

Konishw.

23

Class Test

B.TECH PROGRAMMES

3rd Semester, January 2023

Paper Code: AIDS209 & AIML209

Subject: Probability, Statistic & Linear Algebra

Time: 1½ Hrs.

Max Mark: 30

Note: Attempt Q.No.1 which is compulsory and any TWO more questions from the remaining.

Q1. a) A coin was tossed 400 times and head turns up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance. (2) (CO2)

b) What straight line best fits the following data in the least square sense.

x	1	2	3	4	5
y	14	13	9	5	2

(2) (CO4)

c) The means of two single large samples of 1000 and 2000 members are 67.5 and 68.0

respectively. Can the samples be regarded as drawn from the same population of standard deviation 2.5. (Test at 5% level of significance) (2) (CO2)

d) Determine the value of λ for which the following system of equations fail to have a unique solution (2) (CO1)

$$\lambda x + 3y - z = 1$$

$$x + 2y + z = 2$$

$$-\lambda x + y + 2z = -1$$

e) Prove that the matrix $A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ is unitary. (2) (CO1)

Q2. a) The pressure and volume of a gas are related by the equation $PV^\gamma = k$, where γ and k are constants. Fit this equation to the following set of observation

P 0.5 1.0 1.5 2.0 2.5 3.0

V 1.62 1.00 0.75 0.62 0.52 0.46

(5) (CO4)

b) Fit a second degree parabola to the following data

Mid -Term Examination -Nov 2022

Programme: B.Tech (AIML)
 Paper Code: AIML209
 Time: One and Half hours

Semester: Third Semester(Sept 2022---Dec 2022)
 Paper Name: Probability, Statistics and Linear Algebra
 Maximum Mark: 30

Note:

- Question No 01 is compulsory.
 Attempt any two question from the remaining questions.
 All questions carry equal marks.
 Only scientific calculator is allowed

	Question 1 (Short Note on following questions)	
1(a)	Categorical type of variables	2
1(b)	Conditional probability.	2
1(c)	Skewness and Kurtosis	2
1(d)	Recurrence relation for Prob of Binomial Distribution	2
1(e)	Baye's theorem.	2

	Question 2	
2(a)	A problem in statistics is given to three students A, B and C whose chance of solving it are 0.5 , 0.75 and 0.25 respectively. What is the probability that the problem will not be solved? (7)	(2+6)
2(b)	State and prove Chebyshev's Inequality	

	Question 3	
	Seven coins are tossed and number of heads noted. The experiment is repeated 128 times and following distribution is obtained: No of heads: 0 1 2 3 4 5 6 7 Total Frequencies: 7 6 19 35 30 23 7 1 128	(8)
3(a)	Find expected frequency when coin is unbiased.	
3(b)	Derive Poisson distribution as a limiting form of a binomial distribution. (7)	

	Question 4	
4(a)	X is normally distributed $N(30, 5)$, find the probability of a) $X \geq 45$ b) $26 \leq X \leq 40$ c) $ X-30 > 5$ Given $P(z \leq 0.8) = 0.2881$ $P(z \leq 1) = 0.4772$ $P(z \leq 3) = 0.4986$	(2+2+2)
4(b)	Calculate the Karl Pearson's coefficient of correlation between heights (in inches) of fathers (X) and their sons (Y): X: 65 66 67 67 68 69 70 72 Y: 67 68 65 68 72 72 69 71	(8)

END TERM EXAMINATION

THIRD SEMESTER [B.TECH] FEBRUARY 2023

Paper Code: AIDS209	Subject: Probability, Statistics & Linear Algebra
AIML209	
IOT209	

Time: 3 Hours

Maximum Marks: 75

Note: Attempt five questions including Q.No. 1 which is compulsory.
Select one question from each unit. Assume missing data.

- Q1** ✓ a) The probabilities that students A, B, C and D solve a problem are $\frac{1}{3}$, $\frac{2}{5}$, $\frac{1}{5}$ and $\frac{1}{4}$, respectively. If all of them try to solve the problem, what is the probability that the problem will be solved? (2.5)
- b) Write a short note on skewness of a distribution. (2.5)
- c) Determine the binomial distribution for which mean is 4 and variance is 3. (2.5)
- d) Explain the types of random variables. (2.5)
- e) If A is a Hermitian matrix, then show that iA is a skew-Hermitian matrix. (2.5)
- f) Define Type I and Type II errors. (2.5)

UNIT-I

- Q2** ✓ a) An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter driver, car driver and truck driver is 0.01, 0.03 and 0.15 respectively. One of the insured person meets with an accident. What is the probability that he is a truck driver? (7.5+7.5)
- b) A random variable X takes the values -1, 1, 3 with equal probabilities and 5 with probability $\frac{1}{2}$. Then find probability distribution of X and $P(|X-3|>1)$

- Q3** ✓ a) A random variable X has mean 12 and variance 9. Using Chebyshev's theorem, estimate (i) $P(6 < X < 18)$ and (ii) $P(3 < X < 21)$ (7.5+7.5)
- b) A continuous random variable X has probability density function $f(x) = \begin{cases} \frac{k}{1+x^2}, & -\infty < x < \infty \\ 0, & \text{otherwise} \end{cases}$
Determine the value of k and evaluate $P(X \geq 0)$.

P.T.O.

AIDS/AIML/IOT-209

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[1-2] UNIT-II

- Q4** a) If a random variable X has the M.G.F $M_X(t) = \frac{1}{3-t}$, obtain the standard deviation of X .
 b) Out of 800 families with 5 children each, how many would you expect to have (a) 3 boys (b) 5 girls (c) either 2 or 3 boys? Assume equal probabilities for boys and girls.

- Q5** a) Find the mean and standard deviation of a normal distribution in which 7% of the items are under 35 and 89% are under 63.

Given that $P(Z \leq -1.48) = 0.07$ and $P(Z \leq 1.23) = 0.89$

- b) Compute the Spearman's rank correlation coefficient for the following data set.

Cost	8	7	7	7	6	6	6	5
0	8	5	5	8	7	0	9	
Sale	1	1	1	1	1	1	1	1
Price	2	3	4	4	4	6	5	7

UNIT-III

- Q6** a) Fit a least squares quadratic fit to the following data set. (7.5+7.5)
 Also, estimate $Y(2.4)$

X	1	2	3	4
Y	1.7	1.8	2.3	3.2

- b) Random samples of 400 male workers and 600 female workers were asked about their opinion of a project proposal on quality improvement. 200 male workers and 325 female workers were in favor of the proposal. Test the hypothesis that the proportions of men and women in favor of the proposal are the same at a $\alpha = 5\%$ level of significance. (Use $z_{\alpha} = z_{0.025} = 1.96$)

- Q7** a) Fit an exponential curve of the form $Y = Ae^{BX}$ for the (7.5+7.5) following data.

X	1	2	3	4
Y	7	11	17	27

- b) A study of TV viewers was conducted to find the opinion about the mega serial 'Ramayana'. If 56% of a sample of 300 viewers from south and 48% of 200 viewers from north preferred the serial, test the claim at $\alpha = 5\%$ level of significance that 'Ramayana' is preferred in the south. (Use $z_{\alpha} = z_{0.05} = 1.645$)

P.T.O.

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[1-3] UNIT-IV

- Q8** a) Use the Gram-Schmidt process to produce an orthogonal basis for the set of vectors $\{(3, 0, -1), (8, 5, -6)\}$

$$\begin{aligned} 3x + y + 2z &= 3 \\ x + 2y + z &= 4 \end{aligned}$$

- b) Solve the system of equations $2x - 3y - z = -3$ using Cramer's rule.

- Q9** a) Find a singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$

- b) Find an LU factorization of the matrix $A = \begin{bmatrix} 2 & -4 & 2 \\ 1 & 5 & -4 \\ -6 & -2 & 4 \end{bmatrix}$

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Total No. of Pages: 1

Student Name: KASAIK
Enrolment No. 02715611922

Mid-Term Examination – Nov 2023

Programme: B.Tech (AIML/AI&DS)

Paper Code: AIML/AI&DS-209

Time: One and Half Hours

Semester: Third Semester

Paper Name: Probability, Statistics & Linear Algebra

Maximum Marks: 30

Note:

Question no. 1 is compulsory

Attempt any two questions from the remaining questions

All questions carry equal marks

Only scientific calculator is allowed

Question 1	
1 (a)	Define probability space with example. 2
1 (b)	A die is thrown twice and the sum of the number appearing is observed to be 6. What is the probability that the number 4 has appeared at least once. 2
1 (c)	Write a short note on skewness of a distribution. 2
1 (d)	X is a Poisson variable and it is found that the probability at X=2 is 2/3 of the probability at X=1. Find the probability at X=3. 2
1 (e)	Find the moment generating function (m.g.f. about origin) of the exponential distribution $f(x) = \frac{1}{c} e^{-x/c}$, $0 \leq x \leq \infty, c > 0$. 2

Question 2	
2 (a)	A factory has 2 plants. Records shows that the plant 1 produces 30% and plant 2 produces 70% items. Further 5% of the items produced by plant 1 are defective while 1% produced by plant 2 are defective. If a defective item is drawn at random, find the probability that the defective item was produced by: (i) Plant 1 (ii) Plant 2 5
2 (b)	A random variable X has the following probability mass function: $p(x) = \begin{cases} x/15 & x = 1,2,3,4,5 \\ 0 & \text{otherwise} \end{cases}$ Find: (i) $P\{X=1 \text{ or } 2\}$ (ii) $P\{1/2 < X < 5/2 X > 1\}$ 5

Question 3															
3 (a)	Fit a binomial distribution for the following data and compare the theoretical frequencies with actual once. <table border="1" style="margin-left: auto; margin-right: auto;"><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>f</td><td>2</td><td>14</td><td>20</td><td>34</td><td>22</td><td>8</td></tr></table> 5	x	0	1	2	3	4	5	f	2	14	20	34	22	8
x	0	1	2	3	4	5									
f	2	14	20	34	22	8									
3 (b)	For a distribution, the mean is 10, variance is 16, γ_1 is 1 and β_2 is 4. Obtain the first four moments about origin. Comment upon the nature of distribution. 5														

Question 4		
4 (a)	If the heights of 300 students are normally distributed with mean 68 inch and standard deviation 3 inch. How many students have height: (i) Greater than 72 Inches (ii) Between 65 and 71 inches Given $P(z \leq 1.33) = 0.4082$, $P(z \leq 1) = 0.3413$, $P(z \geq -1) = 0.3413$	5
4 (b)	Obtain the rank correlation coefficient between the variables X and Y from the following pairs of observed data.	5

X	50	55	65	50	55	60	50	65	70	75
Y	110	110	115	125	140	115	130	120	115	160

Class Test - Dec 2023

Programme: B.Tech (AIML/AI&DS)
 Paper Code: AIML/AI&DS-209
 Time: One Hour

Semester: Third Semester
 Paper Name: Probability, Statistics & Linear Algebra
 Maximum Marks: 30

Note:

Question no. 1 is compulsory

Attempt any two questions from the remaining questions

All questions carry equal marks

Only scientific calculator is allowed

Question 1														
1 (a)	Find the value of β for which the homogenous system of equation has non trivial solution. $\begin{aligned} 2x + 3y - 2z &= 0 \\ 2x - y + 3z &= 0 \\ 7x + \beta y - z &= 0 \end{aligned}$	2.5												
1 (b)	Show that matrix $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$ is a unitary matrix, where ω is a complex cube root of unity.	2.5												
1 (c)	What straight line best fits the data in the least square sense. <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr> <td>y</td><td>14</td><td>13</td><td>9</td><td>5</td><td>2</td></tr> </table>	x	1	2	3	4	5	y	14	13	9	5	2	2.5
x	1	2	3	4	5									
y	14	13	9	5	2									
1 (d)	A die is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Show that the die cannot be regarded as an unbiased one.	2.5												

Question 2														
2 (a)	Fit a second degree parabola to the following data	5												
	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr> <td>y</td><td>1</td><td>1.8</td><td>1.3</td><td>2.5</td><td>6.3</td></tr> </table>	x	0	1	2	3	4	y	1	1.8	1.3	2.5	6.3	
x	0	1	2	3	4									
y	1	1.8	1.3	2.5	6.3									
2 (b)	Fit an exponential curve of the form $y = Ae^{Bx}$ for the following data.	5												
	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr> <td>y</td><td>7</td><td>11</td><td>17</td><td>27</td></tr> </table>	x	1	2	3	4	y	7	11	17	27			
x	1	2	3	4										
y	7	11	17	27										

Question 3		
3 (a)	A sample of 900 members has a mean 3.4 and standard deviation 2.61. Is the sample from a large population of mean 3.25 and standard deviation 2.61? If the population is normal and its mean is unknown, find the 95% and 98% fiducial limits of true mean.	5
3 (b)	The mean height of 50 male students who showed above average participation in college athletics was 68.2 inches with a standard deviation of 2.5 inches, while 50 male students who showed no interest in such participation has a mean height of 67.5 inches with standard deviation 2.8 inches. (i) Test the hypothesis that male students who participate in college athletics are taller than other male students.	5

	(ii) By how much should the sample size of each of 2 groups be increased in order that the observed difference of 0.7 inches in the mean height be significant at the 5% level of significance.	
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Question 4		
4 (a)	Find LU factorization of the matrix A= $\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix}$	5
4 (b)	Express the matrix A= $\begin{bmatrix} 1 - 2i & -4 + 3i & 2 - 5i \\ 1 + i & 3 - 2i & 7 - 4i \\ 4 - 2i & 7 & 2 + 3i \end{bmatrix}$ as P+iQ, where P and Q are Hermitian matrix.	5

END TERM EXAMINATION

THIRD SEMESTER [B.TECH] JANUARY 2024

Paper Code: AIDS/AIML/TOT-209

Subject: Probability, Statistics
& Linear Algebra

Time: 3 Hours

Maximum Marks: 75

Note: Attempt five questions in all from each unit including Q.No.1 which is compulsory. Select one question from each unit. Assume missing data, if any.

- Q1 a) State Chebyshev's inequality. (2.5)
- b) If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cup B) = \frac{1}{2}$ then find the conditional probability $P(B|A)$. (2.5)
- c) Write a short note on Kurtosis. (2.5)
- d) Find the projection of vector $v = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ along vector $w = \begin{bmatrix} 1 & 1 \\ 5 & 5 \end{bmatrix}$ in a vector space M of all 2×2 matrices with inner product $\langle A, B \rangle = \text{trace}(B^T A)$. (2.5)
- e) Show that the matrix $A = \begin{bmatrix} \frac{1}{3} - \frac{2}{3}i & \frac{2}{3}i \\ -\frac{2}{3}i & -\frac{1}{3} - \frac{2}{3}i \end{bmatrix}$ is Unitary matrix. (2.5)
- f) If X is normally distributed with mean 30 and standard deviation 5, find the probability $P(|X - 30| > 5)$. (Use $P(0 < z < 1) = 0.3413$) (2.5)

UNIT-I

- Q2 a) A player tossed 2 coins. If two heads show he wins Rs. 40. If one head shows he wins Rs. 20, but if two tail show he pays Rs. 30 as penalty. Calculate the expected value of money he will win in the game. (7.5)
- b) If $P(x) = ke^{-|x|}$ is the probability density function for $-\infty < x < \infty$, find the value of k and the probability $P(0 < x < 4)$. (7.5)
- Q3 (a) A factory has three machines X, Y and Z producing 1000, 2000 and 3000 bolts per day respectively. The machine X produces 1% defective bolts, Y produces 1.5% and Z produces 2% defective bolts. At the end of the day, a bolt is drawn at random and it is found to be defective. What is the probability that this defective bolt has been produced by machine X? (7.5)
- (b) A problem in mathematics is given to three students X, Y, Z whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. What is the probability that (i) problem will be solved, (ii) problem will not be solved (7.5)

P.T.O.

UNIT-II

- Q4 a) Find the Karl Pearson's coefficient of corelation r for the following data (7.5)
- | | | | | | | |
|---|---|---|---|---|---|---|
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| y | 6 | 4 | 3 | 5 | 4 | 2 |
- b) Find the regression coefficient of y on x for the following data: (7.5)
- | | | | | | | |
|---|----|----|----|----|----|----|
| x | 12 | 10 | 14 | 11 | 12 | 9 |
| y | 18 | 17 | 23 | 19 | 20 | 15 |
- Q5 a) In a normal distribution 31% of items are under 45 and 8% are over 64. Find the mean and standard deviation of the normal distribution.
[Use $P(-\infty < z < -0.495) = 0.31$ and $P(0 < z < 1.405) = 0.42$] (7.5)
- b) Determine the Binomial distribution for which mean is 4 and variance is $\frac{4}{3}$.
Also, find the probability $P(X > 1)$. (7.5)

UNIT-III

- Q6 a) Fit the parabolic curve of regression of y on x for the following data (8)
- | | | | | |
|---|---|----|----|----|
| x | 1 | 2 | 3 | 4 |
| y | 6 | 11 | 18 | 27 |
- b) 40 people were attacked by a disease and only 36 survived. Will you reject the hypothesis that the survival rate if attacked by this disease is 85% in the favour of the hypothesis that is more at 5% level?
(Use table value of $Z_\alpha = 1.645$ for the right tail and $|Z_\alpha| = 1.96$ for two tail test if $\alpha = 5\%$). (7)

- Q7 a) Intelligent test of two groups of boys and girls gave the following results:

	Mean	S. D	Size
Girls	84	10	121
Boys	81	12	81

- (i) Is the difference between mean scores of girls and boys significant?
(ii) Is the difference between the standard deviations of scores of girls and boys significant? (5+5)

- b) In a sample of 1000 people, 540 are rice eaters and the rest are wheat eaters.
Can we assume that both rice eater and wheat eater are equally popular at 1% level of significance. (Table value $|Z_\alpha| = 2.58$ for $\alpha = 1\%$ for two tail) (5)

UNIT-IV

- Q8 a) Find the LU decomposition of the matrix $A = \begin{bmatrix} 1 & -3 & 5 \\ 2 & -4 & 7 \\ -1 & -2 & 1 \end{bmatrix}$. (7.5)
- b) Find the singular value decomposition of the matrix $A = \begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$. (7.5)
- Q9 Let U be the subspace of Euclidean space \mathbb{R}^4 spanned by the vectors:
 $v_1 = (1, 1, 1, 1), v_2 = (1, -1, 2, 2), v_3 = (1, 2, -3, -4)$.
- a) Apply Gram Schmidt orthogonalization process to find orthogonal and orthonormal basis of U . (8)
- b) Find the projection of vector $v = (1, 2, -3, 4)$ onto U . (7)
