Deep Unsupervised Learning (Overview)

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Unsupervised Learning - Definition

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- We have a dataset without labels. Out goal is to learn something interesting about the underlying structure of the data:
  - Clusters hidden in the dataset.
  - Outliers: particularly unusual and/or interesting data points.
  - Useful signals hidden in the noise, e.g., human speech over a noisy background.

Components of Unsupervised Learning

- **Data**: Unlabeled data, e.g., images, text, or sensor readings.
- **Model**: A mathematical representation of the data, e.g., a mixture model or a neural network.
- **Objective function**: A measure of how well the model fits the data, e.g., likelihood or reconstruction error.
- **Optimization algorithm**: An algorithm to minimize the objective function, e.g., gradient descent or expectation-maximization.
- **Evaluation metrics**: Measures to assess the quality of the learned model, e.g., silhouette score or clustering accuracy.
- **Applications**: Use cases for unsupervised learning, e.g., clustering, dimensionality reduction, or anomaly detection.

Supervised vs Unsupervised Learning

Aspect	Supervised Learning	Unsupervised Learning
Objective	Learn a function f from labeled input–output pairs.	Discover structure or representations in unlabeled data.
Evaluation	Accuracy, precision/recall on held-out labels.	Clustering validity indices (e.g. silhouette), reconstruction error.
Cost	Methods range from $\mathcal{O}(n)$ to $\mathcal{O}(n^3)$ per fit.	k-means $\mathcal{O}(nkd)$, hierarchical $\mathcal{O}(n^2)$, PCA $\mathcal{O}(nd^2)$.
Labels/Clusters	Fixed, known set of classes.	Number of clusters unknown; must be chosen or inferred.
Output	Classifier or regressor for new inputs.	Cluster assignments, embeddings, density models, or generative samples.

Table 1: Key differences between Supervised and Unsupervised Learning

Unsupervised Learning - Applications

Unsupervised learning is used in various fields and applications, including:

- **Visualisation**: Identifying and making accessiblge useful hidden structures in the data.
- **Anomaly Detection**: Identifying factory components that are likely to break soon.
- **Signal denoising**: Extracting human speech from a noisy recording.
- **Generative Models**: Learning to generate new data points similar to the training data.
- **Feature Learning**: Automatically discovering useful representations of the data.
- **Data Preprocessing**: Cleaning and transforming data for better performance in supervised learning tasks.

Application: Discovering Structure in Digits

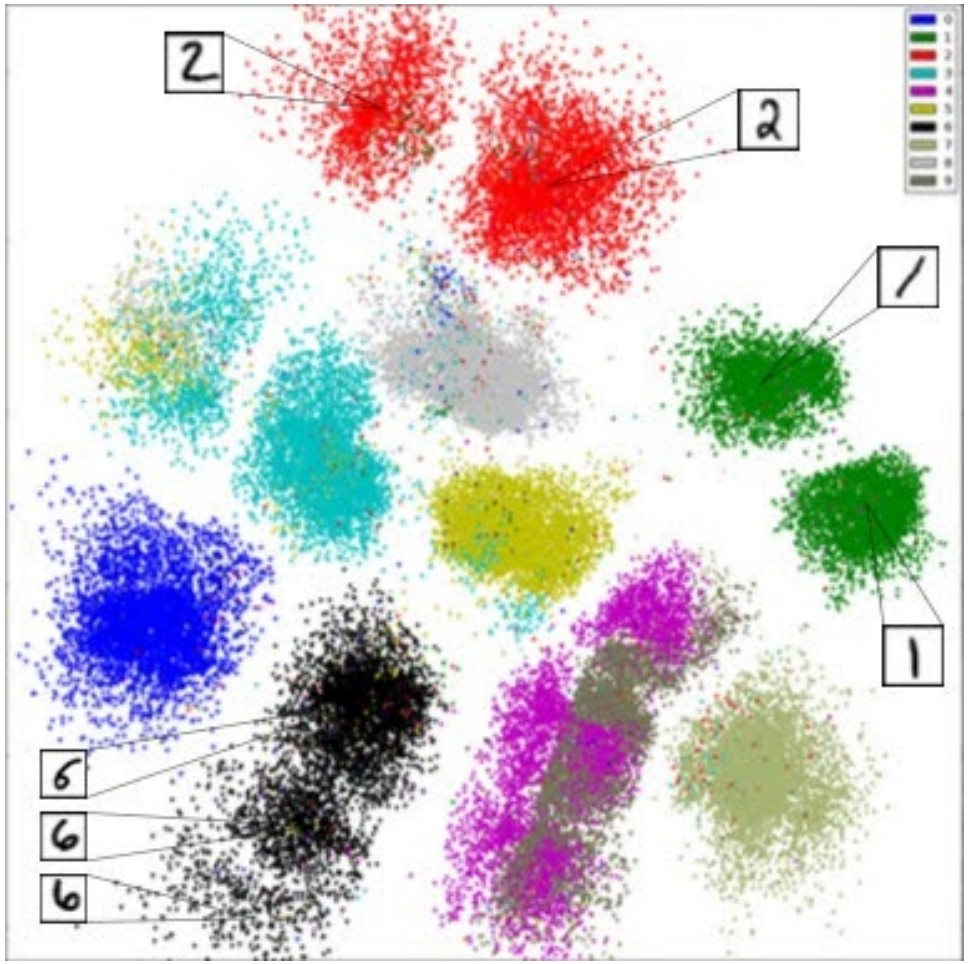


Figure 2: Unsupervised learning can discover structure in digits without any labels.

Application: DNA Analysis

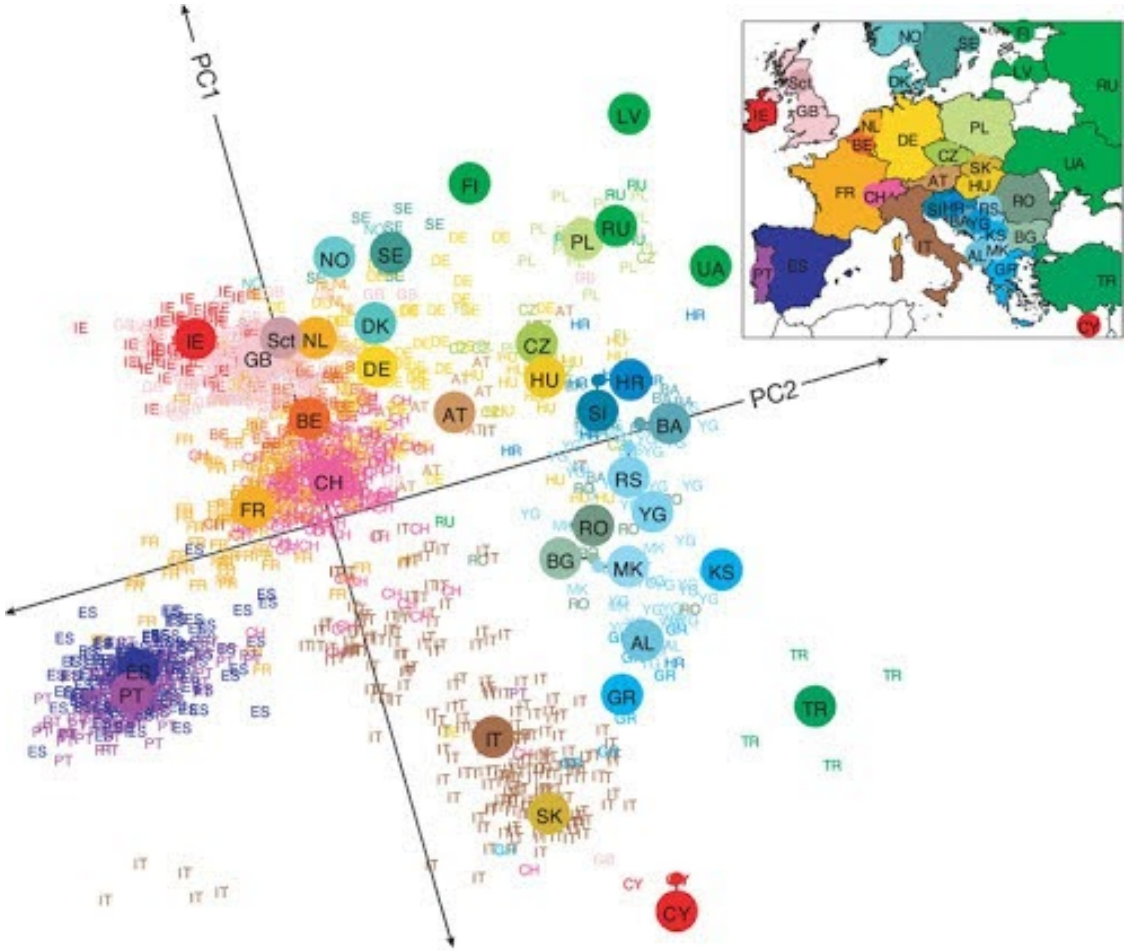


Figure 3: Dimensionality reduction applied to DNA reveal the geography of European countries.

What is Deep Unsupervised Learning?

What is Deep Unsupervised Learning? (cont.)

- Capturing rich patterns in raw data with deep networks in a label-free way.

What is Deep Unsupervised Learning? (cont.)

- Capturing rich patterns in raw data with deep networks in a label-free way.
  - **Generative Models**: Recreate raw data distribution.

Unsupervised Learning - Challenges

Why is unsupervised learning challenging?

- **Exploratory data analysis**: Unsupervised learning is often used for exploratory data analysis, where the goal is to discover patterns or structures in the data without any prior knowledge of the labels.
- **Difficult to assess performance**: Evaluating the performance of unsupervised learning algorithms can be challenging, as there are no ground truth labels to compare against ("right answer" unknown).
- **Sensitivity to noise**: Unsupervised learning algorithms can be sensitive to noise and outliers in the data, which can lead to misleading results.
- **Curse of dimensionality**: As the number of features increases, the data becomes sparse, making it difficult to find meaningful patterns.

Unsupervised Learning - Types

- **Cluster Analysis**:
  - For identifying homogenous subgroups of samples.
  - **Examples**: K-means, hierarchical clustering, DBSCAN.
- **Dimensionality Reduction**:
  - For finding a low-dimensional representation to characterize and visualize the data.
  - Reducing the number of features in a dataset while preserving important information.
  - **Examples**: PCA, t-SNE, UMAP.
- **Anomaly Detection**:
  - **Finding outliers in the dataset**: Identifying unusual (rare items, events, or observations) data points that do not conform to expected patterns.
  - **Examples**: Isolation Forest, One-Class SVM, Autoencoders.

Clustering

A set of methods for finding subgroups within the dataset.

- Observations should share common characteristics within the same group, but differ across groups.
- Groupings are determined from attributes of the data itself — differs from classification.

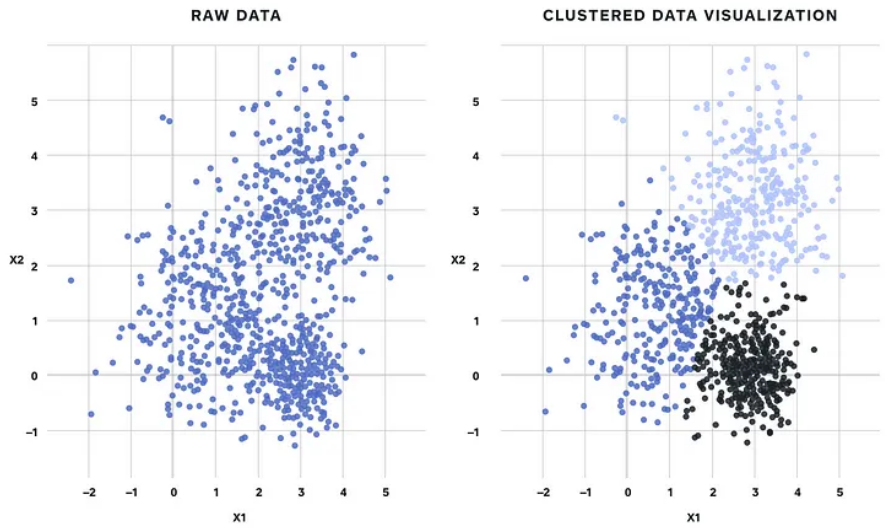


Figure 4: Taking a 2 dimensional dataset and separating it into 3 distinct clusters. [Source]

Clustering (cont.)

Input: Dataset $D = \{x_1, x_2, \ldots, x_n\}$, number of clusters k
Output: Cluster assignments for each data point

Initialization: Randomly initialize k cluster centroids or seeds;

repeat

> **Assignment Step:** Assign each data point x_i to the nearest cluster based on a distance metric;
>
> **Update Step:** Recompute cluster centroids using current assignments;

until *convergence or maximum iterations reached*;

return Final cluster assignments;

Algorithm: Generic Clustering Algorithm

Clustering Vs Classification

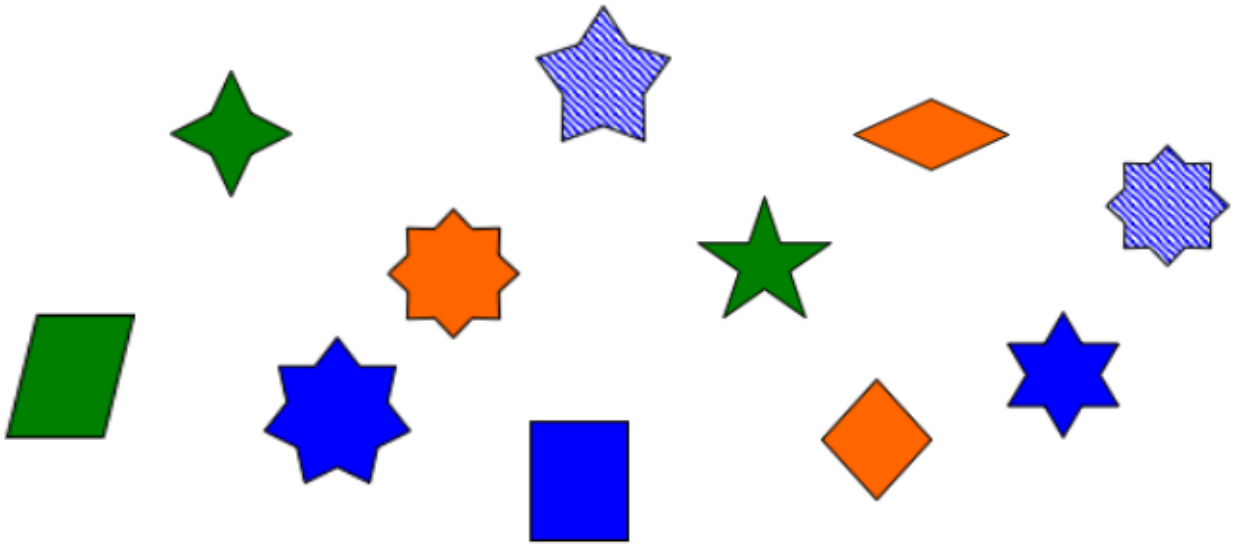


Figure 5: Sample data points.

Clustering Vs Classification (cont.)

Classification

- Labels available
- Assigning to known classes
- Supervised

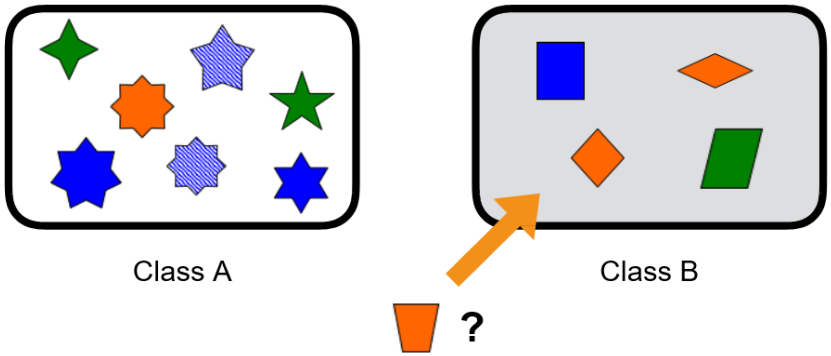


Figure 6: Classification result.

Clustering

- No labels
- Grouping based on similarity
- Unsupervised

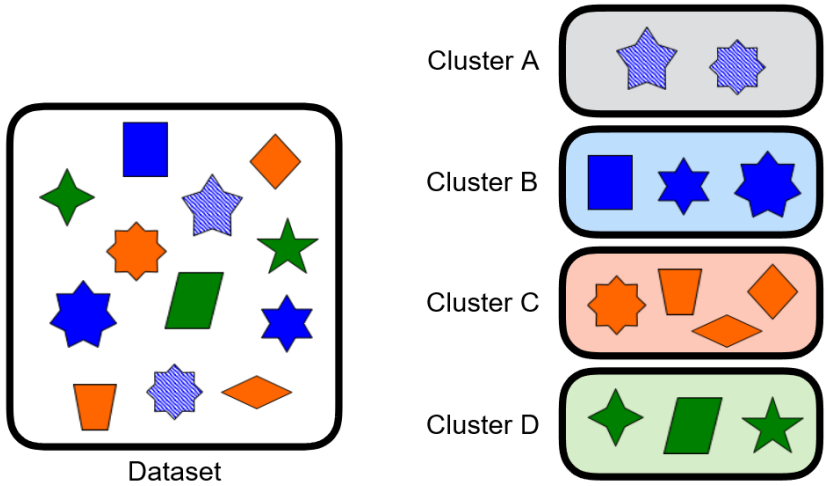


Figure 7: Clustering result.

Clustering: Types

- **Centroid-Based Clustering**: Groups data points based on their proximity to a central point, such as K-means or K-medoids.
- **Hierarchical Clustering**: Builds a hierarchy of clusters using either agglomerative (bottom-up) or divisive (top-down) approaches.
- **Model-Based Clustering**:
  - Each cluster is represented by a parametric distribution.
  - Dataset is a mixture of distributions.
  - Assumes a probabilistic model for the data and uses statistical methods to identify clusters, such as Gaussian Mixture Models (GMM).
- **Hard Clustering**:
  - Each data point is assigned exclusively to exactly one cluster.
  - **Example algorithms**: K-means, Hierarchical clustering.

Clustering: Types (cont.)

- **interpretation**: No ambiguity — clusters are crisp and non-overlapping.

- **Soft/Fuzzy Clustering**:
  - Each data point can belong to multiple clusters simultaneously with varying degrees of membership (probabilities or weights).
  - **Example algorithms**: Gaussian Mixture Models (GMM), Fuzzy C-means.
  - **interpretation**: Reflects uncertainty or mixed membership — clusters can overlap.

Clustering - K-means

Groups data into K clusters that satisfy two properties.

1. Each observation belongs to at least one of the K clusters.
2. Clusters are non-overlapping. No observation belongs to more than one cluster.

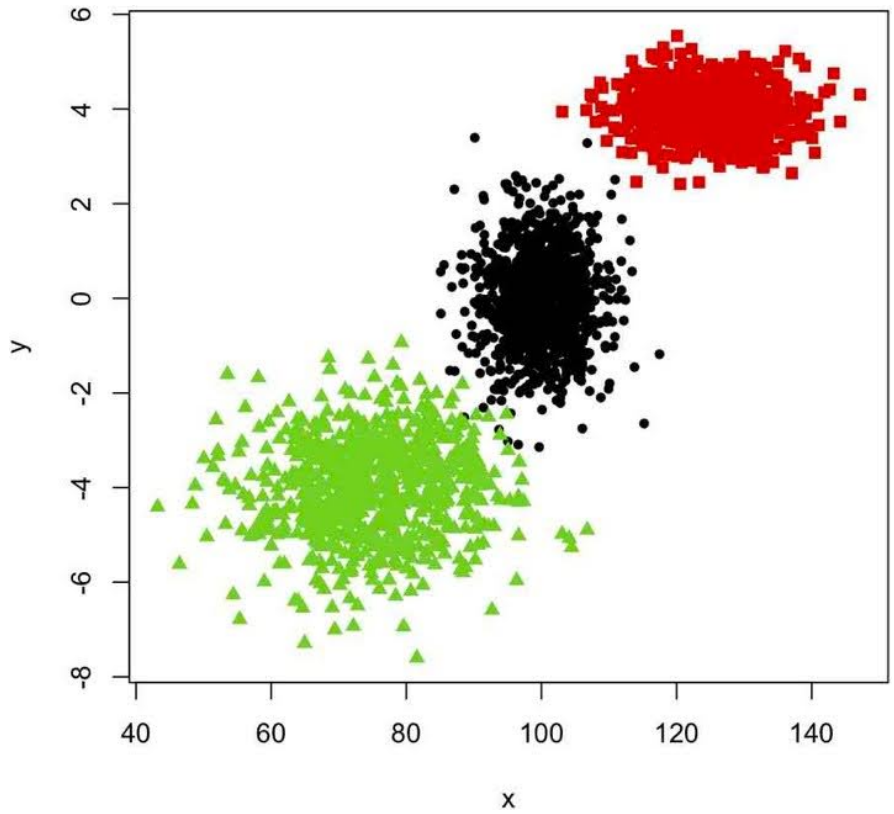


Figure 8: Clusters.

Clustering - K-means (cont.)

A good clustering is one for which the *within-cluster variation* is as small as possible.

Denote each cluster by C_k, and let $W(C_k)$ be a measure of the within-cluster variation.

K-means aims to solve the following optimization problem:

$$\underset{C_1,\ldots,C_k}{\text{minimise}} \left\{ \sum_{k=1}^{K} W(C_k) \right\} \quad (1)$$

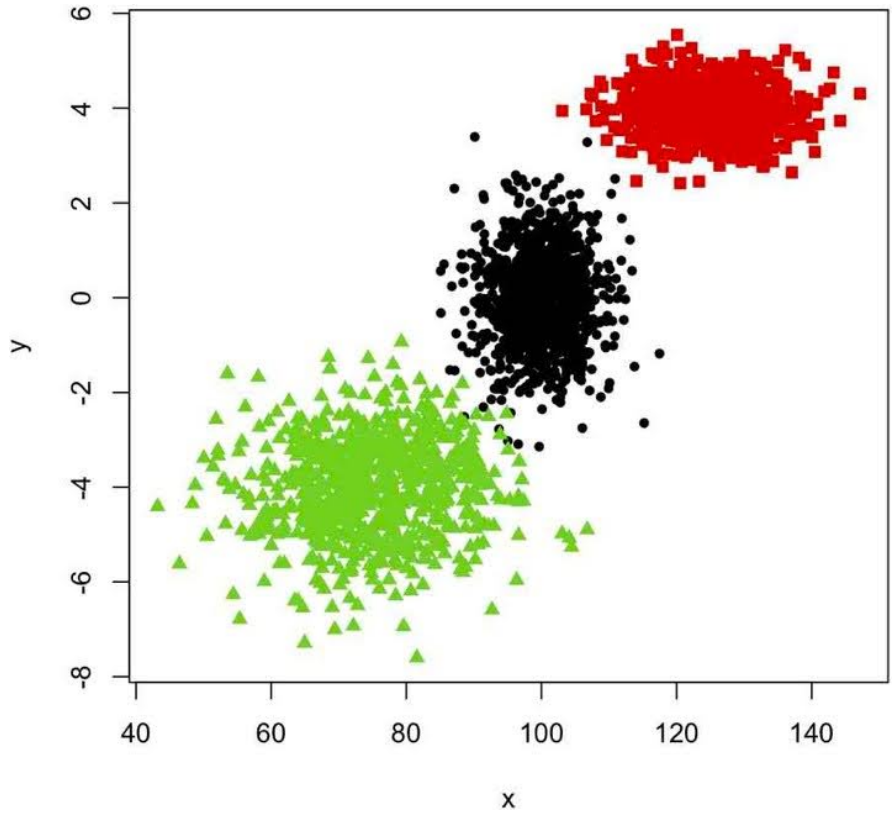


Figure 9: Clusters.

Clustering - K-means (cont.)

How to measure within-cluster variation?

The most common choice is squared Euclidean distance:

$$W(C_k) = \frac{1}{|C_k|} \sum_{i,i' \in C_k} \sum_{j=1}^{p} (x_{ij} - x_{i'j})^2 \tag{2}$$

where $|C_k|$ is the number of points in cluster C_k and x_{ij} is the j^{th} feature of the i^{th} point.

Which means overall we solve:

$$\underset{C_1,\ldots,C_k}{\text{minimise}} \left\{ \sum_{k=1}^{K} \frac{1}{|C_k|} \sum_{i,i' \in C_k} \sum_{j=1}^{p} (x_{ij} - x_{i'j})^2 \right\} \tag{3}$$

Clustering - K-means (cont.)

- It turns out that this optimization problem is difficult to solve, as it is discrete and there are nearly K^n ways to split n samples into K clusters.
- In practice, use an iterative algorithm that finds a local minimum to this optimization.

Clustering - K-means (cont.)

Input: Dataset $D = \{x_1, x_2, \ldots, x_n\}$, number of clusters k
Output: Cluster assignments for each data point

Initialization: Randomly initialize k cluster centroids or seeds;

Repeat until convergence:

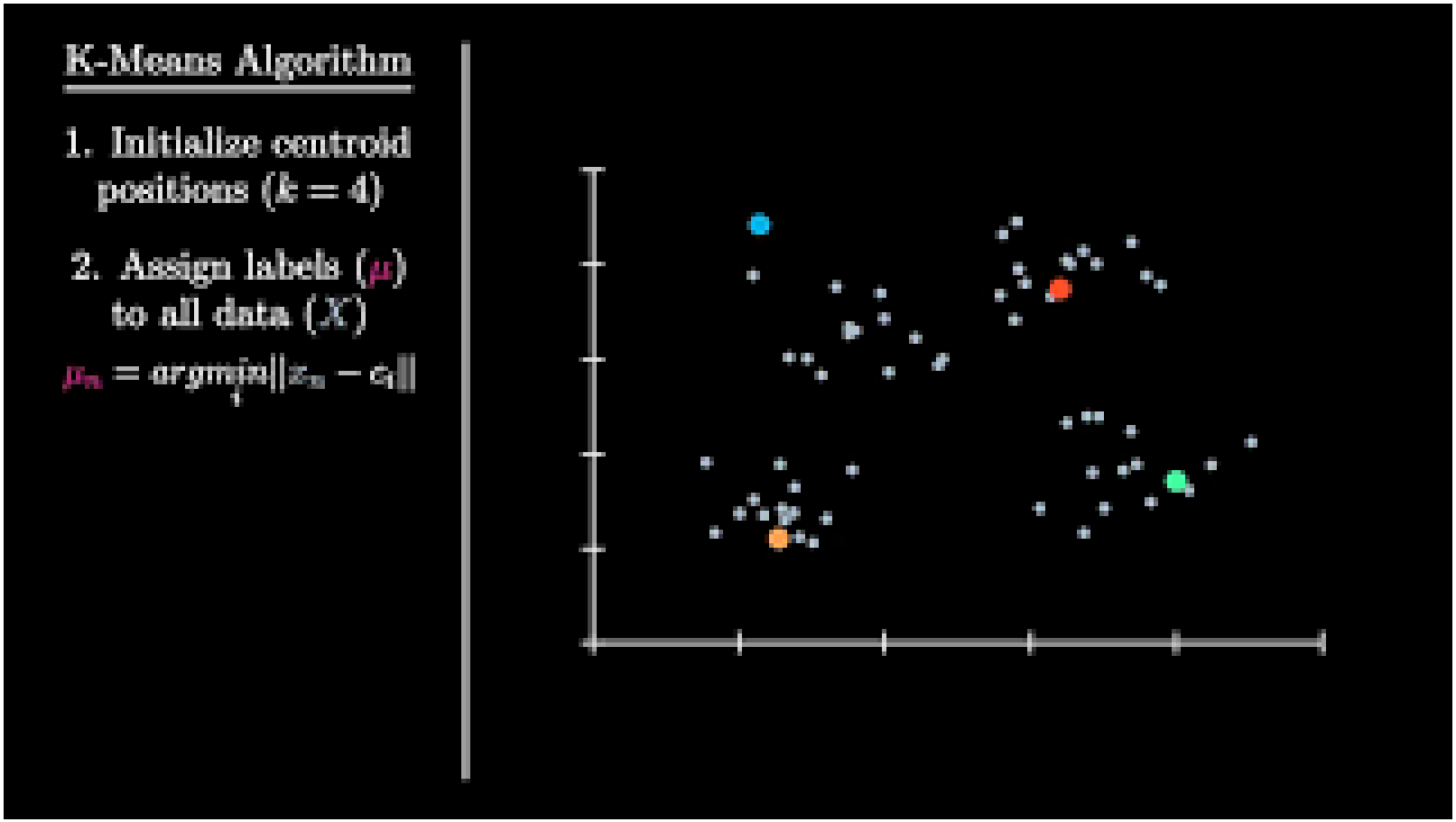
- **Assignment Step:** Assign each data point x_i to the nearest cluster based on a distance metric;
- **Update Step:** Recompute cluster centroids using current assignments;
- **Convergence Check:** Check if cluster assignments have changed or if centroids have stabilized;

Return: Final cluster assignments and centroids;

Algorithm: K-means Clustering Algorithm

Clustering - K-means (cont.)

Watch the K-means clustering algorithm in action:



Clustering - K-means (cont.)

1. It can be shown that the value of the objective function will never increase at each iteration of k-means.

2. Since the algorithm finds local minima, however, it will result in different clusters with different initializations.

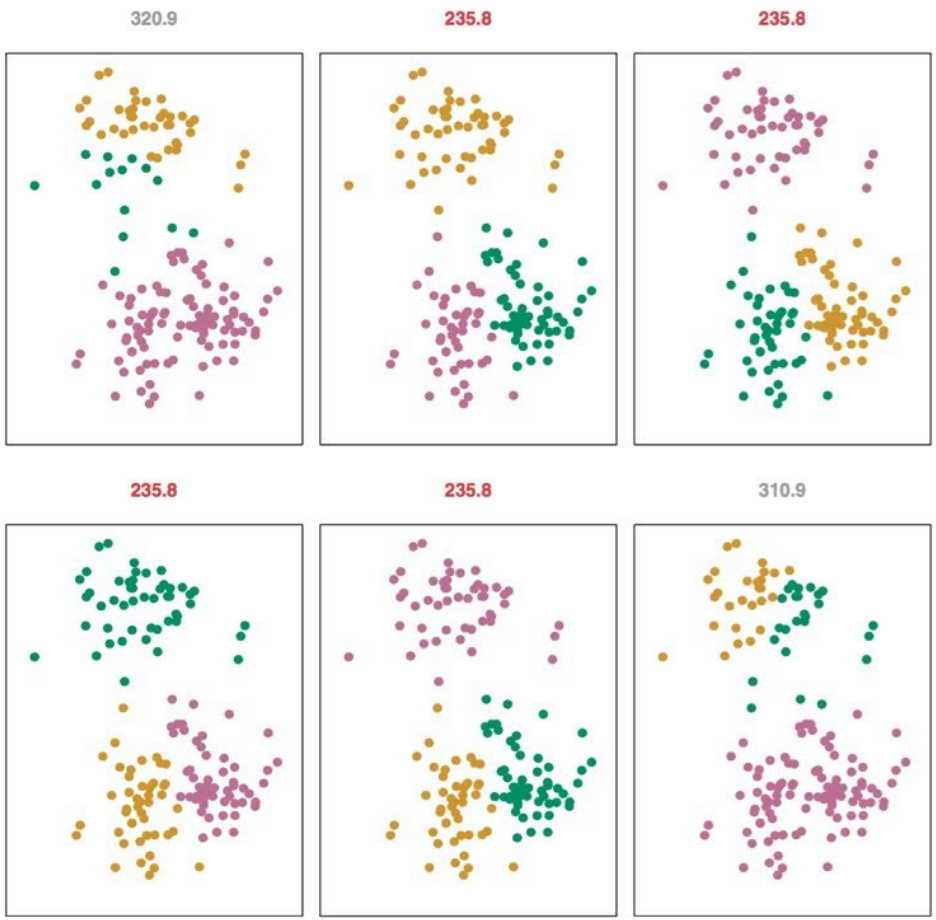


Figure 10: Different initializations of K-means.

K-means - Pros and Cons

Pros

- Simple and easy to implement
- Efficient for large datasets
- Works well with spherical clusters
- Scalable to large datasets

Cons

- Not robust to data perturbations and different initializations
- Sensitive to initial centroid placement
- Assumes spherical clusters
- Requires specifying the number 'K' of clusters in advance
- Sensitive to outliers
- May converge to local minima
- Not suitable for non-convex shapes