PSLA

Previous year Question Paper solution.

Q1. (a) Given
$P(A) = \frac{1}{3}$ $P(B) = \frac{2}{5}$ $P(C) = \frac{1}{5}$ $P(D) = \frac{1}{4}$

To find - P(Problem will be solved)

P(Problem will be Solved) = 1 - P(Problem will not be Solved)

$= 1 - P(\bar{A})P(\bar{B})P(\bar{C})P(\bar{D})$

$P(\bar{A}) = \frac{2}{3}$ $P(\bar{B}) = \frac{3}{5}$ $P(\bar{C}) = \frac{4}{5}$ $P(\bar{D}) = \frac{3}{4}$

P(Problem will be solved) $= 1 - \frac{2}{\cancel{3}} \times \frac{\cancel{3}}{5} \times \frac{\cancel{4}}{5} \times \frac{3}{\cancel{4}}$

$= 1 - \frac{6}{25}$

$\boxed{\text{P(Problem will be solved)} = \frac{19}{25}}$

(b) Shortnote on Skewness - "lack of symmetry

-ve Skewed. Symmetrical +ve Skewed

(f) Type I Error - Hypothesis is Rejected by its trueness

Type II Error - Hypothesis is accepted by it is false

(c) Given
$\mu = 4 = np$, $\sigma^2 = 3$

$np = 4$, $npq = 3$

$4q = 3$

$\boxed{n = 16}$ $\boxed{p = \frac{1}{4}}$. $q = \frac{3}{4}$

Binomial distribution $= \left(\frac{1}{4} + \frac{3}{4}\right)^{16} = (1)^{16} = 1$

(d) Types of Random variable

Discreate — continous.

(e) A → hermition matrix — To show iA → skew hermition

$A = A^*$ — To show $iA = -iA^*$

LHS — RHS

LHS iA

Step 1 Do conjugate, {iota ka sign change Baki sb as it is}

$-i\bar{A}$

Step 2 Now Do the transpos.

$(-i\bar{A})^T$

$-i(\bar{A})^T$ $\{A^* = \bar{A}^T\}$

$-iA^*$

= R.H.S// Hence proved//

Qus 2. (a)

$P(E_1) = \frac{2000}{12000}$ $P(A/E_1) = 0.01$

$P(E_2) = \frac{4000}{12000}$ $P(A/E_2) = 0.03$

$P(E_3) = \frac{6000}{12000}$ $P(A/E_3) = 0.15$

A : accident occur

E_1: insured person scooter driver
E_2: insured person car driver
E_2: insured person truck driver

$$P(E_3/A) = \frac{P(A/E_3)\,P(E_3)}{P\left(\frac{A}{E_1}\right)P(E_1) + P\left(\frac{A}{E_2}\right)P(E_2) + P\left(\frac{A}{E_3}\right)P(E_3)}$$

$$= \frac{\frac{0.15}{100} \times \frac{6000}{12000}}{\frac{2000}{12000} \times \frac{0.01}{100} + \frac{4000}{12000} \times \frac{0.03}{100} + \frac{6000}{12000} \times \frac{0.15}{100}}$$

$$= \frac{90000}{2000 + 12000 + 90000}$$

$$\begin{array}{r} 90000 \\ 12000 \\ 2000 \\ \hline 104000 \end{array}$$

$$= \frac{15}{26}\ \frac{30}{52}\ \frac{90000}{104000}.$$

$$= \frac{15}{26}$$ Ans.

(b) $X = -1, 1, 3, 5$

$3x + \frac{1}{2} = 1$

$P(-1) = \frac{1}{6}$ $\quad P(1) = \frac{1}{6}$ $\quad P(3) = \frac{1}{6}$ $\quad P(5) = \frac{1}{2}$

X	-1	1	3	5
P(X)	1/6	1/6	1/6	1/2

$F(0) = P(X \le 0)$
$= \frac{1}{6}$

$F(1) = P(X \le 1)$
$= \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

$F(3) = P(X \le 3)$
$= \frac{1}{2}$

$F(5) = P(X \le 5)$
$= 1$

$$F(x) = \begin{cases} 0 & -\infty < x < -1 \\ 1/6 & -1 \le x < 1 \\ 1/3 & 1 \le x < 3 \\ 1/2 & 3 \le x < 5 \\ 1 & x \ge 5 \end{cases}$$

~~[illegible]~~

x	-1	1	3	5
$P(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$

Now mean $= E(X) = \Sigma x f(x)$

$$= (-1)\frac{1}{6} + 1\times\frac{1}{6} + 3\times\frac{1}{6} + 5\times\frac{1}{2}$$

$$= 3$$

$$E(x^2) = \Sigma x^2 f(x)$$

$$= (-1)^2\frac{1}{6} + 1\left(\frac{1}{6}\right) + (3)^2\left(\frac{1}{6}\right) + (5)^2\frac{1}{2}.$$

$$= \frac{43}{3}$$

$$\text{var}(X) = E(x^2) - (E(X))^2 = \frac{43}{3} - 9 = \frac{16}{3} \text{ Ans}$$

By Chebyshev's inequality,

$$P(|X-3| \geq 1) \leq \frac{\sigma^2}{c^2}.$$

$$\boxed{P(|X-3| \geq 1) \leq \frac{16}{3}}$$

Q3 (a) $\mu = 12$; $\sigma^2 = 9$; $\boxed{\sigma = 3}$

(i) $P(6 < X < 18)$ (ii) $P(3 < X < 21)$

Chebyshev Inequality

$$P\{|X-\mu| \geq k\sigma\} \leq 1/k^2 \; ; \; P\{|X-\mu| < k\sigma\} \leq 1-\frac{1}{k^2}$$

(i) $P(6 < X < 18)$

$$P\{(6-12 < X-12 < 18-12)\} \leq 1-\frac{1}{k^2}$$

$$P(-6 < X-12 < 6) \leq 1-\frac{1}{k^2}$$

$$P(|X-12| < 6) \leq 1-\frac{1}{k^2}$$

Since, $\boxed{K=2}$.

$P(|X-12|<6) \leq 1-\frac{1}{4}$

$$\boxed{P(|X-12|<6) \leq \frac{3}{4}}$$

(ii) $P(3<X<21)$

$P(3-12 < X-12 < 21-12) \leq 1-\frac{1}{k^2}$

$P(-9 < X-12 < 9) < 1-\frac{1}{k^2}$

Here $\boxed{k=3}$

$P(-9 < X-12 < 9) < 1-\frac{1}{3^2}$

$$\boxed{P(|X-12|<9) < \frac{8}{9}}$$

(b) $f(x) = \begin{cases} \frac{k}{1+x^2} & -\infty < x < \infty \\ 0 & \text{otherwise} \end{cases}$

Find: k ? $\quad P(X \geq 0)$

$\int_{-\infty}^{\infty} \frac{k}{1+x^2} dx = 1$

$k \int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$

$\tan^{-1}(-x) = -\tan^{-1} x$

$k[\tan^{-1} x]_{-\infty}^{\infty} = 1$

$k[\tan^{-1}(+\infty) - \tan^{-1}(-\infty)] = 1$

$k[\frac{\pi}{2} + \frac{\pi}{2}] = 1$

$$\boxed{k = \frac{1}{\pi}}$$

$$P(X \geq 0) = \int_0^{\infty} f(x)\,dx$$

$$= \int_0^{\infty} \frac{1}{\pi} \frac{1}{1+x^2}\,dx$$

$$= \frac{1}{\pi}\left[\tan^{-1}(x)\right]_0^{\infty}$$

$$= \frac{1}{\pi}\left[\frac{\pi}{2} - 0\right]$$

$$= \frac{1}{\pi} \times \frac{\pi}{2}$$

$$= \frac{1}{2}$$ Ans,,

unit - II

Q4 (a) $M_x(t) = \frac{3}{3-t}$

$\frac{d}{dt} M_x(t) = \frac{d}{dt}\left(\frac{3}{3-t}\right) = \frac{-3\,(-1)}{(3-t)^2}$

$= \frac{3}{(3-t)^2}$

$3(3-t)^{-1}$

$\cancel{3(3-t)^{-2}}$

$\cancel{3(-2(3-t)^{-1})}$

$\frac{3}{(3-t)^2}$

$3(3-t)^{-2}$

$3[-2(3-t)]$

3

$3[-2(3-t)^{-3}(-1)$

$3[2(3-t)^{-3}]$

6

$E(x) = \left|\frac{d}{dt} M(x)t\right|_{t=0} = \frac{3}{(3-0)^2} = \frac{3}{9} = \frac{1}{3}$

$E(x^2) = \left|\frac{d^2}{dt^2}(M(x))t\right|_{t=0} = +3(3-t)^{-2}$

$= \frac{6}{(3-t)^3}$

at $t=0$

$= \frac{6}{(3)^3}$

$= \frac{6}{27}$

$= \frac{2}{9}$

$Var(x) = E(x^2) - [E(x)]^2$

$= \frac{2}{9} - \left(\frac{1}{3}\right)^2$

$var(x) = \frac{2}{9} - \frac{1}{9} = \frac{1}{9}$ Ans,,

$\boxed{S.D = \sqrt{\frac{1}{9}} = \frac{1}{3}}$

(b) Total = 800 families $p = \frac{1}{2}$ $q = \frac{1}{2}$

$n = 5$

(a) <u>3 boys</u>.

$$= 5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$$

$$= \frac{5 \times 4 \times 3!}{3! \times 2 \times 1} \left(\frac{1}{2}\right)^5$$

$$= \frac{10}{32} = \frac{5}{16}$$

No. of families with boy (3) $= \frac{5}{16} \times 800 = 250$

(b) <u>5 Girls</u>.

$$= 5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0$$

$$= \frac{5!}{5!} \left(\frac{1}{32}\right)$$

$$= \frac{1}{32}$$

Expected No. of families with 5 Girls $= \frac{1}{32} \times 800$

$= 25$

(c) Either 2 boys or 3 ~~girls~~ boys -

$$= 5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 + 5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$$

$$= \left(\frac{1}{2}\right)^5 \left[\frac{5 \times 4}{2 \times 1} + \frac{5 \times 4}{2 \times 1}\right]$$

$$= \frac{1}{32}[20] = \frac{10}{16} = \frac{5}{8}$$

No. of families $= 800 \times \frac{5}{8} = 500$

Scanned with OKEN Scanner

Ques 5
(b)

X Cost	Y Sale price	Rx	Ry	d	d^2
80	12	1	8	-7	49
78	13	2	7	-5	25
75	14	3.5	5	-1.5	2.25
75	14	3.5	5	-1.5	2.25
68	14	5	5	0	0
67	16	6	3	3	9
60	15	7	2	5	25
59	17	8	1	7	49
					161.5

$\frac{3+4}{2}$

3.5

$m_1 = 2 \qquad m_2 = 3$

$$f = \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12}$$

$$= \frac{2(3)}{12} + \frac{3(8)}{12}$$

$$= 0.5 + 2$$

$$= 2.5$$

Rank correlation $\rho = 1 - \frac{6(\Sigma d^2 + F)}{n(n^2-1)}$

$$= 1 - \frac{6(161.5 + 2.5)}{8(63)}$$

$$= 1 - \frac{\cancel{6}(\cancel{164})}{\cancel{8} \times \cancel{63}}$$

82 41

21

$$= -0.952.$$

$$\begin{array}{r} 161.5 \\ 2.5 \\ \hline 164.0 \end{array}$$

unit - 3 Q6 - (a)

X	y	x^2	x^3	x^4	xy	x^2y
1	1.7	1	1	1	1.7	1.7
2	1.8	4	8	16	3.6	7.2
3	2.3	9	27	81	6.9	20.7
4	3.2	16	64	256	12.8	51.2
10	9	30	100	354	25	80.8

Let Quadratic Equation be $y = a + bx + cx^2$

Nomal Equations - $\Sigma y = na + b\Sigma x + c\Sigma x^2$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$

$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$

$9 = 4a + 10b + 30c$ → ①

$25 = 10a + 30b + 100c \Rightarrow 5 = 2a + 6b + 20c$ → ②

$80.8 = 30a + 100b + 354c$ → ③

on solving ① & ②

$2b + 10c = 1$ → ④

on solving ② & ③

$10b + 54c = 5.8$ → ⑤

Solving ④ & ⑤

$$\begin{array}{r} 10b + 50c = 5 \\ 10b + 54c = 5.8 \\ \hline -4c = -0.8 \end{array}$$

$2b + 2 = 1$

$2b = -1$

$\boxed{c = 0.2}$

~~$c = \frac{8}{40} = \frac{1}{20} = 0.05$~~

$b = 0.5$

$\boxed{b = -0.5}$

~~$9 = 4(a) + 5 + 15$~~

$\boxed{a = 2}$

~~eq 4 ...~~

So, the eqn is

$5 = 2a + (-3) + 4$

$5 = 2a + 1$

$a = 2$

$$\boxed{y = 2 - 0.5x + 0.2x^2}$$

Scanned with OKEN Scanner

Ans 6
(b)

men	(n_1)	= 400	$X_1 = 200$
women	(n_2)	= 600	$X_2 = 325$

$$p_1 = \frac{X_1}{n_1} = \frac{200}{400} = 0.5 \qquad p_2 = \frac{325}{600} = 0.541$$

H_0 :

$$P = \frac{p_1 n_1 + p_2 n_2}{n_1 + n_2} = \frac{400 \times 0.50 + 0.541 \times 600}{1000}$$

$$P = \frac{524}{1000} = 0.524$$

$$\boxed{Q = 1 - P = 0.475}$$

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.5 - 0.541}{\sqrt{0.525 \times 0.475\left(\frac{1}{400} + \frac{1}{600}\right)}}$$

$$= -1.269$$

Since $Z < 1.96$

So, Hypothesis is accepted.

Q7 (a)

x	y	$y = \log_{10} Y$	x^2	xy
1	7	0.845	1	0.845
2	11	1.041	4	2.082
3	17	1.2304	9	3.6912
4	27	1.4313	16	5.7252
10		4.5477	30	12.3434

$$Y = Ae^{Bx}$$

Take log on both side

$$\underbrace{\log_{10} Y}_{} = \log_{10} A + Bx \log_{10} e$$

$$\boxed{y = a + bx}$$

$\log_{10} Y = y$

$\log_{10} A = a$

$B \log_{10} e = b$

Normal Equations are

$$\Sigma y = na + b\Sigma x$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

$$4.5477 = 4a + 10b \Rightarrow 2.27385 = 2a + 5b \rightarrow (1)$$

$$12.3434 = 10a + 30b \Rightarrow [illegible] \rightarrow (2)$$

Solving (1) & (2)

$$11.36925 = 10a + 25b$$

$$12.3434 = 10a + 30b$$

$$-0.97415 = -5b$$

$$\boxed{b = 0.19482} \quad \boxed{a = 0.64985}$$

$B \log_{10} e = b$

$B = 12.72$

$$\boxed{A = 4.46}$$

$$Y = 4.46\, e^{12.72\, x}$$

56% - 300 from South

48% of 200

1.645

(b) 56% of 300 → South

48% of 200 → North

$x_1 = \frac{56}{100} \times 300 = 168$

$x_2 = \frac{48}{100} \times 200 = 96$

$p_1 = 0.56 \qquad p_2 = 0.48$

H_0 : Let Ramayna is preffered in south.

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$P = \frac{p_1 n_1 + p_2 n_2}{n_1 + n_2}$$

$$= \frac{0.56 \times 300 + 0.48 \times 200}{500}$$

$$= \frac{168 + 96}{500}$$

$$= \frac{264}{500}$$

$$P = 0.528$$

$$Q = 1 - 0.528 = 0.472$$

$$Z = \frac{0.56 - 0.48}{\sqrt{0.528 \times 0.472\left(\frac{1}{300} + \frac{1}{200}\right)}}$$

$$= \frac{0.08}{\sqrt{0.0021}}$$

$$= \frac{0.08}{\sqrt{0.0458}}$$

$$= 1.7467$$

$Z = 1.7467 > 1.645$ (Given in Ques)

So, Hypothesis is Rejected.

Hence, Ramayna is not preffered in south.

Ques 8

(a) $\beta_1 = (3,0,-1)$ $\quad\quad$ $\beta_2 = (8,5,-6)$

$\alpha_1 = ?$ $\quad\quad$ $\alpha_2 = ?$

$$\alpha_1 = \frac{\beta_1}{\|\beta_1\|}$$

$$\|\beta_1\|^2 = \langle \beta_1 \cdot \beta_1 \rangle$$

$$\|\beta_1\|^2 = \langle (3,0,-1)(3,0,-1) \rangle$$

$$\|\beta\| = \sqrt{9+0+1}$$

$$\boxed{\|\beta\| = \sqrt{10}}$$

$$\boxed{\alpha_1 = \frac{(3,0,-1)}{\sqrt{10}}}$$

$$\alpha_2 = \frac{V_2}{\|V_2\|}$$

$$V_2 = \beta_2 - \langle \beta_2 \alpha_1 \rangle \alpha_1$$

$$= (8,5,-6) - \langle (8,5,-6)\left(\frac{3}{\sqrt{10}}, 0, \frac{-1}{\sqrt{10}}\right) \rangle \left(\frac{3}{\sqrt{10}}, 0, \frac{-1}{\sqrt{10}}\right)$$

$$= (8,5,-6) - \left(\frac{24}{\sqrt{10}} + 0 + \frac{6}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{10}}, 0, \frac{-1}{\sqrt{10}}\right)$$

$$V_2 = (8,5,-6) - (9, 0, -3)$$

$$V_2 = (-1, 5, -3)$$

$$\|V_2\|^2 = \langle V_2 \cdot V_2 \rangle$$

$$= \langle (-1,5,-3)(-1,5,-3) \rangle$$

$$= (1+25+9)$$

$$\boxed{\|V_2\| = \sqrt{35}}$$

$$\boxed{\alpha_2 = \frac{(-1,5,-3)}{\sqrt{35}}}$$

Scanned with OKEN Scanner

(b)

$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$

} Non homogenous Equation

$$D = \begin{vmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= 3[-3-(-2)] - 1[2-(-1)] + 2[4-(-3)]$$
$$= 3[-3+2] - 1[2+1] + 2[7]$$
$$= -3 - 3 + 14$$

$$\boxed{D = 8}$$

Since $D \neq 0$

So, ~~it may be~~ Equations are consistent ~~or inconsistent~~

So, let's find out,

D_1, D_2, D_3

$$D_1 = \begin{vmatrix} 3 & 1 & 2 \\ -3 & -3 & -1 \\ 4 & 2 & 1 \end{vmatrix}$$

$$= 3[-3-(-2)] - 1[-3-(-4)] + 2[-6-(-3 \times 4)]$$
$$= 3[-1] - 1[1] + 2[6]$$
$$= -3 - 1 + 12$$

$$\boxed{D_1 = 8}$$

$$D_2 = \begin{vmatrix} 3 & 3 & 2 \\ 2 & -3 & -1 \\ 1 & 4 & 1 \end{vmatrix}$$

$$= 3[-3-(-4)] - 3[2-(-1)] + 2[8-(-3)]$$
$$= 3[-3+4] - 3[2-1] + 2[8+3]$$
$$= 3 - 3 + 22$$

$$\boxed{D_2 = 22}$$

$$D_3 = \begin{bmatrix} 3 & 1 & 3 \\ 2 & -3 & -3 \\ 1 & 2 & 4 \end{bmatrix}$$

$$= 3[-3(4) - (-3)(2)] - 1[8 - (-3)] + 3[4 - (-3)]$$

$$= 3[-12 + 6] - 1[11] + 3[7]$$

$$= 18 - 11 + 21$$

$$\boxed{D_3 = 28}$$

$$\begin{array}{r} 21 \\ -11 \\ \hline 10 \\ +18 \\ \hline 28 \end{array}$$

$$x = \frac{D_1}{D} = \frac{8}{8} = 1 \qquad y = \frac{D_2}{D} = \frac{22}{8} \qquad z = \frac{D_3}{D} = \frac{28}{8}$$

~~Solution~~ Equations are consistent & have unique solution.

Qus 9 (a)

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & -2 & 2 \\ -1 & 2 & -2 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 1 & -2 & 2 \\ -1 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 1+1 & -2-2 & 2+2 \\ -2-2 & 4+4 & -4-4 \\ 2+2 & -4-4 & 4+4 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 2 & -4 & 4 \\ -4 & 8 & -8 \\ 4 & -8 & 8 \end{bmatrix}$$

$$|AA^T - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -4 & 4 \\ -4 & 8-\lambda & -8 \\ 4 & -8 & 8-\lambda \end{vmatrix} = 0$$

on solving determinant

$$\lambda = 18, \ \lambda = 0, \ \lambda = 0$$

for $\lambda = 18$

$$\begin{bmatrix} -16 & -4 & 4 \\ -4 & -10 & -8 \\ 4 & -8 & -10 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$-16x_1 - 4x_2 + 4x_3 = 0$$
$$-4x_1 - 10x_2 - 8x_3 = 0$$
$$4x_1 - 8x_2 - 10x_3 = 0$$

$$\frac{x_1}{(-4)(-8)-(-10)(4)} = \frac{x_2}{-[(-16)(-8)-(4)(-4)]} = \frac{x_3}{[(-16)(-10)-(-4)(-4)]}$$

$$\frac{x_1}{32+40} = \frac{x_2}{-[128+16]} = \frac{x_3}{[160-16]}$$

$$\frac{x_1}{72} = \frac{x_2}{-144} = \frac{x_3}{144}$$

$$\frac{x_1}{1} = \frac{x_2}{-2} = \frac{x_3}{2}$$

$\boxed{x_1 = 1}$ $\boxed{x_2 = -2}$ $\boxed{x_3 = 2}$

(ii) Eigen vector $= \begin{bmatrix} +1 \\ -2 \\ 2 \end{bmatrix}$

Normalized Eigen vector $= \begin{bmatrix} 1/3 \\ -2/3 \\ 2/3 \end{bmatrix}$

For $\boxed{\lambda = 0}$

$$\begin{bmatrix} 2 & -4 & 4 \\ -4 & 8 & -8 \\ 4 & -8 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$2x_1 - 4x_2 + 4x_3 = 0$$
$$-4x_1 + 8x_2 - 8x_3 = 0$$
$$4x_1 - 8x_2 + 8x_3 = 0$$

$$\frac{x_1}{(-4)(-8)-(8\times 4)} = \frac{x_2}{2(-8)-(-4)(4)} = \frac{x_3}{2(8)-(-4)(-4)}$$

$$\frac{x_1}{32-32} = \frac{x_2}{0} = \frac{x_3}{0}$$

$$x_1 = 0 \qquad x_2 = 0 \qquad x_3 = 0$$

Eigen vector $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Diagnol matrix $= \begin{bmatrix} \sqrt{18} & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 3\sqrt{2} & 0 \\ 0 & 0 \end{bmatrix}$

$$A^T A = \begin{bmatrix} 1 & -2 & 2 \\ -1 & 2 & -2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+4 & -1-4-4 \\ -1-4-4 & 1+4+4 \end{bmatrix} = \begin{bmatrix} 9 & -9 \\ -9 & 9 \end{bmatrix}$$

$$|A^T A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 9 & -9 \\ -9 & 9 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 9-\lambda & -9 \\ -9 & 9-\lambda \end{bmatrix} \right| = 0$$

$$(9-\lambda)(9-\lambda) - 81 = 0$$

$$\cancel{81} + \lambda^2 - 9\lambda - 9\lambda - \cancel{81} = 0$$

$$\lambda^2 - 18\lambda = 0$$

$$\lambda(\lambda - 18) = 0$$

$$\lambda = 0 \qquad \lambda = 18$$

for $\lambda = 18$

$$\begin{bmatrix} -9 & -9 \\ -9 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-9x_1 - 9x_2 = 0$$

$$-9x_1 - 9x_2 = 0$$

$$-9x_1 = 9x_2$$

$$\frac{x_1}{1} = \frac{x_2}{-1}$$

Eigen vector = $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Normalised

$$\begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

for $\lambda = 0$

$$\begin{bmatrix} +9 & -9 \\ -9 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$9x_1 - 9x_2 = 0$$

$$x_1 = x_2$$

vector

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$V = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad U = \begin{bmatrix} 1/3 \\ -2/3 \\ 2/3 \end{bmatrix} \qquad D = \begin{bmatrix} 3\sqrt{2} \end{bmatrix}$$

$$A = UDV^T$$

$$\boxed{A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1/3 \\ -2/3 \\ 2/3 \end{bmatrix}_{3\times 1} \begin{bmatrix} 3\sqrt{2} \end{bmatrix}_{1\times 1} \begin{bmatrix} 1, -1 \end{bmatrix}_{1\times 2}}$$

A

Q9 (b) $A = \begin{bmatrix} 2 & -4 & 2 \\ 1 & 5 & -4 \\ -6 & -2 & 4 \end{bmatrix}$

$A = LU$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} 2 & -4 & 2 \\ 1 & 5 & -4 \\ -6 & -2 & 4 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & u_{12}l_{21} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

on comparing both sides,

$\boxed{u_{11} = 2}$ $\boxed{u_{12} = -4}$ $\boxed{u_{13} = 2}$

$l_{21}u_{11} = 1$ $l_{31}u_{11} = -6$

$\boxed{l_{21} = 1/2}$ $\boxed{l_{31} = -3}$

$u_{12}l_{21} + u_{22} = 5$

$(-4)\left(\frac{1}{2}\right) + u_{22} = 5$

$\boxed{u_{22} = 7}$

$l_{21}u_{13} + u_{23} = -4$

$\frac{1}{2}(2) + u_{23} = -4$

$\boxed{u_{23} = -5}$

$l_{31}u_{12} + l_{32}u_{22} = -2$

$(-3)(-4) + l_{32}(7) = -2$

$12 + 7l_{32} = -2$

$\boxed{l_{32} = -2}$

$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 4$

$(-3)(2) + (-2)(-5) + u_{33} = 4$

$-6 + 10 + u_{33} = 4$

$\boxed{u_{33} = 0}$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -4 & 2 \\ 0 & 7 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$