CO542 - Neural Networks and Fuzzy Systems
Lab 08: Fuzzy sets and membership functions

E/19/129
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Task 01

Fuzzy Rule Base Table

Input (Speed)	Output (Voltage Action)	Rule Description
Too slow	More voltage (Speed up)	Increase voltage to speed up motor
Just right	No change	Maintain current voltage
Too fast	Less voltage (Slow down)	Decrease voltage to slow down motor

Formal Rule Representation:

- Rule 1: IF Speed is "Too slow" THEN Voltage is "More voltage"
- Rule 2: IF Speed is "Just right" THEN Voltage is "No change"
- Rule 3: IF Speed is "Too fast" THEN Voltage is "Less voltage"

Task 02

Define Membership Functions

Input (Speed) Membership Functions:

- Slow: Triangle [2304, 2362, 2420]
- Desired: Triangle [2362, 2420, 2478]
- Fast: Triangle [2420, 2478, 2536]

Output (Voltage) Membership Functions:

- Low: Triangle [2.32, 2.36, 2.40]
- Medium: Triangle [2.38, 2.40, 2.42]
- High: Triangle [2.40, 2.44, 2.48]

Calculate Input Membership Degrees

For speed = 2437.4 RPM, we need to find membership in each input fuzzy set:

Membership in "Slow" [2304, 2362, 2420]
Since 2437.4 > 2420 (right boundary), membership = 0

Membership in "Desired" [2362, 2420, 2478]

- Peak at 2420
- Right slope: from 2420 to 2478
- Formula: $\mu = (2478 - x) / (2478 - 2420)$

- μ_desired = (2478 - 2437.4) / (2478 - 2420) = 40.6 / 58 = 0.700

Membership in "Fast" [2420, 2478, 2536]
- Peak at 2478
- Left slope: from 2420 to 2478
- Formula: μ = (x - 2420) / (2478 - 2420)
- μ_fast = (2437.4 - 2420) / (2478 - 2420) = 17.4 / 58 = 0.300

Verification: 0.700 + 0.300 = 1.000

Apply Fuzzy Rules
Rule 1: IF speed is slow THEN voltage is high
- Activation level: μ_slow = 0
- Output: High voltage with strength 0

Rule 2: IF speed is desired THEN voltage is medium
- Activation level: μ_desired = 0.700
- Output: Medium voltage with strength 0.700

Rule 3: IF speed is fast THEN voltage is low
- Activation level: μ_fast = 0.300
- Output: Low voltage with strength 0.300

Create Clipped Output Sets
High Voltage (not activated)
- Strength: 0
- No contribution to final output

Medium Voltage [2.38, 2.40, 2.42]
- Clipped at height: 0.700
- Active region: [2.386, 2.414]
- Left boundary: 2.38 + (2.40 - 2.38) × 0.700 = 2.386
- Right boundary: 2.42 - (2.42 - 2.40) × 0.700 = 2.414

Low Voltage [2.32, 2.36, 2.40]
- Clipped at height: 0.300
- Active region: [2.326, 2.394]
- Left boundary: 2.32 + (2.36 - 2.32) × 0.300 = 2.326
- Right boundary: 2.40 - (2.40 - 2.36) × 0.300 = 2.394

Calculate Areas and Centroids
Medium Voltage Trapezoid
- Parallel sides: 0.014 (top) and 0.042 (base width if not clipped)
- Height: 0.700

- Area = 0.700 × (0.014 + 0.028) / 2 = 0.700 × 0.021 = 0.0147
- Centroid = (2.386 + 2.414) / 2 = 2.400

Low Voltage Trapezoid
- Parallel sides: 0.018 (top) and 0.080 (base width if not clipped)
- Height: 0.300
- Area = 0.300 × (0.018 + 0.068) / 2 = 0.300 × 0.043 = 0.0129
- Centroid = (2.326 + 2.394) / 2 = 2.360

Calculate Final Output (Centroid Defuzzification)
Formula: Output = (Σ Area_i × Centroid_i) / (Σ Area_i)

Calculation:
- Numerator = (0.0147 × 2.400) + (0.0129 × 2.360) = 0.03528 + 0.03044 = 0.06572
- Denominator = 0.0147 + 0.0129 = 0.0276
- Output Voltage = 0.06572 / 0.0276 = 2.381 V

Therefore,

The defuzzified output voltage is **2.381 V**

Use the Mamdani model and maximum defuzzification method:

Aggregate Output Sets (Mamdani Union)
Combine the clipped output sets using maximum operator:
- Medium voltage: Active in [2.386, 2.414] with height 0.700
- Low voltage: Active in [2.326, 2.394] with height 0.300

Combined output membership function:
- [2.326, 2.386): Height = 0.300 (from Low only)
- [2.386, 2.394]: Height = max(0.700, 0.300) = 0.700 (overlap region)
- (2.394, 2.414]: Height = 0.700 (from Medium only)

Maximum Defuzzification
Find the voltage value where the aggregated membership function reaches its maximum.
Maximum membership value: 0.700
Region with maximum membership: [2.386, 2.414]

For maximum defuzzification, we take the center of the region with maximum membership:
Output Voltage = (2.386 + 2.414) / 2 = 2.400 V

Therefore:

The defuzzified output voltage using Maximum method is **2.400 V**

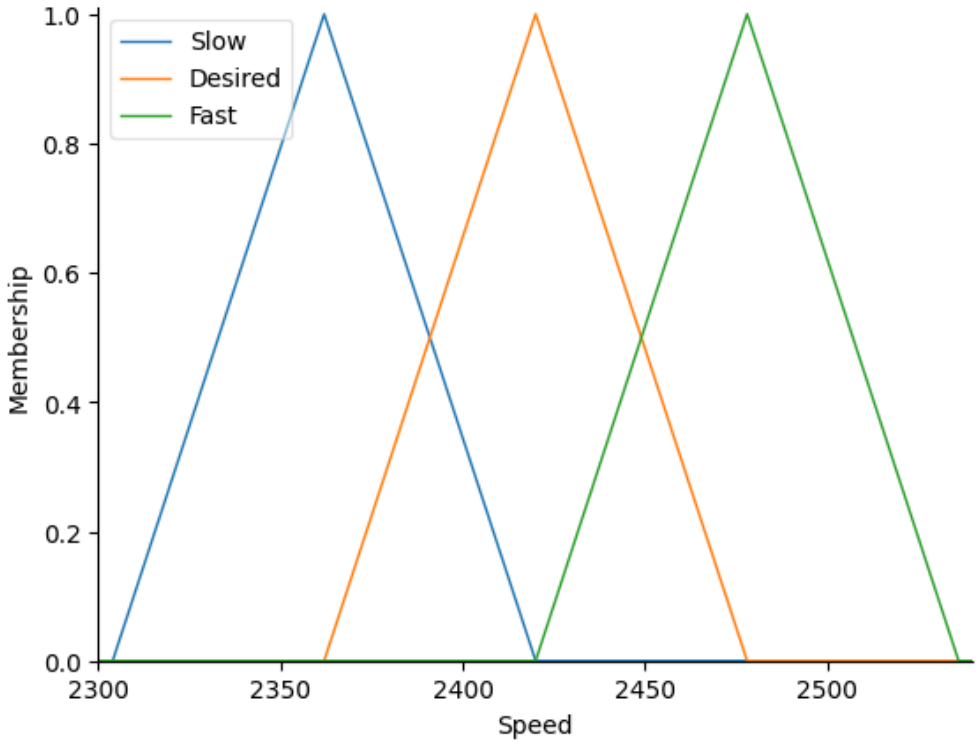
Task 03

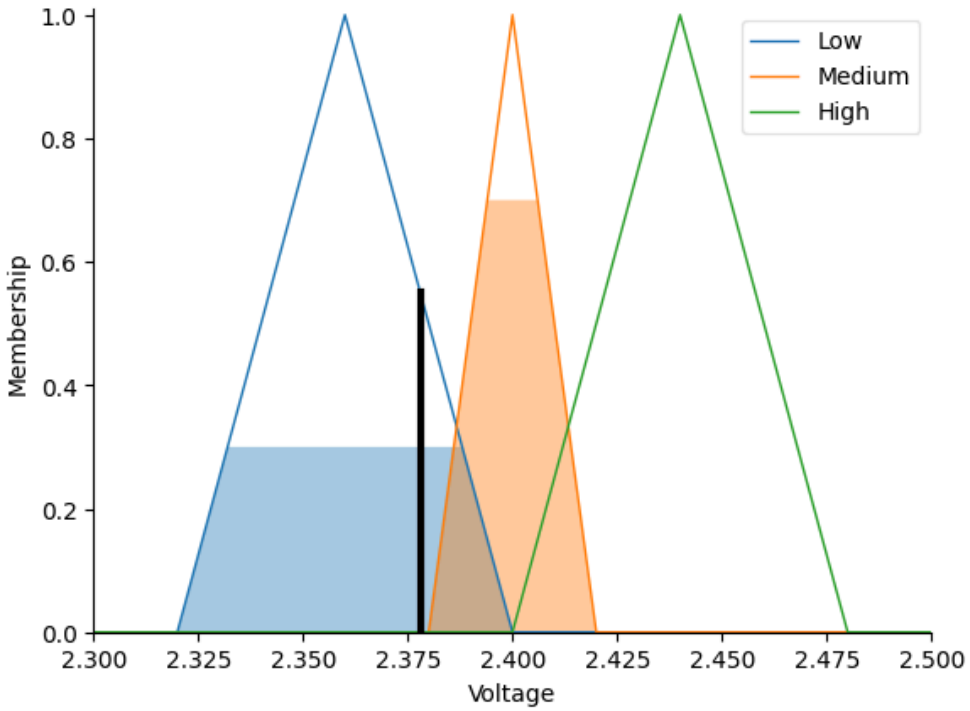
Code:

```
import numpy as np
import skfuzzy as fuzz
from skfuzzy import control as ctrl
import matplotlib.pyplot as plt

# --- Define fuzzy variables with appropriate ranges ---
speed = ctrl.Antecedent(np.arange(2300, 2540, 0.1), 'Speed')       # from
2300 to 2536
voltage = ctrl.Consequent(np.arange(2.3, 2.5, 0.001), 'Voltage')   # from
2.32 to 2.48

# --- Define Input Membership Functions ---
speed['Slow'] = fuzz.trimf(speed.universe, [2304, 2362, 2420])
speed['Desired'] = fuzz.trimf(speed.universe, [2362, 2420, 2478])
speed['Fast'] = fuzz.trimf(speed.universe, [2420, 2478, 2536])

# --- Define Output Membership Functions ---
voltage['Low'] = fuzz.trimf(voltage.universe, [2.32, 2.36, 2.40])
voltage['Medium'] = fuzz.trimf(voltage.universe, [2.38, 2.40, 2.42])
voltage['High'] = fuzz.trimf(voltage.universe, [2.40, 2.44, 2.48])

# --- Define Fuzzy Rules ---
rule1 = ctrl.Rule(speed['Slow'], voltage['High'])       # Rule: Slow →
High Voltage
rule2 = ctrl.Rule(speed['Desired'], voltage['Medium'])  # Rule: Desired →
Medium Voltage
rule3 = ctrl.Rule(speed['Fast'], voltage['Low'])        # Rule: Fast → Low
Voltage

# --- Create control system and simulation ---
speed_ctrl = ctrl.ControlSystem([rule1, rule2, rule3])
sim = ctrl.ControlSystemSimulation(speed_ctrl)

# --- Provide input speed value ---
sim.input['Speed'] = 2437.4
sim.compute()
```

```
# --- Get and print output voltage ---
output_voltage = sim.output['Voltage']
print(f"Input Speed: 2437.4 RPM")
print(f"Output Voltage (scikit-fuzzy centroid method):
{output_voltage:.3f} V")

# --- Optional: Visualize membership functions and result ---
# Input Memberships
speed.view()

# Output Memberships with result
voltage.view(sim=sim)
```

Output:
Input Speed: 2437.4 RPM
Output Voltage (scikit-fuzzy centroid method): 2.378 V





Although the manual defuzzification process yielded a value of 2.381 V using the centroid (center of gravity) method, the scikit-fuzzy simulation produced a slightly different result of 2.378 V. This difference arises due to the level of precision in the defuzzification process. In the manual method, we approximated the fuzzy output areas as simple trapezoids and calculated the centroid using only a few geometric values (e.g., top and bottom widths and central points). However, the scikit-fuzzy library performs defuzzification over a finely sampled universe of discourse—typically with steps as small as 0.001 V—allowing it to compute a more accurate numerical centroid of the aggregated output membership function. As a result, the simulation's output reflects a more precise representation of the system's behavior. Therefore, 2.378 V is considered the correct and final defuzzified output voltage for the given input of 2437.4 RPM.