# A discussion of filtering technique using SIMP method: filter radius

**Chen Kaiwen**[1]

**Keywords: filtering technique, SIMP, Heaviside projection, filter radius, topology optimization**

To ensure existence of solutions to the topology optimization problem and to avoid the formation of checkerboard patterns, some restriction of the design must be imposed.

The restriction methods for density based topology optimization problems can roughly be divided into three categories

1. Mesh-independent filtering methods, constituting sensitivity filters and density filters
2. Constraint methods such as perimeter control, global gradient control, regularized penalty methods and integral filtering
3. Other methods like wavelet parameterization, phase-field approaches and level-set methods

The filtering methods in group 1 are the most popular one due to their ease of implementation and their efficiency. The constraint methods in group 2 may be difficult to use because they require a problem and geometry dependent choice and tuning of the constraint value. The alternative methods in group 3 are mostly still in research and have yet to be successfully applied to advanced problems with many constraints. There are also methods that are hybrids between categories 1 and 2. [1]

A common approach is the application of a filter to either the sensitivities or the densities. The density filter is quite useful when different modern filters are implemented.

1. The sensitivity filter modifies the sensitivities $\frac{\partial c}{\partial x_e}$ as

$$\frac{\widehat{\partial c}}{\partial x_e} = \frac{1}{\max(\gamma, x_e)\sum_{i\in N_e} H_{ei}} \sum_{i\in N_e} H_{ei}\, x_i \frac{\partial c}{\partial x_i}$$

$N_e$ is the set of elements i for which the center-to-center distance $\Delta(e,i)$ to element e is smaller than the radius $r_{min}$ and $H_{ei}$ is a weight factor defined as:

$$H_{ei} = \max(0, r_{min} - \Delta(e,i))$$

The term $\gamma$ is a small positive number introduced in order to avoid division by zero.

2. The density filter transforms the original densities $x_e$ as follows:

$$\widetilde{x_e} = \frac{1}{\sum_{i\in N_e} H_{ei}} \sum_{i\in N_e} H_{ei}\, x_i$$

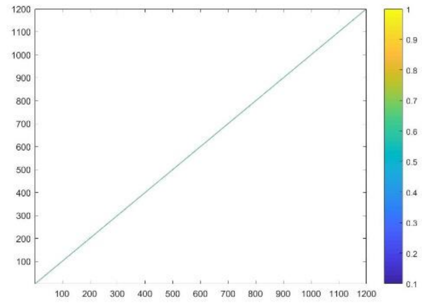
[1]**Department of Mechanical and Aerospace Engineering**
**Master of Science in Aeronautical Engineering**
**Hong Kong University of Science and Technology**
**Clear Water Bay, Kowloon, Hong Kong**
**Email:kchenau@connect.ust.hk**

It causes the original densities $x_e$ to lose their physical meaning. The use of a density filter not only implies filtering of densities but also a chain rule modification of the sensitivities of the objective function and the volume constraint. Both operations involve a weighted average over different elements.

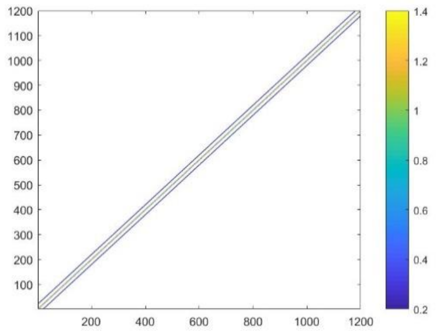
In the optimization loop, the initially corresponding physical densities are not totally identical to design variables, i.e. the prescribed volume fraction f. It holds if the design variables represent a homogeneous field. For other non-volume-preserving filters, it may be necessary to compute the initial physical densities by explicit application of the filter to the initial design variables $x_e$ and to adjust the initial design variables in such a way that the volume constraint is satisfied since the design variables $x_e$ has lost their physical meaning(as this constraint is specified in terms of the physical densities, not the design variables)[2]

Using the same command code top88(60,20,0.5,3, rmin,1) which set the nelx = 60, nely = 20, volume fraction = 0.5, penalization = 3, ft = 1, change different rmin and we can see the matrix Hs and Hs has different values.

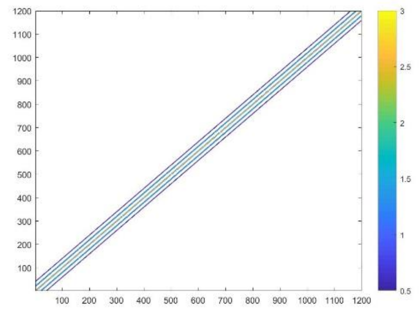
Change the rmin as 1,1.5,3 and 5. Plot the contour of matrix H, respectively.



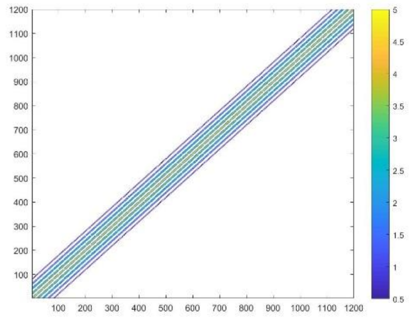

Rmin = 1




Rmin = 1.5



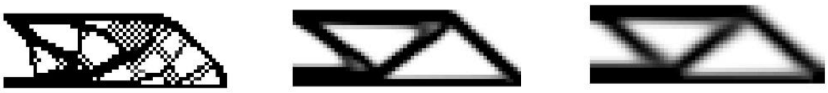

Rmin = 3




Rmin = 5

As the parameter rmin increase, the number of line increases, which means the filter circle cover more elements. For the same x, the corresponding points mean the distance between two certain elements are less than rmin and H is greater than 0.

Here we can see that by introducing the filter the checkerboard problem can be solved quite well. The following pictures are equals 1, 1.5 and 3. (MBB beam example)



Rmin = 1 Rmin = 1.5 Rmin = 3

However, after using density filter, the boundary of structure becomes vague. The density value along the structure's boundary is not clearly 0 or 1 numbers. Most of the time, the density of boundary element is a decimal number between 0 and 1. In the result picture, the index 0 means exist and the index 1 means void. Density value along the boundary can be 0.33, 0.54, 0.87 etc. This side-effect is also introduced by the density and sensitivity filter as the rmin=0 figure has a quite clear boundary.

The filtering methods intended for regularization of topology optimization problems can be divided into density and sensitivity based method. Several alternate filtering schemes that claim black and white solution have been published. These methods are also categorized in density and sensitivity based methods. In [2] the density filter used modified density filtering.

1. Density filters
    a. Modified density filtering
    b. Bi-lateral density filtering
    c. Density filtering with a Heaviside step function[3]
    d. Modified density filtering with a Heaviside step function
2. Sensitivity filters
    a. Alternative sensitivity filter
    b. Sensitivity filter without density weighting
    c. Mean sensitivity filter
    d. Bi-lateral sensitivity filter

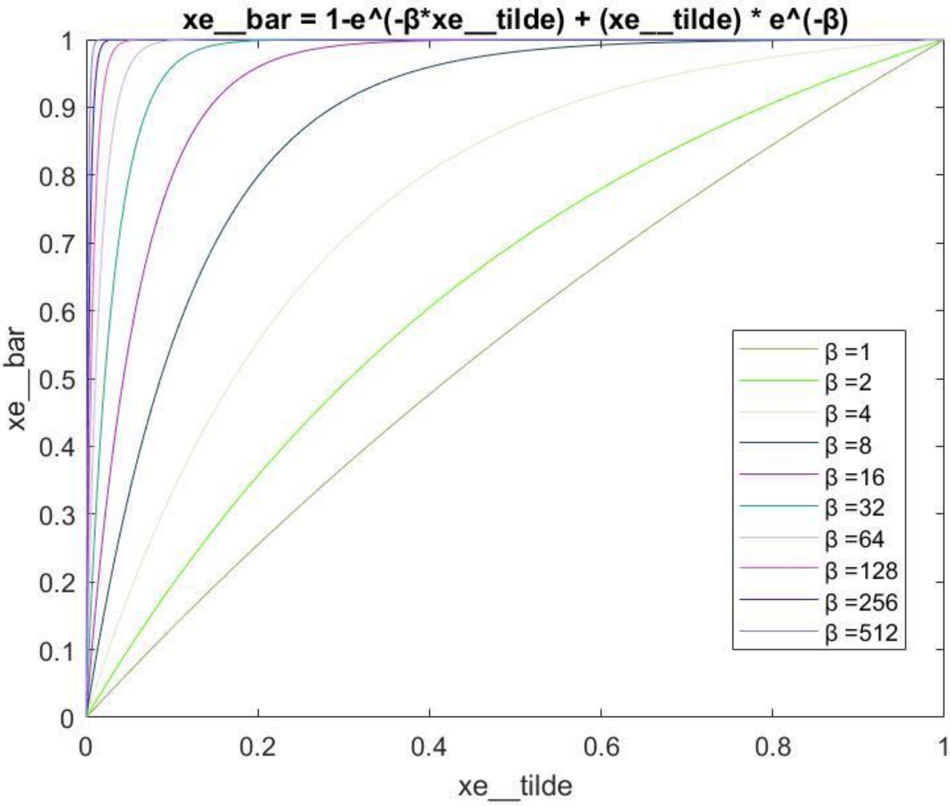
The Heaviside filter is modification of the original density filter with a Heaviside step function that projects the density $\widetilde{x_e}$ (intermediate density) to a physical density $\overline{x_e}$. The physical density $\overline{x_e}$ equals one if $\widetilde{x_e} > 0$ and zero if $\widetilde{x_e} = 0$.

To allow for the use of a gradient-based optimization scheme, Heaviside function is replaced with a smoother function.

For the same command top88(60,20,0.5,3, rmin,1), implement the smoother function. i.e.

$$\overline{x_e} = 1 - e^{-\beta \widetilde{x_e}} + \widetilde{x_e} e^{-\beta}$$

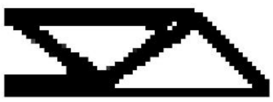
The parameter β controls the smoothness of approximation. When β approaches infinity, the approximation approaches the true Heaviside step function and when β equal to 0, the Heaviside filter is not used.




We tried update the parameter β in the loop from 1 to 512. The upper bound of β could be set larger in the further study.

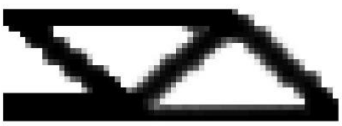
After we use the Heaviside function, the radius of filter area becomes a key parameter. In the [2], the author recommends the 0.03 times the width of design domain. i.e. rmin=1.8. For the same problem, use the command line top88_Black_and_white_solution(60,20,0.5,3, rmin,3). Compare different radius(rmin).

Rmin=1.5

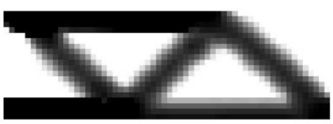


Rmin=1.8

Rmin=2



Rmin=3



Rmin=4

We can see that rmin=1.8 gives us the best result. It doesn't have any unreality structure, which happened in the case rmin=1.5. Also, it has a quite clear boundary compared with rmin=3 or 4, in other words, the boundary is distinct. The two holes in the structure (rmin=1.8 or 2) disappear when we use better mesh. [2] The problem now becomes that after we use this kind of black and white solution, the boundary need to be refined for manufacturing since it is quite zigzag.

Also, one thing should be reminded is the factor of penalization power p. In the former cases, the penalization power p is set as 3. However, for the real case the relation between the $E/E_0$ and density is hard to define theoretically. We already know that increasing the penalization power is helpful to get a solid/void solution. However, it will also have a side effect. The computation time becomes longer. The picture showed is using penalization power as 5. top88_Black_and_white_solution

(60,20,0.5,5,1.8,3)

In the following term, the focus is shoot in density filters, especially the density filtering with a Heaviside step function and the modified density filtering with a Heaviside step function. The aim is to achieve a proper length scale of the filter in the optimized design and to obtain black-and-white solutions.

**Mid-Term Report Label:**
Project Topic: Fileting technique
Project Scope: Regularization Scheme for SIMP
Specific Aspects to be Studied: The filtering techniques
Work to be Completed by the Term End: Black-and-white solutions

**Reference Note:**

[1]: Sigmund, O., 2007. Morphology-based black and white filters for topology optimization. *Structural and Multidisciplinary Optimization*, *33*(4-5), pp.401-424.

[2]: Andreassen, E., Clausen, A., Schevenels, M., Lazarov, B.S. and Sigmund, O., 2011. Efficient topology optimization in MATLAB using 88 lines of code. *Structural and Multidisciplinary Optimization*, *43*(1), pp.1-16.

[3]: Guest, J. K., Prévost, J. H., & Belytschko, T. (2004). Achieving minimum length scale in topology optimization using nodal design variables and projection functions. *International journal for numerical methods in engineering*, *61*(2), 238-254.

[4]: A survey of structural and multidisciplinary continuum topology optimization: post 2000, Joshua D. Deaton, Ramana V. Grandhi, Structural and Multidisciplinary Optimization, January 2014, Volume 49, Issue 1, pp 1-38.