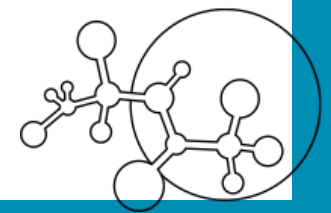




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A 99 Line topology optimization code written by Ole Sigmund in Matlab

Feng Jia

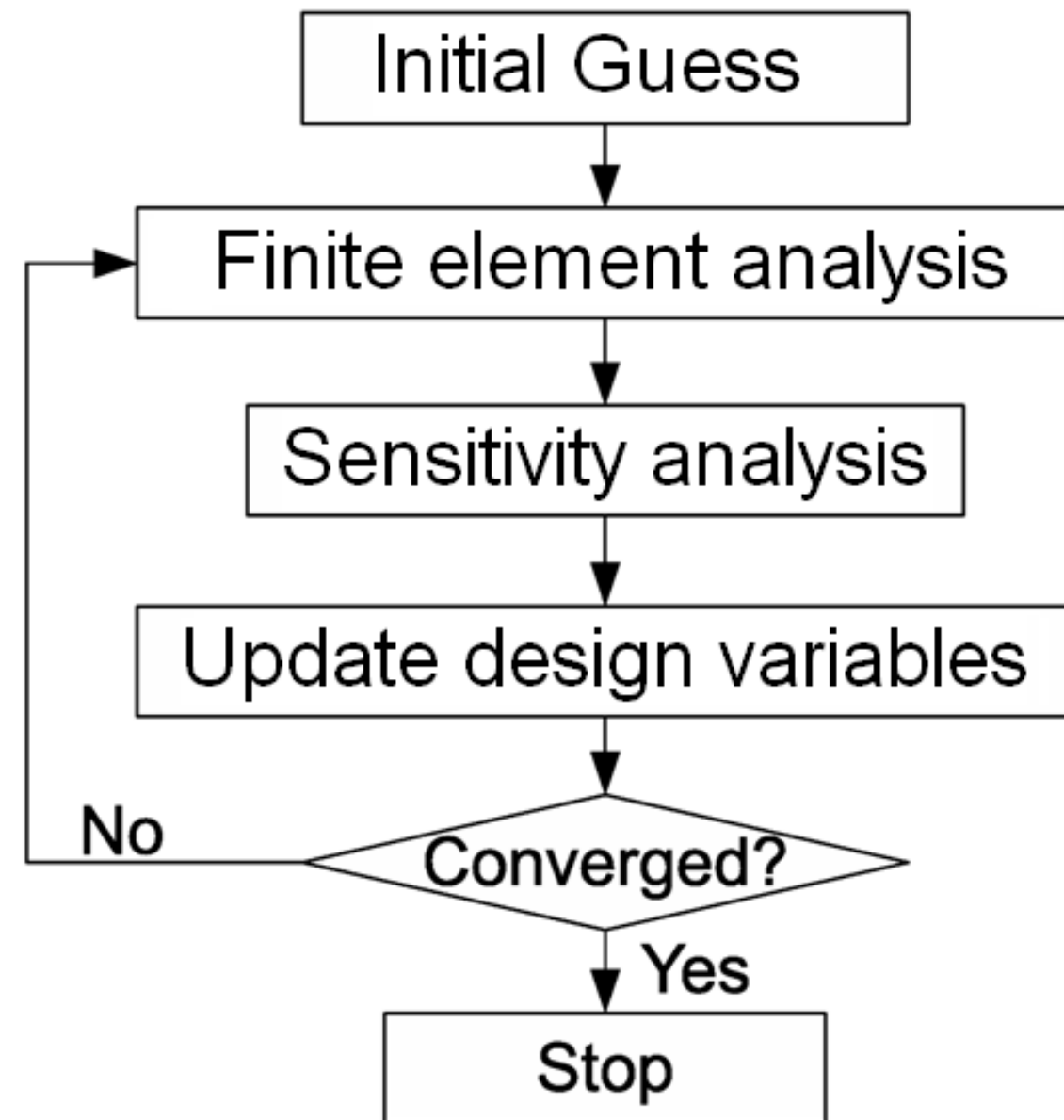
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Outline

- Three components of an optimization problem
- What is topology optimization?
- Which problem is the code used to solve?
- Optimal condition
- MATLAB implementation
- Numerical examples and Numerical instability
- Mathematical programming methods
- An efficient 88-line code

Three components of an optimization problem

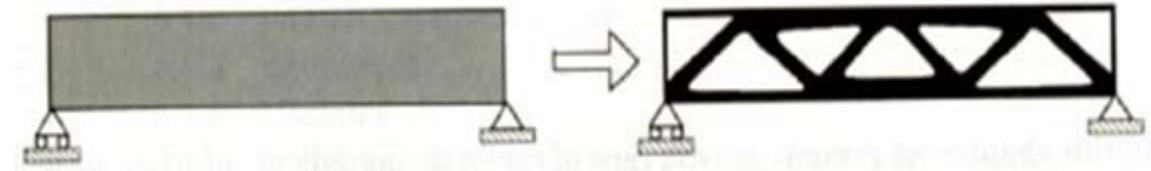
Objective function
Design variable
constraints



What is topology optimization?

Purpose:

To find the optimal lay-out of a structure within a specified region.



Known Quantity:

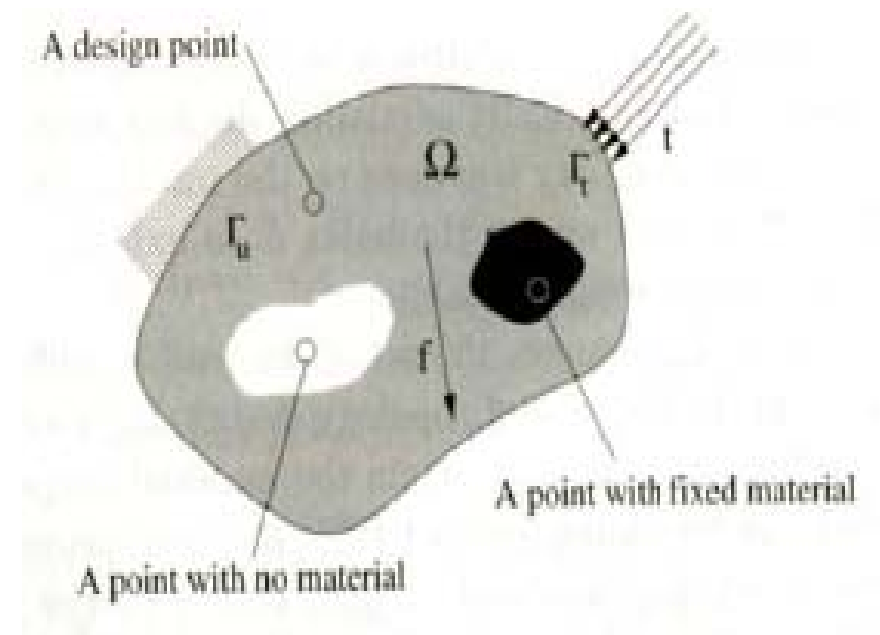
Applied loads

Possible support conditions

Volume constraint of the structure

Additional design restrictions

Location and size of prescribed holes or solid areas.



Advantage:

The physical size and the shape and connectivity of the structure do not necessary provided in advance.

Which problem is the code used to solve? -

- Minimum Compliance Design

energy bilinear form:

$$a(u, v) = \int_{\Omega} E_{ijkl}(x) \varepsilon_{ij}(u) \varepsilon_{kl}(v) d\Omega$$

Linearized strain:

$$\varepsilon_{ij}(u) = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

Load linear form:

$$l(u) = \int_{\Omega} f u d\Omega + \int_{\Gamma_T} t u ds$$

Minimum compliance design:

$$\begin{aligned} & \min_{u, E_e} l(u) \\ & s.t.: \quad a_E(u, v) = l(v), \quad v \in U, E \in E_{ad} \end{aligned}$$

State variable and design variable: u, E

Discretized form of compliance design:

$$\begin{aligned} & \min_{u, E_e} \frac{1}{2} u^T K u \\ & s.t.: \quad K(E_e) u = f, \quad E_e \in E_{ad} \end{aligned}$$

Minimum Compliant Design – Design variable

An optimal distribution of a given isotropic material in space

$$E_{ijkl} = l_{\Omega^{mat}} E_{ijkl}^0, \quad l_{\Omega^{mat}} = \begin{cases} 1 & \text{if } x \in \Omega^{mat} & \text{Material point} \\ 0 & \text{if } x \in \Omega^{void} & \text{Void (no material)} \end{cases}$$

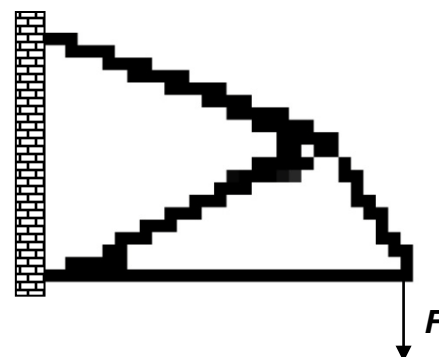
$$\int_{\Omega} l_{\Omega^{mat}} d\Omega = Vol(\Omega^{mat}) \leq V_0$$

This is integral programming problem.

Nonsmoothness of the objective functions and design variable cannot use the gradient information.

Many times objective function evaluation is computationally prohibitive.

Only works for a small scale problem with simulated annealing / genetic algorithms.



Black-White raster representation of the geometry
with *Pixels* when using finite element discretization

Minimum Compliant Design – Design variable

Replace integral variable with continuous variables

$$E_{ijkl}(x) = \rho(x)^p E_{ijkl}^0, \quad p > 1, \quad \rho \rightarrow \text{Design variable}$$

$$\int_{\Omega} \rho(x) d\Omega \leq V; \quad 0 \leq \rho(x) \leq 1, \quad x \in \Omega$$

Penalty method to steers the final solution to discrete integral variable

$$E_{ijkl}(x) = E_{ijkl}^0 \leftarrow \rho = 1$$

$$E_{ijkl}(x) = E_{ijkl}^{\min} \leftarrow \rho < 1$$

Implementation of Penalty: SIMP Method

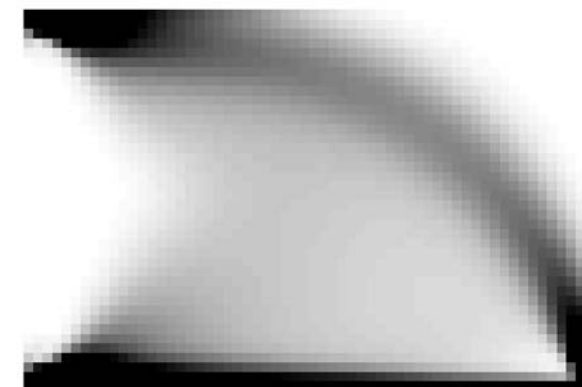
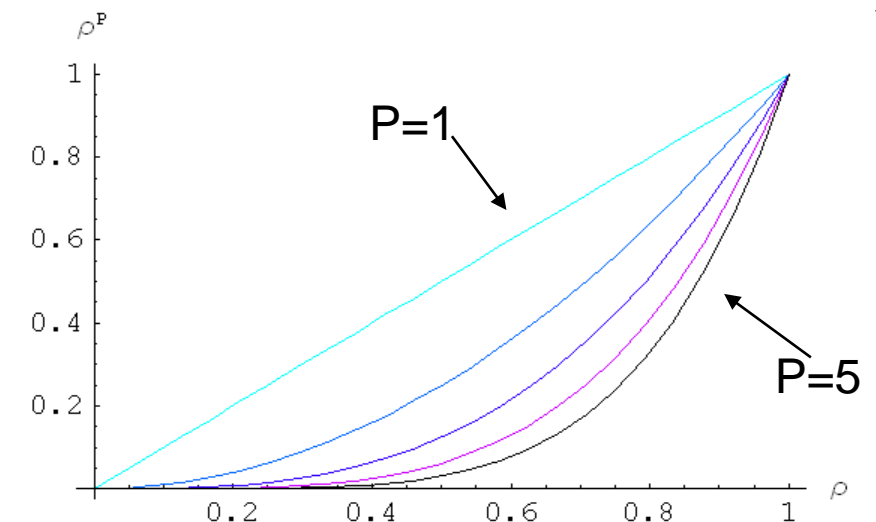
(Solid Isotropic Material with Penalization)

Intermediate density value are unfavourable

Penalization is achieved without using any explicit penalization term.

$$p \geq \max \left\{ \frac{2}{1-\nu^0}, \frac{4}{1+\nu^0} \right\} \quad (\text{in } 2-D),$$

$$p \geq \max \left\{ 15 \frac{1-\nu^0}{7-5\nu^0}, \frac{3}{2} \frac{1-\nu^0}{1-2\nu^0} \right\} \quad (\text{in } 3-D),$$



Optimal condition

For structural optimization with continuous design variable

$$\begin{aligned} & \underset{u, \rho}{Min} \quad l(u) \\ & s.t. \quad a_E(u, v) = l(v) \quad \text{for all } v \in U, \\ & \quad E_{ijkl}(x) = \rho(x)^p E_{ijkl}^0, \\ & \quad \int_{\Omega} \rho(x) d\Omega \leq V_0; \quad 0 < \rho_{\min} \leq \rho \leq 1 \end{aligned}$$

Necessary condition for SIMP Method.

$\rho_{\min} = 10^{-3} \sim 10^{-6}$ Low bound of density in order to prevent possible singularity of the equilibrium equation.

With Lagrange multiplier Λ for area constraint and λ for continuous design variable constraint

$$\begin{aligned} L = & l(u) - \{a_E(u, \bar{u}) - l(\bar{u})\} + \Lambda(\int_{\Omega} \rho(x) d\Omega - V_0) \\ & + \int_{\Omega} \lambda^+(x)(\rho(x) - 1) d\Omega + \int_{\Omega} \lambda^-(x)(\rho_{\min} - \rho(x)) d\Omega \end{aligned}$$

Optimal condition(2)

The optimal condition for continuous design variable ρ is

$$\frac{\partial E_{ijkl}}{\partial \rho} \varepsilon_{ij}(u) \varepsilon_{kl}(u) = \Lambda + \lambda^+ - \lambda^-$$

With the switch condition of the Lagrange multiplier

$$\lambda^- \geq 0, \quad \lambda^+ \geq 0, \quad \lambda^-(\rho_{\min} - \rho(x)) = 0, \quad \lambda^+(\rho(x) - 1) = 0$$

For all intermediate densities $\rho_{\min} < \rho < 1$

$$p\rho(x)^{p-1} E_{ijkl}^0 \varepsilon_{ij}(u) \varepsilon_{kl}(u) = \Lambda$$

Update scheme for the continuous design variable – Optimality criteria method

$$\rho_{K+1} = \begin{cases} \max\{(1-\zeta)\rho_K, \rho_{\min}\} & \text{if } \rho_K B_K^\eta \leq \max\{(1-\zeta)\rho_K, \rho_{\min}\}, \\ \min\{(1+\zeta)\rho_K, 1\} & \text{if } \min\{(1+\zeta)\rho_K, 1\} \leq \rho_K B_K^\eta, \\ \rho_K B_K^\eta & \text{otherwise.} \end{cases}$$

$$B_K = \Lambda_K^{-1} p\rho(x)^{p-1} E_{ijkl}^0 \varepsilon_{ij}(u_K) \varepsilon_{kl}(u_K)$$

$\zeta, \eta \rightarrow$ Control parameter, A typical value is 0.2 and 0.5

Local optimal $\rightarrow B_K=1$

MATLAB implementation

Optimization model

$$\left. \begin{array}{l} \min_{\mathbf{x}} : c(\mathbf{x}) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^N (x_e)^p \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e \\ \text{subject to: } \frac{V(\mathbf{x})}{V_0} \leq 1 \\ : \mathbf{K} \mathbf{U} = \mathbf{F} \\ : \mathbf{0} < \mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{1} \end{array} \right\}$$

Sensitivity

$$\frac{\partial c}{\partial x_e} = -p(x_e)^{p-1} \mathbf{u}_e^T \mathbf{k}_0 \mathbf{u}_e$$

Update rule

- OPTIMALITY CRITERIA METHOD

$$\left\{ \begin{array}{l} \max(x_{\min}, x_e - m) \\ \text{if } x_e B_e^\eta \leq \max(x_{\min}, x_e - m), \\ x_e B_e^\eta \\ \text{if } \max(x_{\min}, x_e - m) < x_e B_e^\eta < \min(1, x_e + m) \\ \min(1, x_e + m) \\ \text{if } \min(1, x_e + m) \leq x_e B_e^\eta, \end{array} \right.$$

$$B_e = \frac{-\frac{\partial c}{\partial x_e}}{\lambda \frac{\partial V}{\partial x_e}}$$

Matlab Code – Main Code

```
x(1:nely,1:nelx) = volfrac;
loop = 0; change = 1.;
while change > 0.01
    loop = loop + 1;
    xold = x;
    [U]=FE(nelx,nely,x,penal);
    [KE] = lk;
    c = 0.;

    for ely = 1:nely
        for elx = 1:nelx
            n1 = (nely+1)*(elx-1)+ely;
            n2 = (nely+1)* elx +ely;
            Ue = U([2*n1-1;2*n1; 2*n2-1;2*n2; 2*n2+1;2*n2+2; 2*n1+1;2*n1+2],1);
            c = c + x(ely,elx)^penal*Ue'*KE*Ue;
            dc(ely,elx) = -penal*x(ely,elx)^(penal-1)*Ue'*KE*Ue;
        end
    end

    [dc] = check(nelx,nely,rmin,x,dc);
    [x] = OC(nelx,nely,x,volfrac,dc);
    change = max(max(abs(x-xold)));
end
```

% INITIALIZE

% START ITERATION

% FE-ANALYSIS

% OBJECTIVE FUNCTION

% SENSITIVITY ANALYSIS

% FILTERING OF SENSITIVITIES

% OPTIMALITY CRITERIA METHOD

Matlab Code – Element Stiffness Matrix

Element Stiffness Matrix

```
function [KE]=lk
```

```
E = 1.;
```

```
nu =1/3.;
```

```
k=[ 1/2-nu/6    1/8+nu/8  -1/4-nu/12 -1/8+3*nu/8 ...  
   -1/4+nu/12  -1/8-nu/8    nu/6      1/8-3*nu/8];
```

```
KE = E/(1-nu^2)* ...
```

```
    [ k(1) k(2) k(3) k(4) k(5) k(6) k(7) k(8)  
      k(2) k(1) k(8) k(7) k(6) k(5) k(4) k(3)  
      k(3) k(8) k(1) k(6) k(7) k(4) k(5) k(2)  
      k(4) k(7) k(6) k(1) k(8) k(3) k(2) k(5)  
      k(5) k(6) k(7) k(8) k(1) k(2) k(3) k(4)  
      k(6) k(5) k(4) k(3) k(2) k(1) k(8) k(7)  
      k(7) k(4) k(5) k(2) k(3) k(8) k(1) k(6)  
      k(8) k(3) k(2) k(5) k(4) k(7) k(6) k(1)];
```

Matlab Code – FEM ANALYSIS

```
function [U]=FE(nelx,nely,x,penal)
[KE] = lk;
K = sparse(2*(nelx+1)*(nely+1), 2*(nelx+1)*(nely+1));
F = sparse(2*(nely+1)*(nelx+1),1); U = zeros(2*(nely+1)*(nelx+1),1);
for elx = 1:nelx
    for ely = 1:nely
        n1 = (nely+1)*(elx-1)+ely;
        n2 = (nely+1)* elx  +ely;
        edof = [2*n1-1; 2*n1; 2*n2-1; 2*n2; 2*n2+1; 2*n2+2; 2*n1+1; 2*n1+2];
        K(edof,edof) = K(edof,edof) + x(ely,elx)^penal*KE;
    end
end

F(2*(nelx+1)*(nely+1),1)=-1;
fixeddofs=union([1,2],[2*nely+1:2*(nely+1)]);

alldofs    = [1:2*(nely+1)*(nelx+1)];
freedofs   = setdiff(alldofs,fixeddofs);
% SOLVING
U(freedofs,:) = K(freedofs,freedofs) \ F(freedofs,:);
U(fixeddofs,:)= 0;
```

Matlab Code – OPTIMALITY CRITERIA

```
function [xnew]=OC(nelx,nely,x,volfrac,dc)
```

```
l1 = 0; l2 = 100000; move = 0.2;
```

```
while (l2-l1 > 1e-4)
```

```
    lmid = 0.5*(l2+l1);
```

```
    xnew = max(0.001,max(x-move,min(1.,min(x+move,x.*sqrt(-dc./lmid)))));
```

```
    if sum(sum(xnew)) - volfrac*nelx*nely > 0;
```

```
        l1 = lmid;
```

```
    else
```

```
        l2 = lmid;
```

```
    end
```

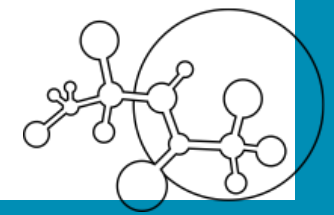
```
end
```

lmid: 50.0000	l1: 0.0000	l2: 50.0000
lmid: 25.0000	l1: 0.0000	l2: 25.0000
lmid: 12.5000	l1: 0.0000	l2: 12.5000
lmid: 6.2500	l1: 6.2500	l2: 12.5000
lmid: 9.3750	l1: 6.2500	l2: 9.3750
lmid: 7.8125	l1: 6.2500	l2: 7.8125
lmid: 7.0313	l1: 7.0313	l2: 7.8125
lmid: 7.4219	l1: 7.0313	l2: 7.4219
lmid: 7.2266	l1: 7.2266	l2: 7.4219
lmid: 7.3242	l1: 7.3242	l2: 7.4219
lmid: 7.3730	l1: 7.3242	l2: 7.3730
lmid: 7.3486	l1: 7.3242	l2: 7.3486
lmid: 7.3364	l1: 7.3364	l2: 7.3486
lmid: 7.3425	l1: 7.3425	l2: 7.3486
lmid: 7.3456	l1: 7.3425	l2: 7.3456
lmid: 7.3441	l1: 7.3441	l2: 7.3456
lmid: 7.3448	l1: 7.3448	l2: 7.3456
lmid: 7.3452	l1: 7.3448	l2: 7.3452
lmid: 7.3450	l1: 7.3450	l2: 7.3452
lmid: 7.3451	l1: 7.3450	l2: 7.3451

Matlab code command

`top(nelx, nely, volfrac, penal, rmin)`

- ▶ `nelx` and `nely`: number of elements in the horizontal and vertical directions,
- ▶ `volfrac`: volume fraction,
- ▶ `penal`: penalization power,
- ▶ `rmin`: filter size(divided by element size).



Example 1

`top(40, 20, 0.5, 3, 1.0)`

Example 1

top(40, 20, 0.5, 3, 1.0)



Checkerboard Pattern

Example 1 -- Checkerboard Pattern Problem

Solution: Low-pass filter

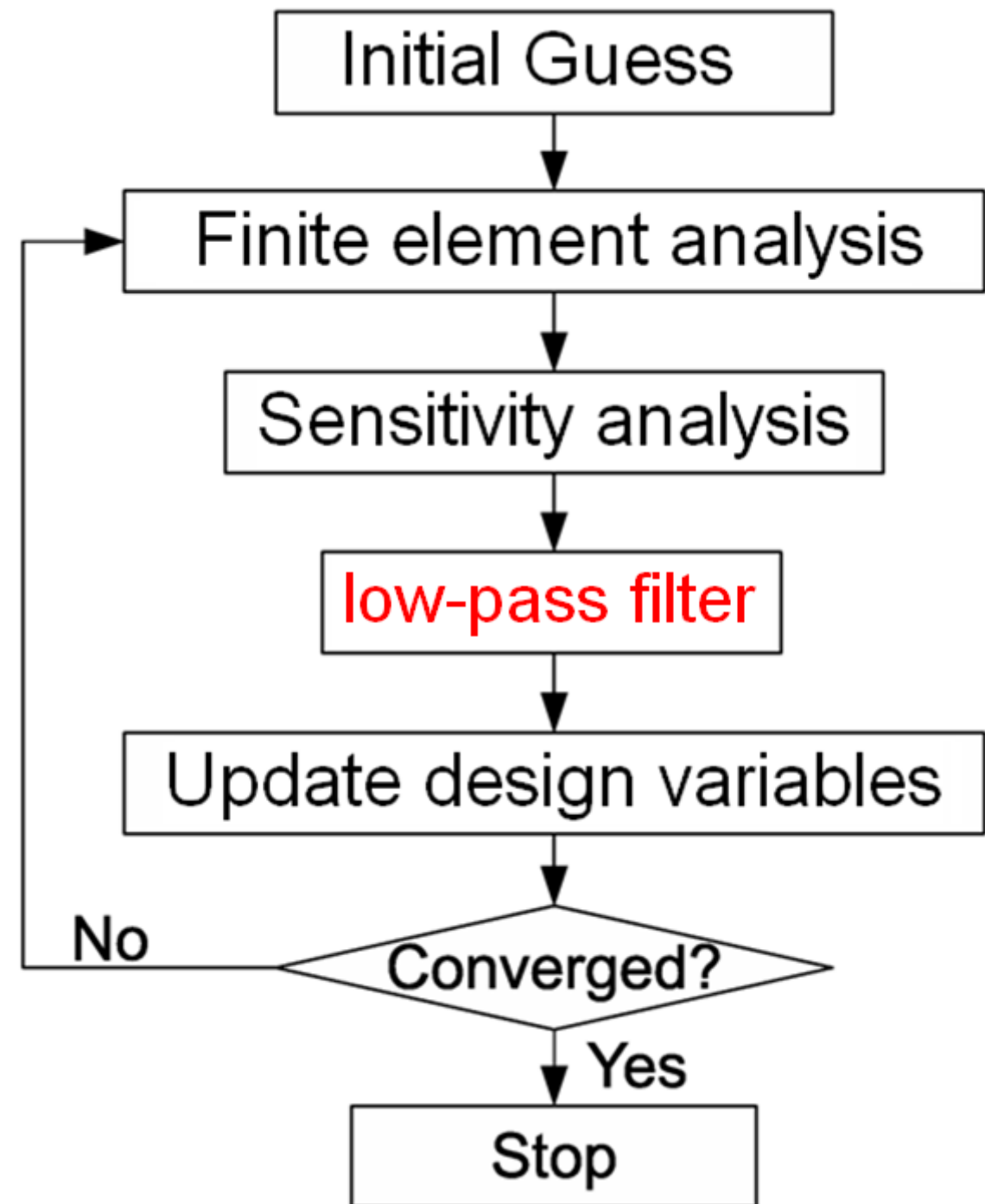
$$\frac{\widehat{\partial c}}{\partial x_e} = \frac{1}{x_e \sum_{f=1}^N \hat{H}_f} \sum_{f=1}^N \hat{H}_f x_f \frac{\partial c}{\partial x_f}.$$

$$\hat{H}_f = r_{\min} - \text{dist}(e, f),$$

$$\{f \in N \mid \text{dist}(e, f) \leq r_{\min}\}, \quad e = 1, \dots, N$$

[dc] = check(nelx,
nely, rmin, x, dc);

*% FILTERING OF
SENSITIVITIES*

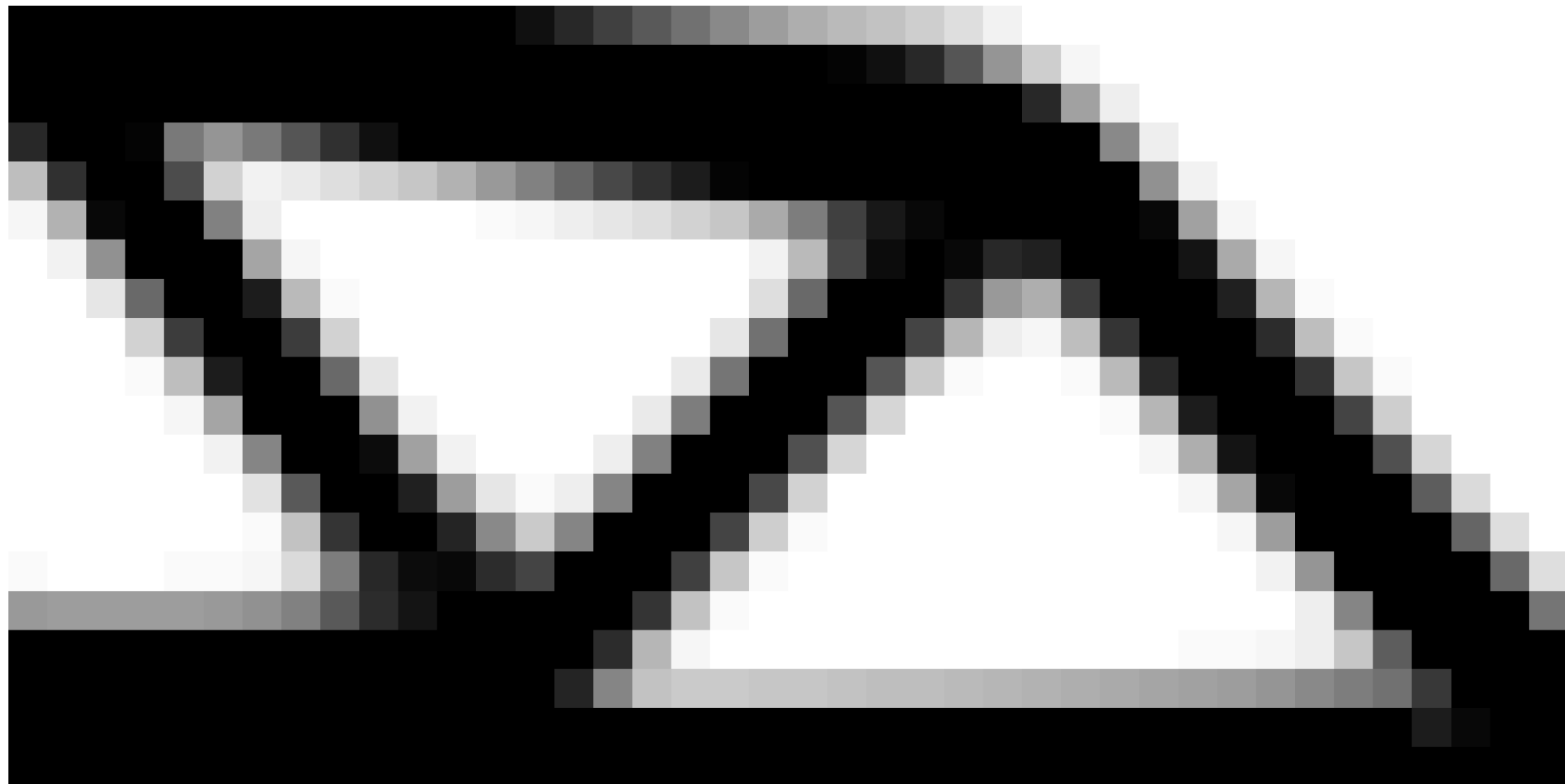


Example 2

`top(40, 20, 0.5, 3, 1.5)`

Example 2

`top(40, 20, 0.5, 3, 1.5)`



`top(40, 20, 0.5, 3, 3)` what will happen?

Example 2

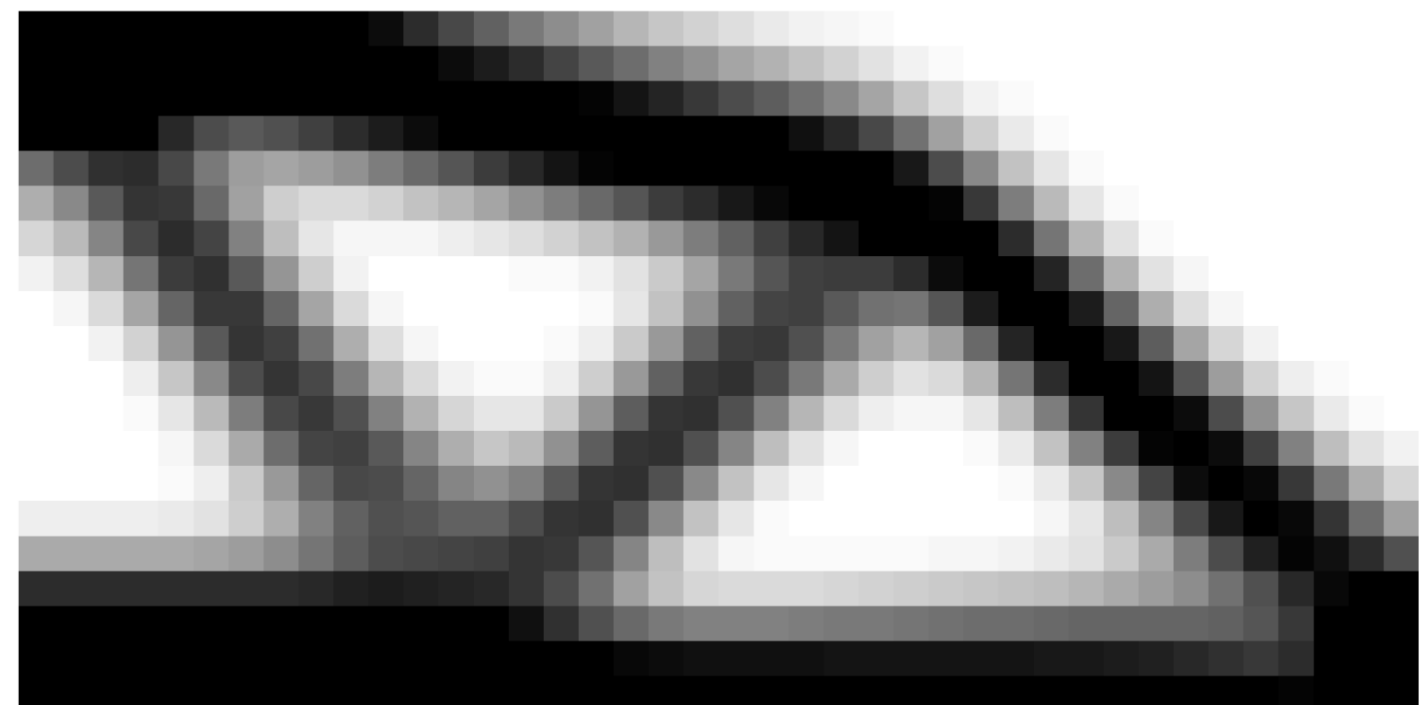
$r_{\min}=1.5$

$\text{Obj}=82.7562;$



$r_{\min}=3$

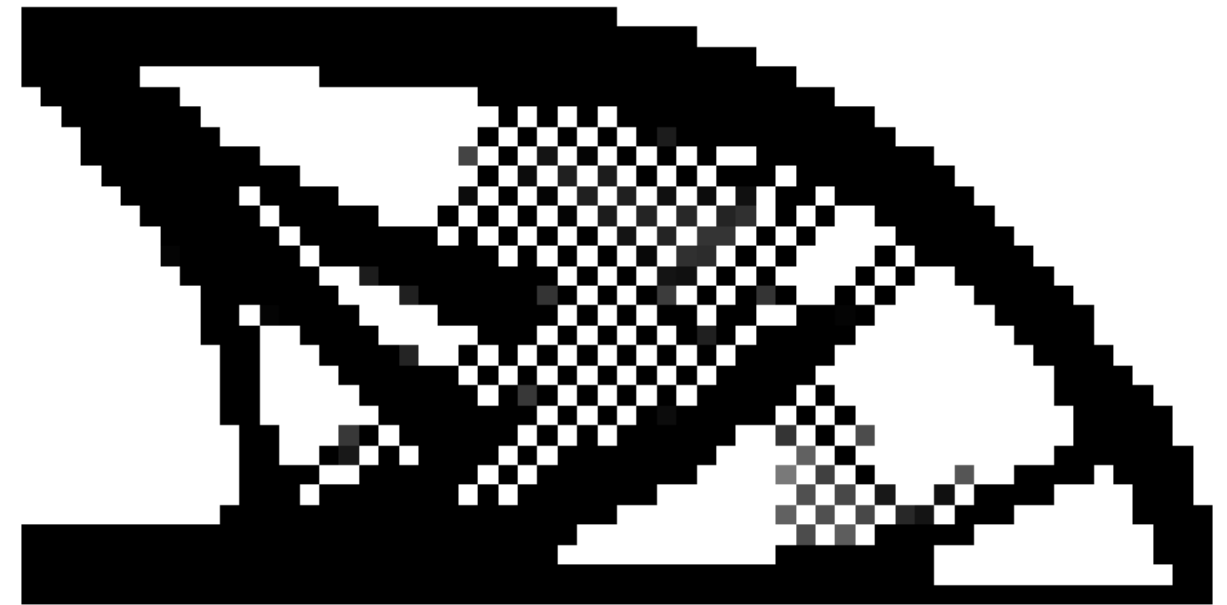
$\text{Obj}=99.1929;$



Example 3

top(60, 30, 0.5, 3, 1.0)

Obj: 83.0834



top(40, 20, 0.5, 3, 1.0)

Obj: 80.4086;

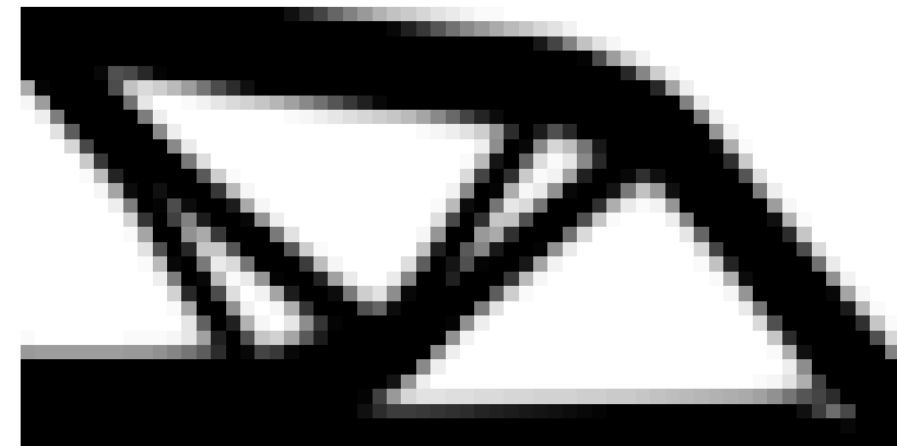


Mesh dependency

Example 3

`top(60, 30, 0.5, 3, 1.5)`

Obj: 81.3491



`top(60, 30, 0.5, 3, 2.25)`

Obj: 83.5963



`top(40, 20, 0.5, 3, 1.5)`

Obj=82.7562;



Numerical instability

Numerical Instabilities in Structural Topology Optimization

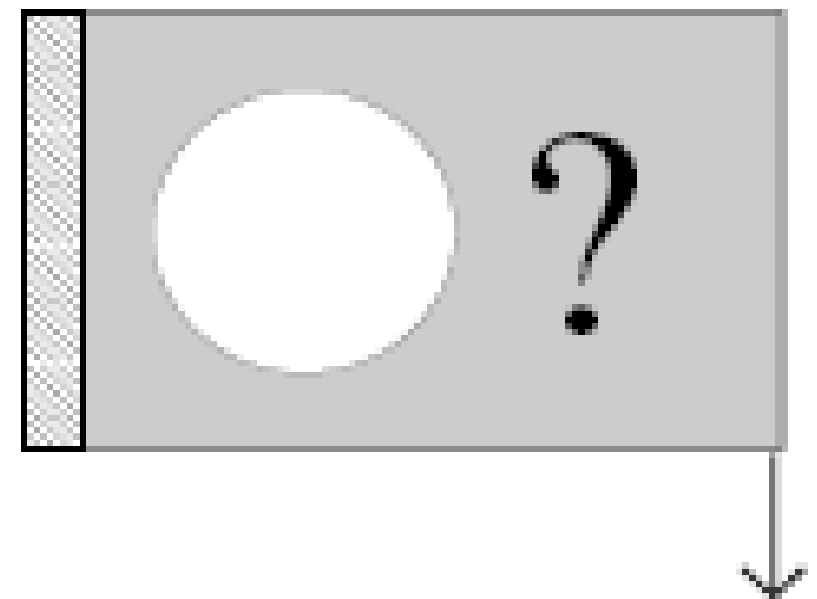
- ▶ Checkerboard Pattern
- ▶ Mesh – dependency
- ▶ Local Minimal solution (non-convex optimization)

Remedy

- ▶ Low-pass filter.
- ▶ Add other constraints to the optimization problem [1].

Example 4 – passive elements

- ▶ Add passive to the call in lines 29 and 39.
- ▶ Add line after line 43
 `xnew(find(passive))=0.001;`
- ▶ Add the following 10 lines to the main program (after line 5)
 for ely= 1:nely
 for elx=1:nelx
 if $\sqrt{(\text{ely}-\text{nely}/2.)^2+(\text{elx}-\text{nelx}/3.)^2} < \text{nely}/3.$
 `passive(ely,elx)=1;`
 `x(ely,elx)=0.001;`
 else
 `passive(ely,elx)=0;`
 end
 end
 end
end



`top (45,30,0.5,3,1.5)`

Update design variables – mathematical programming methods

At each iteration step, updated design variables can be obtained by solving a simpler approximate optimization subproblem.

These subproblems are constructed based on sensitivity information at the current iteration step as well as some iteration history. [1]

Our optimization problem can be transformed into the following general form

$$\min_{\mathbf{x}} f_0(\mathbf{x})$$

$$\text{s.t. } f_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, m$$

$$\mathbf{x} \in \mathcal{X} = \{\mathbf{x} \in \mathbb{R}^n, x_j^{\min} \leq x_j \leq x_j^{\max}, j = 1, \dots, n\}.$$

Sequential linear programming (SLP)

The subproblem at iteration k is as follows [2]

$$\min_{\mathbf{x}} f_0(\mathbf{x}^k) + \nabla f_0(\mathbf{x}^k)^T (\mathbf{x} - \mathbf{x}^k)$$

$$\text{s.t. } f_i(\mathbf{x}^k) + \nabla f_i(\mathbf{x}^k)^T (\mathbf{x} - \mathbf{x}^k) \leq 0, \quad i = 1, \dots, m$$

$$\mathbf{x} \in \mathcal{X},$$

$$l_j^k \leq x_j - x_j^k \leq u_j^k, \quad j = 1, \dots, n.$$

Here, l_j^k and u_j^k are move limits.

This subproblem may be solved by the simplex algorithm.

Sequential quadratic programming (SQP)

The subproblem at iteration k is as follows [2]

$$\min_{\mathbf{x}} f_0(\mathbf{x}^k) + \nabla f_0(\mathbf{x}^k)^T (\mathbf{x} - \mathbf{x}^k) + \frac{1}{2} (\mathbf{x} - \mathbf{x}^k)^T H(\mathbf{x}^k) (\mathbf{x} - \mathbf{x}^k)$$

$$\text{s.t. } f_i(\mathbf{x}^k) + \nabla f_i(\mathbf{x}^k)^T (\mathbf{x} - \mathbf{x}^k) \leq 0, \quad i = 1, \dots, m$$

$$\mathbf{x} \in \mathcal{X},$$

Here, $H(\mathbf{x}^k)$ is a positive definite first order approximation of the Hessian of f_0 at \mathbf{x}^k .

SLP and SQP are designed to solve general nonlinear optimization problems.

Convex linearization (CONLIN)

The subproblem at iteration k is as follows [2]

$$\min_{\mathbf{x}} f_0^{C,k}(\mathbf{x})$$

$$\text{s.t. } f_i^{C,k}(\mathbf{x}) \leq 0, \quad i = 1, \dots, m$$

$$\mathbf{x} \in \chi := \{\mathbf{x} \in \mathbb{R}^n, 0 < x_j^{\min} \leq x_j \leq x_j^{\max}, j = 1, \dots, n\}.$$

Where

$$f_i^{C,k}(\mathbf{x}) := f_i(\mathbf{x}^k) + \sum_{j \in \Omega_+} f_{ij}^{L,k}(\mathbf{x}) + \sum_{j \in \Omega_-} f_{ij}^{R,k}(\mathbf{x}),$$

Here, $\Omega_+ := \{j : \partial f_i(\mathbf{x}^k) / \partial x_j > 0\}$ and $\Omega_- := \{j : \partial f_i(\mathbf{x}^k) / \partial x_j \leq 0\}$,

$$f_{ij}^{L,k}(\mathbf{x}) = \partial f_i(\mathbf{x}^k) / \partial x_j (x_j - x_j^k) \text{ and}$$

$$f_{ij}^{R,k}(\mathbf{x}) = \frac{\partial f_i(\mathbf{x}^k)}{\partial x_j} \frac{x_j^k (x_j - x_j^k)}{x_j}.$$

Method of moving asymptotes (MMA)

The subproblem at iteration k is as follows [2]

$$\min_{\mathbf{x}} f_0^{M,k}(\mathbf{x})$$

$$\text{s.t. } f_i^{M,k}(\mathbf{x}) \leq 0, \quad i = 1, \dots, m$$

$\alpha_j^k \leq x_j \leq \beta_j^k, j = 1, \dots, n$. Here, α_j^k and β_j^k are move limits.

Where

$$f_i^{M,k}(\mathbf{x}) := f_i(\mathbf{x}^k) - \sum_{j=1}^n \left(\frac{p_{ij}^k}{U_j^k - x_j^k} + \frac{q_{ij}^k}{x_j^k - L_j^k} \right) + \sum_{j=1}^n \left(\frac{p_{ij}^k}{U_j^k - x_j} + \frac{q_{ij}^k}{x_j - L_j^k} \right),$$

Here, L_j^k and U_j^k are moving asymptotes that satisfy $L_j^k < x_j^k < U_j^k$,

$$p_{ij}^k = \begin{cases} (U_j^k - x_j^k)^2 \partial f_i(\mathbf{x}^k) / \partial x_j & \text{if } \partial g_i(\mathbf{x}^k) / \partial x_j > 0 \\ 0 & \text{otherwise,} \end{cases}$$

$$q_{ij}^k = \begin{cases} 0 & \text{if } \partial g_i(\mathbf{x}^k) / \partial x_j \geq 0 \\ -(x_j^k - L_j^k)^2 \partial f_i(\mathbf{x}^k) / \partial x_j & \text{otherwise.} \end{cases}$$

Developed by Svanberg [3]

MMA codes

Svanberg's MMA codes are used to solve the following

$$\left. \begin{array}{ll} \min_{\mathbf{x}, \mathbf{y}, z} : & f_0(\mathbf{x}) + a_0 z + \sum_{i=1}^m (c_i y_i + \frac{1}{2} d_i y_i^2) \\ \text{subject to :} & f_i(\mathbf{x}) - a_i z - y_i \leq 0, \quad i = 1, \dots, m \\ & : x_j^{\min} \leq x_j \leq x_j^{\max}, \quad j = 1, \dots, n \\ & : y_i \geq 0, \quad i = 1, \dots, m \\ & : z \geq 0 \end{array} \right\},$$

Our problem is

$$\left. \begin{array}{ll} \min_{\mathbf{x}} : & f_0(\mathbf{x}) \\ \text{subject to :} & f_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, m \\ & : x_j^{\min} \leq x_j \leq x_j^{\max}, \quad j = 1, \dots, n \end{array} \right\},$$

Svanberg's suggestion for our problem

$$a_0 = 1, \quad a_i = 0, \quad c_i = 1000, \quad d = 0.$$

MMA codes

The MMA call is

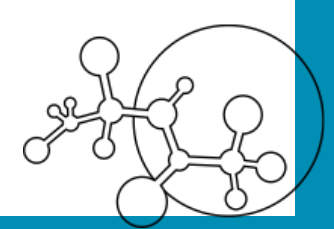
```
function [xmma,ymma,zmma,lam,xsi,eta,mu,zet,s,low,upp] = ...  
    mmasub(m,n,iter,xval,xmin,xmax,xold1,xold2, ...  
    f0val,df0dx,df0dx2,fval,dfdx,dfdx2,low,upp,a0,a,c,d);
```

Hints:

- ▶ Check the definition of the MMA variables in the mmasub.m file.
- ▶ Be careful of the difference between row and column vectors.
- ▶ Normalize constraints and objective function. i.e. use $V(x)/V_0 - 1 \leq 0$ instead of $V(x) \leq V_0$.

Efficient topology optimization in MATLAB using 88 lines of code [4,5]

- ▶ A valuable successor to the 99 line code.
- ▶ A speed improvement.
- ▶ Other low-pass filter methods.



Summary

Three components of an optimization problem.

Numerical instability is discussed.

Update design variables by mathematical programming methods.

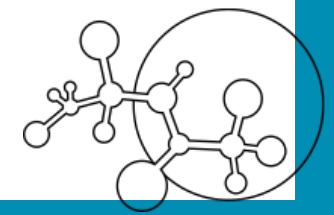
reference

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Thanks for your attention