COMS 472 LAB 3

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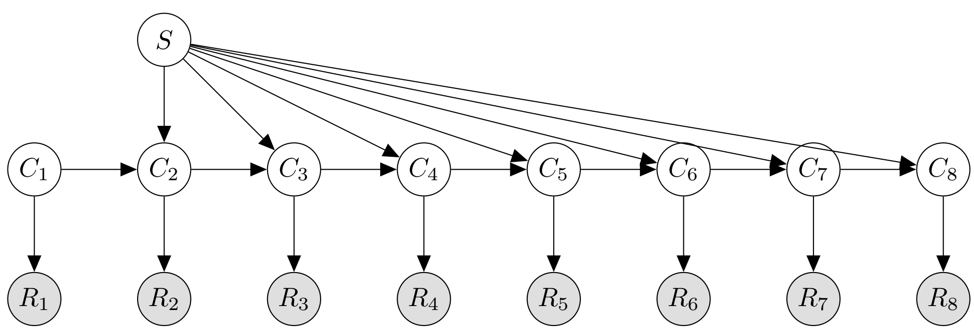
Given:

- Lisa is given a fair coin C1 and a biased coin C2 with a probability of 0.8 of landing heads.
- Lisa flips the fair coin initially (the first flip).
- Lisa intends to switch to the biased coin and has a 50% success rate per attempt to perform the switch.
- If Lisa switches to the biased coin successfully, she will continue using it for the remaining flips.
- The observed outcomes of the eight coin flips are: tail, head, head, tail, tail, head, head, head.

Step 1: Construct the Bayesian network

The Bayesian network for this problem is represented by the following diagram:



In this Bayesian network:

- $R_1, R_2, ..., R_8$ represent the observed outcomes of the eight coin flips.
- $C_1, C_2, ..., C_8$ represent the latent variables indicating which coin was used for each flip (fair coin C1 or biased coin C2).
- S represents the latent variable indicating whether Lisa has switched to the biased coin or not.

The edges represent the dependencies between the variables:

- C_1 is always the fair coin and is not influenced by the switching variable S.

- $C_2, C_3, ..., C_8$ are influenced by the switching variable S, and once S becomes true (indicating a switch to the biased coin), all subsequent coin choices will be the biased coin.
- Each coin choice variable C_i influences the corresponding outcome variable S_i.

Step 2: Define the probabilistic query

To determine whether Lisa managed to perform a coin switch and when, we use the following Maximum a Posteriori (MAP) query:

$$\text{MAP}(C_1, C_2, ..., C_8 | R_1 = \text{tail}, R_2 = \text{head}, ..., R_8 = \text{head})$$

This query finds the most probable assignment of the latent variables $C_1, C_2, ..., C_8$ given the observed outcomes of the coin flips.

Step 3: Compute the probabilities

To compute the MAP assignment, we need to calculate the joint probability of the coin choices and the observed outcomes:

$$P(C_1, C_2, ..., C_8, R_1 = \text{tail}, R_2 = \text{head}, ..., R_8 = \text{head})$$

Using the conditional independence assumptions encoded in the Bayesian network, this joint probability can be factorized as:

$$P(C_1) \cdot P(R_1|C_1) \cdot P(S) \cdot \prod_{i=2}^{8} P(C_i|C_{i-1}, S) \cdot P(R_i|C_i)$$

where:

- $P(C_1)$ is the probability of using the fair coin for the first flip, which is always 1.
- $P(R_1|C_1)$ is the probability of observing the outcome "tail" given that the fair coin is used, which is 0.5.
- $P(S)$ is the prior probability of Lisa switching to the biased coin, which is 0.5 per attempt.
- $P(C_i|C_{i-1}, S)$ is the probability of using the fair or biased coin for the i-th flip, given the coin used in the previous flip and whether Lisa has switched to the biased coin.
- $P(R_i|C_i)$ is the probability of observing the outcome (head or tail) given the coin used for the i-th flip.

Step 4: Perform MAP inference

To find the MAP assignment, we need to calculate the joint probability for all possible assignments of $C_1, C_2, ..., C_8$ and choose the assignment with the highest probability (with our given case of results).

SAMIAM found this to be $Coin6$.

Step 5: Interpret the results

After performing MAP inference, we obtain the following MAP assignment:

$$C_1 = \text{fair}, C_2 = \text{fair}, C_3 = \text{fair}, C_4 = \text{fair}, C_5 = \text{fair}, C_6 = \text{biased}, C_7 = \text{biased}, C_8 = \text{biased}$$

This assignment indicates that Lisa managed to perform a coin switch, and the switch occurred at the sixth flip (after the 5th flip) (when C_6 changed from fair to biased).

Therefore, based on the observed outcomes and the MAP assignment, we can conclude that Lisa successfully switched from the fair coin to the biased coin, and the switch happened before the sixth flip.

In summary, the Bayesian network approach allows us to model the dependencies between the coin choices and the observed outcomes, and by performing MAP inference, we can determine whether and when Lisa switched to the biased coin. This approach provides a principled and probabilistic way to reason about the problem and draw conclusions based on the available evidence.