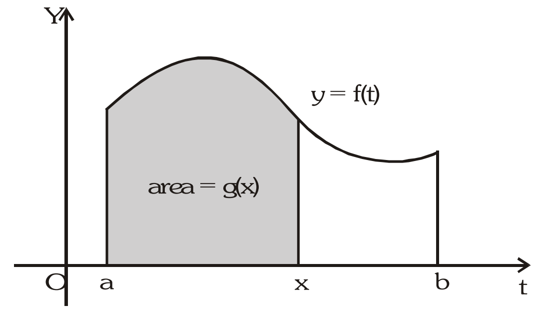
DEFINITE INTEGRATION

A definite integral is denoted by $\int_a^b f(x)dx$ which represent the algebraic area bounded by the curve y = f(x), the ordinates x = a, x = b and the x axis.

1. THE FUNDAMENTAL THEOREM OF CALCULUS :

The Fundamental Theorem of Calculus is appropriately named because it establishes a connection between the two branches of calculus : differential calculus and integral calculus. Differential calculus arose from the tangent problem, whereas integral calculus arose from a seemingly unrelated problem, the area problem. Newton's teacher at Cambridge, Isaac Barrow (1630-1677), discovered that these two problems are actually closely related. In fact, he realized that differentiation and integration are inverse processes. The Fundamental Theorem of Calculus gives the precise inverse relationship between the derivative and the integral. It was Newton and Leibnitz who exploited this relationship and used it to develop calculus into a systematic mathematical method. In particular, they saw that the Fundamental Theorem enabled them to compute areas and integrals very easily without having to compute them as limits of sums.



The Fundamental Theorem of Calculus, Part 1 If f is continuous on [a, b], then the function g defined by

$$g(x) = \int_a^x f(t)dt \qquad a \le x \le b$$

is continuous on [a, b] and differentiable on (a, b), and g'(x) = f(x).

The Fundamental Theorem of Calculus, Part 2 If f is continuous on [a, b], then

$$\int_a^b f(x)dx = F(b) - F(a)$$

where F is any antiderivative of f, that is, a function such that F '= f.

Note : If $\int_a^b f(x)dx = 0 \Rightarrow$ then the equation f(x) = 0 has atleast one root lying in (a, b) provided f is a continuous function in (a,b).

3. PROPERTIES OF DEFINITE INTEGRAL :

(a) $\int_a^b f(x)dx = \int_a^b f(t)\,dt$ provided f is same

(b) $\int_a^b f(x)dx = -\int_b^a f(x)\,dx$

(c) $\int_a^b f(x)dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, where c may lie inside or outside the interval [a,b]. This property is to be used when f is piecewise continuous in (a, b).

Illustration 1 : If $f(x)=\begin{cases}x^2, & 0<x<2\\ 3x-4, & 2\le x<3\end{cases}$ then evaluate $\int_0^3 f(x)dx$

Solution : $\int_0^3 f(x)dx=\int_0^2 f(x)dx+\int_2^3 f(x)dx=\int_0^2 x^2dx+\int_2^3(3x-4)dx$

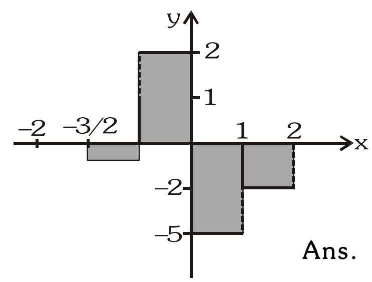
$$=\left(\frac{x^3}{3}\right)_0^2+\left(\frac{3x^2}{2}-4x\right)_2^3=\frac{8}{3}+\frac{27}{2}-12-6+8=37/6$$ **Ans.**

Illustration 2 : If $f(x)=\begin{cases}3[x]-5\dfrac{|x|}{x}, & x\ne 0\\ 2, & x=0\end{cases}$ then $\int_{-3/2}^{2} f(x)dx$ is equal to ([.] denotes the greatest integer function)

(A) $-\frac{11}{2}$ (B) $-\frac{7}{2}$ (C) -6 (D) $-\frac{17}{2}$

Solution : $3[x]-5\frac{|x|}{x}=3[x]-5$, if $x>0$

$=3[x]+5$, if $x<0$



$$\Rightarrow \int_{-3/2}^{2} f(x)dx=\int_{-3/2}^{-1}(-1)dx+\int_{-1}^{0}(2)dx+\int_{0}^{1}(-5)dx+\int_{1}^{2}(-2)dx$$

$$=-1\left(-1+\frac{3}{2}\right)+2(1)+1(-5)+(-2)=-\frac{1}{2}+2-5-2=-\frac{11}{2}$$ **Ans. (A)**

Illustration 3 : The value of $\int_1^2 (x^{[x^2]}+[x^2]^x)dx$, where [.] denotes the greatest integer function, is equal to -

(A) $\frac{5}{4}+\sqrt{3}+(2^{\sqrt{3}}-2^{\sqrt{2}})+\frac{1}{\log 3}(9-3^{\sqrt{3}})$

(B) $\frac{5}{4}+\sqrt{3}+\frac{\sqrt{2}}{3}+\frac{1}{\log 2}(2^{\sqrt{3}}-2^{\sqrt{2}})+\frac{1}{\log 3}(9-3^{\sqrt{3}})$

(C) $\frac{5}{4}+\frac{\sqrt{2}}{3}+\frac{1}{\log 2}(2^{\sqrt{3}}-2^{\sqrt{2}})+\frac{1}{\log 3}(9-3^{\sqrt{3}})$

(D) none of these

Solution : We have, $I=\int_1^2(x^{[x^2]}+[x^2]^x)dx=\int_1^{\sqrt{2}}(x+1)dx+\int_{\sqrt{2}}^{\sqrt{3}}(x^2+2^x)dx+\int_{\sqrt{3}}^{2}(x^3+3^x)dx$

$$=\left(\frac{x^2}{2}+x\right)_1^{\sqrt{2}}+\left(\frac{x^3}{3}+\frac{2^x}{\log 2}\right)_{\sqrt{2}}^{\sqrt{3}}+\left(\frac{x^4}{4}+\frac{3^x}{\log 3}\right)_{\sqrt{3}}^{2}$$

$$=\frac{5}{4}+\sqrt{3}+\frac{\sqrt{2}}{3}+\frac{1}{\log 2}(2^{\sqrt{3}}-2^{\sqrt{2}})+\frac{1}{\log 3}(3^2-3^{\sqrt{3}})$$ **Ans. (B)**

Illustration 4 : Evaluate : $\int_{-10}^{20}[\cot^{-1}x]dx$. Here [.] is the greatest integer function.

Solution : $I=\int_{-10}^{20}[\cot^{-1}x]dx$, we know $\cot^{-1}x\in(0,\pi)\ \forall\ x\in R$

$$\text{Thus } [\cot^{-1}x]=\begin{cases}3, & x\in(-\infty,\cot 3)\\ 2, & x\in(\cot 3,\cot 2)\\ 1, & x\in(\cot 2,\cot 1)\\ 0 & x\in(\cot 1,\infty)\end{cases}$$

Hence $I=\int_{-10}^{\cot 3}3dx+\int_{\cot 3}^{\cot 2}2dx+\int_{\cot 2}^{\cot 1}1dx+\int_{\cot 1}^{20}0dx=30+\cot 1+\cot 2+\cot 3$ **Ans.**

Do yourself -1 :

Evaluate :

(i) $\int_0^3 |x^2 - x - 2|\,dx$

(ii) $\int_0^4 \{x\}dx$, where {.} denotes fractional part of x.

(iii) $\int_0^{\pi/2} |\sin x - \cos x|\,dx$

(iv) If $f(x) = \begin{cases} 2 & 0 \le x \le 1 \\ x + [x] & 1 \le x < 3 \end{cases}$, where [.] denotes the greatest integer function. Evaluate $\int_0^2 f(x)dx$

(d) $\int_{-a}^{a} f(x)\,dx = \int_0^a [f(x) + f(-x)]dx = \begin{bmatrix} 0 & ;\text{if } f(x) \text{ is an odd function} \\ 2\int_0^a f(x)dx & ;\text{if } f(x) \text{ is an even function} \end{bmatrix}$

Illustration 5 : Evaluate $\int_{-1/2}^{1/2} \cos x\,\ell n\left(\frac{1+x}{1-x}\right)dx$

Solution : $f(-x) = \cos(-x)\,\ell n\left(\frac{1-x}{1+x}\right) = -\cos\,\ell n\left(\frac{1+x}{1-x}\right) = -f(x)$

$\Rightarrow$ f(x) is odd

Hence, the value of the given integral = 0. **Ans.**

Illustration 6 : If $f(x) = \begin{vmatrix} \cos x & e^{x^2} & 2x\cos^2 x/2 \\ x^2 & \sec x & \sin x + x^3 \\ 1 & 2 & x + \tan x \end{vmatrix}$, then the value of $\int_{-\pi/2}^{\pi/2} (x^2+1)(f(x) + f''(x))dx$

(A) 1 (B) −1 (C) 2 (D) none of these

Solution : As, $f(x) = \begin{vmatrix} \cos x & e^{x^2} & 2x\cos^2 x/2 \\ x^2 & \sec x & \sin x + x^3 \\ 1 & 2 & x + \tan x \end{vmatrix}$

$\Rightarrow$ $f(-x) = -f(x)$ $\Rightarrow$ f(x) is odd

$\Rightarrow$ f'(x) is even $\Rightarrow$ f''(x) is odd

Thus, f(x) + f''(x) is odd function let,

$\phi(x) = (x^2 + 1).\{f(x) + f''(x)\}$

$\Rightarrow$ $\phi(-x) = -\phi(x)$

i.e. $\phi(x)$ is odd

$\therefore$ $\int_{-\pi/2}^{\pi/2} \phi(x)dx = 0$ **Ans. (D)**

Do yourself -2 :

Evaluate :

(i) $\int_{-\pi/2}^{\pi/2} (x^2\sin^3 x + \cos x)dx$ (ii) $\int_{-\pi/2}^{\pi/2} \ell n\left[2\left(\frac{4-\sin\theta}{4+\sin\theta}\right)\right]d\theta$

(e) $\int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx$, In particular $\int_0^a f(x)dx = \int_0^a f(a-x)\,dx$

Illustration 7 : If f, g, h be continuous functions on [0, a] such that f(a – x) = –f(x), g(a – x) = g(x) and 3h(x) – 4h(a – x) = 5, then prove that $\int_0^a f(x)g(x)h(x)dx = 0$

Solution : $I = \int_0^a f(x)g(x)h(x)dx = \int_0^a f(a-x)g(a-x)h(a-x)dx = -\int_0^a f(x)g(x)h(a-x)dx$

7I = 3I + 4I

$= \int_0^a f(x)g(x)\{3h(x)-4h(a-x)\}\,dx = 5\int_0^a f(x)g(x)dx = 0$

(since f(a – x) g(a – x) = –f(x)g(x))

$\Rightarrow$ I = 0 **Ans.**

Illustration 8 : Evaluate $\int_{-\pi}^{\pi} \frac{x\sin x}{e^x+1}dx$

Solution : $I = \int_{-\pi}^{0} \frac{x\sin x}{e^x+1}dx + \int_0^{\pi} \frac{x\sin x}{e^x+1}dx = I_1 + I_2$

where $I_1 = \int_{-\pi}^{0} \frac{x\sin x}{e^x+1}dx$

Put x = –t $\Rightarrow$ dx = – dt

$\Rightarrow \quad I_1 = \int_{\pi}^{0} \frac{(-t)\sin(-t)(-dt)}{e^{-t}+1} = \int_0^{\pi} \frac{t\sin t\,dt}{e^{-t}+1} = \int_0^{\pi} \frac{e^t t\sin t\,dt}{e^t+1} = \int_0^{\pi} \frac{e^x x\sin x\,dx}{e^x+1}$

Hence $I = I_1 + I_2 = \int_0^{\pi} \frac{e^x x\sin x}{e^x+1}dx + \int_0^{\pi} \frac{x\sin x}{e^x+1}dx$

$I = \int_0^{\pi} x\sin x dx = \int_0^{\pi} (\pi-x)\sin(\pi-x)dx = \pi\int_0^{\pi} \sin x dx - I$

$\Rightarrow \quad 2I = \pi\int_0^{\pi} \sin x dx = \pi\left|-\cos x\right|_0^{\pi} = 2\pi \Rightarrow I = \pi$ **Ans.**

Illustration 9 : Evaluate $\int_0^2 \frac{dx}{(17+8x-4x^2)[e^{6(1-x)}+1]}$

Solution : Let $I = \int_0^2 \frac{dx}{(17+8x-4x^2)[e^{6(1-x)}+1]}$

Also $I = \int_0^2 \frac{dx}{(17+8x-4x^2)[e^{-6(1-x)}+1]}$ $\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

Adding, we get

$2I = \int_0^2 \frac{1}{17+8x-4x^2}\left(\frac{1}{e^{6(1-x)}+1} + \frac{1}{e^{-6(1-x)}+1}\right)dx$

$= \int_0^2 \frac{1}{17+8x-4x^2}dx = -\frac{1}{4}\int_0^2 \frac{dx}{x^2-2x-17/4}$

$$= -\frac{1}{4}\int_0^2 \frac{dx}{(x-1)^2 - 21/4} = -\frac{1}{4}\times\frac{1}{2\times\frac{\sqrt{21}}{2}}\left[\log\left|\frac{x-1-\frac{\sqrt{21}}{2}}{x-1+\frac{\sqrt{21}}{2}}\right|\right]_0^2$$

$$= -\frac{1}{4\sqrt{21}}\left[\log\left|\frac{2x-2-\sqrt{21}}{2x-2+\sqrt{21}}\right|\right]_0^2 \quad \Rightarrow \quad I = -\frac{1}{8\sqrt{21}}\left[\log\left|\frac{2-\sqrt{21}}{2+\sqrt{21}}\right| - \log\left|\frac{2+\sqrt{21}}{\sqrt{21}-2}\right|\right]$$

$$= -\frac{1}{4\sqrt{21}}\left[\log\left|\frac{\sqrt{21}-2}{2+\sqrt{21}}\right|\right]$$ **Ans.**

Illustration 10 : Evaluate $\int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}}$

Solution :

$$I = \int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}} = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx \quad .....(i)$$

then $$I = \int_0^{\pi/2} \frac{\sqrt{\cos(\pi/2-x)dx}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}} = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \quad .....(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}.dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}.dx = \int_0^{\pi/2} 1.dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0$$

$$2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$ **Ans.**

Illustration 11 : $\int_0^1 \cot^{-1}(1-x+x^2)dx$ equals -

(A) $\frac{\pi}{2}+\log 2$ (B) $\frac{\pi}{2}-\log 2$ (C) π – log 2 (D) none of these

Solution :

$$I = \int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx = \int_0^1 \tan^{-1}\left(\frac{x+(1-x)}{1-x(1-x)}\right)dx$$

$$= \int_0^1 [\tan^{-1}x + \tan^{-1}(1-x)]\ dx = \int_0^1 \tan^{-1}x\ dx + \int_0^1 \tan^{-1}(1-x)\ dx$$

$$= 2\int_0^1 \tan^{-1}x\ dx = 2\left[x\ \tan^{-1}x - \frac{1}{2}\log(1+x^2)\right]_0^1 = 2\frac{\pi}{4} - \log 2 = \frac{\pi}{2} - \log 2$$ **Ans. (B)**

Illustration 12 : $\int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x}dx$

Solution :

$$I = \int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x}dx \quad .....(i)$$

$$I = \int_0^{\pi/2} \frac{a\sin(\pi/2-x) + b\cos(\pi/2-x)}{\sin(\pi/2-x)+\cos(\pi/2-x)}dx = \int_0^{\pi/2} \frac{a\cos x + b\sin x}{\sin x + \cos x}dx \quad .....(ii)$$

$$\therefore\ 2I = \int_0^{\pi/2} \frac{(a+b)(\sin x+\cos x)}{\sin x + \cos x}dx = \int_0^{\pi/2}(a+b)dx = (a+b)\pi/2 \Rightarrow I = (a+b)\pi/4$$ **Ans.**

Illustration 13 : $\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}}dx$ equals -

(A) 2 (B) π (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

Solution : $I = \int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}}dx = \int_0^{\pi/2} \frac{2^{\sin(\pi/2-x)}}{2^{\sin x(\pi/2-x)}+2^{\cos(\pi/2-x)}}dx = \int_0^{\pi/2} \frac{2^{\cos x}}{2^{\cos x}+2^{\sin x}}dx$

$2I = \int_0^{\pi/2} dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$ **Ans. (C)**

Do yourself -3 :

Evaluate :

(i) $\int_1^5 \frac{\sqrt{x}}{\sqrt{6-x}+\sqrt{x}}dx$ (ii) $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\tan^5 x}$

(f) $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx = \begin{cases} 2\int_0^a f(x)dx \ ; \text{ if } f(2a-x)=f(x) \\ 0 \ ; \text{ if } f(2a-x)=-f(x) \end{cases}$

Illustration 14 : Evaluate $\int_0^{\pi} \frac{xdx}{1+\cos^2 x}$

Solution : Let $I = \int_0^{\pi} \frac{xdx}{1+\cos^2 x} = \int_0^{\pi} \frac{(\pi-x)dx}{1+\cos^2(\pi-x)} = \int_0^{\pi} \frac{\pi dx}{1+\cos^2 x} - I$

$\Rightarrow 2I = \int_0^{\pi} \frac{\pi dx}{1+\cos^2 x} = 2\pi\int_0^{\pi/2} \frac{dx}{1+\cos^2 x} = 2\pi\int_0^{\pi/2} \frac{\sec^2 xdx}{2+\tan^2 x}$

Let tan x = t so that for $x \to 0$, $t \to 0$ and for $x \to \pi/2$, $t \to \infty$. Hence we can write,

$I = \pi\int_0^{\infty} \frac{dt}{2+t^2} = \pi\frac{1}{\sqrt{2}}\left[\tan^{-1}\frac{t}{\sqrt{2}}\right]_0^{\infty} = \frac{\pi^2}{2\sqrt{2}}$ **Ans.**

Illustration 15 : Prove that $\int_0^{\pi/2} \log(\sin x)dx = \int_0^{\pi/2} \log(\cos x)dx = -\frac{\pi}{2}\log 2$

Solution : Let $I = \int_0^{\pi/2} \log(\sin x)dx$ .... (i)

then $I = \int_0^{\pi/2} \log\sin\left(\frac{\pi}{2}-x\right)dx = \int_0^{\pi/2} \log(\cos x)dx$ .... (ii)

adding (i) and (ii), we get

$2I = \int_0^{\pi/2} \log\sin x\,dx + \int_0^{\pi/2} \log\cos x\,dx = \int_0^{\pi/2} (\log\sin x + \log\cos x)dx$

$\Rightarrow 2I = \int_0^{\pi/2} \log(\sin x\cos x)dx = \int_0^{\pi/2} \log\left(\frac{2\sin x\cos x}{2}\right)dx$

$= \int_0^{\pi/2} \log\left(\frac{\sin 2x}{2}\right)dx = \int_0^{\pi/2} \log(\sin 2x)dx - \int_0^{\pi/2} (\log 2)dx$

$$= \int_0^{\pi/2} \log \sin 2x \,.\, dx - (\log 2)(x)_0^{\pi/2}$$

$$\Rightarrow \quad 2I = \int_0^{\pi/2} \log(\sin 2x)dx - \frac{\pi}{2}\log 2 \quad \ldots\ldots\ldots \text{(iii)}$$

Let $I_1 = \int_0^{\pi/2} \log(\sin 2x)dx$, putting $2x = t$, we get

$$I_1 = \int_0^{\pi} \log(\sin t)\frac{dt}{2} = \frac{1}{2}\int_0^{\pi} \log(\sin t)dt = \frac{1}{2}.2\int_0^{\pi/2} \log(\sin t)dt$$

$$I_1 = \int_0^{\pi/2} \log(\sin x)dx$$

$\therefore$ (iii) becomes ; $2I = I - \frac{\pi}{2}\log 2$

Hence $\int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2}\log 2$ **Ans.**

Illustration 16 : $\int_0^{\pi/2} (2\log \sin x - \log \sin 2x)dx$ equals -

(A) π log 2 (B) $-\pi$ log 2 (C) $(\pi/2)$ log 2 (D) $-(\pi/2)$ log 2

Solution :

$$I = \int_0^{\pi/2} (2\log \sin x - \log 2\sin x \cos x)dx = \int_0^{\pi/2} (2\log\sin x - \log 2 - \log \sin x - \log \cos x)dx$$

$$= \int_0^{\pi/2} \log \sin x dx - \int_0^{\pi/2} \log 2 dx - \int_0^{\pi/2} \log \cos x dx = -(\pi/2)\log 2$$ **Ans. (D)**

Do yourself -4 :

Evaluate :

(i) $\int_{-\sqrt{3}}^{\sqrt{3}} \frac{dx}{(1+e^x)(1+x^2)}$ (ii) $\int_0^{\pi/2} \ell n\left(\sin^2 x \cos x\right)dx$

(iii) $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x}dx$ (iv) $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}\, dx$

(g) $\int_0^{nT} f(x)dx = n\int_0^{T} f(x)dx$, $(n \in I)$; where 'T' is the period of the function i.e. $f(T + x) = f(x)$

Note that : $\int_x^{T+x} f(t)dt$ will be independent of x and equal to $\int_0^T f(t)dt$

(h) $\int_{a+nT}^{b+nT} f(x)dx = \int_a^b f(x)dx$ where f(x) is periodic with period T & $n \in I$.

(i) $\int_{mT}^{nT} f(x)dx = (n-m)\int_0^T f(x)dx$, $(n, m \in I)$ if f(x) is periodic with period 'T'.

Illustration 17 : Evaluate $\int_0^{4\pi} |\cos x|\,dx$

Solution : Note that $|\cos x|$ is a periodic function with period π. Hence the given integral.

$$I = 4\int_0^{\pi} |\cos x|\,dx = 4\left[\int_0^{\frac{\pi}{2}} \cos x\,dx - \int_{\frac{\pi}{2}}^{\pi} \cos x\,dx\right] = 4\left[[\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{\pi}\right] = 4[1+1] = 8$$ **Ans.**

Illustration 18 : The value of x satisfying $\int_0^{2[x+14]} \left\{\frac{x}{2}\right\}dx = \int_0^{\{x\}} [x+14]dx$, is equal to (where [.] and {.} denotes the greatest integer and fractional part of (x)

(A) [–14, –13] (B) (0, 1) (C) (–15, –14] (d) none of these

Solution : $\int_0^{2[x+14]} \left\{\frac{x}{2}\right\}dx = \int_0^{\{x\}} [x+14]dx \Rightarrow \int_0^{28+2[x]} \left\{\frac{x}{2}\right\}dx = \int_0^{\{x\}} (14+[x])dx$

$$\Rightarrow \int_0^{28} \left\{\frac{x}{2}\right\}dx + \int_{28}^{28+2[x]} \left\{\frac{x}{2}\right\}dx = (14+[x])\{x\} \Rightarrow 14\int_0^{2} \left\{\frac{x}{2}\right\}dx + \int_0^{2[x]} \left\{\frac{x}{2}\right\}dx = (14+[x])\{x\}$$

{using $\int_0^{nT} f(x)dx = n\int_0^{T} f(x)dx$ and $\int_a^{a+nT} f(x)dx = \int_0^{nT} f(x)dx$ where T is period of f(x)}

$\Rightarrow$ 14 + [x] = (14 + [x]){x} $\Rightarrow$ (14 + [x])(1 – {x}) = 0

$\Rightarrow$ [x] = –14 $\Rightarrow$ x $\in$ [–14, –13) **Ans. (A)**

Illustration 19 : Evaluate $\int_0^{16\pi/3} |\sin x|\,dx$

Solution : $\int_0^{16\pi/3} |\sin x|\,dx = \int_0^{5\pi} |\sin x|\,dx + \int_{5\pi}^{5\pi+\pi/3} |\sin x|\,dx = 5\int_0^{\pi} |\sin x|\,dx + \int_0^{\pi/3} |\sin x|\,dx$

$$= 5[-\cos x]_0^{\pi} + [-\cos x]_0^{\pi/3} = 10 + \left(-\frac{1}{2}+1\right) = \frac{21}{2}$$ **Ans.**

Illustration 20 : Evaluate : $\int_0^{2n\pi} [\sin x + \cos x]dx$. Here [.] is the greatest integer function.

Solution : Let $I = \int_0^{2n\pi} [\sin x + \cos x]dx = n\int_0^{2\pi} [\sin x + \cos x]dx$

($\because$ [sinx + cosx] is periodic function with period 2π]

$$[\sin x + \cos x] = \begin{cases} 1, & 0 \le x \le \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \le x \le \frac{3\pi}{4} \\ -1, & \frac{3\pi}{4} < x \le \pi \\ -2, & \pi < x \le \frac{3\pi}{2} \\ -1, & \frac{3\pi}{2} < x \le \frac{7\pi}{4} \\ 0, & \frac{7\pi}{4} < x \le 2\pi \end{cases}$$

Hence $I = n\left[\int_0^{\pi/2} 1dx + \int_{\pi/2}^{3\pi/4} 0dx + \int_{3\pi/4}^{\pi} -1dx + \int_{\pi}^{3\pi/2} -2dx + \int_{3\pi/2}^{7\pi/4} -1dx + \int_{7\pi/4}^{2\pi} 0dx\right]$

$I = n\left[\frac{\pi}{2} + 0 - \pi + \frac{3\pi}{4} - 3\pi + 2\pi - \frac{7\pi}{4} + \frac{3\pi}{2} + 0\right] = -n\pi$ **Ans.**

Do yourself -5 :

Evaluate :

(i) $\int_{-1.5}^{10} \{2x\}dx$, where $\{.\}$ denotes fractional part of x.

(ii) $\int_{20\pi+\frac{\pi}{6}}^{20\pi+\frac{\pi}{3}} (\sin x + \cos x)dx$

(iii) $\int_0^{[x]} \frac{3^x}{3^{[x]}}dx$, where [.] denotes greatest integer function.

4. WALLI'S FORMULA :

(a) $\int_0^{\pi/2} \sin^n x\, dx = \int_0^{\pi/2} \cos^n x\, dx = \frac{(n-1)(n-3).....(1 \text{ or } 2)}{n(n-2).....(1 \text{ or } 2)}K$

where $K = \begin{cases} \pi/2 & \text{if n is even} \\ 1 & \text{if n is odd} \end{cases}$

(b) $\int_0^{\pi/2} \sin^n x.\cos^m x\, dx = \frac{[(n-1)(n-3)(n-5)....1 \text{ or } 2][(m-1)(m-3)....1 \text{ or } 2]}{(m+n)(m+n-2)(m+n-4)....1 \text{ or } 2}K$

Where $K = \begin{cases} \frac{\pi}{2} & \text{if both m and n are even } (m, n \in N) \\ 1 & \text{otherwise} \end{cases}$

Illustration 21 : $\int_{-\pi/2}^{\pi/2} \sin^4 x \ \cos^6 x \ dx =$

(A) $\frac{3\pi}{64}$ (B) $\frac{3\pi}{572}$ (C) $\frac{3\pi}{256}$ (D) $\frac{3\pi}{128}$

Solution : $I = \int_{-\pi/2}^{\pi/2} \sin^4 x \ \cos^6 x \ dx = 2\int_0^{\pi/2} \sin^4 x \ \cos^6 x.dx = 2\frac{(3.1)(5.3.1)}{10.8.6.4.2}.\frac{\pi}{2} = \frac{3\pi}{256}$ **Ans. (C)**

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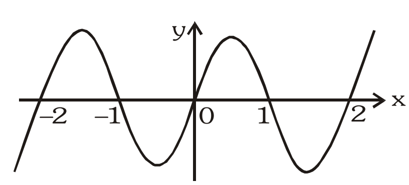
5. DERIVATIVE OF ANTIDERIVATIVE FUNCTION (Newton-Leibnitz Formula) :

If h(x) & g(x) are differentiable functions of x then, $\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)dt = f[h(x)].h'(x) - f[g(x)].g'(x)$

Illustration 22 : Find the points of maxima/minima of $\int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t}dt$

Solution : Let $f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t}dt$

$f'(x) = \frac{x^4 - 5x^2 + 4}{2 + e^{x^2}}2x - 0 = \frac{(x-1)(x+1)(x-2)(x+2)2x}{2 + e^{x^2}}$



Graph of f'(x)

From the wavy curve, it is clear that f'(x) changes its sign at x = ± 2, ±1, 0 and hence the points of maxima are −1, 1 and of the minima are −2, 0, 2.

Illustration 23 : Evaluate $\dfrac{d}{dt}\displaystyle\int_{t^2}^{t^3}\frac{1}{\log x}dx$

Solution : $\dfrac{d}{dt}\displaystyle\int_{t^2}^{t^3}\frac{1}{\log x}dx = \frac{1}{\log t^3}.\frac{d}{dt}(t^3) - \frac{1}{\log t^2}.\frac{d}{dt}(t^2) = \frac{3t^2}{3\log t} - \frac{2t}{2\log t} = \frac{t(t-1)}{\log t}$ **Ans.**

Do yourself - 6 :

(i) If $f(x) = \displaystyle\int_{1/x}^{\sqrt{x}} \sin t\, dt$, then find $f\,'(1)$.

(ii) $\displaystyle\int_{\pi/3}^{x}\sqrt{3-\sin^2 t}\,dt + \int_0^y \cos t\,dt = 0$, then evaluate $\dfrac{dy}{dx}$.

6. DEFINITE INTEGRAL AS LIMIT OF A SUM :

An alternative way of describing $\int_a^b f(x)dx$ is that the definite integral $\int_a^b f(x)dx$ is a limiting case of the summation of an infinite series, provided $f(x)$ is continuous on [a,b]

i.e. $\displaystyle\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh)$ where $h = \dfrac{b-a}{n}$. The converse is also true i.e., if we have an infinite series of the above form, it can be expressed as a definite integral.

Step I : Express the given series in the form $\sum \frac{1}{n} f\left(\frac{r}{n}\right)$

Step II : Then the limit is its sum when $n \to \infty$, i.e. $\lim\limits_{n\to\infty} \frac{1}{n} f\left(\frac{r}{n}\right)$

Step III : Replace $\frac{r}{n}$ by x and $\frac{1}{n}$ by dx and $\lim\limits_{n\to\infty}\sum$ by the sign of $\int$

Step IV : The lower and the upper limit of integration are the limiting values of $\frac{r}{n}$ for the first and the last term of r respectively.

Some particular cases of the above are.

(a) $\displaystyle\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}f\left(\frac{r}{n}\right)$ or $\displaystyle\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{n}f\left(\frac{r}{n}\right) = \int_0^1 f(x)dx$

(b) $\displaystyle\lim_{x\to\infty}\sum_{r=1}^{pn}\frac{1}{n}\left(\frac{r}{n}\right) = \int_\alpha^\beta f(x)dx$

where $\alpha = \lim\limits_{x\to\infty}\dfrac{r}{n} = 0$ (as r = 1)

and $\beta = \lim\limits_{n\to\infty}\dfrac{r}{n} = p$ (as r = pn)

Illustration 24 : Evaluate $\lim\limits_{n\to\infty}\left(\frac{1}{2n+1}+\frac{1}{2n+2}+\dots\dots\dots+\frac{1}{6n}\right)$

Solution : Let $S_n = \frac{1}{2n+1}+\frac{1}{2n+2}+\dots\dots\dots+\frac{1}{6n} = \sum\limits_{r=1}^{4n}\frac{1}{2n+r} = \sum\limits_{r=1}^{4n}\frac{1}{n}.\frac{1}{2+\left(\frac{r}{n}\right)}$

$\Rightarrow \quad S = \lim\limits_{n\to\infty} S_n = \int\limits_0^4 \frac{dx}{2+x} = [\ell n\,|\,2+x\,|\,]_0^4 = \ell n6 - \ell n2 = \ell n3$ **Ans.**

Illustration 25 : Evaluate $\lim\limits_{n\to\infty}\left[\frac{\sqrt{n}}{\left(3+4\sqrt{n}\right)^2}+\frac{\sqrt{n}}{\sqrt{2}\left(3\sqrt{2}+4\sqrt{n}\right)^2}+\frac{\sqrt{n}}{\sqrt{3}\left(3\sqrt{3}+4\sqrt{n}\right)^2}+\dots\dots+\frac{1}{49n}\right]$

Solution : Let $p = \lim\limits_{n\to\infty}\left[\frac{\sqrt{n}}{\left(3+4\sqrt{n}\right)^2}+\frac{\sqrt{n}}{\sqrt{2}\left(3\sqrt{2}+4\sqrt{n}\right)^2}+\dots\dots\dots+\frac{\sqrt{n}}{\sqrt{n}\left(3\sqrt{n}+4\sqrt{n}\right)^2}\right]$

Analyzing the expression with the view of increasing integral value we get the expression in terms of r as

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{\sqrt{n}}{\sqrt{r}\left(3\sqrt{r}+4\sqrt{n}\right)^2} = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n\sqrt{\frac{r}{n}}\left(3\sqrt{\frac{r}{n}}+4\right)^2} = \int_0^1\frac{dx}{\sqrt{x}\left(3\sqrt{x}+4\right)^2}$$

Put $3\sqrt{x}+4=t, \quad \therefore \quad \frac{3}{2\sqrt{x}}dx = dt$

Hence $p = \frac{2}{3}\int\limits_4^7\frac{dt}{t^2} = \frac{2}{3}\left[-\frac{1}{t}\right]_4^7 = \frac{2}{3}\left(-\frac{1}{7}+\frac{1}{4}\right) = \frac{1}{14}$ **Ans.**

Do yourself - 7 :

Evaluate :

(i) $\lim\limits_{n\to\infty}\left[\frac{1}{n+2.1}+\frac{1}{n+2.2}+\frac{1}{n+2.3}\dots\dots\frac{1}{3n}\right]$ (ii) $\lim\limits_{n\to\infty}\sum\limits_{r=0}^{n-1}\frac{1}{\sqrt{n^2-r^2}}$

7. ESTIMATION OF DEFINITE INTEGRAL :

(a) If f(x) is continuous in [a, b] and it's range in this interval is [m, M], then $m(b-a) \le \int\limits_a^b f(x)dx \le M(b-a)$

Illustration 26 : Prove that $4 \le \int\limits_1^3\sqrt{3+x^3}\,dx \le 2\sqrt{30}$

Solution : Since the function $f(x) = \sqrt{3+x^2}$ increases monotonically on the interval [1, 3], m = 2, M = $\sqrt{30}$, b – a =2.

Hence, $2.2 \le \int\limits_1^3\sqrt{3+x^3}\,dx \le 2\sqrt{30} \Rightarrow 4 \le \int\limits_1^3\sqrt{3+x^3}\,dx \le 2\sqrt{30}$ **Ans.**

(b) If $f(x) \le \phi(x)$ for $a \le x \le b$ then $\int\limits_a^b f(x)dx \le \int\limits_a^b \phi(x)\,dx$

Illustration 27 : Prove that $\frac{\pi}{6} \le \int_0^1 \frac{dx}{\sqrt{4-x^2-x^3}} \le \frac{\pi}{4\sqrt{2}}$

Solution : Since $4 - x^2 \ge 4 - x^2 - x^3 \ge 4 - 2x^2 > 0\ \forall\ x \in [0, 1]$

$$\sqrt{4-x^2} \ge \sqrt{4-x^2-x^3} \ge \sqrt{4-2x^2} > 0\ \forall\ x \in [0, 1]$$

$$\Rightarrow \quad 0 < \frac{1}{\sqrt{4-x^2}} \le \frac{1}{\sqrt{4-x^2-x^3}} \le \frac{1}{\sqrt{4-2x^2}}\ \forall\ x \in [0\ 1]$$

$$\Rightarrow \quad \int_0^1 \frac{dx}{\sqrt{4-x^2}} \le \int_0^1 \frac{dx}{\sqrt{4-x^2-x^3}} \le \int_0^1 \frac{dx}{\sqrt{4-2x^2}}\ \forall x \in [0, 1]$$

$$\Rightarrow \quad \left[\sin^{-1}\frac{x}{2}\right]_0^1 \le \int_0^1 \frac{dx}{\sqrt{4-x^2-x^3}} \le \frac{1}{\sqrt{2}}\left[\sin^{-1}\frac{x}{\sqrt{2}}\right]_0^1 \Rightarrow \frac{\pi}{6} \le \int_0^1 \frac{dx}{\sqrt{4-x^2-x^3}} \le \frac{\pi}{4\sqrt{2}}$$ **Ans.**

(c) $\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|dx$.

Illustration 28 : Prove that $\left|\int_{10}^{19} \frac{\sin x}{1+x^8}dx\right| < 10^{-7}$

Solution : To find $I = \left|\int_{10}^{19} \frac{\sin x}{1+x^8}dx\right| \le \int_{10}^{19}\left|\frac{\sin x}{1+x^8}\right|dx$ .... (i)

Since $|\sin x| \le 1$ for $x \ge 10$

The inequality $\left|\frac{\sin x}{1+x^8}\right| \le \frac{1}{|1+x^8|}$ .... (ii)

also, $10 \le x \le 19$

$\Rightarrow$ $1 + x^8 > 10^8$

$$\Rightarrow \quad \frac{1}{1+x^8} < \frac{1}{10^8} \text{ or } \frac{1}{|1+x^8|} < 10^{-8} \quad .... (iii)$$

from (ii) and (iii) ;

$$\left|\frac{\sin x}{1+x^8}\right| < 10^{-8}$$

$$\left|\int_{10}^{19} \frac{\sin x}{1+x^8}dx\right| < \int_{10}^{19} 10^{-8}dx$$

$$\therefore \quad \left|\int_{10}^{19} \frac{\sin x}{1+x^8}dx\right| < (19-10).10^{-8} < 10^{-7}$$ **Ans.**

Illustration 29 : If $f(x)$ is integrable function such that $|f(x) - f(y)| \le |x^2 - y^2|$, $\forall\ x, y \in [a,b]$ then prove that $\left|\int_a^b \frac{f(x)-f(a)}{x+a}dx\right| \le \frac{(a-b)^2}{2}$.

Solution : Given, $\left|\int_a^b \frac{f(x)-f(a)}{x+a}dx\right| \le \int_a^b \left|\frac{f(x)-f(a)}{x+a}\right| dx$

$$\le \int_a^b \left|\frac{x^2-a^2}{x+a}\right| dx = \int_a^b |x-a|\,dx = \int_a^b (x-a)dx = \frac{(a-b)^2}{2}$$

(d) If $f(x) \ge 0$ on the interval [a,b], then $\int_a^b f(x)dx \ge 0$.

Illustration 30: If f (x) is a continous function such that f (x) ≥ 0 $\forall$ x $\in$ [2,10] and $\int_4^8 f(x)\,dx = 0$, then find f (6).

Solution : f (x) is above the x-axis or on the x-axis for all x $\in$ [2,10]. If f(x) is greater than zero for any sub interval of [4,8], then $\int_4^8 f(x)dx$ must be greater than zero. But $\int_4^8 f(x)dx = 0 \Rightarrow f(x) = 0\ \forall\ x \in [4,8]$
$\Rightarrow f(6) = 0$.

Do yourself - 8 :

(i) Prove that $4 \le \int_1^3 \sqrt{3+x^2}\,dx \le 4\sqrt{3}$

(ii) Prove that $\frac{\pi}{4} \le \int_0^{2\pi} \frac{dx}{5+3\sin x} \le \pi$.

(iii) Show that $\frac{3}{5}\left(2^{1/3}-1\right) \le \int_0^1 \frac{x^4}{(1+x^6)^{2/3}}dx \le 1$

<u>Miscelleneous Illustrations</u> :

Illustration 31 : Evaluate : $\int_0^{\pi} \frac{x^3\cos^4 x\sin^2 x}{(\pi^2-3\pi x+3x^2)}dx$

Solution : Let $I = \int_0^{\pi} \frac{x^3\cos^4 x\sin^2 x}{(\pi^2-3\pi x+3x^2)}dx$ .... (i)

$$= \int_0^{\pi} \frac{(\pi-x)^3\cos^4(\pi-x)\sin^2(\pi-x)dx}{\pi^2-3\pi(\pi-x)+3(\pi-x)^2} \qquad \text{(By. Prop.)}$$

$$= \int_0^{\pi} \frac{(\pi^3-x^3-3\pi^2x+3\pi x^2)\cos^4 x\sin^2 x}{(\pi^2-3\pi x+3x^2)}dx \qquad \text{......... (ii)}$$

Adding (i) and (ii) we have

$$2I = \int_0^{\pi} \frac{(\pi^3-3\pi^2x+3\pi x^2)\cos^4 x\sin^2 x}{(\pi^2-3\pi x+3x^2)}dx$$

$$\Rightarrow \quad 2I = \pi\int_0^{\pi} \cos^4 x\sin^2 x\,dx \quad \Rightarrow \quad 2I = 2\pi\int_0^{\pi/2} \cos^4 x\sin^2 x\,dx$$

$$\therefore \quad I = \pi\int_0^{\pi/2} \cos^4 x\sin^2 x\,dx$$

Using walli's formula, we get $I = \pi\frac{(3.1)(1)}{6.4.2}\frac{\pi}{2} = \frac{\pi^2}{32}$ **Ans.**

Illustration 32 : Let f be an injective function such that f(x) f(y) + 2 = f(x) + f(y) + f(xy) for all non negative real x and y with f(0) = 1 and f'(1) = 2 find f(x) and show that $3\int f(x)dx - x(f(x)+2)$ is a constant.

Solution : We have f(x)f(y) + 2 = f(x) + f(y) + f(xy)

Putting x = 1 & y = 1

then f(1)f(1) + 2 = 3f(1)

we get f(1) = 1,2

f(1) ≠ 1 (∵ f(0) = 1 & function is injective)

then f(1) = 2

Replacing y by $\frac{1}{x}$ in (1) then

$$f(x)f\left(\frac{1}{x}\right)+2 = f(x)+f\left(\frac{1}{x}\right)+f(1) \quad \Rightarrow \quad f(x)f\left(\frac{1}{x}\right) = f(x)+f\left(\frac{1}{x}\right)$$

Hence f(x) is of the type

$f(x) = 1 \pm x^n$

∵ f(1) = 2

∴ $f(x) = 1 + x^n$

and $f'(x) = nx^{n-1} \Rightarrow f'(1) = n = 2$

$f(x) = 1 + x^2$

$$\therefore \quad 3\int f(x)dx - x(f(x)+2) = 3\int (1+x^2)dx - x(1+x^2+2)$$

$$= 3\left(x+\frac{x^3}{3}\right) - x(3+x^2)+c = c = \text{constant}$$

Illustration 33 : Evaluate : $\int_{-1}^{1}[x[1+\sin \pi x]+1]dx$, [.] is the greatest integer function.

Solution : Let $I = \int_{-1}^{1}[x[1+\sin \pi x]+1]dx = \int_{-1}^{0}[x[1+\sin \pi x]+1]dx + \int_{0}^{1}[x[1+\sin \pi x]+1]dx$

Now $[1 + \sin\pi x] = 0$ if $-1 < x < 0$

$[1 + \sin\pi x] = 1$ if $0 < x < 1$

$$\therefore \quad I = \int_{-1}^{0} 1.dx + \int_{0}^{1}[x+1]dx = 1 + 1\int_{0}^{1} dx = 1 + 1 = 2.$$ **Ans.**

Illustration 34 : Find the limit, when $n \to \infty$ of

$$\frac{1}{\sqrt{(2n-1^2)}} + \frac{1}{\sqrt{(4n-2^2)}} + \frac{1}{\sqrt{(6n-3^2)}} + \ldots.. + \frac{1}{n}$$

Solution : Let $P = \lim_{n\to\infty}\left[\frac{1}{\sqrt{2n-1^2}} + \frac{1}{\sqrt{4n-2^2}} + \frac{1}{\sqrt{6n-3^2}} + \ldots. + \frac{1}{n}\right]$

$$= \lim_{n\to\infty}\left[\frac{1}{\sqrt{1(2n)-1^2}} + \frac{1}{\sqrt{2(2n)-2^2}} + \frac{1}{\sqrt{3(2n)-3^2}} + \ldots. + \frac{1}{\sqrt{n(2n)-n^2}}\right]$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{\sqrt{r(2n)-r^2}} = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n.\sqrt{2\frac{r}{n}-\left(\frac{r}{n}\right)^2}} = \int_{0}^{1}\frac{dx}{\sqrt{(2x-x^2)}}$$

Put $x = t^2 \Rightarrow dx = 2t\ dt$

$$\therefore \quad P = \int_0^1 \frac{2t\,dt}{t\sqrt{2-t^2}} = \left[2\sin^{-1}\left(\frac{t}{\sqrt{2}}\right)\right]_0^1 = 2\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 2\left(\frac{\pi}{4}\right)$$

Hence $\quad P = \pi/2.$ **Ans.**

Illustration 35 : If $f(x) = \begin{cases} 1-|x|, & |x| \le 1 \\ |x|-1, & |x| > 1 \end{cases}$, and $g(x) = f(x-1) + f(x+1)$. Find the value of $\int_{-3}^{5} g(x)\,dx$.

Solution : Given,

$$f(x) = \begin{cases} -x-1, & x < -1 \\ 1+x, & -1 \le x < 0 \\ 1-x, & 0 \le x \le 1 \\ x-1, & x > 1 \end{cases} \quad ; \quad f(x-1) = \begin{cases} -x, & x-1 < -1 & \Rightarrow x < 0 \\ x, & -1 \le x-1 < 0 & \Rightarrow 0 \le x < 1 \\ 2-x, & 0 \le x-1 \le 1 & \Rightarrow 1 \le x \le 2 \\ x-2, & x-1 > 1 & \Rightarrow x > 2 \end{cases}$$

Similarly

$$f(x+1) = \begin{cases} -x-2, & x+1 < -1 & \Rightarrow x < -2 \\ x+2, & -1 \le x+1 < 0 & \Rightarrow -2 \le x < -1 \\ -x, & 0 \le x+1 \le 1 & \Rightarrow -1 \le x \le 0 \\ x, & x+1 > 1 & \Rightarrow x > 0 \end{cases}$$

$$\Rightarrow \quad g(x) = f(x-1) + f(x+1) = \begin{cases} -2x-2 & x < -2 \\ 2, & -2 \le x < -1 \\ -2x, & -1 \le x \le 0 \\ 2x, & 0 < x < 1 \\ 2, & 1 < x \le 2 \\ 2x-2, & 2 < x \end{cases}$$

Clearly g(x) is even,

$$\text{Now} \quad \int_{-3}^{5} g(x)dx = 2\int_0^3 g(x)\,dx + \int_3^5 g(x)\,dx = 2\left(\int_0^1 2x\ dx + \int_1^2 2dx + \int_2^3 (2x-2)\,dx\right) + \int_3^5 (2x-2)\,dx = 24$$

ANSWERS FOR DO YOURSELF

1 :	(i)	$\frac{31}{6}$	(ii)	2	(iii)	$2(\sqrt{2}-1)$	(iv)	$\frac{9}{2}$
2 :	(i)	2	(ii)	$\pi\ell n2$				
3 :	(i)	2	(ii)	$\pi/12$				
4 :	(i)	$\frac{\pi}{3}$	(ii)	$-\left(\frac{3\pi}{2}\right)\ell n2$	(iii)	0	(iv)	$\frac{4}{3}$
5 :	(i)	$\frac{23}{4}$	(ii)	$\left(\sqrt{3}-1\right)$	(iii)	$\frac{2[x]}{\ell n3}$		
6 :	(i)	$\frac{3}{2}\sin 1$	(ii)	$\frac{dy}{dx} = \frac{-\sqrt{3-\sin^2 x}}{\cos y}$				
7 :	(i)	$\frac{1}{2}\ell n3$	(ii)	$\frac{\pi}{2}$				

EXERCISE - 01 CHECK YOUR GRASP

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. If $\int_0^{\pi/3} \frac{\cos x}{3+4\sin x} dx = k \log\left(\frac{3+2\sqrt{3}}{3}\right)$ then k is-

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$

2. $\int_{e^{e^{e}}}^{e^{e^{e^{e}}}} \frac{dx}{x\ell nx.\ell n(\ell nx).\ell n(\ell n(\ell nx))}$ equals -

(A) 1 (B) 1/e (C) e − 1 (D) 1 + e

3. The value of the definite integral $\int_1^{\infty} (e^{x+1} + e^{3-x})^{-1} dx$ is

(A) $\frac{\pi}{4e^2}$ (B) $\frac{\pi}{4e}$ (C) $\frac{1}{e^2}\left(\frac{\pi}{2} - \tan^{-1}\frac{1}{e}\right)$ (D) $\frac{\pi}{2e^2}$

4. The value of the definite integral $\int_1^e \left((x+1)e^x.\ell nx\right) dx$ is -

(A) e (B) e^{e+1} (C) $e^e(e-1)$ (D) $e^e(e-1)+e$

5. Let a, b, c be non-zero real numbers such that ; $\int_0^1 (1+\cos^8 x)(ax^2+bx+c)dx = \int_0^2 (1+\cos^8 x)(ax^2+bx+c)dx$,

then the quadratic equation $ax^2 + bx + c = 0$ has -

(A) no root in (0,2) (B) atleast one root in (0,2)

(C) a double root in (0,2) (D) none

6. If f(x) = A sin $\left(\frac{\pi x}{2}\right)$ + B, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)dx = \frac{2A}{\pi}$, then the constant A and B are-

(A) $\frac{\pi}{2}$ and $\frac{\pi}{2}$ (B) $\frac{2}{\pi}$ and 3π (C) 0 and $\frac{-4}{\pi}$ (D) $\frac{4}{\pi}$ and 0

7. If $I_n = \int_0^{\pi/4} \tan^n x dx$ then $\lim_{n\to\infty} n\,(I_n + I_{n-2}) =$

(A) 1 (B) 1/2 (C) ∞ (D) 0

8. $\int_0^{\infty} \frac{x\tan^{-1}x}{(1+x^2)^2} dx$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{8}$

9. Suppose f, f' and f'' are continuous on [0, e] and that f'(e) = f(e) = f(1) = 1 and $\int_1^e \frac{f(x)}{x^2} dx = \frac{1}{2}$, then the value of

$\int_1^e f''(x)\ell n\,x\,dx$ equals -

(A) 0 (B) 1 (C) 2 (D) none of these

10. $\int_{1/2}^{2} \frac{1}{x}\sin\left(x-\frac{1}{x}\right)dx$ has the value equal to -

(A) 0 (B) $\frac{3}{4}$ (C) $\frac{5}{4}$ (D) 2

11. $\int_{2}^{4}\left[\log_x 2-\frac{(\log_x 2)^2}{\ell n 2}\right]dx =$

(A) 0 (B) 1 (C) 2 (D) 4

12. Suppose that F(x) is an antiderivative of f(x) = $\frac{\sin x}{x}$, x > 0 then $\int_{1}^{3}\frac{\sin 2x}{x}dx$ can be expressed as -

(A) F(6) – F(2) (B) $\frac{1}{2}$(F(6) – F(2)) (C) $\frac{1}{2}$(F(3) – F(1)) (D) 2(F(6) – F(2))

13. $\int_{0}^{\infty} f\left(x+\frac{1}{x}\right).\frac{\ell n x}{x}dx$

(A) is equal to zero (B) is equal to one (C) is equal to $\frac{1}{2}$ (D) can not be evaluated

14. Integral $\int_{0}^{1}|\sin 2\pi x|\,dx$ is equal to -

(A) 0 (B) $-\frac{1}{\pi}$ (C) $\frac{1}{\pi}$ (D) $\frac{2}{\pi}$

15. $\int_{2}^{3}\frac{\sqrt{x}}{\sqrt{(5-x)}+\sqrt{x}}dx =$

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{5}$ (D) none

16. For any integer n the integral $\int_{0}^{\pi} e^{\cos^2 x}\cos^3(2n+1)x\,dx$ has the value **[JEE 1985]**

(A) π (B) 1 (C) 0 (D) none of these

17. $\int_{2}^{3}\frac{(x+2)^2}{2x^2-10x+53}dx$ is equal to -

(A) 2 (B) 1 (C) 1/2 (D) 5/2

18. The value of the definite integral $\int_{0}^{1}(1+e^{-x^2})\,dx$ is- **[JEE 1981]**

(A) –1 (B) 2 (C) 1 + e^{-1} (D) none of these

19. $\int_{-\pi}^{\pi}(\cos ax-\sin bx)^2\,dx$ where a and b are integer is equal to -

(A) $-\pi$ (B) 0 (C) π (D) 2π

20. The value of $\int_{-\pi}^{\pi}(1-x^2)\sin x\cos^2 x\,dx$ is -

(A) 0 (B) $\pi-\frac{\pi^3}{3}$ (C) $2\pi - \pi^3$ (D) $\frac{7}{2}-2\pi^3$

21. The value of $\int_{\pi}^{2\pi}[2\sin x]dx$, where [ ] represents the greatest integer function is -

(A) $-\frac{5\pi}{3}$ (B) $-\pi$ (C) $\frac{5\pi}{3}$ (D) -2π

22. If $\int_{0}^{f(x)} t^2 dt = x\cos \pi x$, then f'(9)

(A) is equal to $-\frac{1}{9}$ (B) is equal to $-\frac{1}{3}$ (C) is equal to $\frac{1}{3}$ (D) is non existent

23. Let $f : R \to R$ be a differentiable function and $f(1) = 4$. Then the value of $\lim_{x\to 1}\int_{4}^{f(x)}\frac{2t}{x-1}dt$ is - **[JEE 1990]**

(A) $8f'(1)$ (B) $4f'(1)$ (C) $2f'(1)$ (D) $f'(1)$

24. If $g(x) = \int_{0}^{x}\cos^4 t\, dt$, then $g(x + \pi)$ equals - **[JEE 1997]**

(A) $g(x) + g(\pi)$ (B) $g(x) - g(\pi)$ (C) $g(x)g(\pi)$ (D) $\frac{g(x)}{g(\pi)}$

25. For $n \in N$, the value of the definite integral $\int_{0}^{n\pi+V}\sqrt{\frac{1+\cos 2x}{2}}dx$ where $\frac{\pi}{2} < V < \pi$ is -

(A) $2n + 1 - \cos V$ (B) $2n - \sin V$ (C) $2n + 2 - \sin V$ (D) $2n + 1 - \sin V$

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

26. $\int_{0}^{\infty}\frac{x}{(1+x)(1+x^2)}dx$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$

(C) is same as $\int_{0}^{\infty}\frac{dx}{(1+x)(1+x^2)}$ (D) cannot be evaluated

27. Which of the following are true ?

(A) $\int_{a}^{\pi-a} x.f(\sin x)dx = \frac{\pi}{2}.\int_{a}^{\pi-a} f(\sin x)dx$ (B) $\int_{-a}^{a} f(x^2)dx = 2.\int_{0}^{a} f(x^2)dx$

(C) $\int_{0}^{n\pi} f(\cos^2 x)dx = n.\int_{0}^{\pi} f(\cos^2 x)dx$ (D) $\int_{0}^{b-c} f(x+c)dx = \int_{c}^{b} f(x)dx$

28. Let $f(x) = \int_{2}^{x}\frac{dt}{\sqrt{1+t^4}}$ and g be the inverse of f. Then the value of g'(0) is -

(A) 1 (B) 17 (C) $\sqrt{17}$ (D) none of these

29. If $f(x) = \int_{1}^{x}\frac{\ell nt}{1+t}dt$ where $x > 0$ then the value(s) of x satisfying the equation, $f(x) + f(1/x) = 2$ is -

(A) 2 (B) e (C) e^{-2} (D) e^2

30. The value of $\lim_{n\to\infty}\sum_{r=1}^{r=4n}\frac{\sqrt{n}}{\sqrt{r}\left(3\sqrt{r}+4\sqrt{n}\right)^2}$ is equal to -

(A) $\frac{1}{35}$ (B) $\frac{1}{14}$ (C) $\frac{1}{10}$ (D) $\frac{1}{5}$

31. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$ $I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$ then -

(A) $I_3 > I_4$ (B) $I_3 = I_4$ (C) $I_1 > I_2$ (D) $I_2 > I_1$

32. Let $S_n = \frac{n}{(n+1)(n+2)} + \frac{n}{(n+2)(n+4)} + \frac{n}{(n+3)(n+6)} + \ldots\ldots + \frac{1}{6n}$, then $\lim_{n\to\infty} S_n$ is -

(A) $\ell n\frac{3}{2}$ (B) $\ell n\frac{9}{2}$ (C) greater than one (D) less than two

33. The value of the integral $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}} dx$ is- [JEE 1983]

(A) $\pi/4$ (B) $\pi/2$ (C) $\int_{\pi/8}^{3\pi/8} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}} dx$ (D) $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x}$

34. Let f be a positive function, let $I_1 = \int_{1-k}^{k} x f[x(1-x)]dx$, $I_2 = \int_{1-k}^{k} f[x(1-x)]dx$, where $2k - 1 > 0$. Then $\frac{I_1}{I_2}$ is - [JEE 1997]

(A) 2 (B) k (C) $\frac{1}{2}$ (D) less than 1

CHECK YOUR GRASP — ANSWER KEY — EXERCISE-1

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	A	A	D	B	D	A	D	D	A
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A	A	A	D	A	C	C	D	D	A
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	A	A	A	A	C	A,C	A,B,C,D	C	C,D	C
Que.	31	32	33	34						
Ans.	C	A,D	A,D	C,D						

EXERCISE - 02

BRAIN TEASERS

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THEN ONE CORRECT ANSWERS)

1. The value of $\int_0^1 \frac{2x^2+3x+3}{(x+1)(x^2+2x+2)}dx$ is -

(A) $\frac{\pi}{4}+2\ell n2-\tan^{-1}2$ (B) $\frac{\pi}{4}+2\ell n2-\tan^{-1}\frac{1}{3}$ (C) $2\ell n2-\cot^{-1}3$ (D) $-\frac{\pi}{4}+\ell n4+\cot^{-1}2$

2. If $I_n=\int_0^1 \frac{dx}{(1+x^2)^n}$; $n \in N$, then which of the following statements hold good ?

(A) $2nI_{n+1}=2^{-n}+(2n-1)I_n$ (B) $I_2=\frac{\pi}{8}+\frac{1}{4}$ (C) $I_2=\frac{\pi}{8}-\frac{1}{4}$ (D) $I_3=\frac{\pi}{16}-\frac{5}{48}$

3. If a, b, c $\in$ R and satisfy $3a + 5b + 15c = 0$, the equation $ax^4 + bx^2 + c = 0$ has -

(A) atleast one root in (–1, 0) (B) atleast one root in (0, 1)

(C) atleast two roots in (–1, 1) (D) no root in (–1, 1)

4. Let $u=\int_0^\infty \frac{dx}{x^4+7x^2+1}$ & $v=\int_0^\infty \frac{x^2dx}{x^4+7x^2+1}$ then -

(A) $v > u$ (B) $6v = \pi$ (C) $3u + 2v = 5\pi/6$ (D) $u + v = \pi/3$

5. Let f(x) be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and g be the function satisfying $f(x) + g(x) = x^2$. The value of the integral $\int_0^1 f(x)g(x)dx$ is -

(A) $e-\frac{1}{2}e^2-\frac{5}{2}$ (B) $e - e^2 - 3$ (C) $\frac{1}{2}(e-3)$ (D) $e-\frac{1}{2}e^2-\frac{3}{2}$

6. For $f(x) = x^4 + |x|$, let $I_1=\int_0^\pi f(\cos x)dx$ and $I_2=\int_0^{\pi/2} f(\sin x)dx$ then $\frac{I_1}{I_2}$ has the value equal to -

(A) 1 (B) 1/2 (C) 2 (D) 4

7. Number of values of x satisfying the equation $\int_{-1}^{x}\left(8t^2+\frac{28}{3}t+4\right)dt=\frac{\left(\frac{3}{2}\right)x+1}{\log_{(x+1)}\sqrt{x+1}}$, is -

(A) 0 (B) 1 (C) 2 (D) 3

8. The value of definite integral $\int_\infty^0 \frac{ze^{-z}}{\sqrt{1-e^{-2z}}}dz$

(A) $-\frac{\pi}{2}\ell n2$ (B) $\frac{\pi}{2}\ell n2$ (C) $-\pi\ell n2$ (D) $\pi\ell n\frac{1}{\sqrt{2}}$

9. $\int_0^{\pi/4}(\cos 2x)^{3/2}.\cos x\,dx=$

(A) $\frac{3\pi}{16}$ (B) $\frac{3\pi}{32}$ (C) $\frac{3\pi}{16\sqrt{2}}$ (D) $\frac{3\pi\sqrt{2}}{16}$

10. The value of $\int_0^1\left(\prod_{r=1}^{n}(x+r)\right)\left(\sum_{k=1}^{n}\frac{1}{x+k}\right)dx$ equals

(A) n (B) n! (C) (n+1)! (D) n.n!

11. If the value of the integral $\int_1^2 e^{x^2}dx$ is α, then the value of $\int_e^{e^4}\sqrt{\ell nx}\,dx$ is -

(A) $e^4 - e - \alpha$ (B) $2e^4 - e - \alpha$ (C) $2(e^4 - e) - \alpha$ (D) $2e^4 - 1 - \alpha$

12. The value of $\lim\limits_{x\to\infty}\frac{d}{dx}\int_{\sqrt{3}}^{\sqrt{x}}\frac{r^3}{(r+1)(r-1)}dr$ is -

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) non existent

13. $\int_0^{\infty}[2e^{-x}]dx$ where [x] denotes the greatest interger function is -

(A) 0 (B) $\ell n2$ (C) e^2 (D) $2/e$

14. Let $f(x) = \frac{\sin x}{x}$, then $\int_0^{\pi/2} f(x)f\left(\frac{\pi}{2}-x\right)dx =$

(A) $\frac{2}{\pi}\int_0^{\pi} f(x)dx$ (B) $\int_0^{\pi} f(x)dx$ (C) $\pi\int_0^{\pi} f(x)dx$ (D) $\frac{1}{\pi}\int_0^{\pi} f(x)dx$

15. If for a non-zero x, $af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$, where $a \neq b$, then $\int_1^2 f(x)dx =$

(A) $\frac{1}{a^2+b^2}\left(a\log 2 + 5a + \frac{7b}{2}\right)$ (B) $\frac{1}{a^2-b^2}\left(a\log 2 - 5a + \frac{7b}{2}\right)$

(C) $-\frac{1}{a^2+b^2}\left(a\log 2 + 5a - \frac{7b}{2}\right)$ (D) none of these

16. If a, b and c are real numbers then the value of $\lim\limits_{t\to 0}\ell n\left(\frac{1}{t}\int_0^t (1 + a\sin bx)^{c/x}dx\right)$ equals -

(A) abc (B) $\frac{ab}{c}$ (C) $\frac{bc}{a}$ (D) $\frac{ca}{b}$

17. Let $y = f(x)$ be a differentiable curve satisfying $\int_2^x f(t)dt = \frac{x^2}{2} + \int_x^2 t^2 f(t)dt$, then $\int_{-\pi/4}^{\pi/4}\frac{f(x) + x^9 - x^3 + x + 1}{\cos^2 x}dx$ equals -

(A) 0 (B) 1 (C) 2 (D) 4

18. If $y = f(x)$ is a linear function satisfying the relation $f(xy) = f(x).f(y)$ $\forall\, x, y \in R$, then the curve $y^2 + \int_0^x (\sin t + a^2t^3 + bt)dt = \alpha,\ \alpha \in R^+$ cuts $y = f^{-1}(x)$ at -

(A) no point (B) exactly one point (C) atleast two points (D) infinite points

19. If $f(8 - t) = f(t)$ and $\int_0^4 f(\alpha)d\alpha = 8$, then $\int_0^8 f(\gamma)d\gamma$ is -

(A) 4 (B) 8 (C) 16 (D) 32

20. If $x = \int_0^{t^2} e^{\sqrt{z}} \left\{\frac{2\tan\sqrt{z} + 1 - \tan^2\sqrt{z}}{2\sqrt{z}\sec^2\sqrt{z}}\right\} dz$ & $y = \int_0^{t^2} e^{\sqrt{z}} \left\{\frac{1 - \tan^2\sqrt{z} - 2\tan\sqrt{z}}{2\sqrt{z}\sec^2\sqrt{z}}\right\} dz$.

Then the inclination of the tangent to the curve at $t = \frac{\pi}{4}$ is -

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{4}$

21. The value of integral $\int_0^{\pi} x f(\sin x)dx =$

(A) $\frac{\pi}{2}\int_0^{\pi} f(\sin x)dx$ (B) $\pi\int_0^{\pi/2} f(\sin x)dx$ (C) $\pi\int_0^{\pi/2} f(\cos x)dx$ (D) $\frac{\pi}{2}\int_0^{\pi} f(\cos x)dx$

BRAIN TEASERS — ANSWER KEY — EXERCISE-2

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A,C,D	A,B	A,B,C	B,C,D	D	C	B	A,D	C	D
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	C	B	A	B	A	C	C	C	D
Que.	21									
Ans.	A,B,C									

EXERCISE - 03

MISCELLANEOUS TYPE QUESTIONS

TRUE / FALSE

1. The value of the integral $\int_{-\infty}^{0} x.e^{x}\,dx$ is not finite.

2. If n is a positive integer then $\int_{0}^{1} (\ell nx)^{n}\,dx = (-1)^{n} n!$.

3. $\int_{1}^{\infty} \frac{1}{x^{p}}\,dx = \frac{1}{p-1}$, where $p \in R - \{1\}$

4. The average value of the function $f(x) = \sin^2 x \cos^3 x$ on the interval $[-\pi, \pi]$ is 0.

5. If $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec} x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$. Then $\int_{0}^{\pi/2} f(x)\,dx = -\left(\frac{15\pi+32}{60}\right)$ [JEE 1987]

6. For $n > 0$, $\int_{0}^{2\pi} \frac{x \sin^{2n} x}{\sin^{2n} x + \cos^{2n} x}\,dx = \pi$ [JEE 1996]

MATCH THE COLUMN

Following questions contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE** statement in **Column-II**.

1.

	Column-I		Column-II
(A)	$\int_{4}^{10} \frac{[x^2]dx}{[x^2 - 28x + 196] + [x^2]} =$ {where [.] denotes greatest integer function}	(p)	$\frac{1}{100}$
(B)	$\int_{-1}^{2} \frac{\lvert x \rvert}{x}\,dx =$	(q)	3
(C)	$\lim_{n\to\infty} \frac{1^{99} + 2^{99} + \ldots.. + n^{99}}{n^{100}} =$	(r)	$\frac{1}{3}$
(D)	$5050 \int_{-1}^{1} \sqrt{x^{200}}\,dx = \frac{1}{\alpha}$, then $\alpha =$	(s)	1

2.

	Column-I		Column-II
(A)	$\int_{-1}^{1} \frac{3x^2}{1+4^{\tan x}}\,dx =$	(p)	7
(B)	$\int_{6}^{8} \frac{\sin x^2 dx}{\sin x^2 + \sin(x-14)^2} =$	(q)	$\frac{1}{2}$
(C)	$\frac{1}{156} \int_{1}^{13} [x]dx =$ {where [.] denotes greatest integer function}	(r)	1
(D)	$\frac{1}{\pi \ell n 2} \int_{\pi/2}^{0} \ell n \sin 2x dx =$	(s)	2

3.

	Column-I		Column-II
(A)	If [ ] denotes the greatest integer function and $f(x)=\begin{cases}3[x]-\dfrac{5\lvert x\rvert}{x}; & x\neq 0\\ 2 & ; x=0\end{cases}$, then is equal to $\int_{-3/2}^{2} f(x)dx$	(p)	1
(B)	The value of $\int_{-\pi/2}^{\pi/2}\frac{\cos x}{1+e^x}dx$ of is	(q)	$-\frac{11}{2}$
(C)	If $I_1=\int_1^{\sin\theta}\frac{x}{1+x^2}dx$ and $I_2=\int_1^{\mathrm{cosec}\,\theta}\frac{1}{x(x^2+1)}dx$ then the value of $\begin{vmatrix} I_1 & I_1^2 & I_2\\ e^{I_1+I_2} & I_2^2 & -1\\ 1 & I_1^2+I_2^2 & -1\end{vmatrix}$, is	(r)	$\frac{3}{2}$
(D)	If f(x) and g(x) are two continuous functions defined on R, then the value of $\int_{-a}^{a}\{f(x)+f(-x)\}\{g(x)-g(-x)\}dx$, is	(s)	0

ASSERTION & REASON

These questions contains, Statement-I (assertion) and Statement-II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement-I :** The equation $4x^3 - 9x^2 + 2x + 1 = 0$ has atleast one real root in (0, 1).

because

Statement-II : If 'f' is a continuous function such that $\int_a^b f(x)=0$, then the equation f(x) = 0 has atleast one real root in (a, b).

(A) A (B) B (C) C (D) D

2. **Statement-I :** $\int_0^{\pi} x\tan x\cos^3 x dx=\frac{\pi}{2}\int_0^{\pi}\tan x\cos^3 x dx$.

because

Statement-II : $\int_a^b x f(x)dx=\frac{a+b}{2}\int_a^b f(x)dx$.

(A) A (B) B (C) C (D) D

3. **Statement-I :** If $f(x)=\int_1^x\frac{\ell n t dt}{1+t+t^2}$ (x > 0), then $f(x)=-f\left(\frac{1}{x}\right)$

because

Statement-II : If $f(x)=\int_1^x\frac{\ell n t dt}{t+1}$, then $f(x)+f\left(\frac{1}{x}\right)=\frac{1}{2}(\ell n x)^2$.

(A) A (B) B (C) C (D) D

4. Let $f(x) = x - x^2 + 1$.

Statement-I : $g(x)=\max\{f(t) : 0\le t\le x\}$, then $\int_0^1 g(x)dx=\frac{29}{24}$

because

Statement-II : f(x) is increasing in $\left(0,\frac{1}{2}\right)$ and decreasing in $\left(\frac{1}{2},1\right)$.

(A) A (B) B (C) C (D) D

5. Statement-I : Let m & n be positive integers. $a = \cos\left\{\int_{-\pi}^{\pi}(\sin mx.\sin nx)dx\right\}$, if $m \neq n$ & $b = \cos\left\{\int_{-\pi}^{\pi}(\sin mx.\sin nx)dx\right\}$ if m = n, then a + b = 2.

because

Statement-II : $\int_{-\pi}^{\pi}(\sin mx.\sin nx)dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$, where m & n are positive integers.

(A) A (B) B (C) C (D) D

6. Statement-I : $\int_{1/3}^{3}\frac{1}{x}\text{cosec}^{99}\left(x - \frac{1}{x}\right)dx = 0$.

because

Statement-II : $\int_{-a}^{a} f(x)dx = 0$ if $f(-x) = -f(x)$.

(A) A (B) B (C) C (D) D

7. Statement-I : $\sum_{r=0}^{n-1}\frac{1}{n}\left(\sqrt{\frac{r}{n}}+1\right) < \int_0^1(\sqrt{x}+1)\,dx < \sum_{r=1}^{n}\frac{1}{n}\left(\sqrt{\frac{r}{n}}+1\right)$, $n \in N$.

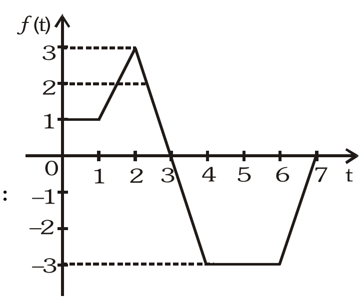
because

Statement-II : If $f(x)$ is continuous and increasing in [0, 1], then $\sum_{r=0}^{n-1}\frac{1}{n}f\left(\frac{r}{n}\right) < \int_0^1 f(x)dx < \sum_{r=1}^{n}\frac{1}{n}f\left(\frac{r}{n}\right)$, where $n \in N$

(A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1



Let $g(x) = \int_0^x f(t)dt$, where f is a function whose graph is show adjacently.

On the basis of above information, answer the following questions :

1. Maximum value of g(x) in $x \in [0, 7]$ is -
(A) 3 (B) 9/2
(C) 3/2 (D) 6

2. Value of x at which g(x) becomes zero, is -
(A) 3 (B) 4 (C) 5 (D) 6

3. Set of values of x in [0, 7] for which g(x) is negative is -
(A) (2, 7) (B) (3, 7) (C) (4, 6) (D) (5, 7)

Comprehension # 2

The average value of a function f(x) over the interval, [a, b] is the number

$$\mu = \frac{1}{b-a}\int_a^b f(x)dx$$

The square root $\left\{\frac{1}{b-a}\int_a^b [f(x)]^2\,dx\right\}^{1/2}$ is called the root mean square of f on [a, b]. The average value of μ is attained if f is continuous on [a, b].

On the basis of above information, answer the following questions :

1. The average ordinate of y = sin x over the interval $[0, \pi]$ is -
(A) $1/\pi$ (B) $2/\pi$ (C) $4/\pi^2$ (D) $2/\pi^2$

2. The average value of the pressure varying from 2 to 10 atm if the pressure p and the volume v are related by $pv^{3/2} = 160$ is -

(A) $\frac{20}{\sqrt[3]{20}\left(\sqrt[3]{10}+\sqrt[3]{2}\right)}$ (B) $\frac{10}{\sqrt[3]{10}+\sqrt[3]{2}}$ (C) $\frac{40}{\sqrt[3]{20}\left(\sqrt[3]{10}+\sqrt[3]{2}\right)}$ (D) $\frac{160}{\sqrt[3]{20}\left(\sqrt[3]{10}+\sqrt[3]{2}\right)}$

3. The average value of $f(x) = \frac{\cos^2 x}{\sin^2 x + 4\cos^2 x}$ on $[0, \pi/2]$ is -

(A) $\pi/6$ (B) $4/\pi$ (C) $6/\pi$ (D) $1/6$

Comprehension # 3

Consider $g(x) = \begin{cases} \left\{\frac{\max.(f(t)) + \min.(f(t))}{2},\ 0 \le t \le x\right\} & 0 \le x \le 4 \\ |x-5| + |x-4| & 4 < x < 5 \\ \tan\left(\sin^{-1}\left(\frac{6-x}{\sqrt{x^2-12x+37}}\right)\right) & x \ge 5 \end{cases}$

where $f(x) = x^2 - 4x + 3$.

On the basis of above information, answer the following questions :

1. $\int_2^5 g(x)\,dx$ is equal to

(A) 5/3 (B) 3 (C) 13/3 (D) 3/2

2. If $h(x) = \int_0^{x^2} g(t)dt$, then complete set of values of x in the interval [0, 7] for which h(x) is decreasing, is -

(A) (6, 7] (B) (5, 7] (C) $\left(\sqrt{6}, \sqrt{7}\right]$ (D) $\left(\sqrt{6}, 7\right]$

3. $\lim\limits_{x\to 4} \frac{g(x)-g(2)}{\ell n(\cos(4-x))}$ is equal to -

(A) 0 (B) 1 (C) 2 (D) does not exist

MISCELLANEOUS TYPE QUESTION — **ANSWER KEY** — EXERCISE-3

- *True / False*

  1. F 2. T 3. F 4. T 5. T 6. F

- *Match the Column*

  1. (A)→(q), (B)→(s), (C)→(p), (D)→(p) 2. (A)→(r), (B)→(r), (C)→(q), (D)→(q)
  3. (A)→(q), (B)→(p), (C)→(s), (D)→(s)

- *Assertion & Reason*

  1. A 2. C 3. D 4. A 5. D 6. A 7. A

- *Comprehension Based Questions*

  Comprehension # 1 : 1. B 2. C 3. D
  Comprehension # 2 : 1. B 2. C 3. D
  Comprehension # 3 : 1. B 2. D 3. A

EXERCISE - 04 [A] CONCEPTUAL SUBJECTIVE EXERCISE

1. Compute the integrals :

(a) $\int_{2}^{-13} \frac{dx}{\sqrt[5]{(3-x)^4}}$ (b) $\int_{0}^{1} (e^x - 1)^4 e^x dx$ (c) $\int_{\pi/4}^{\pi/3} \frac{x\,dx}{\sin^2 x}$ (d) $\int_{0}^{1} \frac{\sqrt{x}\,dx}{1+x}$

(e) $\int_{1}^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x^2} dx$ (f) $\int_{\sqrt{2}}^{2} \frac{dx}{x^5\sqrt{x^2-1}}$ (g) $\int_{0}^{\frac{1}{\sqrt{3}}} \frac{dx}{(2x^2+1)\sqrt{x^2+1}}$

2. Prove that : (a) $\int_{\alpha}^{\beta} \sqrt{(x-\alpha)(\beta-x)}\, dx = \frac{(\beta-\alpha)^2 \pi}{8}$ (b) $\int_{\alpha}^{\beta} \sqrt{\frac{x-\alpha}{\beta-x}}\, dx = (\beta-\alpha)\frac{\pi}{2}$

(c) $\int_{\alpha}^{\beta} \frac{dx}{x\sqrt{(x-\alpha)(\beta-x)}} = \frac{\pi}{\sqrt{\alpha\beta}}$ where $\alpha, \beta > 0$ (d) $\int_{\alpha}^{\beta} \frac{x\,.\,dx}{\sqrt{(x-\alpha)(\beta-x)}} = (\alpha+\beta)\frac{\pi}{2}$ where $\alpha < \beta$

3. Evaluate : (a) $\int_{0}^{3} |(x-1)(x-2)|\, dx$ (b) $\int_{0}^{\pi} |\cos x|\, dx$

4. Given function $f(x) = \begin{cases} x^2, & \text{for } 0 \le x < 1 \\ \sqrt{x}, & \text{for } 1 \le x \le 2 \end{cases}$. Evaluate $\int_{0}^{2} f(x)dx$

5. Evaluate : (a) $\int_{0}^{2} [x^2]dx$ (b) $\int_{-1}^{1} [\cos^{-1} x]dx$, where [.] represents the greatest integer function

6. Evaluate : (a) $\int_{0}^{\pi} \log(1+\cos x)dx$ (b) $\int_{0}^{2t} \frac{f(x)}{f(x)+f(2t-x)}dx$

(c) $\int_{0}^{\frac{\pi}{4}} \log(1+\tan x)dx$ (d) $\int_{0}^{\frac{\pi}{2}} \frac{x \sin 2x\,dx}{\cos^4 x + \sin^4 x}$

7. Prove that for any positive integer k, $\frac{\sin 2kx}{\sin x} = 2[\cos x + \cos 3x + \ldots\ldots + \cos(2k-1)x]$

Hence prove that $\int_{0}^{\pi/2} \sin 2kx \cot x\, dx = \frac{\pi}{2}$ [JEE 1990]

8. Evaluate : $\int_{0}^{1} \frac{x^4(1-x)^4}{1+x^2}dx$

9. Evaluate : $\int_{0}^{\pi/2} \frac{a \sin x + b \cos x}{\sin\left(\frac{\pi}{4}+x\right)}dx$

10. Evaluate : $\int_{0}^{2\pi} \frac{dx}{2+\sin 2x}$

11. Evaluate : $\int_{-\sqrt{2}}^{\sqrt{2}} \frac{2x^7+3x^6-10x^5-7x^3-12x^2+x+1}{x^2+2}dx$

12. Evaluate : (a) $\int_0^1 \frac{1-x}{1+x} \cdot \frac{dx}{\sqrt{x+x^2+x^3}}$ (b) $\int_1^{\frac{1+\sqrt{5}}{2}} \frac{x^2+1}{x^4-x^2+1} \ell n\left(1+x-\frac{1}{x}\right) dx$

13. Integrate : $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}} dx$ [JEE 1999]

14. Evaluate : $\int_0^1 \frac{\sin^{-1}\sqrt{x}}{x^2-x+1} dx$

15. Evaluate : $\int_0^{\pi/4} \frac{\cos x - \sin x}{10+\sin 2x} dx$

16. Evaluate $\int_0^{\pi} \frac{x \sin 2x \sin\left(\frac{\pi}{2}\cos x\right)}{2x-\pi} dx$ [JEE 1991]

17. Evaluate : $\int_0^{2\pi} e^x \cos\left(\frac{\pi}{4}+\frac{x}{2}\right) dx$

18. $\int_1^2 \frac{(x^2-1)dx}{x^3.\sqrt{2x^4-2x^2+1}} = \frac{u}{v}$ where u and v are in their lowest form. Find the value of $\frac{(1000)u}{v}$

19. Prove that if $J_m = \int_1^e \ell n^m x dx$, then $J_m = e - mJ_{m-1}$ (m a positive integer).

20. Prove the inequalities :

(a) $\frac{\pi}{6} < \int_0^1 \frac{dx}{\sqrt{4-x^2-x^3}} < \frac{\pi\sqrt{2}}{8}$ (b) $2\,e^{-1/4} < \int_0^2 e^{x^2-x} dx < 2e$ (c) $\frac{1}{2} \le \int_0^2 \frac{dx}{2+x^2} \le \frac{5}{6}$

21. Suppose g(x) is the inverse of f(x) and f(x) has a domain $x \in [a, b]$. Given $f(a) = \alpha$ and $f(b) = \beta$, then find the value of $\int_a^b f(x)dx + \int_{\alpha}^{\beta} g(y)dy$ in terms of a, b, α and β.

22. Let α, β be the distinct positive roots of the equation tan x = 2x then evaluate $\int_0^1 (\sin \alpha x . \sin \beta x) dx$ independent of α and β.

23. Let h(x) = (fog)(x) + K where K is any constant. If $\frac{d}{dx}(h(x)) = \frac{\sin x}{\cos^2(\cos x)}$ then compute the value of j(0) where $j(x) = \int_{g(x)}^{f(x)} \frac{f(t)}{g(t)} dt$, where f and g are trigonometric functions.

24. (a) $f(x) = \int_0^{\sin^2 x} \sin^{-1}\sqrt{t}dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}dt,\ x \in \left[0, \frac{\pi}{2}\right]$ determine f(x)

(b) $f(x) = \int_{e^x}^{e^{3x}} \frac{tdt}{\ell nt},\ x > 0$ find differential coefficient of f(x) w.r.t. ℓnx when $x = \ell n2$

25. Given a function f(x) such that **[JEE 1984]**

(a) it is integrable over every interval on the real line and

(b) $f(T + x) = f(x)$, for every x and a real T, then show that the integral $\int_a^{a+T} f(x)dx$ is independent of a.

26. Find the limits : (a) $\lim_{x\to 0^+} \dfrac{\int_0^{x^2} \sin\sqrt{x}\,dx}{x^3}$ (b) $\lim_{x\to +\infty} \dfrac{\left(\int_0^x e^{x^2}dx\right)^2}{\int_0^x e^{2x^2}dx}$

27. Evaluate : (a) $\lim_{n\to\infty}\left(\dfrac{1}{n+1}+\dfrac{1}{n+2}+.....+\dfrac{1}{6n}\right)$ **[JEE 1981]**

(b) $\underset{n\to\infty}{\text{Limit}}\ \dfrac{1}{n}\left[\dfrac{1}{n+1}+\dfrac{2}{n+2}+.....+\dfrac{3n}{4n}\right]$

(c) $\underset{n\to\infty}{\text{Limit}}\left[\dfrac{n!}{n^n}\right]^{1/n}$

CONCEPTUAL SUBJECTIVE EXERCISE — **ANSWER KEY** — EXERCISE-4(A)

1. (a) $-5\left(\sqrt[5]{16}-1\right)$ (b) $0.2\ (e-1)^5$ (c) $\dfrac{\pi(9-4\sqrt{3})}{36}+\dfrac{1}{2}\ell n\dfrac{3}{2}$ (d) $2-\dfrac{\pi}{2}$ (e) $\sqrt{2}-\dfrac{2}{\sqrt{3}}+\ell n\dfrac{2+\sqrt{3}}{1+\sqrt{2}}$

(f) $\dfrac{1}{32}\left(\pi+\dfrac{7\sqrt{3}}{2}-8\right)$ (g) $\text{arc}\tan\dfrac{1}{2}$ **3.** (a) $\dfrac{11}{6}$ (b) 2 **4.** $\dfrac{1}{3}\left(4\sqrt{2}-1\right)$

5. (a) $5-\sqrt{2}-\sqrt{3}$ (b) cos1 + cos2 + cos3 + 3 **6.** (a) $-\pi\log 2$ (b) t (c) $\dfrac{\pi}{8}\log 2$ (d) $\dfrac{\pi^2}{8}$

8. $\left(\dfrac{22}{7}-\pi\right)$ **9.** $\dfrac{\pi(a+b)}{2\sqrt{2}}$ **10.** $\dfrac{2\pi}{\sqrt{3}}$ **11.** $\dfrac{\pi}{2\sqrt{2}}-\dfrac{16\sqrt{2}}{5}$ **12.** (a) $\dfrac{\pi}{3}$ (b) $\dfrac{\pi}{8}\ell n2$

13. $\dfrac{\pi}{2}$ **14.** $\dfrac{\pi^2}{6\sqrt{3}}$ **15.** $\dfrac{1}{3}\left(\text{arc}\tan\dfrac{\sqrt{2}}{3}-\text{arc}\tan\dfrac{1}{3}\right)$ **16.** $\dfrac{8}{\pi^2}$ **17.** $-\dfrac{3\sqrt{2}}{5}\left(e^{2\pi}+1\right)$

18. 125 **21.** $b\beta - a\alpha$ **22.** 0 **23.** sec(1) – 1 **24.** (a) $f(x) = \pi/4$ (b) 60 **26.** (a) $\dfrac{2}{3}$; (b) 0

27. (a) log6; (b) $3-\ell n\ 4$ (c) $\dfrac{1}{e}$

EXERCISE - 04 [B]

BRAIN STORMING SUBJECTIVE EXERCISE

1. Prove that : $\int_a^b \frac{x^{n-1}\left((n-2)x^2+(n-1)(a+b)x+nab\right)}{(x+a)^2(x+b)^2}dx = \frac{b^{n-1}-a^{n-1}}{2(a+b)}$

2. If a_1, a_2 and a_3 are the three values of a which satisfy the equation $\int_0^{\pi/2}(\sin x + a\cos x)^3 dx - \frac{4a}{\pi-2}\int_0^{\pi/2} x\cos x\, dx = 2$ then find the value of $1000\left(a_1^2+a_2^2+a_3^2\right)$.

3. Show that the sum of the two integrals $\int_{-4}^{-5} e^{(x+5)^2}\,dx + 3\int_{1/3}^{2/3} e^{9(x-2/3)^2}\,dx$ is zero.

4. Show that $\int_0^{\infty}\frac{dx}{x^2+2x\cos\theta+1} = 2\int_0^1\frac{dx}{x^2+2x\cos\theta+1} = \begin{bmatrix} \dfrac{\theta}{\sin\theta} & \text{if } \theta\in(0,\pi) \\ \dfrac{\theta-2\pi}{\sin\theta} & \text{if } \theta\in(\pi,2\pi)\end{bmatrix}$

5. Evaluate : $\int_0^{\pi/4}\frac{x^2(\sin 2x-\cos 2x)}{(1+\sin 2x)\cos^2 x}\,dx$

6. Comment upon the nature of roots of the quadratic equation $x^2 + 2x = k + \int_0^1 |t+k|\,dt$ depending on the value of $k \in R$.

7. If the derivative of f(x) wrt x is $\frac{\cos x}{f(x)}$ then show that f(x) is a periodic function.

8. Determine a positive integer $n \le 5$, such that $\int_0^1 e^x(x-1)^n\,dx = 16 - 6e$.

9. (a) If $|x| < 1$ prove that $\frac{1-2x}{1-x+x^2}+\frac{2x-4x^3}{1-x^2+x^4}+\frac{4x^3-8x^7}{1-x^4+x^8}+\ldots\ldots\infty = \frac{1+2x}{1+x+x^2}$.

   (b) Prove the identity f(x) = $\tan x+\frac{1}{2}\tan\frac{x}{2}+\frac{1}{2^2}\tan\frac{x}{2^2}+\ldots\ldots.+\frac{1}{2^{n-1}}\tan\frac{x}{2^{n-1}} = \frac{1}{2^{n-1}}\cot\frac{x}{2^{n-1}}-2\cot 2x$

10. If $f(x) = x + \int_0^1 [xy + x\,y]\,f(y)\,dy$ where x and y are independent variables. Find f(x).

11. Given that $U_n = \{x(1-x)\}^n$ & $n \ge 2$ prove that $\frac{d^2U_n}{dx^2} = n(n-1)U_{n-2} - 2n(2n-1)U_{n-1}$,

    further if $V_n = \int_0^1 e^x \cdot U_n\,dx$, prove that when $n \ge 2$, $V_n + 2n(2n-1).V_{n-1} - n(n-1)V_{n-2} = 0$

12. Evaluate :

    (a) $\underset{n\to\infty}{\text{Limit}}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\left(1+\frac{3^2}{n^2}\right)\ldots..\left(1+\frac{n^2}{n^2}\right)\right]^{1/n}$

    (b) For potitive integers n, let $A_n = \frac{1}{n}\{(n+1)+(n+2)+\ldots\ldots.(n+n)\}$, $B_n = \{(n+1)(n+2)\ldots\ldots.(n+n)\}^{1/n}$.

    If $\underset{n\to\infty}{\text{Lim}}\frac{A_n}{B_n} = \frac{ae}{b}$ where $a, b \in N$ and relatively prime find the value of (a + b).

13. Prove that : (a) $I_{m,n} = \int_0^1 x^m . (1-x)^n\, dx = \dfrac{m!\, n!}{(m+n+1)!}$ m , n $\in$ N.

(b) $I_{m,n} = \int_0^1 x^m . (\ell n\, x)^n\, dx = (-1)^n \dfrac{n!}{(m+1)^{n+1}}$ m , n $\in$ N.

14. Prove that the sum to (n +1) terms of $\dfrac{C_0}{n(n+1)} - \dfrac{C_1}{(n+1)(n+2)} + \dfrac{C_2}{(n+2)(n+3)} - \ldots\ldots$ equals $\int_0^1 x^{n-1}.(1-x)^{n+1}\, dx$

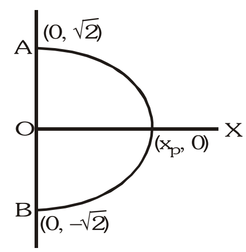
& evaluate the integral.

15. Evaluate the integral : $\int_3^5 \left(\sqrt{x+2\sqrt{2x-4}} + \sqrt{x-2\sqrt{2x-4}}\right)dx$

16. Prove that $\int_0^x \left(\int_0^u f(t)dt\right) du = \int_0^x f(u).(x-u)du$

17. Evaluate $\int_0^1 (tx+1-x)^n\, dx$, where n is a positive integer and t is a parameter independent of x. Hence show that $\int_0^1 x^k (1-x)^{n-k}\, dx = \dfrac{1}{[{}^nC_k(n+1)]}$ for k = 0, 1, ... n. [JEE 1981]

18. If 'f' is a continuous function with $\int_0^x f(t)\, dt \to \infty$ as $|x| \to \infty$, then show that every line y = mx intersects the curve $y^2 + \int_0^x f(t)\, dt = 2$! [JEE 1991]



BRAIN STORMING SUBJECTIVE EXERCISE	ANSWER KEY	EXERCISE-4(B)

2. 5250 **5.** $\dfrac{\pi^2}{16} - \dfrac{\pi}{4}\ell n2$ **6.** real & distinct $\forall$ k $\in$ R **8.** n = 3

10. $f(x) = x + \dfrac{61}{119}x + \dfrac{80}{119}x$ **12.** (a) $2\, e^{(1/2)(\pi-4)}$ (b) 11 **14.** $\dfrac{(n-1)!(n+1)!}{(2n+1)!}$

15. $2\sqrt{2} + \dfrac{4}{3}\left(3\sqrt{3} - 2\sqrt{2}\right)$ **17.** $\dfrac{t^{n+1}-1}{(t-1)(n+1)}$

EXERCISE - 05 [A]

JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

1. If $I_n = \int_0^{\pi/4} \tan^n x\, dx$ then the value of $n(I_{n-1} + I_{n+1})$ is- [AIEEE-2002]

(1) 1 (2) $\pi/2$ (3) $\pi/4$ (4) n

2. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} =$ [AIEEE-2002]

(1) π^2 (2) $\pi^2/4$ (3) $\pi/8$ (4) $\pi^2/8$

3. $\int_{\pi}^{10\pi} |\sin x|\, dx =$ [AIEEE-2002]

(1) 9 (2) 10 (3) 18 (4) 20

4. $\int_0^{\sqrt{2}} [x^2]\, dx$ is equal to (where [.] denotes greatest integer function) [AIEEE-2002]

(1) $\sqrt{2} - 1$ (2) $2(\sqrt{2} - 1)$ (3) $\sqrt{2}$ (4) none of these

5. $\lim_{n\to\infty} \frac{1^P + 2^P + 3^P + \dots + n^P}{n^{P+1}}$ equals - [AIEEE-2002]

(1) 1 (2) $\frac{1}{P+1}$ (3) $\frac{1}{P+2}$ (4) P^2

6. Let $\frac{d}{dx}F(x) = \left(\frac{e^{\sin x}}{x}\right)$, x > 0. If $\int_1^4 \frac{3}{x} e^{\sin x^3}\, dx = F(k) - F(1)$, then one of the possible values of k, is- [AIEEE-2003]

(1) 64 (2) 15 (3) 16 (4) 63

7. If $f(a + b - x) = f(x)$, then $\int_a^b x f(x)\, dx$ is equal to- [AIEEE-2003]

(1) $\frac{a+b}{2}\int_a^b f(a+b-x)\, dx$ (2) $\frac{a+b}{2}\int_a^b f(b-x)\, dx$ (3) $\frac{a+b}{2}\int_a^b f(x)\, dx$ (4) $\frac{b-a}{2}\int_a^b f(x)\, dx$

8. The value of the integral $I = \int_0^1 x(1-x)^n\, dx$ is- [AIEEE-2003]

(1) $\frac{1}{n+1} + \frac{1}{n+2}$ (2) $\frac{1}{n+1}$ (3) $\frac{1}{n+2}$ (4) $\frac{1}{n+1} - \frac{1}{n+2}$

9. The value of $\lim_{x\to 0} \frac{\int_0^{x^2} \sec^2 t\, dt}{x \sin x}$ is - [AIEEE-2003]

(1) 0 (2) 3 (3) 2 (4) 1

10. $\lim_{n\to\infty} \frac{(1)^4 + 2^4 + 3^4 + \dots + n^4}{n^5} - \lim_{n\to\infty} \frac{(1)^3 + 2^3 + 3^3 + \dots + n^3}{n^5}$ is equal to - [AIEEE-2003]

(1) 1/5 (2) 1/30 (3) zero (4) 1/4

11. If $f(y) = e^y$, $g(y) = y$; $y > 0$ and $F(t) = \int_0^t f(t-y)\, g(y)\, dy$, then- [AIEEE-2003]

(1) $F(t) = te^{-t}$ (2) $F(t) = 1 - e^{-1}(1 + t)$
(3) $F(t) = e^t - (1 + t)$ (4) $F(t) = te^t$

12. Let f(x) be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and g(x) be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)\, g(x)\, dx$ is - [AIEEE-2003]

(1) $e + \frac{e^2}{2} + \frac{5}{2}$ (2) $e - \frac{e^2}{2} - \frac{5}{2}$ (3) $e + \frac{e^2}{2} - \frac{3}{2}$ (4) $e - \frac{e^2}{2} - \frac{3}{2}$

13. $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} e^{r/n}$ is- [AIEEE-2004]

(1) e (2) e – 1 (3) 1 – e (4) e + 1

14. The value of $\int_{-2}^{3} |1-x^2|\, dx$ is- [AIEEE-2004]

(1) 28/3 (2) 14/3 (3) 7/3 (4) 1/3

15. The value of $I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}}\, dx$ is- [AIEEE-2004]

(1) 0 (2) 1 (3) 2 (4) 3

16. If $\int_0^{\pi} x\, f(\sin x)\, dx = A \int_0^{\pi/2} f(\sin x)\, dx$, then A is - [AIEEE-2004]

(1) 0 (2) π (3) $\pi/4$ (4) 2π

17. If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} x g\{x(1-x)\}dx$ and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$, then the value of $\frac{I_2}{I_1}$ is- [AIEEE-2004]

(1) 2 (2) –3 (3) –1 (4) 1

18. $\lim_{n\to\infty} \left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + \ldots + \frac{1}{n}\sec^2 1\right]$ equals- [AIEEE-2005]

(1) $\frac{1}{2}\sec 1$ (2) $\frac{1}{2}\operatorname{cosec} 1$ (3) $\tan 1$ (4) $\frac{1}{2}\tan 1$

19. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$ then- [AIEEE-2005]

(1) $I_2 > I_1$ (2) $I_1 > I_2$ (3) $I_3 = I_4$ (4) $I_3 > I_4$

20. Let $f : R \to R$ be a differentiable function having $f(2) = 6$, $f'(2) = \left(\frac{1}{48}\right)$. Then $\lim_{x\to 2} \int_6^{f(x)} \frac{4t^3}{x-2}\, dt$ equals - [AIEEE-2005]

(1) 24 (2) 36 (3) 12 (4) 18

21. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}\, dx$, $a > 0$ is- [AIEEE-2005]

(1) $a\pi$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{a}$ (4) 2π

22. The value of the integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}\,dx$ is - [AIEEE-2006]

(1) $\frac{3}{2}$ (2) 2 (3) 1 (4) $\frac{1}{2}$

23. $\int_{-3\pi/2}^{-\pi/2} [(x+\pi)^3 + \cos^2(x+3\pi)]\,dx$ is equal to- [AIEEE-2006]

(1) $(\pi^4/32) + (\pi/2)$ (2) $\pi/2$ (3) $(\pi/4) - 1$ (4) $\pi^4/32$

24. $\int_0^{\pi} x\, f(\sin x)\,dx$ is equal to- [AIEEE-2006]

(1) $\pi\int_0^{\pi} f(\sin x)\,dx$ (2) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$ (3) $\pi\int_0^{\pi/2} f(\cos x)\,dx$ (4) $\pi\int_0^{\pi} f(\cos x)dx$

25. The value of $\int_1^a [x]f'(x)\,dx$, a > 1, where [x] denotes the greatest integer not exceeding x is- [AIEEE-2006]

(1) [a] f(a) − {f(1) + f(2) + ... + f([a])} (2) [a] f([a]) − {f(1) + f(2) + ... + f(a)}

(3) a f([a]) − {f(1) + f(2) + ... + f(a)} (4) a f(a) − {f(1) + f(2) + ... + f([a])}

26. Let F(x) = f(x) + $f\left(\frac{1}{x}\right)$, where f(x) = $\int_1^x \frac{\log t}{1+t}\,dt$. Then F(e) equals- [AIEEE-2007]

(1) $\frac{1}{2}$ (2) 0 (3) 1 (4) 2

27. The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{12}$ is- [AIEEE-2007]

(1) 2 (2) π (3) $\sqrt{3}/2$ (4) $2\sqrt{2}$

28. Let I = $\int_0^1 \frac{\sin x}{\sqrt{x}}\,dx$ and J = $\int_0^1 \frac{\cos x}{\sqrt{x}}\,dx$. Then which one of the following is true ? [AIEEE-2008]

(1) I > $\frac{2}{3}$ and J > 2 (2) I < $\frac{2}{3}$ and J < 2 (3) I < $\frac{2}{3}$ and J > 2 (4) I > $\frac{2}{3}$ and J < 2

29. $\int_0^{\pi} [\cot x]\,dx$, where [.] denotes the greatest integer function, is equal to - [AIEEE-2009]

(1) −1 (2) $-\frac{\pi}{2}$ (3) $\frac{\pi}{2}$ (4) 1

30. Let p(x) be a function defined on R such that p'(x) = p'(1 − x), for all x ∈ [0, 1], p(0) = 1 and p(1) = 41. Then $\int_0^1$ p(x) dx equals :- [AIEEE-2010]

(1) $\sqrt{41}$ (2) 21 (3) 41 (4) 42

31. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}\,dx$ is :- [AIEEE-2011]

(1) $\frac{\pi}{2}\log 2$ (2) log 2 (3) π log 2 (4) $\frac{\pi}{8}\log 2$

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32. Let [.] denote the greatest integet function then the value of $\int_0^{1.5} x[x^2]dx$ is :- [AIEEE-2011]

(1) $\frac{5}{4}$ (2) 0 (3) $\frac{3}{2}$ (4) $\frac{3}{4}$

33. If $g(x) = \int_0^x \cos 4t \, dt$, then $g(x + \pi)$ equals : [AIEEE-2012]

(1) $g(x) \cdot g(\pi)$ (2) $\frac{g(x)}{g(\pi)}$ (3) $g(x) + g(\pi)$ (4) $g(x) - g(\pi)$

34. **Statement-I :** The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$. [JEE-MAIN-2013]

Statement-II : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.

(1) Statement-I is true, Statement-II is true; Statement-II is a **correct** explanation for Statement-I.

(2) Statement-I is true, Statement-II is true; Statement-II is **not** a correct explanation for Statement-I.

(3) Statement-I is true, Statement-II is false.

(4) Statement-I is false, Statement-II is true.

PREVIOUS YEARS QUESTIONS — **ANSWER KEY** — EXERCISE-5 [A]

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans	1	1	3	1	2	1	3	4	4	1	3	4	2	1	3
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans	2	1	4	2	4	2	1	2	3	1	1	1	2	2	2
Que.	31	32	33	34											
Ans	3	4	3,4	4											

EXERCISE - 05 [B]

JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS

1. (a) The value of the integral $\int_{e^{-1}}^{e^2}\left|\frac{\log_e x}{x}\right| dx$ is -

(A) 3/2 (B) 5/2 (C) 3 (D) 5

(b) Let $g(x) = \int_0^x f(t)\, dt$, where f is such that $\frac{1}{2} \le f(t) \le 1$ for $t \in (0, 1]$ and $0 \le f(t) \le \frac{1}{2}$ for $t \in (1, 2]$. Then g(2) satisfies the inequality -

(A) $-\frac{3}{2} \le g(2) < \frac{1}{2}$ (B) $0 \le g(2) < 2$ (C) $\frac{3}{2} < g(2) \le \frac{5}{2}$ (D) $2 < g(2) < 4$

(c) If $f(x) = \begin{cases} e^{\cos x} . \sin x & \text{for } |x| \le 2 \\ 2 & \text{otherwise} \end{cases}$. Then $\int_{-2}^{3} f(x)dx$ -

(A) 0 (B) 1 (C) 2 (D) 3

[JEE 2000, Screening, 1+1+1+1M out of 35]

(d) For x > 0, let $f(x) = \int_1^x \frac{\ell nt}{1+t}dt$. Find the function f(x) + f(1/x) and show that, f(e) + f(1/e) = 1/2.

[JEE 2000, (Mains) 5M out of 100]

2. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}dx, a > 0$ is - [JEE 2001]

(A) π (B) $a\pi$ (C) $\frac{\pi}{2}$ (D) 2π

3. Let $f : (0, \infty) \to R$ and $F(x) = \int_0^x f(t)dt$. If $F(x^2) = x^2 (1 + x)$, then f(4) equals - [JEE 2001]

(A) $\frac{5}{4}$ (B) 7 (C) 4 (D) 2

4. (a) Let $f(x) = \int_1^x \sqrt{2-t^2}\, dt$. Then the real roots of the equation $x^2 - f'(x) = 0$ are -

(A) ± 1 (B) $\pm\frac{1}{\sqrt{2}}$ (C) $\pm\frac{1}{2}$ (D) 0 and 1

(b) Let T > 0 be a fixed real number. Suppose f is a continuous function such that for all $x \in R$ f (x + T) = f(x). If $I = \int_0^T f(x)\, dx$ then the value of $\int_3^{3+3T} f(2x)\, dx$ is -

(A) $\frac{3}{2}I$ (B) 2I (C) 3I (D) 6I

(c) The integral $\int_{-\frac{1}{2}}^{\frac{1}{2}}\left([x] + \ell n\left(\frac{1+x}{1-x}\right)\right) dx$ equals -

(A) $-\frac{1}{2}$ (B) 0 (C) 1 (D) $2\ell n\left(\frac{1}{2}\right)$

[JEE 2002 (Screening) 3+3+3M]

5. (a) If $\ell(m, n) = \int_0^1 t^m (1+t)^n dt$, then the expression for $\ell(m, n)$ in terms of $\ell(m+1, n-1)$ is -

(A) $\frac{m}{n+1}\ell(m+1, n-1)$ (B) $\frac{n}{m+1}\ell(m+1, n-1)$

(C) $\frac{2^n}{m+1}+\frac{n}{m+1}\ell(m+1, n-1)$ (D) $\frac{2^n}{m+1}-\frac{n}{m+1}\ell(m+1, n-1)$

(b) If function f defined by $f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$ increases in the interval -

(A) nowhere (B) $x \le 0$ (C) $x \in [-2, 2]$ (D) $x \ge 0$

[JEE 2003 (Screening) 3+3M]

6. If f(x) is an even function, then prove that $\int_0^{\pi/2} f(\cos 2x)\cos x\, dx = \sqrt{2}\int_0^{\pi/4} f(\sin 2x)\cos x\, dx$

[JEE 2003 (Mains) 2M out of 60]

7. (a) The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x}}\, dx$ is - [JEE 2004]

(A) $\frac{\pi}{2}+1$ (B) $\frac{\pi}{2}-1$ (C) -1 (D) 1

(b) If f(x) is differentiable and $\int_0^{t^2} x\, f(x)\, dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals -

(A) $\frac{2}{5}$ (B) $-\frac{5}{2}$ (C) 1 (D) $\frac{5}{2}$

[JEE 2004 (Screening)]

(c) If $y(x) = \int_{\pi^2/16}^{x^2} \frac{\cos x.\ \cos\sqrt{\theta}}{1+\sin^2\ \sqrt{\theta}}\, d\theta$, then find $\frac{dy}{dx}$ at $x = \pi$.

(d) Evaluate : $\int_{-\pi/3}^{\pi/3} \frac{\pi + 4x^3}{2-\cos\left(|x| + \frac{\pi}{3}\right)} dx$ [JEE 2004 (Mains) 2+4M out of 60]

8. (a) If $\int_{\sin x}^{1} t^2 (f(t))\, dt = (1-\sin x)$ then $f\left(\frac{1}{\sqrt{3}}\right)$ is - [JEE 2005 (Screening) 3+3M]

(A) 1/3 (B) $1/\sqrt{3}$ (C) 3 (D) $\sqrt{3}$

(b) $\int_{-2}^{0} (x^3+3x^2+3x+3+(x+1)\cos(x+1))\, dx$ is equal to -

(A) – 4 (B) 0 (C) 4 (D) 6

9. Evaluate $\int_0^{\pi} e^{|\cos x|}\left(2\sin\left(\frac{1}{2}\cos x\right)+3\cos\left(\frac{1}{2}\cos x\right)\right)\sin x\, dx$ [JEE 2005, (Mains), 2M out of 60]

10 to 12 are based on the following Comprehension

Suppose we define the definite integral using the following formula $\int_a^b f(x)\,dx = \frac{b-a}{2}(f(a)+f(b))$, for more accurate result for $c \in (a, b)$ $F(c) = \frac{c-a}{2}(f(a)+f(c)) + \frac{b-c}{2}(f(b)+f(c))$.

When $c = \frac{a+b}{2}$, $\int_a^b f(x)\,dx = \frac{b-a}{4}(f(a)+f(b)+2f(c))$

10. $\int_0^{\pi/2} \sin x\,dx$ is equal to -

(A) $\frac{\pi}{8}(1+\sqrt{2})$ (B) $\frac{\pi}{4}(1+\sqrt{2})$ (C) $\frac{\pi}{8\sqrt{2}}$ (D) $\frac{\pi}{4\sqrt{2}}$

[JEE 2006, 5M out of 184]

11. If $f''(x) < 0, \forall\ x \in (a, b)$ and c is a point such that a < c < b and (c, f(c)) is the point lying on the curve for which F(c) is maximum then f'(c) is equal to -

(A) $\frac{f(b)-f(a)}{b-a}$ (B) $\frac{2(f(b)-f(a))}{b-a}$ (C) $\frac{2(f(b)-f(a))}{2b-a}$ (D) 0

[JEE 2006, 5M out of 184]

12. If f(x) is a polynomial and if $\lim\limits_{t\to a} \frac{\int_a^t f(x)\,dx - \left(\frac{t-a}{2}\right)(f(t)+f(a))}{(t-a)^3} = 0$ for all a, then the degree of f(x) can atmost be -

(A) 1 (B) 2 (C) 3 (D) 4

[JEE 2006, 5M out of 184]

13. The value of $\frac{5050\int_0^1 (1-x^{50})^{100}\,dx}{\int_0^1 (1-x^{50})^{101}\,dx}$ is. [JEE 2006, 6M]

14. Match the following : [JEE 2006, (1.5, +1.5)M out of 184]

	Column-I		Column-II
(A)	$\int_0^{\pi/2} (\sin x)^{\cos x} \{\cos x \cot x - \sin x \,.\, \ln(\sin x)\}\,dx$	(p)	$4/3$
(B)	$\left\vert\int_0^1 (1-y^2)\,dy\right\vert + \left\vert\int_1^0 (y^2-1)\,dy\right\vert$	(q)	1
		(r)	$\left\vert\int_0^1 \sqrt{1-x}\,dx\right\vert + \left\vert\int_{-1}^0 \sqrt{1+x}\,dx\right\vert$

15. $\lim\limits_{x\to\frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)\,dt}{x^2 - \frac{\pi^2}{16}}$ equals - [JEE 2007]

(A) $\frac{8}{\pi}f(2)$ (B) $\frac{2}{\pi}f(2)$ (C) $\frac{2}{\pi}f\left(\frac{1}{2}\right)$ (D) 4f(2)

16. Match the integrals in Column-I with the values in Column-II [JEE 2007, 6M]

Column-I		Column-II	
(A)	$\int_{-1}^{1}\frac{dx}{1+x^2}$	(p)	$\frac{1}{2}\log\left(\frac{2}{3}\right)$
(B)	$\int_{0}^{1}\frac{dx}{\sqrt{1-x^2}}$	(q)	$2\log\left(\frac{2}{3}\right)$
(C)	$\int_{2}^{3}\frac{dx}{1-x^2}$	(r)	$\frac{\pi}{3}$
(D)	$\int_{1}^{2}\frac{dx}{x\sqrt{x^2-1}}$	(s)	$\frac{\pi}{2}$

17. Let $S_n = \sum_{k=1}^{n}\frac{n}{n^2+kn+k^2}$ and $T_n = \sum_{k=0}^{n-1}\frac{n}{n^2+kn+k^2}$ [JEE 2008, 4M]

for n = 1, 2, 3, ... Then,

(A) $S_n < \frac{\pi}{3\sqrt{3}}$ (B) $S_n > \frac{\pi}{3\sqrt{3}}$ (C) $T_n < \frac{\pi}{3\sqrt{3}}$ (D) $T_n > \frac{\pi}{3\sqrt{3}}$

18. Let f be a non-negative function defined on the interval [0, 1]. If $\int_0^x\sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)dt$, $0 \le x \le 1$, and $f(0) = 0$, then - [JEE 2009, 3M, –1M]

(A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$ (B) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(C) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$ (D) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

19. If $I_n = \int_{-\pi}^{\pi}\frac{\sin nx}{(1+\pi^x)\sin x}dx$, n = 0, 1, 2, ..., then - [JEE 2009, 4M, –1M]

(A) $I_n = I_{n+2}$ (B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$ (C) $\sum_{m=1}^{10} I_{2m} = 0$ (D) $I_n = I_{n+1}$

20. Let f: R $\to$ R be a continuous function which satisfies $f(x) = \int_0^x f(t)dt$ .Then the value of f(ln 5) is............... [JEE 2009, 4M, –1M]

21. The value of $\lim_{x\to 0}\frac{1}{x^3}\int_0^x\frac{t\ell n(1+t)}{t^4+4}dt$ is [JEE 10, 3M, –1M]

(A) 0 (B) $\frac{1}{12}$ (C) $\frac{1}{24}$ (D) $\frac{1}{64}$

22. The value(s) of $\int_0^1\frac{x^4(1-x)^4}{1+x^2}dx$ is (are) [JEE 10, 3M]

(A) $\frac{22}{7}-\pi$ (B) $\frac{2}{105}$ (C) 0 (D) $\frac{71}{15}-\frac{3\pi}{2}$

23. For any real number x, let [x] denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval [–10, 10] by

$$f(x)=\begin{cases} x-[x] & \text{if } [x] \text{ is odd,} \\ 1+[x]-x & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of $\dfrac{\pi^2}{10}\displaystyle\int_{-10}^{10} f(x)\cos\pi x\,dx$ is **[JEE 10, 3M]**

24. Let f be a real-valued function defined on the interval (–1,1) such that $e^{-x}f(x)=2+\displaystyle\int_0^x \sqrt{t^4+1}\,dt$, for all x ∈ (1,1), and let f^{-1} be the inverse function of f. Then $(f^{-1})'$ (2) is equal to - **[JEE 10, 5M, –2M]**

(A) 1 (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{1}{e}$

25. The value of $\displaystyle\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x\sin x^2}{\sin x^2+\sin(\ln 6-x^2)}dx$ is **[JEE 2011, 3 (–1)M]**

(A) $\dfrac{1}{4}\ell n\dfrac{3}{2}$ (B) $\dfrac{1}{2}\ell n\dfrac{3}{2}$ (C) $\ell n\dfrac{3}{2}$ (D) $\dfrac{1}{6}\ell n\dfrac{3}{2}$

26. Let S be the area of the region enclosed by $y=e^{-x^2}$, y = 0, x = 0, and x = 1. Then - **[JEE 2012, 4M]**

(A) $S\ge\dfrac{1}{e}$ (B) $S\ge 1-\dfrac{1}{e}$

(C) $S\le\dfrac{1}{4}\left(1+\dfrac{1}{\sqrt{e}}\right)$ (D) $S\le\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{e}}\left(1-\dfrac{1}{\sqrt{2}}\right)$

27. The value of the integral $\displaystyle\int_{-\pi/2}^{\pi/2}\left(x^2+\ln\frac{\pi+x}{\pi-x}\right)\cos x\,dx$ is **[JEE 2012, 3M, –1M]**

(A) 0 (B) $\dfrac{\pi^2}{2}-4$ (C) $\dfrac{\pi^2}{2}+4$ (D) $\dfrac{\pi^2}{2}$

28. For a ∈ R (the set of all real numbers), a ≠ –1.

$$\lim_{n\to\infty}\frac{(1^a+2^a+\ldots\ldots+n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+\ldots\ldots+(na+n)]}=\frac{1}{60}$$

Then a = **[JEE(Advanced) 2013, 3, (–1)M]**

(A) 5 (B) 7 (C) $\dfrac{-15}{2}$ (D) $\dfrac{-17}{2}$

PREVIOUS YEARS QUESTIONS	ANSWER KEY	EXERCISE-5 [B]

1. (a) B (b) B (c) C (d) $\dfrac{1}{2}\ell n^2 x$ **2.** C **3.** C **4.** (a) A; (b) C; (c) B **5.** (a) D (b) B

7. (a) B (b) A (c) 2π (d) $\dfrac{4\pi}{\sqrt{3}}\tan^{-1}\dfrac{1}{2}$ **8.** (a) C (b) C **9.** $\dfrac{24}{5}\left(e\cos\left(\dfrac{1}{2}\right)+\dfrac{e}{2}\sin\left(\dfrac{1}{2}\right)-1\right)$

10. A **11.** A **12.** A **13.** 5051 **14.** (A)→(q), (B)→(p, r) **15.** A

16. (A)→(s); (B)→(s); (C)→(p); (D)→(r) **17.** A, D **18.** C **19.** A,B,C **20.** 0

21. B **22.** A **23.** 4 **24.** B **25.** A **26.** A,B,D **27.** B **28.** B