Vector Space Models & Word Embeddings

Naeemullah Khan

naeemullah.khan@kaust.edu.sa

KAUST Academy

King Abdullah University of Science and Technology

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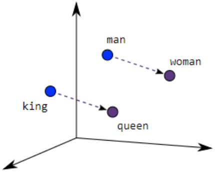
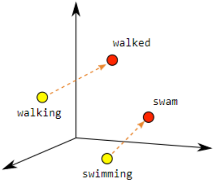
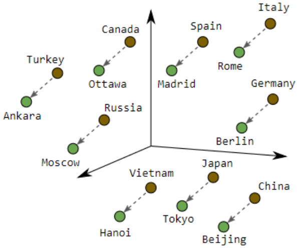


Male-Female



Verb Tense



Country-Capital

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Motivation

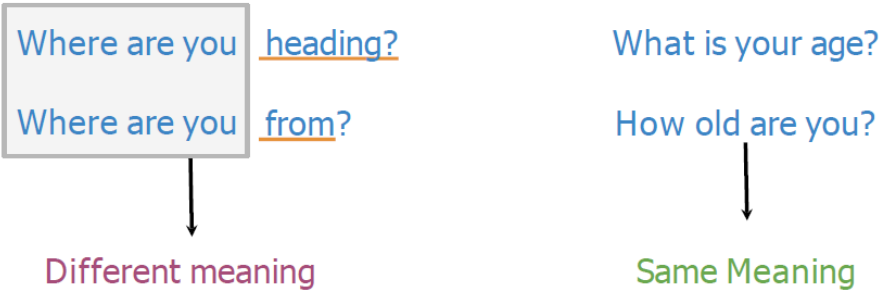
- Text is symbolic; machines require numerical representations to process it.
- Classical NLP struggled with sparse, high-dimensional vectors.
- Vector Space Models (VSMs) and Word Embeddings represent words in a continuous vector space.
- These models capture semantic meaning and relationships geometrically.
- Basis for modern NLP methods including Transformers.

Learning Outcomes

- Understand and implement basic Vector Space Models.
- Differentiate between word-by-word and word-by-document designs.
- Use similarity measures to compute semantic relatedness.
- Explain the rationale behind One-Hot vs. Dense word vectors.
- Understand and implement basic Word2Vec models (CBOW and Skip-gram).
- Appreciate the improvements of GloVe and fastText over earlier models.

Vector Space Model: **Introduction**

Why learn vector space models?



- Words and sentences can have different meanings depending on context.
- Vector space models help capture semantic similarity and differences.
- Useful for tasks like paraphrase detection, question answering, and information retrieval.

"You shall know a word by the company it keeps"

Firth, 1957

(Firth, J. R. 1957:11)

Vector Space Models (VSM)

- VSM represents words or documents as vectors in an n-dimensional space.
- Based on the **distributional hypothesis**: Words that occur in similar contexts have similar meanings.
- Forms the foundation for information retrieval, document classification, and word embeddings.

Vector Space Models – Formal Definition

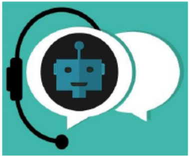
- A **term-document matrix**: Rows represent words (terms), columns represent documents (or vice versa).
- Each cell contains a value such as:
  - Term Frequency (TF)
  - TF-IDF (Term Frequency-Inverse Document Frequency)
  - Co-occurrence count
- The matrix is typically high-dimensional and sparse:
  - Size $= |\text{Vocabulary}| \times |\text{Documents}|$

Vector Space Model Applications

- You eat *cereal* from a *bowl* $\rightarrow$ Capturing semantic similarity (paraphrase understanding)
- You *buy* something and someone else *sells* it $\rightarrow$ Capturing relational meaning (analogies)

Information Extraction

Machine Translation



Chatbots

Word-by-Word vs Word-by-Document Designs

Idea

- Co-occurrence $\rightarrow$ Vector representation
- Relationships between words/documents

Word by Word Design

Number of times they ***occur together within a certain distance*** *k*

I like simple data

I prefer simple raw data

k=2

	simple	raw	like	I
data	2	1	1	0

n

Word by Document Design

Number of times a word *occurs within a certain category*

Corpus

	Entertainment	Economy	Machine Learning
data	500	6620	9320
film	7000	4000	1000

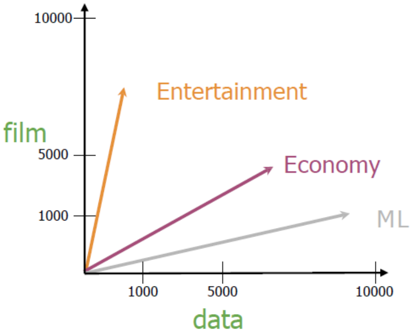
Comparison: Word-by-Word vs Word-by-Document

- **Word-by-Word:**
  - Matrix built from co-occurrence counts between words.
  - Captures semantic similarity directly.
  - Better for word similarity tasks.
- **Word-by-Document:**
  - Matrix built from word frequencies in documents.
  - Useful for document classification and search.
  - Better for document-level tasks.

Tradeoffs: Choice depends on the task: word similarity vs. document analysis.

Similarity Measures

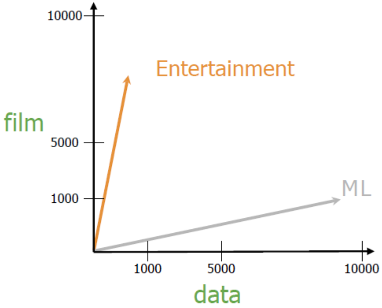
Vector Space



	Entertainment	Economy	ML
data	500	6620	9320
film	7000	4000	1000

Measures of "similarity:"
Angle
Distance

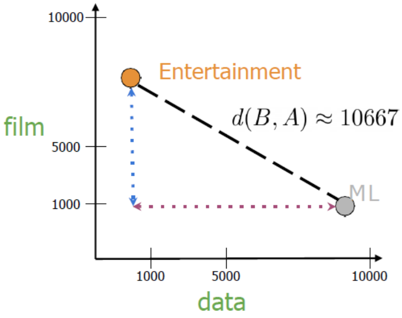
Euclidean Distance



Corpus **A**: (500,7000)

Corpus **B**: (9320,1000)

- Measures the straight-line distance between two points in space.
- Formula: $d(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}$
- Sensitive to the scale (magnitude) of the vectors.



Corpus **A**: (500,7000)

Corpus **B**: (9320,1000)

$$d(B, A) = \sqrt{\underline{(B_1 - A_1)^2} + \underline{(B_2 - A_2)^2}}$$

$$c^2 = a^2 + b^2$$

$$d(B, A) = \sqrt{(-8820)^2 + (6000)^2}$$

Euclidean Distance for n-dimensional vectors

	data	boba ($\vec{w}$)	ice-cream ($\vec{v}$)
AI	6	0	1
drinks	0	4	6
food	0	6	8

$$= \sqrt{(1-0)^2 + (6-4)^2 + (8-6)^2}$$

$$= \sqrt{1+4+4} = \sqrt{9} = 3$$

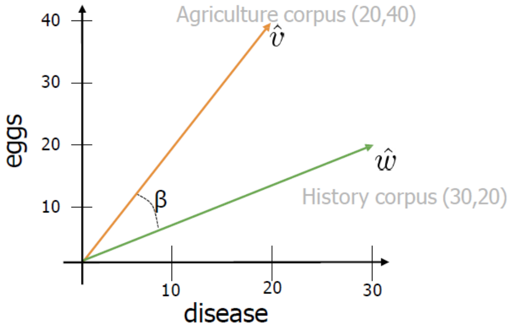
$$d(\vec{v}, \vec{w}) = \sqrt{\sum_{i=1}^{n} (v_i - w_i)^2} \longrightarrow \text{Norm of } (\vec{v} \ \vec{w})$$

Euclidean Distance in Python

```
# Create numpy vectors v and w
v = np.array([1, 6, 8])
w = np.array([0, 4, 6])
# Calculate the Euclidean distance d
d = np.linalg.norm(v-w)
# Print the result
print("The Euclidean distance between v and w is: ", d)
```

```
The Euclidean distance between v and w is: 3
```

Cosine Similarity



$$\hat{v} \cdot \hat{w} = \|\hat{v}\|\|\hat{w}\| \cos(\beta)$$

$$\boxed{\cos(\beta) = \frac{\hat{v} \cdot \hat{w}}{\|\hat{v}\|\|\hat{w}\|}}$$

$$= \frac{(20 \times 30) + (40 \times 20)}{\sqrt{20^2 + 40^2} \times \sqrt{30^2 + 20^2}}$$

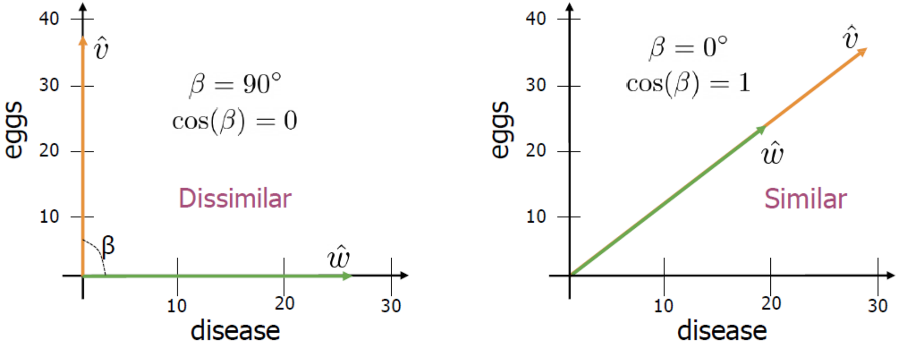
$$= 0.87$$

- Measures the cosine of the angle between two vectors.
- Formula: cosine$(\mathbf{x}, \mathbf{y}) = \frac{\mathbf{x} \cdot \mathbf{y}}{\|\mathbf{x}\|\|\mathbf{y}\|}$
- Ranges from -1 (opposite) to 1 (same direction).

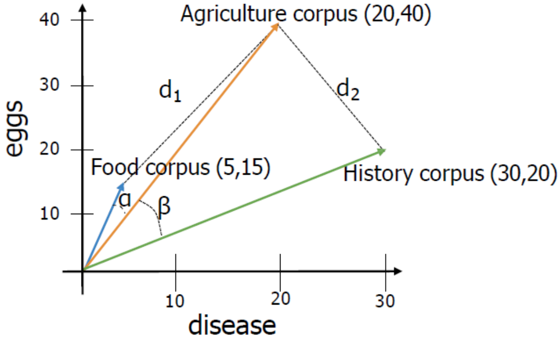
Cosine Similarity (cont.)

- Less sensitive to magnitude, focuses on orientation.

Cosine Similarity (cont.)



- Cosine similarity when corpora are different sizes

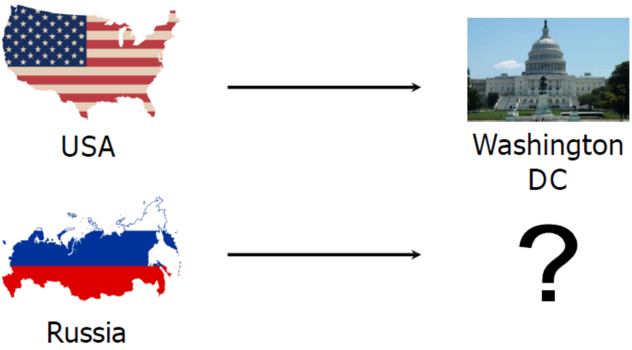


Euclidean distance: $d_2 < d_1$

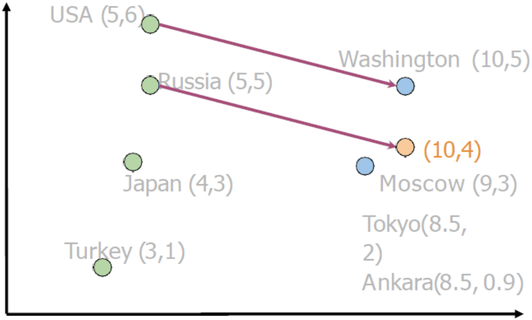
Angles comparison: $\beta > \alpha$

The cosine of the angle between the vectors

Manipulating word vectors



Manipulating word vectors (cont.)



Washington - USA = $[5 \quad -1]$

Russia + $[5 \quad -1]$ = $[10 \quad 4]$

Moscow

[Mikolov et al, 2013, Distributed Representations of Words and Phrases and their Compositionality]

$d > 2$ (brace over the value columns)

oil	0.20	...	0.10
gas	2.10	...	3.40
city	9.30	...	52.1
town	6.20	...	34.3

How can you visualize if your representation captures these relationships?

oil & gas

town & city

Visualization of word vectors (cont.)

$d > 2$

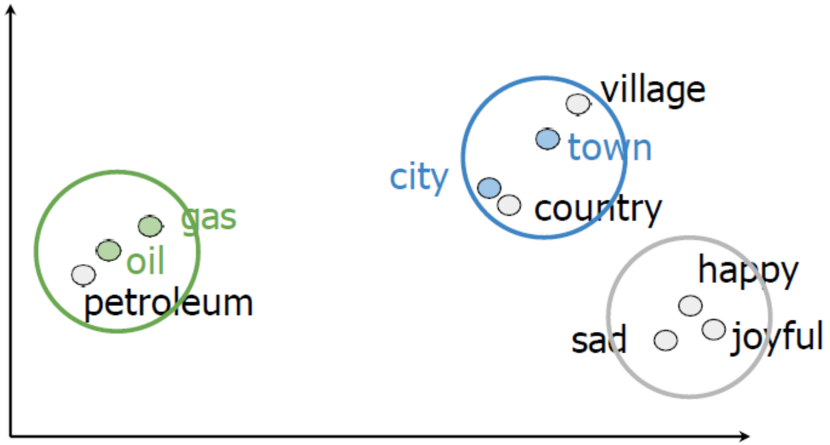
oil	0.20	...	0.10
gas	2.10	...	3.40
city	9.30	...	52.1
town	6.20	...	34.3

$\xrightarrow{\text{PCA}}$

$d = 2$

oil	2.30	21.2
gas	1.56	19.3
city	13.4	34.1
town	15.6	29.8

Visualization of word vectors (cont.)



Word Embedding: **One-Hot vs. Dense Vectors**

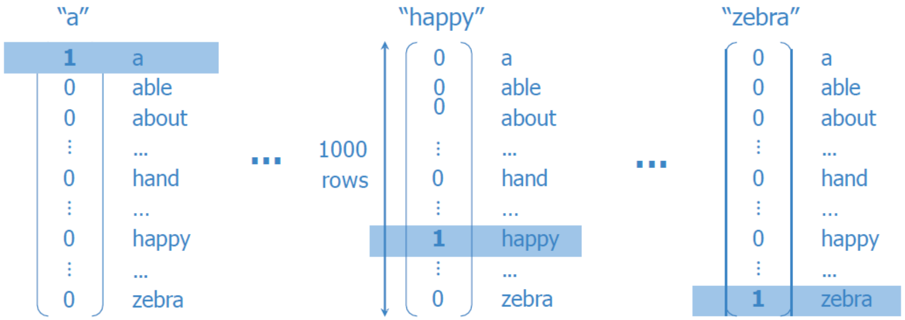
Integer Representation of Words

Word	Number
a	1
able	2
about	3
...	...
hand	615
...	...
happy	621
...	...
zebra	1000

Integer Representation of Words (cont.)

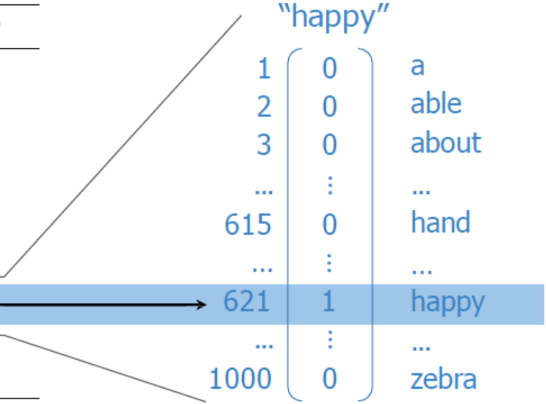
+ Simple
- Ordering: little semantic sense

One-hot vectors



One-hot vectors (cont.)

Word	Number
a	1
able	2
about	3
...	...
hand	615
...	...
happy	621
...	...
zebra	1000



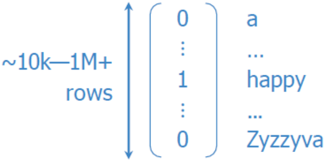
One-hot vectors (cont.)

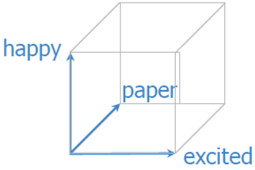
\+ Simple

\+ No implied ordering

\- Huge vectors

\- No embedded meaning





d(paper, excited)
= d(paper, happy)
= d(excited, happy)

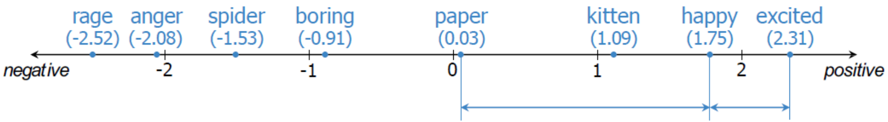
One-Hot Encoding

- Each word is represented as a unique vector.
- Vector has 1 at one position, 0 elsewhere.
- **Problems:**
  - High-dimensional and sparse.
  - No semantic meaning or similarity captured.

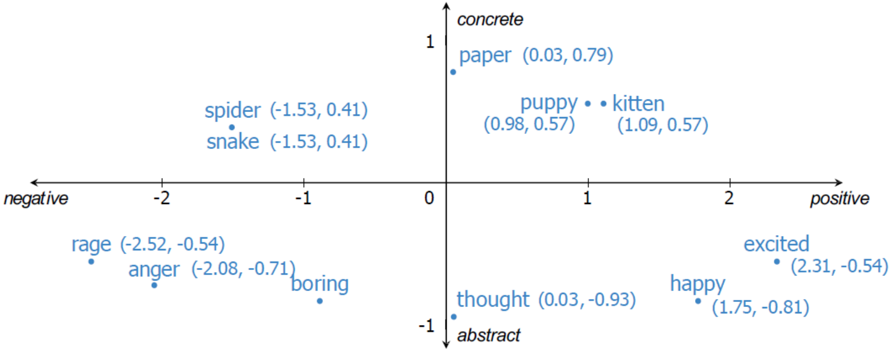
Dense Embeddings

- Vectors are learned from data.
- Low-dimensional (e.g., 100–300).
- Capture semantic relationships.
- Example: $\texttt{"king"} - \texttt{"man"} + \texttt{"woman"} \approx \texttt{"queen"}$

Meaning as vectors



Meaning as vectors (cont.)



Word embedding vectors

$$\begin{array}{cc} & \text{"happy"} \\ \text{\textasciitilde 100—\textasciitilde 1000 rows} \updownarrow & \begin{pmatrix} 0.123 \\ \vdots \\ -4.059 \\ \vdots \\ 1.891 \end{pmatrix} \end{array}$$

+ Low dimension
+ Embed meaning
  - e.g. semantic distance

    forest $\approx$ tree   forest $\not\approx$ ticket

  - e.g. analogies

    Paris:France :: Rome:?

Terminology

integers

word vectors

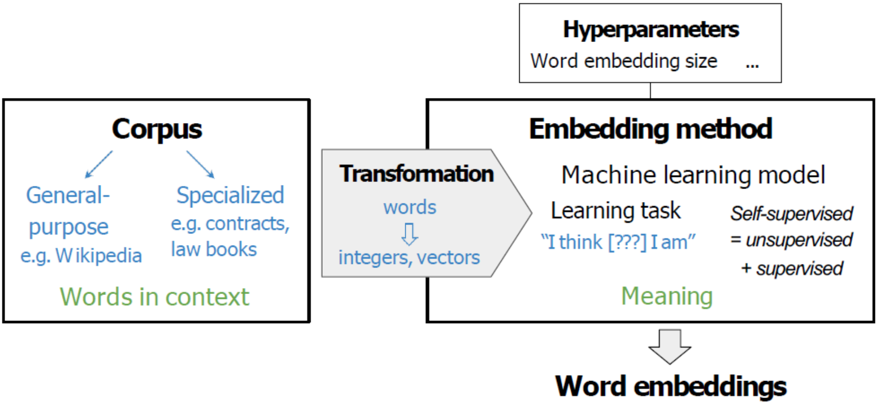
one-hot vectors

word embedding vectors

"word vectors"

word embeddings

Word embedding process



Embedding Method: **Word2Vec**

Word Embeddings – Intuition

- Dense vector representations of words learned from context.
- Encode syntactic and semantic meaning.
- Vectors reflect relationships like synonymy, analogy, etc.

Basic Word Embedding Methods

- **word2vec** (Google, 2013)
  - Continuous bag-of-words (CBOW)
  - Continuous skip-gram / Skip-gram with negative sampling (SGNS)
- **Global Vectors (GloVe)** (Stanford, 2014)
- **fastText** (Facebook, 2016)
  - Supports out-of-vocabulary (OOV) words

Advanced Word Embedding Methods

- Deep learning-based, contextual embeddings:
  - **BERT** (Google, 2018)
  - **ELMo** (Allen Institute for AI, 2018)
  - **GPT-2** (OpenAI, 2018)
- Tunable pre-trained models available

Word2Vec Overview

- Word2Vec learns word embeddings from large text corpora.
- Two main architectures:
  - Continuous Bag-of-Words (CBOW)
  - Skip-gram
- Both models use neural networks to predict words based on context.

Word2Vec – CBOW

- **Goal:** Predict target word from context.
- **Architecture:**
  - **Input:** Surrounding words (context)
  - **Output:** Target word
- Fast and accurate for frequent words.
- **Example:**
  - Context: "the ___ sat on the mat" → Predict "cat"

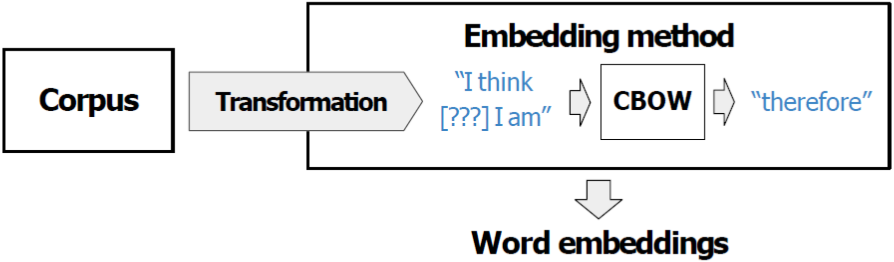


Figure 2: CBOW Architecture

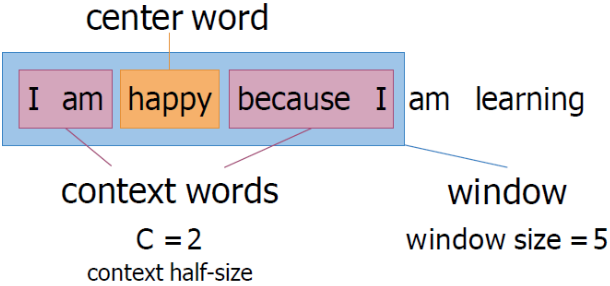
Center word prediction: rationale

Corpus | Transformation | **CBOW**

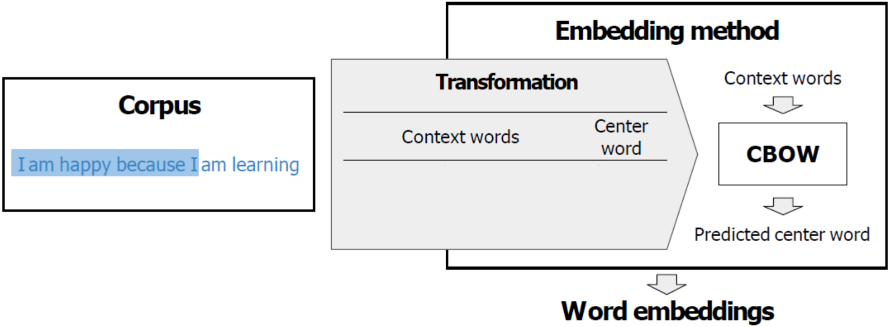
The little ____?____ is barking

dog
puppy
hound
terrier
...

Creating a training example



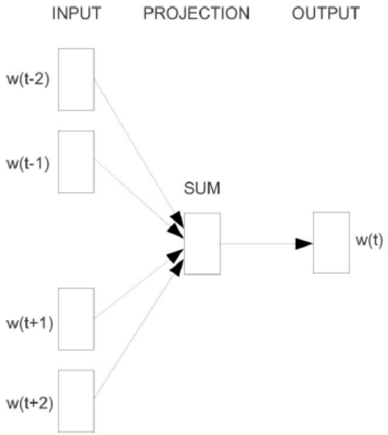
From corpus to training



Word2Vec – Skip-Gram

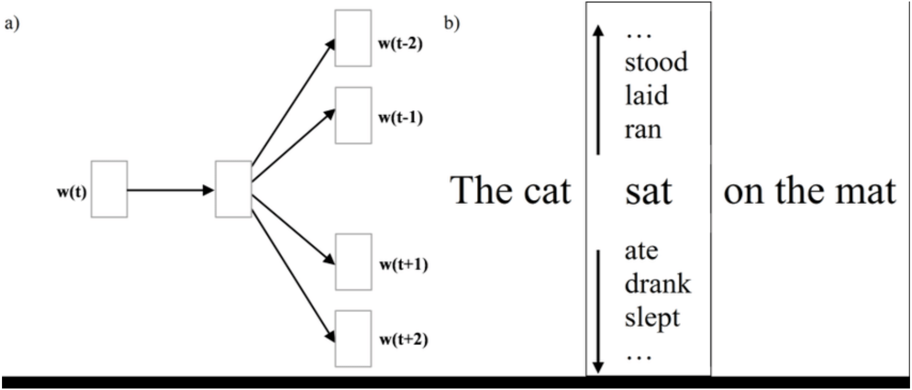
- **Goal:** Predict context words from a target word.
- **Architecture:**
  - **Input:** Target word
  - **Output:** Surrounding words (context)
- Better for rare words.
- **Example:**
  - Input: "cat" $\rightarrow$ Output: "the", "sat", "on", "the", "mat"

Word2Vec – Skip-Gram (cont.)

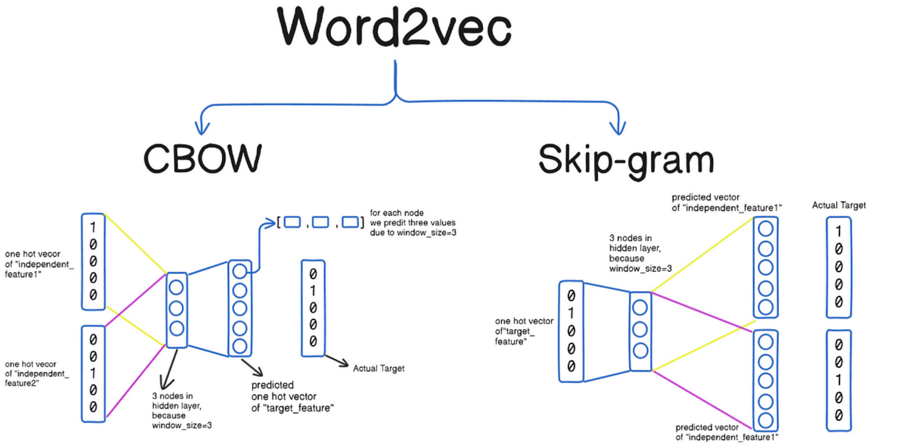


Source: Mikolov, T., Chen, K., Corrado, G.S., & Dean, J. (2013). [Efficient Estimation of Word Representations in Vector Space]

Figure 3: Skip-Gram Architecture


a)
w(t)
w(t-2)
w(t-1)
w(t+1)
w(t+2)
b)
...
stood
laid
ran
The cat
sat
on the mat
ate
drank
slept
...

CBOW vs Skip-Gram



Limitations of Word2Vec

- Does not consider sub-word information.
- Same vector for all senses of a word (polysemy issue).
- Ignores word order within the context window.

GloVe – Global Vectors

GloVe – Global Vectors

Proposed by: Pennington et al., 2014

- Combines global matrix factorization with local context windows.
- Captures co-occurrence statistics of words across entire corpus.
- Embeddings reflect ratios of co-occurrence probabilities.

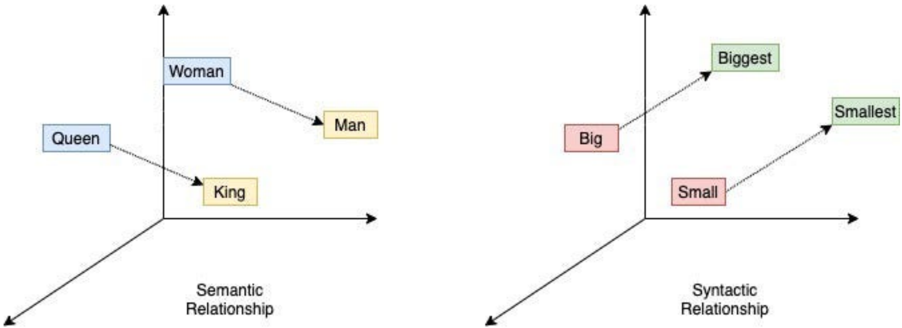
Loss function:

$$J = \sum_{i,j=1}^{V} f(P_{ij}) \left(w_i^\top \tilde{w}_j + b_i + \tilde{b}_j - \log X_{ij} \right)^2$$

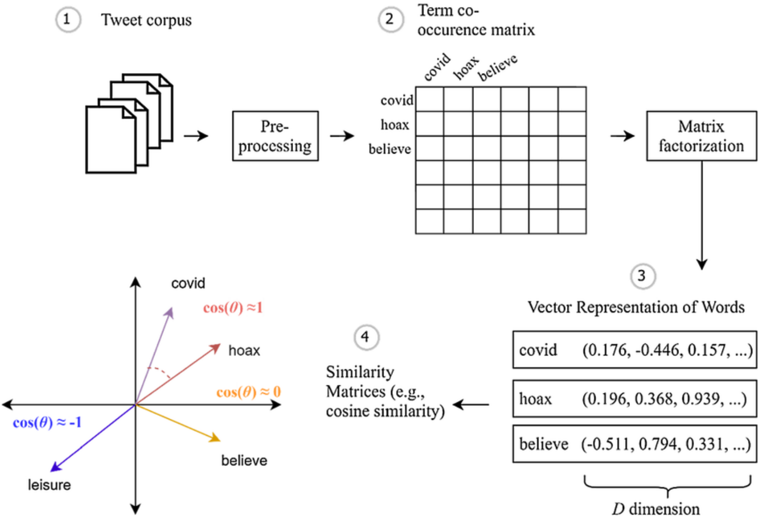
where:

- X_{ij}: number of times word j occurs in the context of word i
- P_{ij}: probability of word j in the context of i
- $w_i, \tilde{w}_j$: word and context word vectors
- $b_i, \tilde{b}_j$: bias terms
- f: weighting function

GloVe – Global Vectors (cont.)



GloVe – Global Vectors (cont.)



fastText – Subword Embeddings

fastText – Key Ideas

- Developed by Facebook AI (2016)
- Builds on Word2Vec by incorporating character n-grams
- Useful for morphologically rich languages and rare words
- Better handling of OOV (out-of-vocabulary) words

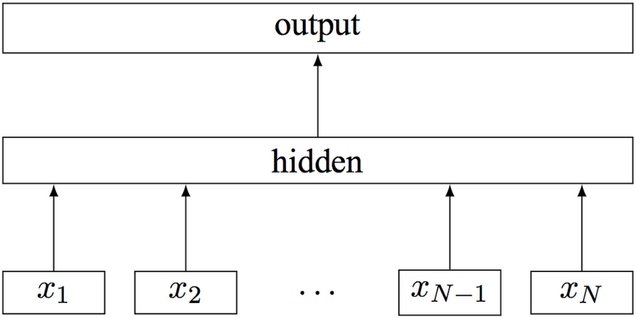
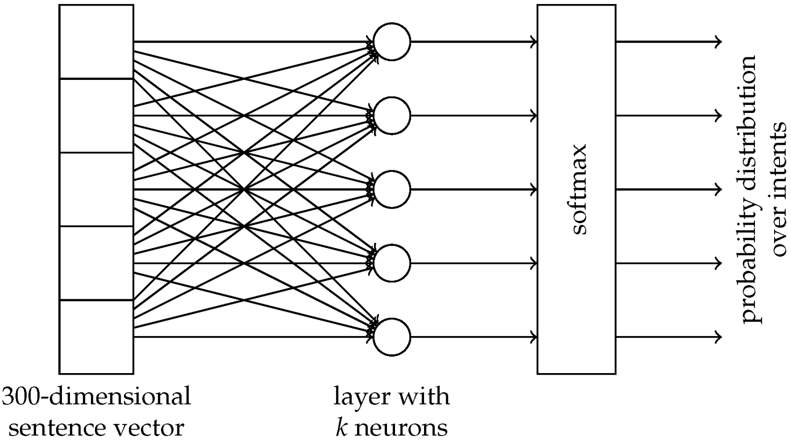


Figure 1: Model architecture of `fastText` for a sentence with N ngram features $x_1, \ldots, x_N$. The features are embedded and averaged to form the hidden variable.



FastText-Based Intent Detection for Inflected Languages

fastText – Example

- Word: `playing`
- Character n-grams (n=3): `["pla", "lay", "ayi", "yin", "ing"]`
- Word vector = sum of n-gram vectors

Feature	Word2Vec	GloVe	fastText
Local Context	✓	–	✓
Global Info	–	✓	✓(partial)
Subword Info	–	–	✓
Handles OOV	–	–	✓

Limitations of Vector Space Models

- **High Dimensionality:**
  - Vectors can become very high-dimensional, leading to computational inefficiency.
  - Curse of dimensionality: distance metrics become less meaningful.
- **Sparsity:**
  - One-hot vectors are sparse, leading to inefficiencies in storage and computation.
  - Dense vectors mitigate this but still require large datasets for effective training.
- **Lack of Context:**
  - Traditional VSMs do not capture word context effectively.
  - Same word can have different meanings in different contexts (polysemy).

Limitations of Vector Space Models (cont.)

- **Semantic Limitations:**
  - Cannot capture complex relationships like negation or antonymy.
  - Similar words may not always be semantically related (e.g., "bank" vs. "river bank").
- **Scalability:**
  - As vocabulary size increases, the term-document matrix becomes larger and more sparse.
  - Requires significant computational resources for training and inference.

Summary

Future Directions

- **Contextual embeddings** (ELMo, BERT, GPT): Word vectors depend on sentence context.
- **Multilingual embeddings** and cross-lingual models.
- **Graph-based embeddings** (e.g., knowledge graph completion).
- **Hybrid embeddings**: Combining structured and unstructured data.

Key Takeaways

- Vector Space Models (VSMs) are foundational for modern NLP.
- Word embeddings capture semantic relationships in a continuous space.
- Word2Vec, GloVe, and fastText are key methods for generating word vectors.
- Dense embeddings outperform one-hot vectors in capturing meaning and relationships.
- Future work focuses on contextual, multilingual, and hybrid embeddings.

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Credits

Dr. Prashant Aparajeya

Computer Vision Scientist — Director(AISimply Ltd)

p.aparajeya@aisimply.uk

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