FUNCTIONS

December 2023

1. The interval in which the function f given by $f(x) = x^2 e^{-x}$ is strictly increasing, is

   (a) $(-\infty, \infty)$

   (b) $(-\infty, 0)$

   (c) $(2, \infty)$

   (d) $(0, 2)$

2. The function $f(x) = \frac{x-1}{x(x^2-1)}$ is discontinuous at

   (a) exactly one point

   (b) exactly two points

   (c) exactly three points

   (d) no ponits

3. The function $f : \mathbb{R} \rightarrow [-1,1]$ defined by $f(x) = \cos x$ is

   (a) both one-one and onto

   (b) not one-one, but onto

   (c) one-one, but onto

   (d) neither one-one, nor onto

4. The range of the principal value branch of the function $y = \sec^{-1} x$ is _____.

5. The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is _____.

6. Find the value of k, so that the function $f(x) = \begin{cases} kx^2 + 5 & \text{if} \quad x \le 1 \\ 2 & \text{if} \quad x > 1 \end{cases}$

   is continuous at $x = 1$.

7. Check whether the relation $\mathbb{R}$ in the set $\mathbb{N}$ of natural numbers given by

$$\mathbb{R} = \{(a, b) : \text{a is divisor of b}\} \tag{1}$$

   is reflexive, symmetric or transitive. Also detemine whether $\mathbb{R}$ is an equivalence relation.

8. Prove that :

$$\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right) \tag{2}$$