 = 1$$

$$\mu_1 = E(x-\mu)$$
$$= E(x) - E(\bar{x})$$
$$= \bar{x} - \bar{x}E(1)$$
$$= \bar{x} - \bar{x}$$
$$= 0$$

I^{st} moment about mean $= 0$

$$\mu_2 = E(x-\mu)^2$$
$$= E(x-\bar{x})^2$$
$$= E(x^2 + \bar{x}^2 - 2x\bar{x})$$
$$= E(x^2) + E(\bar{x}^2) - 2E(x\bar{x})$$
$$= E(x^2) + \bar{x}^2 - 2\bar{x}E(x)$$
$$= E(x^2) + \bar{x}^2 - 2\bar{x}^2$$
$$= E(x^2) - \bar{x}^2$$
$$= E(x^2) - (E(x))^2$$
$$\mu_2 = Var(x)$$

2^{nd} moment about mean is variance.

$$\mu_3 = E(x-\bar{x})^3$$
$$= E(x^3 - \bar{x}^3 - 3x^2\bar{x} + 3x\bar{x}^2)$$
$$= E(x^3) - \bar{x}^3 - 3\bar{x}E(x^2) + 3\bar{x}^2E(x)$$

$$= E(x^3) - (E(x))^3 - 3E(x)E(x^2) + 3(E(x))^3$$
$$= E(x^3) - 3E(x)E(x^2) + 2(E(x))^3$$
$$\mu_3 = \mu_3' + 2(\mu_1')^3 - 3\mu_1'\mu_2'$$

Binomial Expansion

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^n$$

Binomial Expansion of $(x-\bar{x})^4$

$$= (x)^4 - 4x^3\bar{x} + 6x^2(\bar{x})^2 - 4x(\bar{x})^3 + (\bar{x})^4$$

$$E(x-\bar{x})^4 = E(x^4) - 4E(x^3\bar{x}) + 6E(x^2(\bar{x})^2) - 4E(x(\bar{x})^3) + E(\bar{x})^4$$

$$= E(x^4) - 4\bar{x}E(x^3) + 6\bar{x}^2E(x^2) - 4\bar{x}^3E(x) + (\bar{x})^4$$

$$= E(x^4) - 4\bar{x}E(x^3) + 6(\bar{x})^2E(x^2) - 4(\bar{x})^4 + (\bar{x})^4$$

$$E(x-\bar{x})^4 = E(x^4) - 4\bar{x}E(x^3) + 6(\bar{x})^2E(x^2) - 3(\bar{x})^4$$

$$\boxed{\mu_4 = \mu'_4 - 4\mu'_1\mu'_3 + 6\mu'_2\mu'_1 - 3\mu'^4_1}$$

The shape of any distribution can be described by its various moments -

(1) The mean which indicates the central tendency of a distribution.

(2) The second moment is the ~~various~~ varience which indicates the width or deviation.

(3) The third moment is the skewness which indicates any assymetric lining to either left or Right.

(4) The fourth moment is the Kurtosis which indicates the degree of central peakness or Equivalently. The flatness of the outer tales.

Relation
Pearson's β and γ cofficient -

→ Karl pearson defined the four cofficient based upon the first four moments about mean

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \qquad \gamma_1 = \frac{\mu_3}{\sigma^3} = \pm\sqrt{\beta_1}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} \qquad \gamma_2 = \beta_2 - 3$$

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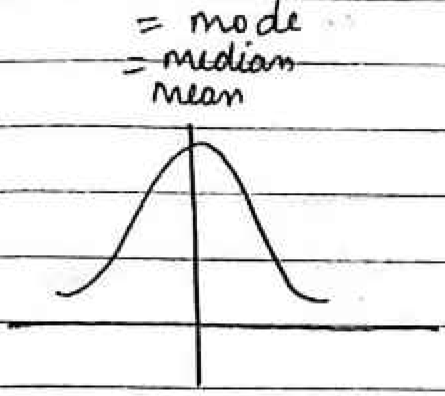
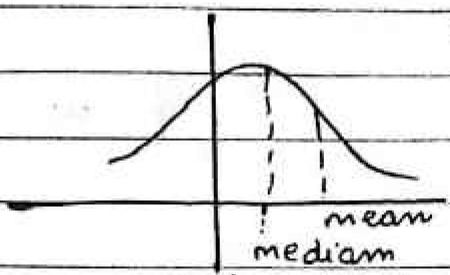
Skewness -

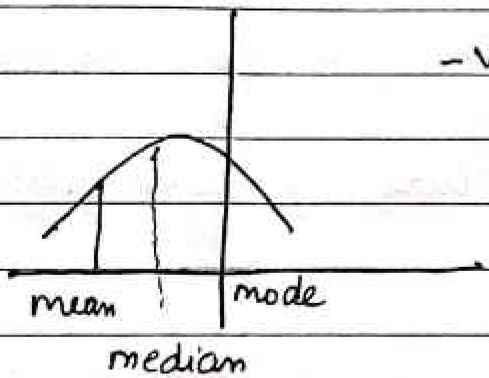
Skewness means - "lack of Symmetry" we study Skewness to have an idea about Shape of curve which can draw with the help of given data.



Symmetrical



mode +ve Skewed.



-ve Skewness.

If

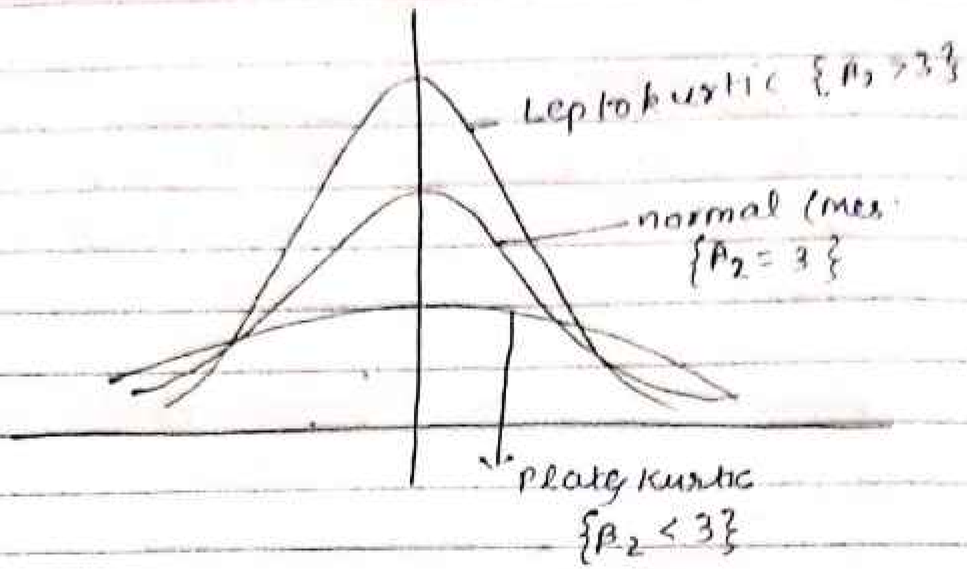
$\beta_1 = 0$ curve will be symmetrical.

$\beta_1 > 0$ curve will be Positive skewed

$\beta_1 < 0$ curve will be negetive Skewed.

Kurtosis -

It Enables us to have an idea about the flatness or peakness of the frequency curve. It is measured by the cofficient β_2 or γ_2 given by $\frac{\mu_4}{\mu_2^2}$ or $\beta_2 - 3$ respectively.



Que for a distribution the mean is 10 varience is 16 $\gamma_1 = 1$, β_2 is 4 obtain the first 4 four moment about the origin i.e., 0 comment upon nature of distribution.

$\mu_1' = 10$

$\mu_2' = ?$

varience $= \mu_2' - (\mu_1')^2$

$16 = \mu_2' - (10)^2$

$116 = \mu_2'$

$\boxed{\mu_2' = 116}$

$$\gamma_1 = \frac{\mu_3}{\sigma^3} = \frac{\mu_3}{(4)^3} = 1$$

$$\boxed{\mu_3 = 64}$$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3$$

$$64 = \mu_3' - 3 \times 10 \times 116 + 2(10)^3$$

$$64 = \mu_3' - 3480 + 2000$$

$$\mu_3' = 1544$$

Relation between rth moment about mean & rth moment about origin

$$\mu_r = \mu_r' - {}^rC_1\, \mu\, \mu_{r-1}' + {}^rC_2\, \mu^2 \mu_{r-2} - {}^rC_3\, \mu^3 \mu_{r-3}' + \cdots + (-1)^r \mu_r$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\cancel{4 = \frac{\mu_4}{(16)^2}}$$

3
16
×16
96
16
256

~~Hence~~. Here $4 = \frac{\mu_4}{256}$

$\mu_4 = 256 \times 4$

$$\boxed{\mu_4 = 1024}$$

2

but we have to find μ_4'

$$\mu_4 = \mu_4' - 4\mu_3\mu_1' + 6\mu_2\mu_1'^2 - 3\mu_1'^4$$

$1024 = \mu_4' - 4 \times 1544 \times 10 + 6 \times 116 \times (10)^2 - 3(10)^4$

$1024 = \mu_4' - 6176 + 69600 - 30000$

$1024 = \mu_4' - 36176 + 69600$

$1024 = \mu_4' + 33424$

$$\boxed{\mu_4' = -32,400}$$

$$\boxed{\mu_4 = \mu_4' - 4\mu_3\mu_1' + 6\mu_2\mu_2' + \mu_1'^4}$$

$1024 = \mu_4' - 4 \times 1544 \times 10 + 6 \times 116 \times (10)^2 - 3(10)^4$

$1024 = \mu_4' - 61760 + 69600 - 30000$

$\cancel{1024 = \mu_4' - 36176 + 69600}$

$\cancel{\boxed{\mu_4' = -32400}}$

$1024 = \mu_4' + 4840$

$$\boxed{\mu_4' = -3816}$$

$\mu_3 = 64 \qquad \mu_2 = \text{var} = 16$

$$\beta_1 = \frac{\cancel{64}^{1} \times \cancel{64}^{4}}{\cancel{16} \times 16 \times 16} = 1$$

Since, $\beta_1 = 1$ so, distribution is +ve skewed
$\beta_1 > 3$ so, distribution is leptokurtic

22/09/23

Ques If the first four moment of a distribution about the value 5 are $-4, 22, -117$ & 560. Determine the corresponding moment about mean and origin?

$\mu_r =$ about mean
$\mu_r' =$ about origin
$\mu_r'' =$ about a point

$\mu_1'' = -4$
$\mu_2'' = 22$
$\mu_3'' = -117$
$\mu_4'' = 560$

$$\mu_r'' = E(x-5)^r$$

$\mu_1'' = E(x-5)^1 = -4$
$= E(x-5) = -4$
$-4 = E(x) - E(5)$
$-4 = E(x) - 5$
$-4 = \mu - 5$
$-4 = \mu_1' - 5$
$\boxed{\mu_1' = 1}$

1^{st} moment about mean $= 0$ | $\mu_1 = E(x-1)$
$= 1-1 = 0$

$\mu_2'' = E(x-5)^2$
$22 = E(x^2) + 25 - 10$
$22 = E(x^2) + 15$
$\boxed{7 = \mu_2'}$

$\mu_2 = E(x-1)^2$
$= E(x^2 + 1 - 2x)$
$= E(x^2) + 1 - 2$
$= 7 + 1 - 2$
$\boxed{\mu_2 = 6}$

$125 + 105 = 230$, $230 - 75 = 155$, $155 - 117 = 38$

$\mu_3'' = E(x-5)^3$
$-117 = E(x^3 - 125 - 15x^2 + 75x)$
$= E(x^3) - 125 - 7 \times 15 + 75 \times 1$
$= E(x^3) - 125 - 105 + 75$
$-117 = E(x^3) - 155$
$\boxed{E(x^3) = 38}$

$38 - 19 = 19$

$\mu_3 = E(x-1)^3$
$= E(x^3 - 3x^2 - 1 + 3x)$
$= E(x^3) - 3E(x^2) - 1 + 3E(x)$
$= 38 - 21 - 1 + 3$
$\boxed{\mu_3 = 19}$

$64 \times 4 = 256$, $16 \times 6 = 96$

~~$\mu_4'' = E(x-4)^4$~~
~~$= E\{(x)^4 - 4 \times 4 \times x^3 + 6x^2(4)^2 - 4 \times x \times (4)^3 + (4)^4\}$~~
~~$\mu_4'' = E(x^4) - 16E(x^3) + 96E(x^2) - 256E(x) + 256$~~

$$\mu_4'' = E(x^4) - 16 \times 38 + 96 \times 7$$
$$560 = E(x^4) - 608 + 672$$
$$560 = E(x^4) + 64$$
$$\boxed{E(x^4) = 496}$$

$$\mu_4'' = E(x-5)^4$$
$$560 = E\{x^4 - 4 \times 5 \times x^3 + 6 \times x^2 \times 25 - 4 \times x \times (5)^3 + (5)^4\}$$
$$= E(x^4) - 20E(x^3) + 150E(x^2) - 500E(x) + 625 \}$$
$$560 = E(x^4) - 20 \times 38 + 150 \times 7 - 500 + 625$$
$$= E(x^4) - 760 + 1050 - 500 + 625$$
$$560 = E(x^4) - 1260 + 1050 + 625$$
$$560 = E(x^4) - 210 + 625$$
$$560 = E(x^4) + 415$$
$$\boxed{145 = E(x^4)} \qquad \boxed{145 = \mu_4'}$$

$$\mu_4 = E(x-1)^4$$
$$\mu_4 = E\{x^4 - 4x^3 + 6x^2 - 4x + 1\}$$
$$= E(x^4) - 4E(x^3) + 6E(x^2) - 4E(x) + 1$$
$$= 145 - 4 \times 38 + 6 \times 7 - 4(1) + 1$$
$$= 145 - 152 + 42 - 3$$
$$\boxed{\mu_4 = 32}$$

$\mu_1 = 0$	$\mu_1' = 1$	$\mu_1'' = -4$
$\mu_2 = 6$	$\mu_2' = 7$	$\mu_2'' = 22$
$\mu_3 = 19$	$\mu_3' = 38$	$\mu_3'' = -117$
$\mu_4 = 32$	$\mu_4' = 145$	$\mu_4'' = 560$

Ques The first 4 moment of a distribution about $X = 4$, are 1, 4, 10 & 45 Show that the mean is 5. and the varience is 3.

$\mu_3 = 0$ & $\mu_4 = 26$

$\mu_1'' = 1$

$\mu_2'' = 4$

$\mu_3'' = 10$

$\mu_4'' = 45$

$$\mu_1'' = 1 = E(x-4)^1$$
$$1 = E(x) - E(4)$$
$$1 = E(x) - 4$$
$$\boxed{E(x) = 5}$$
$$\boxed{\mu = 5} \text{ mean}$$

$$\mu_2'' = 4 = E(x-4)^2$$
$$4 = E(x^2 + 16 - 8x)$$
$$4 = E(x^2) + 16 - 8 \times 5$$
$$4 = E(x^2) + 16 - 40$$
$$4 = E(x^2) \cancel{} - 24$$
$$E(x^2) = 28$$
$$\boxed{\mu_2' = 28}$$

$$\begin{array}{r} 40 \\ -16 \\ \hline 24 \end{array}$$

$$\mu_2 = E(x-5)^2$$
$$= E\{x^2 + 25 - 10x\}$$
$$\mu_2 = E\{x^2\} + 25 - 50$$
$$\mu_2 = E\{x^2\} - 25$$
$$\mu_2 = 28 - 25$$
$$\boxed{\mu_2 = 3} \text{ varience.}$$

$\mu_3'' = 10 = E(x-4)^3$

$10 = E\{x^3 - 64 - 12x^2 + 48x\}$

$10 = E\{x^3\} - 64 - 12(28) + 48(5)$

$10 = E\{x^3\} - 64 - 336 + 240$

$10 = E\{x^3\} - 160$

$E\{x^3\} = 170$

$\mu_3 = E(x-5)^3$

$= E\{x^3 - 125 - 15x^2 + 75x\}$

$= 170 - 125 - 15(28) + 375$

$= 170 - 125 - 420 + 375$

$\boxed{\mu_3 = 0}$ Ans//

$\mu_4'' = 45 = E(x-4)^4$

$45 = E\{x^4 - 4(4)x^3 + 6\cdot(4)^2x^2 - 4(4)^3x + (4)^4\}$

$45 = \{E(x^4) - 16E(x^3) + 96E(x^2) - 256E(x) + 256\}$

$45 = E(x^4) - 16(170) + 96(28) - 256(5) + 25$

$45 = E(x^4) - 2720 + 2688 - 1280 + 25$

$2740 = E(x^4)$

$\mu_4 = E(x-5)^4$

$\mu_4 = E\{x^4 - 20x^3 + 150x^2 - 500x + 625\}$

$\mu_4 = 2740 - 20(170) + 150(28) - 500(5) + 625$

$\mu_4 = 2740 - 3400 + 4200 - 2500 + 625$

$\mu_4 = 7565 - 5900$

$\mu_4'' = 45 = E(x-4)^4$

$45 = E\{x^4 - 4x^3(4) + 6x(4)^2x^2 - 4(4)^3x + (4)^4\}$

$45 = E(x^4) - 16E(x^3) + 96E(x^2) - 256E(x) + 256$

$45 = E(x^4) - 16(170) + 96(28) - 256(5) + 256$

$45 = E(x^4) - 2720 + 2688 - 1280 + 256$

$E(x^4) = 45 + 2720 + 1280 - 2688 - 256$

$E(x^4) = 1101$

$\mu_4 = E(x-5)^4$

$= E\{x^4 - 20x^3 + 6(5)^2x^2 - 4(5)^3x + (5)^4\}$

$= E(x^4) - 20E(x^3) + 150E(x^2) - 500x + 625$

$= E(x^4) - 20(170) + 150(28) - 500(5) + 625$

$= 1101 - 3400 + 4200 - 2500 + 625$

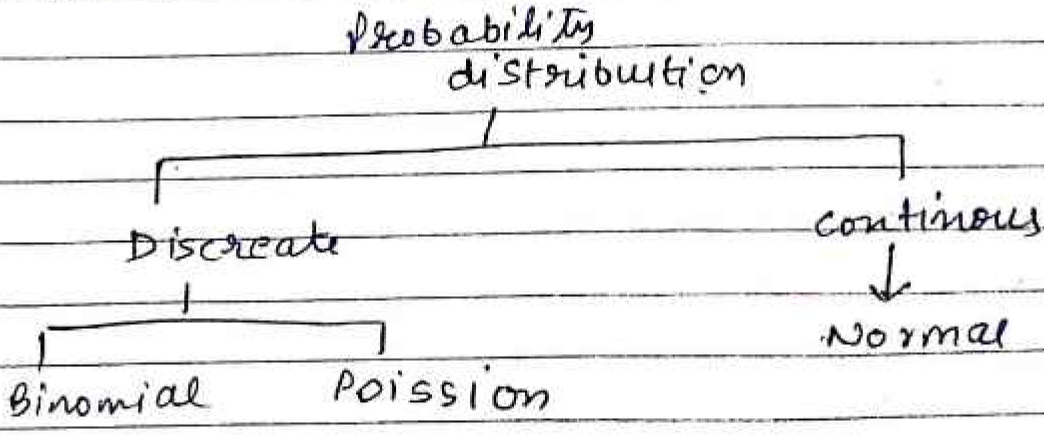
$\boxed{\mu_4 = 26}$

Hence proved//

25/09/23

Probability distribution

Distribution which are not obtained by actual observations or Experiments but are deduced mathematically, under centrain predetermined hypothesis or assumption.
Following certain probability Rules are called probability distribution or theortical probability distribution.



Moment genrating function - (MGF)
The moment generating function of the discreate probability distri of the variate x about the value $x = a$ is defined as "Expected value of $e^{t(x-a)}$ & is denoted by $M_a(t)$

$$M_a(t) = E\left(e^{t(x-a)}\right)$$

$$\boxed{M_a(t) = \sum p(x) e^{t(x-a)}}$$

If $x \to$ continous

$$\boxed{M_a(t) = \int_{-\infty}^{\infty} e^{t(x-a)} f(x)\,dx}$$

Binomial distribution

A Binomial distribution can be probability of a success or failure outcome, occurence or Non occurence, yes or No outcome in an Experiment i.e., Repeated Multiple times.

→ The Binomial distribution is a type of distribution that has two possible outcome It is also known as Bernoulli's distribution.

A Random variable X is Said to follow Binomial distribution if it assumes only Non negetive values and its probability Mass function is given by-

10/.

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$\rightarrow$ Probab. Mass function

$$P(X=x) = p(x) = \begin{cases} {}^{n}C_{x} \cdot p^{x} q^{n-x} & x = 0, 1, 2 \ldots \\ 0 & \text{otherwise} \end{cases}$$

$p \rightarrow$ Probability of sucess

$q \rightarrow$ Probability of failure.

Q 10 coins are thrown simultaneously, find the probability of getting 7 head.

p = getting head = $1/2$

q = getting tail = $1/2$

$$p(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$= {}^{10}C_{x}\, p^{x} q^{10-x}$$

$$= {}^{10}C_{7}\, p^{7} q^{3} + {}^{10}C_{8}\, p^{8} q^{2} + {}^{10}C_{9}\, p^{9} q^{1} + {}^{10}C_{10}\, p^{10}$$

$$= {}^{10}C_{7} \left(\tfrac{1}{2}\right)^{7} \left(\tfrac{1}{2}\right)^{3} + {}^{10}C_{8} \left(\tfrac{1}{2}\right)^{8} \left(\tfrac{1}{2}\right)^{2}$$

$$+ {}^{10}C_{9} \left(\tfrac{1}{2}\right)^{9} \left(\tfrac{1}{2}\right) + {}^{10}C_{10} \left(\tfrac{1}{2}\right)^{10}$$

$$= \left(\frac{1}{2}\right)^{10} \left[{}^{10}C_{7} + {}^{10}C_{8} + {}^{10}C_{9} + {}^{10}C_{10} \right]$$

$$= \left(\frac{1}{2}\right)^{10} \left[\frac{\lfloor 10}{\lfloor 7 \lfloor 3} + \frac{\lfloor 10}{\lfloor 8 \lfloor 2} + \frac{\lfloor 10}{\lfloor 9 \lfloor 1} + \frac{\lfloor 10}{\lfloor 10} \right]$$

$$= \frac{1}{1024} \left[\frac{10 \times \overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}}}{\cancel{3} \times \cancel{2}} + \frac{10 \times 9}{\cancel{2}} + \frac{10}{1} + 1 \right]$$

$$= \frac{1}{1024} \left[120 + 45 + 10 + 1 \right]$$

$$= \frac{176}{1024}$$

Q A & B play a game in which there chances of winning are in Ratio 3:2 find A's chance wining atleast 3 games out of 5 games player?

~~P(A) = 3/8~~ ~~P(B) =~~

$$p = \frac{3}{5} \qquad q = \frac{2}{5}$$

$$P(x \geq 3) = {}^nC_x\, p^x q^{n-x}$$

$$= {}^5C_3 p^3 q^2 + {}^5C_4 p^4 q + {}^5C_5 p^5$$

$$= {}^5C_3 \left(\frac{3}{5}\right)^3 \left(\frac{2}{5}\right)^2 + {}^5C_4 \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right) + {}^5C_5 \left(\frac{3}{5}\right)^5$$

$$= \left(\frac{1}{5}\right)^5 \left[\frac{\lfloor 5}{\lfloor 3 \lfloor 2}(3)^3(2)^2 + \frac{\lfloor 5}{\lfloor 4}(3)^4(2) + 1(3)^5\right]$$

$$= \frac{(3)^3}{3125}\left[10 \times 4 + 5 \times 6 + 9\right]$$

$$= \frac{27}{3125}\left[40 + 30 + 9\right]$$

$$= \frac{2133}{3125} = 0.682$$

$(a+b)^n = {}^nC_0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \dots {}^nC_n b^n$

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Q Show that $P(x)$ is probability mass function?

T.P : $p(x) \geq 0$

$\sum p(x) = 1$

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$\sum px = \sum_{x=0}^{n} {}^nC_x p^x q^{n-x}$$

$$= {}^nC_0 q^n + {}^nC_1 q^{n-1} p^1 + {}^nC_2 p^2 q^{n-2} + \dots + {}^nC_n p^n$$

$$= (p+q)^n = 1^n = 1$$ {By Binomial Expansion}

23/10/23

Moment genrating function for Binomial distribution

Ques Find moment genrating function of Binomial distribution?

m.g.f $= M_x t$
$= E(e^{xt})$

$$M_x(t) = E(e^{xt})$$

$$= \sum p(x) e^{xt}$$

$p(x) \rightarrow$ Probability density function.

$$= \sum_{x=0}^{n} e^{xt}\, {}^nC_x p^x q^{n-x}$$

$$= {}^nC_0 q^n + e^t\, {}^nC_1 p q^{n-1} + e^{2t} p^2 q^{n-2}\, {}^nC_2 + \dots + {}^nC_n p^n e^{nt}$$

$$= {}^nC_0 q^n (e^t p)^0 + {}^nC_1 (e^t p)^1 q^{n-1} + {}^nC_2 (e^t p)^2 q^{n-1} + \dots + {}^nC_n (pe^t)^n$$

$$= (e^t p + q)^n$$

$\hookrightarrow$ By Binomial theorem

$$(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_n a^0 b^n$$

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Mean and variance of Binomial distribution -

Mean = np

$n \rightarrow$ no. of times Experiment Performed
$p \rightarrow$ probability of success.

Variance = npq

$q \rightarrow$ Probability of failure.

$S.D = \sqrt{npq}$

$\mu_3 = npq(q-p)$
$\mu_4 = npq(1+3(n-2)pq)$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(npq(q-p))^2}{(npq)^3}$$

$$= \frac{(npq)^2(q-p)^2}{(npq)^3}$$

$$= \frac{(q-p)^2}{npq}$$

$$\sqrt{\beta_1} = \gamma_1 = \frac{q-p}{\sqrt{npq}} \quad \{ \text{since, } q = 1-p$$

$$= \frac{1-2p}{\sqrt{npq}}$$

* If p is less than $1/2$ $(p < 1/2)$ then skewness is +ve
* If p is greater than $1/2$ $(p > 1/2)$ then skewness is -ve
* If p is $1/2$ then skewness is symmetrical.

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{npq(1+3(n-2)pq)}{(npq)^2}$$

$$= \frac{(1+3(n-2)pq)}{npq}$$

$$= \frac{1}{npq} + \frac{3npq}{npq} - \frac{6pq}{npq}$$

$$\boxed{\beta_2 = 3 + \frac{(1-6pq)}{npq}}$$

$$\gamma_2 = \beta_2 - 3 = 3 + \frac{(1-6pq)}{npq} - 3$$

$$\boxed{\gamma_2 = \frac{1-6pq}{npq}}$$

<u>Ques</u> If the chance that one of 10 telephones lines are Busy at instant is 0.2 ~~what~~ then

(i) what is the probability that 5 of lines are busy

(ii) what is the probability all lines are busy.

<u>Solution</u> $n = 10$; $p = 0.2$ $q = 0.8$

$$p(x) = {}^nC_x\, p^x q^{n-x}$$

(i) $p(\text{5 lines are busy}) = {}^{10}C_5 (0.2)^5 (0.8)^5$

$= \dfrac{\lfloor 10}{\lfloor 5 \lfloor 5} (32 \times 10^{-5}) \times (32768 \times 10^{-5})$

$= \dfrac{\cancel{2}\,\cancel{10} \times \cancel{9}^{3} \times \cancel{8}^{2} \times 7 \times 6}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \times 32 \times 32768 \times 10^{-10}$

$= 3 \times 2 \times 7 \times 6 \times 32 \times 32768 \times 10^{-10}$

$= 264241156 \times 10^{-10}$

$P(\text{5 lines are Busy}) = 0.0264241156$

(ii) $P(\text{all lines are busy}) = {}^{10}C_{10} (0.2)^{10} (0.8)^0$

$= 1 \times (2 \times 10^{-1})^{10}$

$= 1024 \times 10^{-10}$

$= 0.0000001024$

Ques If one out of Every 10 bulb is defective find
(i) mean and S.D for distribution of defective Bulb in a total of 500 Bulbs.
(ii) The cofficient of Skewness γ_1 & cofficient of Kurtosis β_2?

$p = 0.1 = \dfrac{1}{10}$; $q = 1 - p = 0.9$

(i) mean $= np = 500 \times 0.1$

$\boxed{\text{mean} = 50}$

(ii) $S.D = \sqrt{npq} = \sqrt{500 \times 0.1 \times 0.9}$

$= \sqrt{45}$

$= 6.71$

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(ii) $\gamma_1 = \frac{1-2p}{\sqrt{npq}} = \frac{1-2\times0.1}{\sqrt{45}} = \frac{1-0.2}{6.71}$

$$\boxed{\gamma_1 = 0.119}$$

$\beta_2 = 3 + \frac{(1-6pq)}{npq}$

$= 3 + \frac{(1-6\times0.1\times0.9)}{45} = 3 + \frac{0.46}{45}$

$$\boxed{\beta_2 = 3.0102}$$

Que Fit a Binomial distribution for the following Data and compare the theoritical frequencies with actual works

x	0	1	2	3	4	5
f	2	14	20	34	22	8
fx	0	14	40	102	88	40

mean $= \frac{\Sigma fx}{\Sigma f} = \frac{284}{100} = 2.84$

1.00
0.47
0.53

$\boxed{n=5}$

mean $= np$

$p = \frac{\text{mean}}{n} = \frac{2.84}{5} = 0.568$

$\boxed{\cancel{p = 0.47}}$ $\boxed{\cancel{q = 0.53}}$

~~For $x=0$~~

~~Binomial distribution for $n=0$~~ $= \cancel{{}^6C_0\, p^6 q^0}$

$= \cancel{(0.47)^6}$

$\boxed{\cancel{p = 0.56}}$ $\boxed{\cancel{q = 0.432}}$

$p = 0.568 \qquad q = 0.432$

For $\boxed{x = 0}$

Binomial distribution $= N\left[{}^{n}C_{x}\, p^{x}\, q^{n-x}\right]$

$= 100\left[{}^{5}C_{0}\,(.568)^{0}\,(.432)^{5}\right]$

$= 100 \times 0.015$

$= 1.5$

For $\boxed{x = 1}$

$= 100\left[{}^{5}C_{1}\,(.568)^{1}\,(.432)^{4}\right]$

$= 100 \times 5 \times 0.568 \times 0.034$

$= 9.89$

For $\boxed{x = 2}$

$= 100\left[{}^{5}C_{2}\,(.560)^{2}\,(.432)^{3}\right]$

$= 100 \times 10 \times (.568)^{2}\,(.432)^{3}$

$= 1000 \times 0.322 \times 0.080$

$= \cancel{50000}$

$= 26.01$

For $x = 3$

$= 100\left[{}^{5}C_{3}\,(.568)^{3}\,(0.432)^{2}\right]$

$= 100\left[\frac{5 \times \cancel{4}^{\,2}}{\cancel{2}} \times 0.183 \times 0.186\right]$

$= \cancel{10000}\ 34.038$

For $\boxed{x=4}$

$$= {}^5C_4 \times 100 \times (0.568)^4 \times 0.432$$

$$= 5 \times 100 \times 0.044$$

$$= 22.48$$

For $\boxed{x=5}$

$$= {}^5C_5 \times 100 \times (0.568)^5$$

$$= 5.91$$

x	0	1	2	3	4	5
f	2	14	20	34	22	8
Actual f	1.5	9.89	26.01	34.036	22.48	5.91

Ques Assuming that half the population are consumer of chocolates So, that the chance of an individual being a consumer is 1/2 and assuming that each of the 100 investigators takes 10 individuals to see wheather they are consumers How many investigators would you expect to Report that 3 people or less were consumers?

$$p = 1/2 \quad ; \quad q = 1/2$$

Required Probability =

$P(r \le 3)$

$P(r=0) + P(r=1) + P(r=2) + P(r=3)$

$$\frac{10\times 9\times 8}{3\times 2} \; 4$$

$$= {}^{10}C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{10} + {}^{10}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^9 + {}^{10}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^8 + {}^{10}C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^7$$

$$= \left(\frac{1}{2}\right)^{10}\left[1 + 10 + 45 + 120\right]$$

$$\begin{array}{r} 120 \\ 10 \\ 45 \\ +1 \\ \hline 176 \end{array}$$

$$= \left(\frac{1}{2}\right)^{10}[176]$$

$$= \frac{176}{1024}$$

$$= 0.17$$

No. of investegators $= 100 \times 0.17$

$= 17$

Poisson distribution

It is a distribution Related to the probabilities of Events which are extremely Rare but, which have a large no. of independent opportunities for occurence.

This distribution can be defined as a "Limiting case of a Binomial distribution" by making "n" very large & "p" very small keeping "n" & "p" fixed

Limiting conditions -

$n \to \infty \qquad p \to 0$

$np = m$

Proof

The probability of r success in a Binomial distibution is

$$p(r) = {}^nC_r p^r q^{n-r}$$

$$= \frac{n!}{r!(n-r)!} p^r q^{n-r}$$

$$= \frac{n(n-1)\cdots(n-r-1)\cancel{(n-r)!}}{r!\cancel{(n-r)!}} p^r q^{n-r}$$

$$= \frac{np(np-p)(np-2p)\cdots(np-(r-1)p)}{r!} q^{n-r}$$

as $n \to \infty$ $\quad p \to 0$

$$np = m$$

therefore,

$$p(r) = \frac{\overbrace{m \cdot m \cdot m \cdot m \cdots m}^{r \text{ times}}}{r!} \times q^{n-r}$$

$$p(r) = \lim_{n\to\infty} \frac{m^r q^{n-r}}{r!}$$

$$= \lim_{n\to\infty} \frac{m^r}{r!} (1-p)^{n-r}$$

$$= \lim_{n\to\infty} \frac{m^r}{r!} \left(1-\frac{m}{n}\right)^{n-r}$$

$$= \frac{m^r}{r!} \lim_{n\to\infty} \left(1-\frac{m}{n}\right)^{n} \left(1-\frac{m}{n}\right)^{-r}$$

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$$= \frac{m^r}{r!} \lim_{n\to\infty} \left(1 - \frac{m}{n}\right)^n$$

PQR-4

$$\boxed{\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a}$$

$$\boxed{P(r) = \frac{m^r e^{-m}}{r!}}$$

Mean = $np = m$

variance = $npq = m(1-p) = m$

$S.D = \sqrt{m}$

$\mu_3 = npq(q-p)$

$= m$

$\mu_4 = npq[1+3(n-2)pq]$

$= m[1+3npq - 6pq]$

$= m[1+3m - 0]$

$= m+3m^2$

$= 3m^2+m$

$\gamma_1 = \sqrt{\beta_1} = \frac{q-p}{\sqrt{npq}} = \frac{1}{\sqrt{m}}$

$\beta_2 = 3+\frac{1-6pq}{npq} = 3+\frac{1}{m}$

Q A certain screw making machine produces an average of two defective screw out of 100 and packs them in boxes of 500. Find the probability that a box contain 15 defective screw?

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Ans $p = \frac{2}{100}$ $n = 500$

$np = m = 10$

$P(r=15) = \frac{m^r e^{-m}}{r!}$

$= \frac{(10)^{15} e^{-10}}{\lfloor 15}$

$= \frac{(10)^{15} \times 0.0000453}{\lfloor 15}$

$= \frac{(10)^{15} \times \cancel{4.5\times10} \times 4.53 \times 10^{-4}}{1307674368000}$

$= \cancel{\frac{10^8}{1307674368} \times 4.53}$

$= \cancel{3.46 \times 10^{-9} \times 10^8}$

$= \cancel{3.46 \times 10^{-1}}$

$= \cancel{0.346}$

0.0346

$= 3.47 \times 10^{-17} \times 10^{15}$

$= 3.47 \times 10^{-2}$

$= 0.034$

By Binomial,

$P = 0.02$ $q = 0.08$ $n = 500$

$r = 15$

$P(r) = {}^nC_r \cdot p^r q^{n-r}$

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$$P(r=15) = {}^{500}C_{15} (0.02)^{15} (0.08)^{485}$$

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Ques A manufacturer ~~who~~ who produces medicine bottles find that ~~or~~ 0.1% of the Bottles are defective The bottles are packed in boxes containing 500 Bottles. find that in 100 such boxes how many boxes are expected to contain.

(i) No. defective bottle.

(ii) Aleast two defective Bottle.

Sol

$$P(\text{defective bottle}) = p = 0.001 = \frac{.1}{100}$$

$$m = np = 500 \times \frac{(.1)}{100} = .5$$

$$P(r) = \frac{e^{-m} m^r}{r!}$$

(ii) $P(\text{atleast two defective}) = 1 - \{P(0) + P(r=1)\}$

$$= 1 - \left\{ \frac{(.5)^0}{\lfloor 0} e^{-0.5} + \frac{(.5)^1}{\lfloor 1} e^{-.5} \right\}$$

$$= 1 - \{0.606 + (.5)(0.606)\}$$

$$= 1 - \{0.606 + 0.303\}$$

$$= 1 - 0.909$$

$$= 0.091$$

>

No. of boxes containing no defective Bottle $= 100 \times .6065$

$= 60.65$

$\cong 61$

No. of boxes containing 1 defective

$= 100 \times .3033$

$= 30.33$

Atleast two defective $= 100 - (60.65 + 30.33)$

$\cong 9$

Que Fit the possion distribution to following data given the number of yeast cell per squar for 405 squares

No of cell / sq	0	1	2	3	4	5	6	7	8	9
No. of square	103	148	98	42	8	4	2	0	0	0
Σfx	0	148	196	126	32	20	12	0	0	0

Mean $= \dfrac{534}{405}$

$= 1.31 = m$

$P(r) = N e^{-m} \dfrac{m^r}{\lfloor r}$

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$$P(r=0) = 405 \times e^{(-1.31)} \cdot \frac{(1.31)^0}{\lfloor 0}$$

$$= 405 \times 0.26$$

$$= 105.3$$

$$P(r=1) = 405 \times \left(e^{-1.31}\right) \times \frac{(1.31)^1}{\lfloor 1}$$

$$= 405 \times 0.26 \times 1.31$$

$$= 137.94$$

$$P(r=2) = 405 \times \left(e^{-1.31}\right) \times \frac{(1.31)^2}{\lfloor 2}$$

$$= 90.35$$

$$P(r=3) = 405 \times e^{(-1.31)} \frac{(1.31)^3}{\lfloor 3}$$

$$= 405 \times 0.37 \times 0.26$$

$$= 40.4$$

$$P(r=4) = 405 \times 0.26 \times 0.122$$

$$= 13.313$$

$$P(r=5) = 405 \times 0.26 \times 0.032$$

$$= 3.38$$

$P(r=6) = 405 \times 0.26 \times$
$= 0.73$

$P(r=7) = 405 \times 0.26 \times \frac{6.620}{5040}$

$= 405 \times 0.26 \times 0.001$

$= 0.138$

$P(r=8) = 405 \times 0.26 \times 0.0002$
$= 0.022$

$P(r=9) = 405 \times 0.26 \times \frac{(1.31)^9}{\lfloor 9}$

$= 0.003$

Ques x is a poission variable & it is found that the probability at $x=2$ is $2/3$ of the $P(x=1)$ find the probability at $x=0$, $x=3$

sol
$$P(x=2) = \frac{2}{3} P(x=1)$$

$$\frac{e^{-m} m^2}{2!} = \frac{2}{3} \frac{e^{-m} m}{1!}$$

$$\boxed{m = \frac{4}{3}} = 1.33$$

$$P(X=0) = e^{-m} \cdot \frac{m^0}{\lfloor 0} = e^{-1.33} = 0.26$$

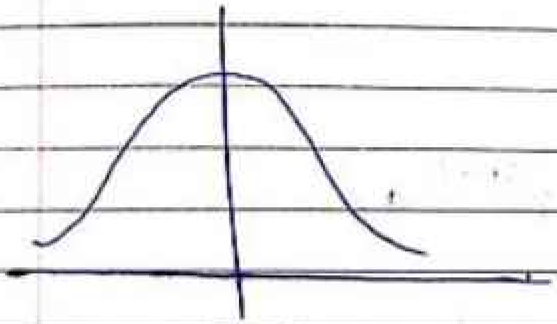
$$P(X=3) = e^{-m} \frac{m^3}{\lfloor 3} = 0.26 \times 0.392$$

$$= 0.10$$

Normal distribution -

continous probability distribution. The limiting form of the Binomial distribution for large value of n where neither p nor q is very small is the normal distribution.

$$z = \frac{x - np}{\sqrt{npq}} = \frac{x - \mu}{\sigma}$$



$z=0$

mean $= \mu =$ median $=$ mode

$$y = P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

$$= \frac{1}{\sigma\sqrt{2\pi}} e^{-z^2/2}$$

✓ $P(x)$ is the probability density function and curve $y = P(x)$ is called Normal curve.

d —4-3

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Moment about mean -

$$\mu_{2n+1} = E(X-\mu)^{2n+1}$$

$$= \int_{-\infty}^{\infty} (x-\mu)^{2n+1} f(x)\, dx$$

$$= \int_{-\infty}^{\infty} (x-\mu)^{2n+1} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu)^{2n+1} e^{-(x-\mu)^2/2\sigma^2} dx$$

In Respect to z

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} z^{2n+1} e^{-z^2/2} \sigma\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \underbrace{\int_{-\infty}^{\infty} e^{-z^2/2} z^{2n+1}\, dz}_{\text{odd}}$$

$$= 0$$

$$\mu_{2n} = E(X-\mu)^{2n}$$

$$= \int_{-\infty}^{\infty} (x-\mu)^{2n} f(x)\, dx$$

μ_{2n}

$$= \int_{-\infty}^{\infty} (x-\mu)^{2n} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} dx$$

$$= \int_{-\infty}^{\infty} (x-\mu)^{2n} \frac{e^{-(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}} dx$$

ILATE

$$= \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} z^{2n} e^{-z^2/2} \sigma\, dz$$

z^{2n} $\quad \frac{z^2}{2} = t$

$z\,dz = dt$

$$= \frac{2}{\cancel{\sigma}\sqrt{2\pi}} \times \cancel{\sigma} \int_{-0}^{\infty} z^{2n} e^{-z^2/2} dz$$

$$= \sqrt{\frac{2}{\pi}} \int_{-0}^{\infty} z^{2n} e^{-z^2/2} dz$$

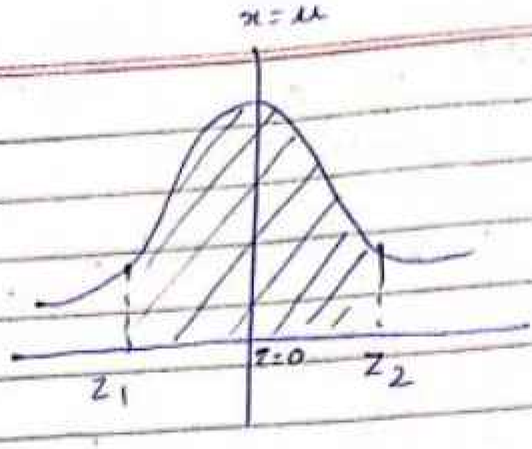
$$= \mu_{2n} = (2n-1)(2n-3)\ldots 5\cdot 3\cdot 1\ \sigma^{2n}$$

$\mu_2 = \cancel{(2(1)-1)(2(1)-3)\ldots 5\cdot 3\cdot 1\,\sigma^2}$

$=$

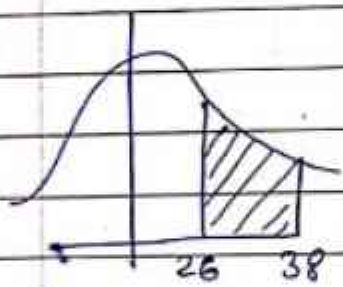
$\boxed{\mu_2 = \sigma^2}$ $\quad$ $\boxed{\mu_4 = 3\cdot 1\cdot \sigma^4}$

$\boxed{\beta_1 = 0}$ $\quad$ $\boxed{\beta_2 = \frac{3\sigma^4}{\sigma^4} = 3}$



Ques A normal curve has mean 20 and variant 100 Find the area between (i) $x = 26$ & $x = 38$ (ii) $x = 15$, $x = 40$

(i)



when $x = 26$

$$z = \frac{26 - 20}{10}$$

$$z = 0.6$$

When $x = 38$

$$z = \frac{38 - 20}{10}$$

$$z = 1.8$$

$P(26 \leq x \leq 38)$

$= P(0.6 \leq z \leq 1.8)$

$= P(z = 1.8) - P(z = 0.6)$

$= 0.4641 - 0.2258$

$= 0.2383$

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(ii) when $x = 15$ when $x = 40$

$z = \frac{15-20}{10}$ $z = \frac{40-20}{10}$

$z = -0.5$ $z = 2$

$P(15 \le X \le 40)$
$= P(-0.5 \le x \le 2)$
$= P(z=2) + P(z=-0.5)$
$= 0.4772 + 0.1915$
$= \cancel{0.2857}\ 0.6687$

Q A sample of 100 ~~...~~ dry cells are tested to find the length of life produce the following Results $\bar{x} = 12$ hrs
$\sigma = 3$ hrs
Assuming the data to be normally distributed what % of cells are expected to have life more than 15 hrs
(ii) less than 6 hrs. (iii) between 10 and 14 hrs.

(i) >15 hrs.
$P(x \ge 15) = P(z \ge 1)$
$= 0.5 - P(z=1)$
$= 0.1587$

% of cells $= 100 \times 0.1587$
$= 15.87$

(ii) $x < 6$ hrs
$z = \frac{6-12}{3} = \frac{-6}{3} = -2$

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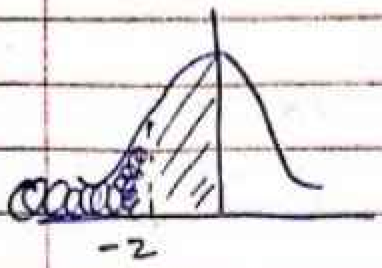


$P(x \le 6)$

$= P(z \le -2)$

$= 0.5 - P(z = -2)$

$= 0.5 - 0.4772$

$= 0.0228$

For 100 $= 100 \times 0.0228$

$= 2.28$

(iii) $10 \le x \le 14$

$\boxed{x = 10}$

$z = \dfrac{10 - 12}{3}$

$= \dfrac{-2}{3} =$

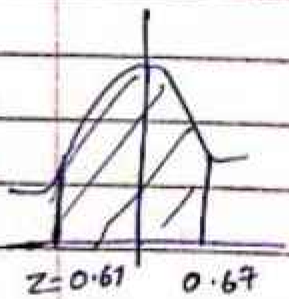
$= -0.67$

$x = 14$

$z = \dfrac{14 - 12}{3}$

$= \dfrac{2}{3}$

$= 0.67$



$P(10 \le x \le 14)$

$= P(-0.67 \le z \le 0.67)$

$= P(z = -0.67) + P(z = 0.67)$

$= 0.2454 + 0.2454$

$= 0.4908$

For 100 cells $= 49.08$

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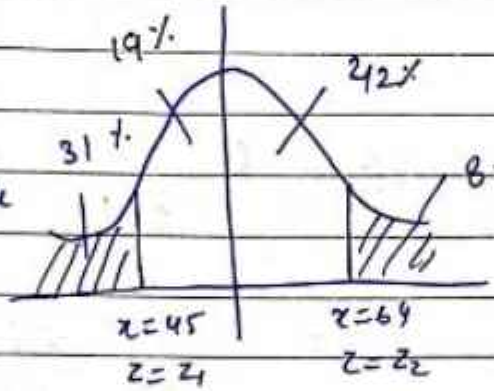
Ques In a Normal distribution 31% of the items under 45 and 8% are over 64. find the mean & SD of distribution?

$$\int_{z_1}^{0} f(z)\,dz = 0.19$$

$z_1 = 0.5$ ← By table

$$\boxed{z_1 = -0.5}$$



$$\int_{0}^{z_2} f(z)\,dz = 0.42$$

$$\boxed{z_2 = 1.4}$$

$$\boxed{z = \frac{x-\mu}{\sigma}}$$

$z_1 = \dfrac{45-\mu}{\sigma}$ | $z_2 = \dfrac{64-\mu}{\sigma}$

$-0.5 = \dfrac{45-\mu}{\sigma}$ | $1.4 = \dfrac{64-\mu}{\sigma}$

$-0.5\sigma = 45-\mu$ $\qquad$ $1.4\sigma = 64-\mu$

$$\begin{aligned} -0.5\sigma &= 45-\cancel{\mu} \\ 1.4\sigma &= 64-\cancel{\mu} \\ \hline -1.9\sigma &= -19 \end{aligned}$$

$$\begin{array}{r} 64 \\ -45 \\ \hline 19 \end{array}$$

$-5 = 45-\mu$

$\mu = 50$

$$\boxed{\sigma = 10} \qquad \boxed{\mu = 50}$$

Ques find the equation of the best fitting Normal curve to the following distribution.

x	0	1	2	3	4	5	
y	13	23	34	15	11	4	100
yx	0	23	68	45	44	20	200

$$\boxed{\mu = 2}$$

$$\sigma = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2}$$

$yx^2 =$ 0 23 136 135 176 100

$$\cancel{\sigma = \sqrt{5.7 - 4} = \sqrt{1.7} = 1.30}$$

$$\sigma = \sqrt{5.7 - 4} = \sqrt{1.7} = 1.3$$

$$y = \frac{1}{1.3\sqrt{2\pi}} e^{-(x-2)^2/3.4}$$

$x = 0$

$$y = \frac{1}{1.3\sqrt{2\pi}} e^{-4/3.4}$$

$$= \frac{1}{1.3\sqrt{2\pi}} e^{-1.17}$$

$$= \frac{e^{-1.17}}{3.25}$$

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Q. For a normally distributed variant with mean 1 & S.D 3. Find the probability that $3.43 \le x \le 6.19$

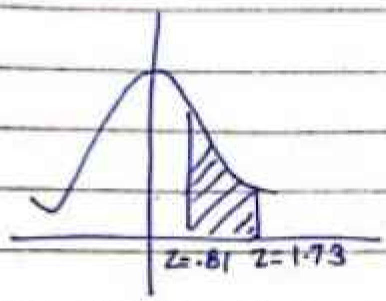
$\mu = 1 \qquad \sigma = 3$

$3.43 \le x \le 6.19$

$$\frac{6.19-1}{3}$$

$.81 \le z \le 1.73$

$P(3.43 \le x \le 6.19)$
$= P(.81 \le z \le 1.73)$
$= P(1.73) - P(.81)$
$= 0.4582 - 0.2910$
$= 0.1672$



Ques The mean height of 500 students is 151 cm & the $\sigma = 15$ cm. Assuming that the ~~heighest~~ heights are Normally distributed. Find how many students lies between 120 & 155 cm height?

Sol $\mu = 151$ cm, $\sigma = 15$ cm
$P(120 \le x \le 155) = ?$

$x = 120$

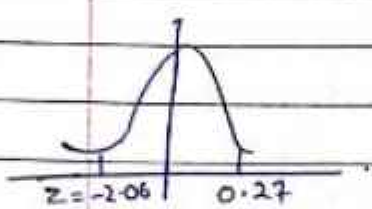
$$z = \frac{120-151}{15} = \frac{-31}{15} = -2.06$$

$x = 155$

$$z = \frac{155-151}{15} = \frac{4}{15} = 0.266 = 0.27$$

$$P(120 \le x \le 155) = P(-2.06 \le z \le 0.27)$$
$$= P(z=-2.06) + P(z=0.27)$$
$$= 0.4803 + 0.1064$$
$$= 0.5867$$

No. of students $= 500 \times 0.5867$
$= 293.35$
$= 293$

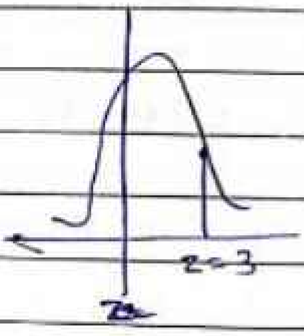


<u>Ques</u> X is normally distributed $\mu = 30$, $\sigma = 5$ find the probability of

(i) $x \geq 45$ (ii) $26 \le x \le 40$ (iii) Mode of $x-30 > 5$

<u>Solution</u> (i) $x = 45$

$$z = \frac{45-30}{5} = \frac{15}{5} = 3$$



$$P(x \geq 45) = 0.5 - P(z=3)$$
$$= 0.5 - 0.4987$$
$$= 0.0013$$

(ii) $x = 26$

$$z = \frac{26-30}{5} = -0.8$$

$x = 40$

$$z = \frac{40-30}{5} = 2$$

$$P(26 \le x \le 40) = P(z=-0.8) + P(z=2)$$
$$= 0.2881 + 0.4772$$
$$= 0.7653$$

(iii) $|x-a| < \delta$ $\qquad$ $|x-a| \geq \delta$

$x \in (a-\delta, a+\delta)$ $\qquad$ $x \leq a-\delta$

$x \geq a+\delta$

$|x-30| > 5$

$x \leq 30-5 = 25$

$x \geq 30+5 = 35$

$35 < x < 25$

$x = 25$	$x = 35$
$z = -1$	$z = 1$

$$P(25 > x > 35) = P(z=-1) + P(z=1)$$
$$= 0.3413 + 0.3413$$
$$= 0.6826.$$

Ques If x is normally distributed with $\mu = 0$, $\sigma^2 = 9$

find z_1 if $P(z \geq z_1) = 0.84$

$$z = \frac{x-\mu}{\sigma}$$

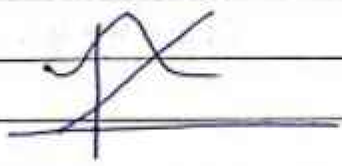
$$z = \frac{x-0}{1}$$

$$\boxed{z = x}$$

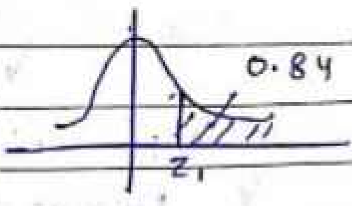
$$P(z \geq z_1) = 0.84$$

$$\int_0^{z_1} f(z)\,dz = 0.5 - 0.84 = -0.34$$

$$\boxed{z_1 = -0.34}$$



$$P(z_1) = 0.1331$$



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Correlation -
when the changes in one variable are associated or followed by the changes in the other variable is called correlation.
If an increase (or decrease) in the value of one variable corresponds to an increase (or decrease) in the other. The correlation is said to be +ve.

If an increase (or decrease) in one variable corresponds to the decrease (or increase) in the other variable the correlation is said to be negative

If there is no relationship between variable they are said to be independent or uncorrelated.

Cofficient of correlation -
The numerical measure of correlation is called the cofficient of correlation and is defined by the Relation

$$\rho \text{ or } \gamma = \frac{\Sigma XY}{n\sigma_x \sigma_y} = \frac{cov(x,y)}{\sigma_x \sigma_y}$$

Karl Pearson's cofficient of correation

$$cov(x,y) = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n}$$

$$\sigma_x = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} \quad \sigma_y = \sqrt{\frac{\Sigma(y-\bar{y})^2}{n}}$$

$X \rightarrow$ Deviation from mean $\bar{x}$) $= x-\bar{x}$
Y Deviation from mean $\bar{y}$) $= y-\bar{y}$

Note The cofficient of correlation lies between -1 & 1. $-1 < r < 1$; $|r| < 1$

σ_x = S.D of x series
σ_y = S.D of y series.

method of calculation -
① Direct method -

$$r \text{ or } \rho = \frac{\Sigma xy}{n\sigma_x\sigma_y}$$

$$= \frac{\frac{1}{n}\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}} = \frac{\frac{1}{n}\Sigma xy}{\sqrt{\Sigma \frac{1}{n}x^2}\sqrt{\frac{1}{n}\Sigma y^2}}$$

$$= \frac{\frac{1}{n}\Sigma xy}{\frac{1}{n}\sqrt{\Sigma x^2 \Sigma y^2}}$$

$$= \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}}$$

$$\rho = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{\{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]\}}}$$

<u>Step deviation method</u> - /<u>indirect method</u>

$$r = \frac{n\Sigma d_x \Sigma d_y - \Sigma d_x \Sigma d_y}{\sqrt{[\{n\Sigma d_x^2 - (\Sigma d_x)^2][n\Sigma d_y^2 - (\Sigma d_y)^2]}}$$

$$d_x = \frac{x-a}{h} \qquad d_y = \frac{y-b}{k}$$

a, b - assumed mean of x & y.
h - diff. between two consecutive term of x
k - " " " " " of y.

Que ~~Ovelogical~~ Psychological test of intelligence and ~~engineering abili~~ Eng. ability were applied to 10 students here is the record of ungrouped data showing intelligence Ratio & Eng. Ratio. calculate the coffient of correlation.

	Stu	A	B	C	D	E	F	G	H	I	J
x	IR	105	104	102	101	100	99	98	96	93	92
y	ER	101	103	100	98	95	96	104	92	97	94

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2 \Sigma Y^2}}$$

$\bar{x} = 99$ $\bar{y} = 98$

		X		Y	XY	X^2	Y^2
A	105	6	101	3	18	36	9
B	104	5	103	5	25	25	25
C	102	3	100	2	6	9	4
D	101	2	98	0	0	4	0
E	100	1	95	-3	-3	1	9
F	99	0	96	-2	0	0	4
G	98	-1	104	6	-6	1	36
H	96	-3	92	-6	+18	9	36
I	93	-6	97	-1	6	36	1
J	92	-7	94	-4	28	49	16
					92	170	140

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$$r = \frac{92}{\sqrt{170 \times 140}}$$

$$\boxed{r = 0.59}$$

~~The cof~~

Q. calculate the Karl pearson's cofficient of correlation for following Data using step deviation method

x	9	8	7	6	5	4	3	2	1
y	15	16	14	13	11	12	10	8	9

x	y	x	y	xy	x^2	y^2
9	15	4	3	12	16	9
8	16	3	4	12	9	16
7	14	2	2	4	4	4
6	13	1	1	1	1	1
5	11	0	-1	0	0	1
4	12	-1	0	0	1	0
3	10	-2	-2	4	4	4
2	8	-3	-4	12	9	16
1	9	-4	-3	12	16	9
				57	60	60

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}} = \frac{57}{\sqrt{60 \times 60}}$$

$$= \frac{57}{60}$$

$$= 0.95$$

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18/10/23.

is a technique of measuring or Estimating the Relationship among variables.

The Regression line describe the average Relationship Existing between x & y.

"Regression line of y on x"

$$\boxed{y-\bar{y} = b_{yx}(x-\bar{x})}$$

y → Dependent

b_{yx} → Regression Cofficient

x → Independent

"Regression line of x on y"

$$\boxed{x-\bar{x} = b_{xy}(y-\bar{y})}$$

$$b_{yx} = \frac{r\sigma_y}{\sigma_x} \quad ; \quad b_{xy} = \frac{r\sigma_x}{\sigma_y}$$

$$= \frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma x^2 - (\Sigma x)^2} \quad ; \quad = \frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$$

Ques Two Random variable have the Regression line with Equation $3x+2y=26 \rightarrow (1)$

$6x+y=31 \rightarrow (2)$

Find the mean values & the cofficient of corelation between x & y.

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$$3x + \cancel{2y} = 26$$
$$12x + \cancel{2y} = 62$$
$$-9x = -36$$

$\boxed{x = 4}$ $\boxed{y = 7}$

$\bar{x} = 4$; $\bar{y} = 7$

$$3x + 2y = 26$$
$$y = \frac{26 - 3x}{2}$$
$$\boxed{b_{yx} = -\frac{3}{2}}$$

$$6x + y = 31$$
$$6x = 31 - y$$
$$x = \frac{31}{6} - \frac{1}{6}y$$
$$\boxed{b_{xy} = -\frac{1}{6}}$$

$$\boxed{b_{yx} = \frac{r\sigma_y}{\sigma_x}}$$ $$\boxed{b_{xy} = \frac{r\sigma_x}{\sigma_y}}$$

Multiply

$$b_{yx} \cdot b_{xy} = r^2$$
$$\frac{+\cancel{3}}{2} \times \frac{+1}{\cancel{6}\,2} = r^2$$
$$\frac{1}{4} = r^2$$
$$r = \pm\frac{1}{2}$$

0.

$$r = -\frac{1}{2}$$

{ byx / bxy jiska value badi hogi uska sign consider karenge }

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Ques Find two lines of Regression and cofficient for correlation for the data given below -

$n = 18$, $\Sigma x = 12$, $\Sigma y = 18$

$\Sigma x^2 = 60$, $\Sigma y^2 = 96$, $\Sigma xy = 48$

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{18 \times 48 - 12 \times 18}{18 \times 60 - 144}$$

$$= \frac{864 - 216}{1080 - 144} = \frac{648}{936}$$

$$= 0.69$$

$$b_{xy} = \frac{648}{18(96) - (18)^2} = \frac{648}{1728 - 324}$$

$$= \frac{648}{1404}$$

$$= 0.46$$

$$\bar{x} = \frac{12}{18} = \frac{2}{3} \qquad \bar{y} = 1$$

(y on x)

$$(y-1) = 0.69(x - 0.67)$$

$$(y-1) = 0.69x - 0.46$$

$$y - 0.69x = -0.46 + 1$$

$$\boxed{y - 0.69x = 0.54}$$

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(x on y)

$$(x - 0.67) = 0.46(y - 0.1)$$

$$x - 0.67 = 0.46y - 0.46$$

$$\boxed{x - 0.46y = 0.21}$$

$$r = \sqrt{b_{yx} \times b_{xy}}$$

$$r = \sqrt{0.46 \times 0.69}$$

$$\boxed{r = 0.56}$$

Ques The following table showing the test scores made by Salesman on an intelligence test and their weekly sales. Calculate the regression line of sales on test scores. and estimate the most probable weekly sales volume. If a salesman makes a score of 70. (y on x)

Salesman	Test Scores (x)	Sales (y)	x^2	xy
1	40	2.5	1600	100
2	70	6.0	4900	420
3	50	4.5	2500	225
4	60	5.0	3600	300
5	80	4.5	6400	360
6	50	2.0	2500	100
7	90	5.5	8100	495
8	40	3.0	1600	120
9	60	4.5	3600	270
10	60	3.0	3600	180
	600	40.5	38400	2570

$$b_{yx} = \frac{10 \times 2570 - 600 \times 40.5}{10 \times 38400 - (600)^2}$$

$$= \frac{25700 - 24300}{384000 - 360000}$$

$$= \frac{1400}{24000}$$

$$= 0.058$$
$$\simeq 0.06$$

$\bar{x} = 60$ $\bar{y} = 4.05$

yra $(y - 4.05) = 0.06(x - 60)$

$$y - 4.05 = 0.06x - 3.6$$

$$\boxed{y = 0.06x + 0.45}$$

Equation of sales on test score

If weekly score is 70

$$y = 0.06(70) + 0.45$$

$$\boxed{y = 4.65}$$

Estimated value

Rank correlation -

is the measure of correlation depending on Rank

$$\rho = 1 - \frac{6 \Sigma d^2}{n(n^2-1)}$$

$d \rightarrow$ diff b/w Rank of two variable

Spearman's Relation of cofficient.

Q Find the Rank correlation for following data ?

x	y	R_x	R_y	$d = R_x - R_y$	d^2
56	147	5	6	-1	1
42	125	10	10	0	0
72	160	1	1	0	0
36	118	12	11	1	1
63	149	3	5	-2	4
47	128	8	9	-1	1
55	150	6	4	2	4
49	145	7	7	0	0
38	115	11	12	-1	1
43	140	9	8	1	1
68	152	2	3	-1	1
60	155	4	2	2	4
					18

$$\cancel{r = 1 - \frac{6(18)}{12(144-1)}} = \cancel{0.068}$$

$$r = 1 - \frac{6(18)}{12(143)} = \frac{143-9}{143}$$

$$r = \frac{134}{143} = 0.937$$

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Ques Calculate the Rank correlation cofficient from the following data showing Rank of 10 Students in 2 subjects :

Maths	Physics	$d = x - y$	d^2
3	5	-2	4
8	9	-1	1
9	10	-1	1
2	1	1	1
7	8	-1	1
10	7	3	9
4	3	1	1
6	4	2	4
1	2	-1	1
5	6	-1	1
			24

$$r = 1 - \frac{6(24)}{10(100-1)}$$

$$r = 1 - \frac{\cancel{6}^{2} \times 24}{\cancel{10}_{5} \times \cancel{99}_{33}}$$

$$= \frac{165 - 24}{165}$$

$$= 0.85$$

- when Ranks are Repeated -

$$\rho = 1 - \frac{6(\Sigma d^2 + F)}{n(n^2 - 1)}$$

$$F = \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12} + \dots$$

Que compute the Rank correlation coefficient for following data of marks obtain by 8 Students in commerce and maths..

comm (x)	Maths (y)	Rx	Ry	d	d^2
15	40	7	3	4	16
20	30	5.5	5	.5	.25
28	50	4	2	2	4
12	30	8	5	3	9
40	20	3	7	-4	16
60	10	2	8	-6	36
20	30	5.5	5	.5	.25
80	60	1	1	0	0
					81.5

$\frac{4+5+6}{3}$

$m_1 = 2 \qquad m_2 = 3$

$$F = \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12} + \dots$$

$$= \frac{2(4-1)}{12} + \frac{3(9-1)}{12}$$

$$= \frac{1}{2} + 2$$

$$= \frac{5}{2} = 2.5$$

$$r = 1 - \frac{6(81.5+2.5)}{8(63)} = 1 - \frac{6 \times 84}{48 \times 63}$$

$$\boxed{r = 0}$$

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Q Rank of 5 students in three subjects are given (computer Phy, & Maths) which two subjects have same approach.

(Rx) computer	(Ry) physics	(Rz) maths	d_{xy}	d_{yz}	d_{zx}	d_{xy}^2	d_{yz}^2	d_{zx}^2
2	5	2	-3	3	0	9	9	0
4	1	3	3	-2	-1	9	4	1
5	2	5	3	-3	0	9	9	0
1	3	4	-2	-1	3	4	1	9
3	4	1	-1	3	-2	1	9	4
						32	32	14

$$r_{xy} = 1 - \frac{\cancel{6}(32)}{5 \times \cancel{24}\,4}$$

$$= \frac{-3}{5}$$

$$= -0.6$$

$$r_{yz} = 1 - \frac{6(32)}{5 \times 24}$$

$$= -\frac{3}{5}$$

$$= -0.6$$

$$r_{zx} = 1 - \frac{\cancel{6}(\cancel{14}\,7)}{5 \times \cancel{24}\,\cancel{4}\,2}$$

$$= \frac{3}{10}$$

$$= 0.3$$

R_{zx} is max.

therefore Computer & maths have same approach

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UNIT-3.

curve fitting -

Several Eqn's of different types can be obtained to Express the ~~dis~~ given data approximately. The process of finding such an Eqn is known as curve fitting.

curve fitting is the process of constructing a curve or mathematical function that has the best fit to a series of Data points possible, subject to constraints.

method of least Square -

(a) To fit the straight line $y = a + bx$

(i) Substitute the observed set of n values in this equation

(ii) form normal Equation for each ~~constraint~~ constant

$$\Sigma y = na + b\Sigma x$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

(iii) Solve these normal Eqn's for a & b.

(iv) Substitute the values of a & b in $y = a+bx$ which is Required line of best fit.

Q what straight line best fits the following data in the least Square sets?

x	1	2	3	4	5
y	14	13	9	5	2

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Let the line ~~orath~~ is $y = a + bx \rightarrow (1)$
Normal Equations are-

x	y	xy	x^2
1	14	14	1
2	13	26	4
3	9	27	9
4	5	20	16
5	2	10	25
15	43	97	55

$$\begin{array}{r} 1 \\ 29 \\ 14 \\ \hline 43 \end{array}$$

~~43 = a+b~~ $43 = 5a + 15b$
$97 = 15a + 55b$

$$129 = 15a + 45b$$
$$\underline{-97 = -15a + 55b}$$
$$32 = -10b$$

$\boxed{b = -3.2}$ $\boxed{a = 18.2}$

$$43 = 5a + 15(-3.2)$$
$$43 + 48 = 5a$$
$$91 = 5a$$
$$\boxed{a = 18.2}$$

Req. line of Best fit $y = 18.2 - 3.2x$

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(b) To fit the parabola $y = a + bx + cx^2$

Normal Eq^n -

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$
$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

Que The velocity "v" of a liq is known to vary with temp "T" Acc. to the Quadratic law $v = a + bT + cT^2$. Find the best fit of a, b & c for the following table

T	V	T^2	T^3	T^4	VT	VT^2
1	2.31	1	1	1	2.31	2.31
2	2.01	4	8	16	4.02	8.04
3	1.80	9	27	81	5.40	16.2
4	1.66	16	64	256	6.64	26.56
5	1.55	25	125	625	7.75	38.75
6	1.47	36	216	1296	8.82	52.92
7	1.41	49	343	2401	9.87	69.09
28	12.21	140	784	4676	44.81	213.87

Normal Eq^n,

$$12.21 = 7a + 28b + 140c$$
$$44.81 = 28a + 140b + 784c$$
$$213.87 = 140a + 784b + 4676c$$

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$48.84 = \cancel{28}a + 112b + 560c$
$44.81 = 28a + 140b + 784c$

$4.03 = -28b - 224c \rightarrow 1$

122V1

$244.2 = \cancel{140}a + 560b + 2800c$
$213.87 = 140a + 784b + 4676c$

$30.33 = -224b - 1876c \rightarrow 2$

$8[4.03 = -28b - 224c]$
$30.33 = -224b - 1876c$

$32.24 = -\cancel{224}b - 1792c$
$30.33 = -224b - 1876c$

$1.91 = 84c$

$\boxed{c = 0.022}$

$4.03 = -28b - 4.9$
$\boxed{-0.31 = b}$

$12.21 = 7a + 28(-0.31) + 140(0.022)$
$12.21 = 7a - 8.68 + 3.08$
$17.81 = 7a$
$\boxed{a = 2.54}$

$y = 2.54 + (-0.31)x + 0.022x^2$
$\boxed{y = 2.54 - 0.31x + 0.022x^2}$

$\boxed{V = 2.54 - 0.31T + 0.022T^2}$

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Fit of some special

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~~Sampling distribution~~ :

Fitting of other curves -

Following curves are some of the special curves which are not linear. but can be Reduced to Linear form by suitable substitution

(1) $y = ax^b$

taking log on both side

$\log_{10} y = \log_{10} a + b \log_{10} x$

$Y = A + bX$

Normal

$\Sigma Y = nA + b\Sigma X$

$\Sigma XY = A\Sigma X + b\Sigma X^2$

(ii) $y = ax^n + b$

$y = aX + b$ $\boxed{\text{Let } x^n = X}$

Normal Eqn's

$\Sigma y = a\Sigma X + nb$

$\Sigma Xy = a\Sigma X^2 + b\Sigma X$

(iii) $y = ae^{bx}$

taking log.

$\log y = \log a + bx\log_{10} e$

$Y = A + \cancel{bx}\ Bx$ $\boxed{B = b\log_{10} e}$

(iv) $xy^a = b$

Taking log

$\log x + a\log y = \cancel{b}\log b$

$\boxed{\cancel{X + aY = B}}$

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Q $\log_{10} y = \cancel{\frac{1}{a}\log_{10} b - \frac{1}{a}\log_{10} x}$

$\log_{10} Y = \frac{1}{a}\log_{10} b \boxed{-\frac{1}{a}} \log_{10} x$

$Y = B + AX$

(v) $xy = ax + by$

Divide by xy

$1 = \frac{a}{y} + \frac{b}{x}$

$1 = aY + bX$

$\frac{1}{a} - \frac{bX}{a} = Y$ $\qquad \frac{1}{a} = A \quad \frac{-b}{a} = B$

$\boxed{A + BX = Y}$

Q using the method of least sq. fit an exponential curve $y = ae^{bx}$ to the data given in followi[ng] table

x	y	$\log_{10} y = Y$	xY	x^2
1	7.209	0.857	0.857	1
2	5.265	0.721	1.442	4
3	3.846	0.585	1.755	9
4	2.809	0.44	1.76	16
5	2.052	0.312	1.56	25
6	1.499	0.175	1.05	36
21		3.09	8.424	91

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$$y = ae^{bx}$$

$$\log_{10} y = \log_{10} a + bx \log_{10} e$$

$$y = A + Bx$$

Eqation A & B $\quad \Sigma y = nA + B\Sigma x$

$$\Sigma xy = A\Sigma x + B\Sigma x^2$$

$$3.09 = 6A + 21B$$
$$8.444 = 21A + 91B$$

$\cancel{21A}$

$$65.016 = 126A + 441B$$
$$50.664 = 126A + 546B$$

$$14.352 = -105B$$

$$\boxed{B = -0.136}$$

$$6A = 3.09 + 21 \times (+0.136)$$
$$A = \frac{5.946}{6}$$

$$\boxed{A = 0.991}$$

$A = \log_{10} a \qquad B = b \log_{10} e$

$\boxed{a = 9.86} \qquad \boxed{b = -0.314}$

$$\boxed{y = 9.86 e^{-0.314x}}$$

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Q The pressure & volume of a gas are Related by Eqn $PV^{\gamma} = K$ where γ & K are constant. Fit this Equation to following set of observation.

p	v	$P = \log_{10} p$	$V = \log_{10} V$	P^2	PV
0.5	1.62	-0.301	0.209	0.090	-0.062
1.0	1.00	0	0	0	0
1.5	0.75	0.176	-0.124	0.030	-0.021
2.0	0.65	0.301	-0.187	0.034	-0.056
2.5	0.52	0.397	-0.283	0.080	-0.112
3.0	0.46	0.477	-0.377	0.113	-0.160
		1.05	-0.722	0.347	-0.411

$$PV^{\gamma} = K$$

$$\log_{10} p + \gamma \log_{10} v = \log_{10} k$$

$$\log_{10} v = \frac{1}{\gamma}\log_{10} k - \frac{1}{\gamma}\log_{10} p$$

$$V = A + BP$$

$$\Sigma V = nA + B\Sigma P \rightarrow (1)$$

$$-0.722 = 6A + 1.05B$$

$$\Sigma PV = A\Sigma P + B\Sigma P^2$$

$$-0.411 = A(1.05) + 0.347B$$

$$-2.466 = -0.7581 - 1.1025B + 2.082B$$

$$\boxed{B = -1.743}$$

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$A = \frac{1.10815}{6} = 0.184$

$p = \log_{10} p$

$A = \frac{1}{V} \log_{10} K$

$0.184 = \frac{1}{V} \log_{10} K$

$\log_{10} K = -0.1055$

$\boxed{K = 0.784}$

$B = \frac{1}{V}$

$\boxed{V = -0.5737}$

$\boxed{pV^{-0.5737} = 0.784}$ - Eqn

sampling distribution -

A small section selected from the population is called a sample and process drawing a sample is called sampling.

Parameter & statistic -

Parameter	statistics	SE
μ	$\bar{x}$	$\sigma/\sqrt{n}$
σ	S	$\sqrt{\sigma^2/2n}$
σ^2	S^2	$\sigma^2/\sqrt{2n}$

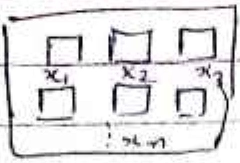
constant of population → Parameter

constant of sample → statistics

No. of possible samples = $^N C_n$

N → size of population

n → size of 1 sample



$\bar{x}_1 = p_1(\bar{x}_1)$
$\bar{x}_2 = p_2(\bar{x}_2)$
$\bar{x}_3 = p_3(\bar{x}_3)$
$\vdots$

$s_1 = p(s_1)$
$s_2 = p(s_2)$
$\vdots$
$s_n = p(s_n)$

The set of values of the statistics so obtained one for each sample is called Sampling distribution of stastics

Standard Error -

The standard deviation of the sampling distribution of a statistics is known as standard Error

$$S.E = \frac{\sigma}{\sqrt{n}}$$

It plays a very important Role in Large sample theory & form the Basis of testing of hypothesis

If t is any statistic than for large sample

$$Z = \frac{t - E(t)}{\sqrt{V(t)}}$$

Expected

$\longrightarrow$ varience , $\sqrt{v} \rightarrow SD$

Null hypothesi's -

The hypothesis formulated for the sake of Rejecting it, under the assumption that it is true. is called NULL hypothesis

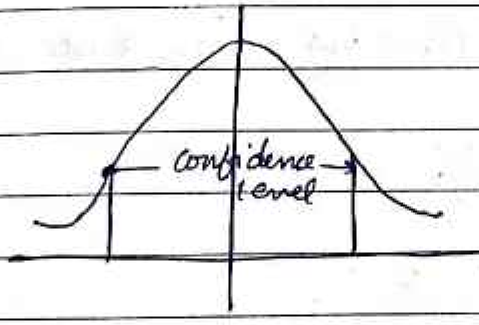
(H_0) - denoted by

Alternate Hypothesis
complementry of Null hypothesis
denoted by $\rightarrow H_a$

Type 1 Error -
Hypothesis is Rejected by it is true

Type 2 Error
Hypothesis is accepted by it is false.

Level of significance :-
The probability level below which we Reject the hypothesis is known as the level of Significance
The Region in which a sample value is falling is Rejected is known as critical Region. ~~with~~



The probability with which we accept ~~and~~ a null hypothesis which is true is the level of confidence

$\boxed{\text{level of significance} + \text{level of confidence} = 1}$

we generally take critical Regions which covers 5% & 1% area of the normal curve.

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Test of significance for large Sample -

The steps to be used in the normal test are as follows -

(i) Compute the test Statistic z under H_0

(ii) If $|z| > 3$ then H_0 is always Rejected.

(iii) If $|z| \leq 3$ we test its significance at certain level of significance ussally at 5% & Sometime at 1% leve of Significance.

If $|z| > 1.96$ hypothesis is Rejected at 5% level of significance.

If $|z| > 2.58$ hypothesis is Rejected at 1% leve of significance.

* formula for finding confidence limit -

~~p ± 3 √pq/n~~

$$\boxed{p \pm 3\sqrt{\frac{pq}{n}}}$$

* test of significance for single proportion -

If X is no. of sucess in n independent trials with constant probability p of sucess for Each trial then

$$E(X) = np$$

$$V(X) = npQ$$

Hence for ~~large~~ N.

$$z = \frac{X - np}{\sqrt{npQ}}$$

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$$\boxed{z = \frac{X/n - P}{\sqrt{PQ/n}}}$$

A die is thrown 9000 times and a throw of 3 or 4 is observed ~~3to~~ 3240 times show that the die cann't be Regarded as an unbaised one & find the limits between which the propabelity of a throw of 3 or 4 lies.

$H_0 \rightarrow$ The die be unbaised.

$X \rightarrow 3240$

$n \rightarrow 9000$

$P = 1/3$ $\quad Q = 2/3$

$$z = \frac{3240 - \overset{3000}{\cancel{9000}}(1/3)}{\sqrt{\underset{1000}{\overset{3000}{\cancel{9000}}} \times \frac{1}{3} \times \frac{2}{3}}}$$

$$= \frac{240}{10\sqrt{10 \times 2}} = \frac{\cancel{24}\ 12}{\sqrt{20}} = \frac{12}{2\sqrt{5}}.$$

$z = 5.36 > 3$

Hypothesis is Rejected.

So, die is baised. $\quad 0.0000256$

$$p = \frac{3240}{9000} = 0.36$$

$q = 1 - 0.36 = 0.64$

$$\text{Limit} = 0.36 \pm 3\sqrt{\frac{0.36 \times 0.64}{9000}}$$

0.36 ± 0.015

$0.375 \quad , \quad 0.345$

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Q 20 people were attacked by a disease & only 18 survive will you. Reject the hypothesis that the survival Rate if attacked by this disease is 85% at 5% level of significance

H_0 = Survival Rate if attacked by this disease. 85%.

$x = 18$ $n = 20$.

$$P = \frac{85}{100} = 0.85$$

$$Q = \frac{15}{100} = 0.15$$

$$z = \frac{18 - 20 \times 0.85}{\sqrt{20 \times .85 \times .15}}$$

$$z = \frac{18 - 17}{1.59}$$

$$z = \frac{1}{1.59}$$

$$z = 0.62.$$

Since $z < 1.96$ therefore hypothesis is accepted at 5% of the significance level.

i.e., survival Rate is 85%.

Q A coin was tossed 400 times and head turns up to 216 times. Test the hypothesis the coin is unbaised at 5% level of significance.

Ho: coin is unbaised.

$x = 216 \qquad n = 400 \qquad P = \frac{1}{2} \qquad Q = \frac{1}{2}$

$$z = \frac{216 - 400 \times \frac{1}{2}}{\sqrt{400 \times \frac{1}{2} \times \frac{1}{2}}} = \frac{16}{10}$$

$z = 1.6$

~~Since $z < 2.58$ hypothesis is accepted at 1% level.~~

$z < 1.96$, hypothesis is ~~Reg~~ accepted at 5%.
Hence coin is unbaised.

Test of significance for difference of proportion

$$z = \frac{(p_1 - p_2) - E(p_1 - p_2)}{\sqrt{V(p_1 - p_2)}}$$

$$\boxed{p_1 = \frac{x_1}{n_1}} \; , \; \boxed{p_2 = \frac{x_2}{n_2}}$$

if $P_1 = P_2$

$$z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

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$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

Q Random sample of 400 men & 600 women were asked weather they would like to have a flyover near their Residence. 200 men & 325 women were in favour of the purposal. Test the hypothesis that propotion of men & women in favour of purposal are same agains that they are not at 5% level of significance?

			x_i	
men	(n_1)	400	x_1	200
women	(n_2)	600	x_2	325

$$p_1 = \frac{x_1}{n_1} = \frac{200}{400} = \frac{1}{2} = 0.50$$

$$p_2 = \frac{x_2}{n_2} = \frac{325}{600} = 0.541$$

$$P = \frac{400 \times 0.50 + 0.541 \times 600}{1000}$$

$$= \frac{200 + 324}{1000} = \frac{524}{1000}$$

$$P = 0.525$$

$$Q = 0.475$$

$$Z = \frac{0.5 - 0.541}{\sqrt{0.525 \times 0.475 \left(\frac{1}{400} + \frac{1}{600}\right)}}$$

0.0025
0.0016

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$$z = \frac{-0.041}{\sqrt{0.249 \times 0.0041}}$$

$$= \frac{-0.041}{\sqrt{0.001039}}$$

$$= \frac{-0.041}{0.0323}$$

$$= \cancel{0.1269}\ -1.269$$

Since $|z| < 1.96$ so,
hypothesis is accepted

2 In a large city A 20% of a Random sample of 900 school children had defected eyesight. In other large city B 15% of a Random Sample 1600 had the same effect. Is this difference b/w two proportion significant. Obtain 95% confidence limit for the difference in population proportion

A $n_1 = 900$ $\qquad x_1 = \frac{20}{100} \times 900 = 180$

B $n_2 = 1600$ $\qquad x_2 = \frac{15}{100} \times 1600 = 240$

$$p_1 = \frac{x_1}{n_1} = \frac{180}{900} = \frac{1}{5} = 0.2$$

$$p_2 = \frac{240}{1600} = 0.15$$

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$$P = \frac{900 \times 0.5 + 1600 \times 0.15}{2500}$$

$$P = \frac{450 + 240}{2500} = 0.276$$

$Z \not< 1.96$

So, hypothesis is accepted.

$$\boxed{p_1 - p_2 \pm 1.96 \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}}$$

0.05 ± 0.03
$(0.019, 0.081)$

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Test of Significance for single mean -
If $x_i (i = 1, 2 \cdots n)$ is a random sample of size n from a normal population with mean μ & variance σ^2 then the sample mean is distributes normally with mean "$\bar{x}$" & variance "σ^2/n"
Thus, for the large sample the standard normal variate corresponding to $\bar{x}$ is

$$z = \frac{\bar{x} - \mu}{\sqrt{\sigma^2/n}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

Confidence limit for μ :-
i) 95% confidience intervel for μ $|z| \leq 1.96$

$$\left| \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \right| \leq 1.96$$

$$|\bar{x} - \mu| \leq 1.96 \sigma/\sqrt{n}$$

$$\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}$$

ii) for 98%. $|z| \leq 2.33$

However in sampling from a finite population of size N the corresponding 95% & 99% Confidence limit for ~~x~~ μ are

$$x \pm 1.96 \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$

for 99%.

$$x \pm 2.58 \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$

The confidence limit for parameter are also known as fiducial limit

Q A Sample of 900 members has a mean 3.4 and standard deviation 2.61 is the sample ~~form~~ from a large population mean 3.25 & standard deviation 2.61. If the population is normal & its mean is unknown. find 95% & 98% Feducial limits of true mean?

H_0 : The sample has been drawn from population 3.25 & SD 2.65

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{3.4 - 3.25}{\frac{2.61}{\sqrt{900}}} = \frac{0.15 \times 30}{2.61}$$

$$= 1.724 < 1.96$$

Hypothesis is accepted, that means sample is drawn from the

$$\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}$$

$$\mu = \left(3.4 \pm \frac{1.96 \times 2.61}{\sqrt{900}}\right)$$

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$\mu = (3.4 \pm 0.17)$

$\mu = (\cancel{3.23}\ 3.22, 3.57)$

$\mu = \left(\bar{x} \pm 2.58 \frac{\sigma}{\sqrt{n}}\right)$

$\mu = \left(3.4 \pm \frac{2.58 \times 2.61}{30}\right)$

$\mu = (3.4 \pm 0.224)$

$\mu = (3.176, 3.62)$

Q A normal population has mean 0.1 & standard deviation 2.1 find the probability mean of sample size 900 will be -ve

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{\bar{x} - 0.1}{2.1/\sqrt{900}}$

$z = \frac{\bar{x} - 0.1}{0.07}$

$0.07z + 0.1 = \bar{x}$

$P(\bar{x} < 0)$

$P(0.1 + 0.07z < 0)$

$\frac{0.100}{0.070}$

$\frac{10}{7}$

$P(z < -1.43)$

$P(z > 1.43)$

$0.5 - 0.4236$

$= 0.0764$

Q The S.D of a population is 2.70 find the probability that in a Random sample of size 66

(i) The sample mean will differ from population mean by 0.75 or more. $P(|\bar{x}-\mu| > 0.75)$

(ii) The sample mean will exceed the population mean by 0.75 or more $P((\bar{x}-\mu) \geq 0.75)$

$\boxed{0.0122}$

$$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

$$(\bar{x}-\mu) = \frac{2.70 \times Z}{\sqrt{66}}$$

$$\bar{x}-\mu = 0.332Z$$

$$P(0.33|Z| \geq 0.75)$$

$$P(|Z| \geq 2.25)$$

$$2(0.5 - P(z \text{ at } 2.25))$$

$$\left\{ \begin{array}{c} \text{If } |x|>1 \\ -\infty < x < -1 \\ 1 < x < \infty \end{array} \right\}$$

$$2(0.5 - 0.4878)$$
$$2(0.0122)$$
$$0.0244$$

Test of significance for difference of mean-

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

If $\sigma_1^2 = \sigma_2^2 = \sigma$

i.e., the sample have been drawn from population with common S.D (σ) then under null hypothesis μ_1 is Equal to μ_2

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

If $\sigma_1^2 \neq \sigma_2^2$ and σ_1 & σ_2 are not known then they are Estimated from Sample values.

$\sigma_1 = S_1$ $\sigma_2 = S_2$

The mean of two single large Samples of 1000 & 2000 members are 67.5 & 68 Respectively can the samples be Regarded as drawn from same population of SD 2.5 (Test at 5% level of significance?

Ho : samples are drawn from same population.

$\bar{x}_1 = 67.5$ $n_1 = 1000$ $\sigma = 2.5$

$\bar{x}_2 = 68$ $n_2 = 2000$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1^2} + \frac{1}{n_2^2}}} = \frac{67.5 - 68}{2.5\sqrt{\frac{1}{(1000)^2} + \frac{1}{(2000)^2}}}$$

$$= \frac{-0.5}{2.5\sqrt{\frac{5}{(2000)^2}}}$$

$$= -\frac{0.5 \times 2000}{2.5\sqrt{5}}$$

$$= \frac{-1000}{2.5\sqrt{5}} =$$

$$Z = -5.9 > 1.96$$

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Hypothesis is Rejected.
So, samples are not drawn from same Population.

Test of significance for Difference of ~~Proportion~~ Standard deviation

If S_1 & S_2 are the S.D of two indepent samples then under null hypothesis H_0 : $\sigma_1 = \sigma_2$ i.e., Sample standard deviation don't differ significantly

$$z = \frac{S_1 - S_2}{S.E(S_1 - S_2)}$$

but in case of large sample the S.E

$$S.E = \sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_2}}$$

$$z = \frac{S_1 - S_2}{\sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_2}}}$$

Q Random Sample drawn from the countries given the following data Relating to the Height of adult males.

	A	B
mean heigh	67.42	67.25
S.D	2.58	2.50
No. of Sample	1000	2000

(i) Is the difference between the mean significant
(ii) Is the difference b/w S.D significant?

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H_0: No significant diff b/w S.D

$$z = \frac{s_1 - s_2}{\sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_2}}}$$

$$= \frac{2.58 - 2.50}{\sqrt{\frac{(2.58)^2}{2 \times 1000} + \frac{(2.50)^2}{2 \times 2000}}}$$

$$= \frac{0.08}{\sqrt{\frac{6.656}{2000} + \frac{6.25}{4000}}}$$

1.03

$$= \frac{0.08 \times \sqrt{4000}}{\sqrt{13.312 + 6.25}}$$

$$= \frac{0.08 \times 20\sqrt{10}}{\sqrt{19.562}}$$

$$= \frac{1.6\sqrt{10}}{4.42}$$

$$= 1.14$$

$ax + by + cz = 0 \rightarrow$ homogenous
$ax + by + cz = d \rightarrow$ Non homogenous

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Linear algebra

Solution of non homogenous linear systems of Equation by determinant

- Cramer's Rule :

The solution of the system of Linear Equation

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

is given by -

$$x = \frac{D_1}{D} \qquad y = \frac{D_2}{D} \qquad z = \frac{D_3}{D}$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad D \neq 0$$

$$D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} \quad D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \quad D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Conditions -

For a system of n simultaneous Linear Eq^n in n unknown -

(1) If $D \neq 0$ then the given system of Equation is consistent and has unique solution is given by

$$x_1 = \frac{D_1}{D}, \quad x_2 = \frac{D_2}{D} \quad \cdots\cdots \quad x_n = \frac{D_n}{D}$$

(2) If $D = 0$ ~~and~~

(i) If $D_1 = D_2 = D_3 = \cdots\cdots = D_n = 0$ then given system of Equation may or may not be consistent

If consisten then Eq^n → Infinetly many Solution

(ii) If Atleast $D_1, D_2, \ldots D_n$ is non zero then the given System of Eq^n is inconsistent

Que solve the following system of Eq^n using cramer's Rule -

$$5x - 7y + z = 11$$
$$6x - 8y - z = 15$$
$$3x + 2y - 6z = 7$$

$$D = \begin{vmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{vmatrix}$$

$$D = 5[48+2] + 7[-36+3] + 1[12+24]$$
$$= 5[50] + 7[-33] + 1[36$$
$$= 250 - 231 + 36$$
$$= 55$$

$$D_1 = \begin{vmatrix} 11 & -7 & 1 \\ 15 & -8 & -1 \\ 7 & 2 & -6 \end{vmatrix}$$

$$= 11[48+2] + 7[-90+7] + 1[30+56]$$
$$= 55$$

$$D_2 = \begin{vmatrix} 5 & 11 & 1 \\ 6 & 15 & -1 \\ 3 & 7 & -6 \end{vmatrix} \qquad D_3 = \begin{vmatrix} 5 & -7 & 11 \\ 6 & -8 & 15 \\ 3 & 2 & 7 \end{vmatrix}$$

$$= -55 \qquad\qquad = -55$$

$\boxed{x = 1}$; $\boxed{y = -1}$ $\boxed{z = -1}$

Q. Using Cramer's Rule / Determinants show that the following System of linear Eq^n is inconsistent

$$x - 3y + 5z = 4$$
$$2x - 6y + 10z = 11$$
$$3x - 9y + 15z = 12$$

$$D = \begin{vmatrix} 1 & -3 & 5 \\ 2 & -6 & 10 \\ 3 & -9 & 15 \end{vmatrix} = 2\begin{vmatrix} 1 & -3 & 5 \\ 1 & -3 & 5 \\ 3 & -9 & 15 \end{vmatrix} = 2(0)$$

$$\boxed{D = 2}$$

$$D_1 = \begin{vmatrix} 4 & -3 & 5 \\ 11 & -6 & 10 \\ 12 & -9 & 15 \end{vmatrix} = 4[-6(15)]+3[$$

$$= 4[-6(15) - (-9)10] + 3[15(11) - 12(10)] + 5[11(-9) - (-6)12]$$
$$= 4[-90 + 90] + 3[165 - 120] + 5[-99 + 72]$$
$$= 0$$
$$D_1 = 0$$

$$D_2 = \begin{vmatrix} 1 & 4 & 5 \\ 2 & 11 & 10 \\ 3 & 12 & 15 \end{vmatrix} = 1[15(11) - 12(10)] - 4[30 - 30] + 5[24 - 33]$$

$$= 165 - 120 + (-45)$$
$$= 0$$

$$D_3 = \begin{vmatrix} 1 & -3 & 4 \\ 2 & -6 & 11 \\ 3 & -9 & 12 \end{vmatrix} = 0$$

Since $D = 0$ as well as $D_1 = D_2 = D_3 = 0$

so, solution is inconsistent

Put $z = k$

$x - 3y = 4 - 5k$

$2x - 6y = 11 - 10k$

$$D = \begin{vmatrix} 1 & -3 \\ 2 & -6 \end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix} 4-5k & -3 \\ 11-10k & -6 \end{vmatrix}$$

$\boxed{D_1 = 9}$

unique solution

Q using Rule solve the following system of Eqn -

$x + y + z = 1$

$x + 2y + 3z = 4$

$x + 3y + 5z = 7$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix}$$

$D = 1[10-9] - 1[5-3] + 1[3-2] = 1 - 2 + 1 = 0$

$$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 3 \\ 7 & 3 & 5 \end{vmatrix} = 1[10-9] - 1[20-21] + 1[12-14]$$

$= 1 + 1 - 2$

$= 0$

$D_2 = 0$

$$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & 3 \\ 1 & 7 & 5 \end{vmatrix} = 1[20-21] - 1[3-5] + 1[7-4]$$

$= -1 - 2 + 3$

$= 0$

$D_3 = 0$

$$D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 7 \end{vmatrix} = 1[14-12] - 1[7-4] + 1(3-2)$$

$= 0$

$x + y = 1 - k$
$x + 2y = 4 - 3k$

$D = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$

$\boxed{D = 1} \neq 0$

So, Consistent unique Solution

$D_1 = \begin{vmatrix} 1-k & 1 \\ 4-3k & 2 \end{vmatrix}$

$= 2 - 2k - 4 + 3k$

$= -2 + k$

$D_2 = \begin{vmatrix} 1 & 1-k \\ 1 & 4-3k \end{vmatrix}$

$= 4 - 3k - 1 + k$

$= 3 - 2k$

$\boxed{x = K - 2}$ $\boxed{y = 3 - 2k}$

$x + 3y + 5z = 7$
$K - 2 + 3(3 - 2K) + 5k = 7$
$K - 2 + 9 - 6k + 5k = 7$
$\boxed{7 = 7}$

Hence the Eqn's have infinetly many sol.

Q Find the values of λ for which the following system of Equation fail to have a unique solution

$\lambda x + 3y - z = 1$
$x + 2y + z = 2$
$-\lambda x + y + 2z = -1$

$D = \begin{vmatrix} \lambda & 3 & -1 \\ 1 & 2 & 1 \\ -\lambda & 1 & 2 \end{vmatrix}$

$D = \lambda[4-1] - 3[2+\lambda] - 1[1+2\lambda]$
$D = 3\lambda - 6 - 3\lambda - 1 - 2\lambda.$
$D = -2\lambda - 7$

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$D = 0$

$\boxed{\lambda = -\frac{7}{2}}$

- Doesn't have any solution for this value of λ.

$$D_1 = \begin{vmatrix} 1 & 3 & -1 \\ 2 & 2 & 1 \\ -1 & 1 & 2 \end{vmatrix}$$

$= 1[4-1] - 3[4+1] - 1[2+2]$

$= 3 - 15 - 4$

$= -16 \neq 0$

So, it has no solution.

Solution of homogenous linear Eq^n System by Cramer's Rule -

① If $D \neq 0$ then the homogenous system of Eq^n have unique solution.

$x_1 = x_2 = \cdots\cdots = x_n = 0$ is called trivial solution

② If $D = 0$ then the homogenous system of Eq^n have infinitely many solution.

Q. Solve the following system of Eq^n using Cramer's Rule

$x + y - z = 0$

$x - 2y + z = 0$

$3x + 6y - 5z = 0$

$$D = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 3 & 6 & -5 \end{vmatrix} = \begin{array}{l} 1[10-6] - 1[-5-3] - 1[6+6] \\ 4 + 8 - 12 \\ = 0 \end{array}$$

$D = 0$

So, infinitely many solution.

$\boxed{z = k}$

$x + y = k$

$x - 2y = -k$

$$D = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -2-1 = -3$$

$$D_1 = \begin{vmatrix} K & 1 \\ -K & 2 \end{vmatrix} = -K$$

$$D_2 = \begin{vmatrix} 1 & K \\ 1 & -K \end{vmatrix} = -K-K = -2K$$

$$x = \frac{D_1}{D} = \frac{K}{3} \qquad y = \frac{D_2}{D} = \frac{2K}{3} \qquad z = k$$

Ques Solve $3x - 4y + 5z = 0$, using cramer's rule.

$$x + y - 2z = 0$$
$$2x + 3y + z = 0$$

$$D = \begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 3(1+6) + 4(1+4) + 5(3-2)$$
$$= 21 + 20 + 5$$
$$= 46 \neq 0$$

unique solution.

$$\boxed{\therefore x = y = z = 0}$$

Ques Find the values of λ for which the homogenous system of Eq^n has non-zero sol^n ?

$$2x + 3y - 2z = 0$$
$$2x - y + 3z = 0$$
$$7x + \lambda y - z = 0$$

$D = 0$

$$\begin{vmatrix} 2 & 3 & -2 \\ 2 & -1 & 3 \\ 7 & \lambda & -1 \end{vmatrix} = 0$$

$$2[1 - 3\lambda] - 3[-2 - 21] + 2[2\lambda + 7] = 0$$
$$2 - 6\lambda + 69 + 4\lambda + 14 = 0$$

23
$\times 3$
69

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$$2[1-3\lambda] - 3[-2-21] - 2[2\lambda+7] = 0$$

$$2 - 6\lambda + 69 - 4\lambda - 14 = 0$$

$$-10\lambda = -57$$

$$\lambda = 5.7$$

$$\begin{array}{r} 69 \\ -14 \\ \hline 55 \end{array}$$

* If $AA' = I$, then given matrix is called as orthogonal matrix.

$$\boxed{A' = \frac{1}{A}} \quad \text{or} \quad \boxed{A' = A^{-1}}$$

Q If A & B are orthogonal matrix of some order Prove AB is also an orthogonal matrix?

To prove $(AB)(AB)' = I$

LHS $(AB)(AB)' = AB(B'A')$

$= A\underbrace{BB'}A'$

$= AIA'$

$= AA'$

$= I = RHS$//

Hence proved//

Conjugate of complex matrix -

Let $A = \begin{bmatrix} 2+i & 2i \\ 1 & 3-i \end{bmatrix}$ $\bar{A} = \begin{bmatrix} 2-i & -2i \\ 1 & 3+i \end{bmatrix}$

Hermitian matrix - (only for complex matrix)

$(\bar{A})' = A$

$(\bar{A})' = -A \rightarrow$ skew hermition matrix

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unitary matrix A^* → Hermittion matrix
$(\bar{A})'A = I$ or $A^*A = I$

Ques Let A be the Square complex matrix then Prove
(1) $A + A^*$ is hermitian matrix.
(2) $A - A^*$ is Skew hermitian matrix.
(3) AA^* & A^* hermition.

Solⁿ. (i) To prove = $(\overline{A+A^*})' = A + A^*$

Let $A + A^* = B$

$$\bar{B} = (\overline{A+A^*}) = \bar{A} + \bar{A^*}$$

$$(\bar{B})' = (\bar{A} + \bar{A^*})'$$
$$= (\bar{A})' + (\bar{A^*})'$$
$$= A^* + (A^*)^*$$
$$= A^* + A$$
$$= B$$

Hence proved //

(ii) $(A - A^*)^* = -(A - A^*)$

$$(A - A^*)^* = A^* - (A^*)^*$$
$$= A^* - A$$
$$\boxed{(A - A^*)^* = -(A - A^*)}$$

Hence proved //

(iii) $(AA^*)^* = AA^*$ & $(A^*)^* = A$ {using property}

$= A^*(A^*)^*$

$= A^*A$

$= AA^*$

Hence proved //

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$1-2i-1-2i$ $\quad -4+3i+1-i$

$-4+3i-1+i$
$-5+4i$

3 Express the matrix $A = \begin{bmatrix} 1-2i & -4+3i & 2-5i \\ 1+i & 3-2i & 7-4i \\ 4-2i & 7 & 2+3i \end{bmatrix}$ as

$P+iQ$ where P & Q are hermition matrix.

$P = \frac{1}{2}(A+A^*)$ $\quad A^* = (\bar{A})'$ $\quad Q = \frac{1}{2i}(A-A^*)$

$$\bar{A} = \begin{bmatrix} 1+2i & -4-3i & 2+5i \\ 1-i & 3+2i & 7+4i \\ 4+2i & 7 & 2-3i \end{bmatrix}$$

$1+2i+1-2i$

$$(\bar{A})' = \begin{bmatrix} 1+2i & 1-i & 4+2i \\ -4-3i & 3+2i & 7 \\ 2+5i & 7+4i & 2-3i \end{bmatrix} = A^*$$

$$P = \frac{1}{2}(A+A^*) = \frac{1}{2}\begin{bmatrix} 2 & -3+2i & 6-3i \\ -3-2i & 6 & 14-4i \\ 6+3i & 14+4i & 4 \end{bmatrix}$$

$$Q = \frac{1}{2i}\begin{bmatrix} -4i & -5+4i & 2+7i \\ 5+4i & -4i & -4i \\ -2+7i & 4i & 6i \end{bmatrix}$$

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Ques Show that matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$ show that matrix is a unitary matrix where ω is the complex cube root of unity?

Ans Given $\omega \rightarrow$ cube root of unity

i.e., $1+\omega+\omega^2 = 0$

$\omega^3 = 1$

now,

$$\bar{A} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$$

$$A^* = (\bar{A})' = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$$

For unitary matrix -

$AA^* = I$

L.H.S

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1+1+1 & 1+\omega+\omega^2 & \omega^2+\omega+1 \\ 1+\omega+\omega^2 & 1+\omega^2+\omega^4 & 1+\omega^3+\omega^3 \\ 1+\omega+\omega^2 & 1+\omega^3+\omega^3 & 1+\omega^4+\omega^2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad R_2 \leftrightarrow R_3$$

$$= \frac{\cancel{3}}{\cancel{3}}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence proved//

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$1-\frac{14}{9} = \frac{-5}{9}$

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LU decompsition of square matrix :-

let $A = LU$

$$\begin{bmatrix} a & b & c \\ p & q & r \\ l & m & n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

Q) Factorise the Matrix $A = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix}$ in LU matrix

$A = LU$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$= \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix}$$

$u_{11} = 1 \qquad u_{12} = 5 \qquad u_{13} = 1$

$u_{11}l_{21} = 2$
$l_{21} = 2$

$u_{11}l_{31} = 3$
$l_{31} = 3$

$l_{21}u_{12} + u_{22} = 1$
$2 \times 5 + u_{22} = 1$
$\boxed{u_{22} = -9}$

$l_{31}u_{12} + l_{32}u_{22} = 1$
$3 \times 5 + l_{32} \times (-9) = 1$
$l_{32} = 14/9$

$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 4$
$3 \times 1 + \frac{14}{9} + u_{33} = 4$
$u_{33} = -5/9$

$l_{21}u_{13} + u_{23} = 3$
$\boxed{u_{23} = 1}$

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$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 14/9 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 & 1 \\ 0 & -9 & 1 \\ 0 & 0 & -5/9 \end{bmatrix}$$

Q factorise $A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$ into LU form

$$\begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$= \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ u_{11}l_{31} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

$u_{11} = 5$ $\quad u_{12} = -2$ $\quad u_{13} = 1$

$l_{21}u_{11} = 7$

$\boxed{l_{21} = \frac{7}{5}}$

$l_{21}u_{12} + u_{22} = 1$

$\frac{7}{5} \times (-2) + u_{22} = 1$

$\frac{1+14}{5}$

$\frac{19}{5}$

$\boxed{u_{22} = \frac{19}{5}}$

$l_{21}u_{13} + u_{23} = -5$

$\frac{7}{5} \times (1) + u_{23} = -5$

$-5-\frac{7}{5}$

$\frac{-25-7}{5}$

$\boxed{u_{23} = \frac{-32}{5}}$

$u_{11}l_{31} = 3$

$5\,l_{31} = 3$

$\boxed{l_{31} = \frac{3}{5}}$

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$l_{31}u_{12} + l_{32}u_{22} = 7$

$\frac{3}{5} \times (-2) + l_{32} \times \frac{19}{5} = 7$

$-\frac{6}{5} + \frac{19}{5} l_{32} = 7$ $\quad \frac{35+6}{5}$ $\quad 41$

$\frac{19}{5} l_{32} = \frac{41}{5}$

$\boxed{l_{32} = \frac{41}{19}}$

$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 4.$

$\frac{3}{5} \times 1 + \frac{41}{19} \times \left(-\frac{32}{5}\right) + u_{33} = 4$

$\cancel{\frac{3}{5} - \frac{1312}{45} + u_{33} = 4}$

$\cancel{\boxed{u_{33} = \frac{1294}{45}}}$

$\cancel{u_{33} = 4 + \frac{1294}{45}}$

$\frac{3}{5} - \frac{1312}{95} + u_{33} = 4$

$u_{33} = 4 - \frac{3}{5} + \frac{1312}{95}$

$= \frac{475 - 57 + 1312}{95}$

$= \frac{1730}{95}$

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Singular value decomposition -

The SVD of a matrix is a factorisation of matrix into 3 matrices thus, the singular value decomposition of Matrix A in terms of factorisation into product of 3 Matrices.

$$\boxed{A = UDV^T}$$

The coloumns of U & V are orthogonal and the matrix D is a diagnol with Real +ve entries.

* Procedure to find SVD of a matrix -

~~mathemo~~ consider a matrix A of order $m \times n$ this can be decomposed as

$$A = UDV^T$$

U is $m \times n$ matrix & the coloumns ~~are~~ orthogonal {that means its coloumns are normalised Eigen vector of AA^T} D is $n \times n$ Diagnol matrix ~~who~~ $D = diag(\sigma_1, \sigma_2 \dots \sigma_n)$

$$\sigma_n = \sqrt{\lambda_n} \quad \lambda_n \rightarrow \text{Eigen value of } A$$

V is $n \times n$ matrix & column orthogonal that means its columns are normalised Eigen vector of A^TA.

If u & D are known we can find V

$$\cancel{u_i = \frac{1}{\sigma_i} AV_i}$$

$$v_i = \frac{1}{\sigma_i} A^T u_i$$

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Q. Find SVD -

$$A = \begin{bmatrix} -4 & -7 \\ 1 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -4 & 1 \\ -7 & 4 \end{bmatrix}$$

step 1 $A \cdot A^T = \begin{bmatrix} -4 & -7 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -4 & 1 \\ -7 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 16+49 & -4-28 \\ -4-28 & 1+16 \end{bmatrix}$$

$$A \cdot A^T = \begin{bmatrix} 65 & -32 \\ -32 & 17 \end{bmatrix}$$

$$|AA^T - \lambda| = 0$$

$$\begin{vmatrix} 65-\lambda & -32 \\ -32 & 17-\lambda \end{vmatrix} = 0$$

$$(65-\lambda)(17-\lambda) - (-32)(-32) = 0$$

$$1105 - 65\lambda - 17\lambda + \lambda^2 \cancel{+1024} - 1024 = 0$$

$$\lambda^2 - 82\lambda + 81 = 0$$

$$\lambda^2 - 81\lambda - \lambda + 81 = 0$$

$$\lambda(\lambda - 81) - 1(\lambda - 81) = 0$$

$$\boxed{\lambda = 81, 1}$$

$$\begin{array}{r} 81 \\ -17 \\ \hline 6 \end{array}$$

$\lambda = 81$. $\begin{bmatrix} -16 & -32 \\ -32 & -64 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$$-16x_1 - 32x_2 = 0$$

$$\cancel{x_3}\; x_1 + 2x_2 = 0$$

$$x_1 = -2x_2 \qquad \frac{x_1}{-2} = \frac{x_1}{1}$$

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Normalised

$$\boxed{X_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}} = \begin{bmatrix} -2/\sqrt{(2)^2+(1)^2} \\ 1/\sqrt{(2)^2+(1)^2} \end{bmatrix} = \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$$

when $\lambda = 1$

$$\begin{bmatrix} 64 & -32 \\ -32 & 16 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$2x_1 - x_2 = 0$$
$$2x_1 = x_2$$
$$\frac{x_1}{1} = \frac{x_2}{2}$$

$$X_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$$

$$U = \begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix}$$

step II $\sigma_1 = \sqrt{\lambda_1}$

$\boxed{\sigma_1 = 9}$ $\boxed{\sigma_2 = 1}$

$$D = \begin{bmatrix} 9 & 0 \\ 0 & 1 \end{bmatrix}$$

step 3

$$A^T A = \begin{bmatrix} -4 & 1 \\ -7 & 4 \end{bmatrix} \begin{bmatrix} -4 & -7 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} +16+1 & 28+4 \\ 28+4 & 49+16 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 32 \\ 32 & 65 \end{bmatrix}$$

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$|A^T A - \lambda I| = 0$

$$\begin{vmatrix} 17-\lambda & 32 \\ 32 & 65-\lambda \end{vmatrix} = 0$$

$(17-\lambda)(65-\lambda) - 32(32) = 0$

$1105 - 17\lambda - 65\lambda + \lambda^2 - 1024 = 0$

$\lambda^2 - 82\lambda + 81 = 0$

$\boxed{\lambda = 81, 1}$

$\lambda = 81$ $\begin{bmatrix} -64 & 32 \\ 32 & -16 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$-64x_1 + 32x_2 = 0.$

$-2x_1 + x_2 = 0$

$\frac{x_1}{1} = \frac{x_2}{+2}$

$X_1 =$ Eigen vector $= \begin{bmatrix} 1 \\ +2 \end{bmatrix}$

Normalised $= \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$

$\lambda = 1$ $\begin{bmatrix} 16 & 32 \\ 32 & 64 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$16x_1 + 32x_2 = 0$

$x_1 + 2x_2 = 0$

$x_1 = -2x_2$

$\frac{x_1}{-2} = \frac{x_2}{1}$

$X_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ Nor. $= \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$

$$V = \begin{bmatrix} 1/\sqrt{5} & -2/\sqrt{5} \\ 2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix}$$

$$\boxed{A = UDV^T}$$

$$V_1 = \frac{1}{\sigma_1} \begin{bmatrix} -4 & 1 \\ -7 & 4 \end{bmatrix} u_1$$

$$= \frac{1}{9} \underbrace{\begin{bmatrix} -4 & 1 \\ -7 & 4 \end{bmatrix}}_{2\times 2} \underbrace{\begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}}_{2\times 1}$$

$$= \frac{1}{9} \begin{bmatrix} 8/\sqrt{5} + \frac{1}{\sqrt{5}} \\ 14/\sqrt{5} + 4/\sqrt{5} \end{bmatrix}$$

$$= \frac{1}{9} \begin{bmatrix} 9/\sqrt{5} \\ 18/\sqrt{5} \end{bmatrix} = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$$

$$V_2 = \frac{1}{1} \begin{bmatrix} -4 & 1 \\ -7 & 4 \end{bmatrix} \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$$

$$= 1 \begin{bmatrix} -4/\sqrt{5} + 2/\sqrt{5} \\ -7/\sqrt{5} + 8/\sqrt{5} \end{bmatrix}$$

$$= \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$$

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Q solve the following system of Eqn ~~in~~ with LU fa.

$$x + 5y + z = 14$$
$$2x + y + 3z = 13$$
$$3x + y + 4z = 17$$

$$A = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 14/9 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 & 1 \\ 0 & -9 & 1 \\ 0 & 0 & -5/9 \end{bmatrix}$$

$$AX = B$$
$$LUX = B$$
$$\text{Let } UX = V$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 14/9 & 01 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 17 \end{bmatrix}$$

$$V_1 = 14$$
$$2V_1 + V_2 = 13$$
$$3V_1 + \frac{14}{9}V_2 + V_3 = 17$$

$$\begin{array}{r} 28 \\ -13 \\ \hline 15 \end{array}$$

$V_2 = -15$ $\boxed{V_3 = -\frac{5}{3}}$

$$3(14) + \frac{14}{9}(-15) + V_3 = 17$$

$$42 + \frac{(-210)}{9} + V_3 = 17$$

$$\frac{378 - 210}{9} + V_3 = 17$$

$$V_3 = 17 - \frac{168}{9}$$

$\alpha_n = \dfrac{V_n}{\|V_n\|}$ $V_n = \beta_n - \langle \beta_n, \alpha_1 \rangle \alpha_1 - \langle \beta_n \alpha_2 \rangle \alpha_2 \ldots\ldots \langle \beta_n, \alpha_{n-1} \rangle \alpha_{n-1}$

$UX = V$

$$\begin{bmatrix} 1 & 5 & 1 \\ 0 & -9 & 1 \\ 0 & 0 & -5/9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ -15 \\ -5/3 \end{bmatrix}$$

$x + 10 + 3 = 14$

$x + 5y + z = 14$

$-9y + z = -15$

$-\frac{5}{9} z = \frac{-5}{3}$

$-9y + 3 = -15$

$-9y = -18$

$z = 3$ $y = 2$ $x = 1$

Gram Schmidt orthogonalisation -

Que Let $\{B = \{\beta_1, \beta_2 \cdots \beta_n\}_V$ of a finite dimensional inner product space V.
Basis

Let $\{\alpha_1, \alpha_2 \ldots \alpha_n\}$ be an orthonormal Basis for V which are required to construct from the Basis B

The vector $\alpha_1 \ldots\ldots \alpha_n$ will be obtain using the following formula?

$$\alpha_1 = \frac{\beta_1}{\|\beta_1\|}$$

$\hookrightarrow$ norm.

$$\alpha_2 = \cancel{\beta}\, \frac{V_2}{\|V_2\|}$$

$$V_2 = \beta_2 - \langle \beta_2, \alpha_1 \rangle \alpha_1$$

$\hookrightarrow$ inner product

$\|\beta_1\| = \langle \beta_1, \beta_1 \rangle$

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Q Apply Gram schmidt process to ~~the~~ ~~se~~ produce an ~~org~~ orthogonal Basis for the set of vectors

$\beta_1 = (1,0,1)$

$\beta_2 = (1,0,-1)$.

$\beta_3 = (0,3,4)$

$$\alpha_1 = \frac{\beta_1}{\|\beta_1\|} = \frac{(1,0,1)}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$$

$$\alpha_2 = \frac{V_2}{\|V_2\|} = \frac{(1,0,-1)}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right)$$

$\cancel{\alpha_3 = \alpha_3}$

$$\boxed{V_2 = \beta_2 - \langle \beta_2, \alpha_1 \rangle \alpha_1}$$

$$\alpha_3 = \frac{V_3}{\|V_3\|}$$

$$= \frac{(0,3,0)}{\sqrt{9}}$$

$$= \frac{3}{\sqrt{9}}\left(0, \frac{3}{\sqrt{9}}, 0\right)$$

$$= 3\,(0,1,0)$$

$$V_3 = \beta_3 - \langle \beta_3 \alpha_1 \rangle \alpha_1 - \langle \beta_3 \alpha_2 \rangle \alpha_2$$

$$= (0,3,4) - \left(\frac{4}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) - \left(\frac{-4}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right)$$

$$= (0,3,4) - (2,0,2) + (2,0,-2)$$

$$= (0,3,0)$$

$\frac{24}{\sqrt{10}}$:

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(ii) $\beta_1 = (3, 0, -1)$
$\beta_2 = (8, 5, -6)$

$\alpha_1 = \frac{\beta_1}{\|\beta_1\|}$ $\qquad \|\beta_1\| = \sqrt{9+1}$

$= \frac{(3, 0, -1)}{\sqrt{10}} = \left(\frac{3}{\sqrt{10}}, 0, \frac{-1}{\sqrt{10}}\right)$

$\alpha_2 = \frac{V_2}{\|V_3\|}$

$V_2 = \beta_2 - \langle \beta_2, \alpha_1 \rangle \alpha_1$

$= (8, 5, -6) - \left(\frac{24}{\sqrt{10}} + 0 + \frac{6}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{10}}, 0, \frac{-1}{\sqrt{10}}\right)$

$= (8, 5, -6) - (9, 0, -3)$

$= (-1, 5, -3)$

$\alpha_2 = \frac{(-1, 5, -3)}{\sqrt{1+25+9}}$

$= \left(\frac{-1}{\sqrt{35}}, \frac{5}{\sqrt{35}}, \frac{-3}{\sqrt{35}}\right)$

Find singular value decomposition of the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$AA^T = \underset{3\times 2}{\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}_{2\times 3}$$

$$= \begin{bmatrix} 1+1 & 1 & -1+1 \\ 1 & 1 & 1 \\ -1+1 & 1 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$|AA^T - \lambda| = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & 0 \\ 1 & 1-\lambda & 1 \\ 0 & 1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)[(1-\lambda)(2-\lambda) - 1] - 1[2-\lambda] = 0$$

$$(2-\lambda)[2-\lambda-2\lambda+\lambda^2-1] - 2 + \lambda = 0$$

$$(2-\lambda)[\lambda^2 - 3\lambda + 1] - 2 + \lambda = 0$$

$$\underline{2\lambda^2} - 6\lambda + \cancel{2} - \lambda^3 + \underline{3\lambda^2} - \cancel{\lambda} - \cancel{2} + \cancel{\lambda} = 0$$

$$-\lambda^3 + 5\lambda^2 - 6\lambda = 0$$

$$\lambda(-\lambda^2 + 5\lambda - 6) = 0$$

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$\lambda = 0$ $\qquad$ $\lambda^2 - 5\lambda + 6 = 0$

$\lambda^2 - 3\lambda - 2\lambda + 6 = 0$

$\lambda(\lambda - 3) - 2(\lambda - 3) = 0$

$\boxed{\lambda = 2, 3}$

For $\lambda = \cancel{0}\ 3$

$$\begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Cross mul.

$-x_1 + x_2 + 0 = 0 \rightarrow (1)$

$x_1 - 2x_2 + x_3 = 0 \rightarrow (2)$

$0 + x_2 - x_3 = 0 \rightarrow (3)$

$x_2 = x_3$

cross mult.

$$\frac{x_1}{1-0} = \frac{x_2}{1} = \frac{x_3}{1}$$

$$X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \text{Normalised} \quad \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

For $\lambda = 2$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$0x_1 + x_2 + 0x_3 = 0$

$x_1 - x_2 + x_3 = 0$

$0x_1 + x_2 + 0x_3 = 0$

$$\frac{x_1}{1} = \frac{x_2}{-(0)} = \frac{x_3}{-1}$$

$$X_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} \\ 0/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

For $\lambda = 0$

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\left\{ \begin{array}{l} 2x_1 + x_2 + 0x_3 = 0 \\ x_1 + x_2 + x_3 = 0 \\ 0x_1 + x_2 + 2x_3 = 0 \end{array} \right.$$

$$\frac{x_1}{1} = \frac{x_2}{-(2-0)} = \frac{x_3}{2-1}$$

$$\frac{x_1}{1} = \frac{x_2}{-2} = \frac{x_3}{1}$$

$$x_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1/\sqrt{6} \\ -2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$$

$$U = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}$$

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$$v_1 = \frac{1}{\sigma_1} A^T . u_1$$

$$= \frac{1}{\sqrt{3}} \underbrace{\begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}}_{2\times 3} \underbrace{\begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}}_{3\times 1}$$

$$= \frac{1}{\sqrt{3}} \begin{bmatrix} 1/\sqrt{3} - 1/\sqrt{3} \\ 1/\sqrt{3} + 1/\sqrt{3} + 1/\sqrt{3} \end{bmatrix}$$

$$= \frac{1}{\sqrt{3}} \begin{bmatrix} 0 \\ 3/\sqrt{3} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$v_2 = \frac{1}{\sigma_2} A^T u_2$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$D = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$