

Reinforcement Learning

Policy Gradient, REINFORCE & Actor-Critic Methods

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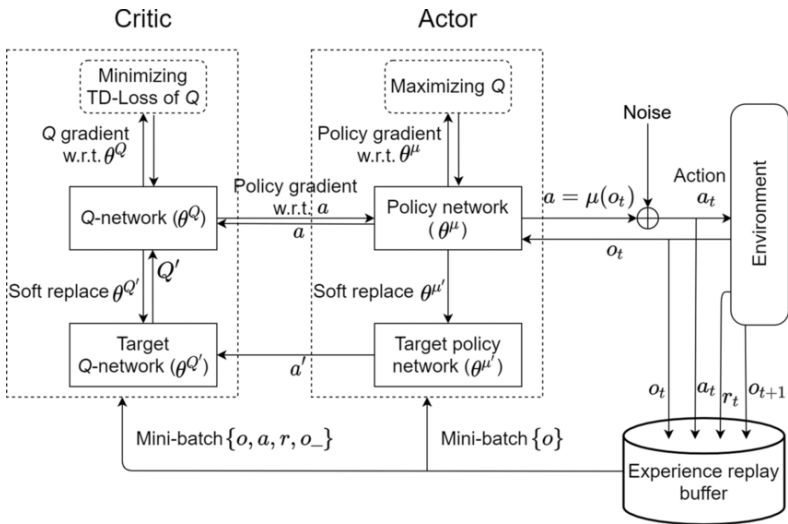


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- ▶ A Markov Decision Process (MDP) defines the RL problem using:
 - States \mathcal{S}
 - Actions \mathcal{A}
 - Rewards \mathcal{R}
 - Transition probabilities \mathbb{P}
 - Discount factor γ
- ▶ Markov property: The future depends only on the present state, not the past.
- ▶ Agent and environment interact in a loop.
- ▶ Policy π decides the agent's actions.

- ▶ Value function: Measures how good a state is.
- ▶ Q-value function: Measures how good a state-action pair is.
- ▶ Bellman equation: Recursively defines value and Q-value functions.
- ▶ Q-learning: Updates Q-values to reduce Bellman error.
- ▶ Deep Q-Learning: Uses neural networks to approximate Q-values.
- ▶ SARSA: On-policy version of Q-learning.

By the end of this session, you will be able to:

- ▶ Understand the **limitations of value-based methods** like Q-learning.
- ▶ Formally define and derive the **Policy Gradient** objective.
- ▶ Implement and interpret the **REINFORCE algorithm**.
- ▶ Explain **Actor-Critic architectures** and their benefits.
- ▶ Evaluate policy-based methods in different environments.

- ▶ Value-based methods learn Q-values and derive the policy indirectly.
 - Inefficient in continuous or large action spaces
 - Can't represent stochastic policies
 - May lead to high variance and instability

Policy Gradient Methods:

- Learn policy parameters directly to maximize expected return:

$$J(\theta) = \mathbb{E}_{\pi_{\theta}}[R]$$

Reinforcement Learning: **Policy Gradients**

- ▶ **What is the problem with Q-learning?**
- ▶ The Q-function can be very complex.
- ▶ For example, a robot grasping an object may have a very high-dimensional state space. It can be difficult to learn the exact Q-value for every (state, action) pair.

- ▶ **What is the problem with Q-learning?**
- ▶ The Q-function can be very complex.
- ▶ For example, a robot grasping an object may have a very high-dimensional state space. It can be difficult to learn the exact Q-value for every (state, action) pair.
- ▶ However, the policy itself can be much simpler; for instance, just closing the robot's hand.
- ▶ Can we learn a policy directly, i.e., find the best policy from a set of possible policies?

- Formally, let us define a class of parameterized policies:

$$\Pi = \{\pi_{\theta} \mid \theta \in \mathbb{R}^m\}$$

- For each policy, we can define its expected return:

$$\mathcal{J}(\theta) = \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t \mid \pi_{\theta} \right]$$

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- Our goal is to find the optimal policy: $\theta^* = \arg \max_{\theta} \mathcal{J}(\theta)$
- How can we achieve this?

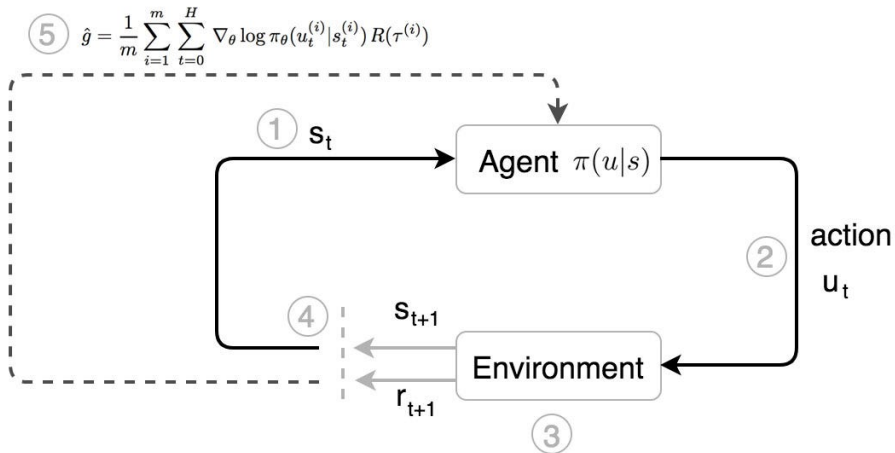
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- How can we achieve this?
- **Solution:** Perform gradient ascent on the policy parameters!



Reinforcement Learning: **REINFORCE**

- ▶ REINFORCE is an elegant algorithm for maximizing the expected return.
- ▶ Intuition: trial and error.
- ▶ Sample a trajectory τ . If you get a high reward, try to make it more likely; if you get a low reward, try to make it less likely.
- ▶ A trajectory is a sequence of states, actions, and rewards:
 $\tau = (s_0, a_0, r_0, s_1, a_1, \dots)$.

- Expected reward:

$$\begin{aligned}\mathcal{J}(\theta) &= \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau)] \\ &= \int_{\tau} r(\tau) p(\tau; \theta) d\tau\end{aligned}$$

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- Now let's differentiate this:

$$\nabla_{\theta} \mathcal{J}(\theta) = \int_{\tau} r(\tau) \nabla_{\theta} p(\tau; \theta) d\tau$$

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- But this is **intractable**!

- ▶ However, we can use a useful trick:

$$\begin{aligned}\nabla_{\theta} p(\tau; \theta) &= p(\tau; \theta) \frac{\nabla_{\theta} p(\tau; \theta)}{p(\tau; \theta)} \\ &= p(\tau; \theta) \nabla_{\theta} \log p(\tau; \theta)\end{aligned}$$

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- ▶ Now, if we substitute this back:

$$\begin{aligned}\nabla_{\theta} \mathcal{J}(\theta) &= \int_{\tau} (r(\tau) \nabla_{\theta} \log p(\tau; \theta)) p(\tau; \theta) d\tau \\ &= \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau) \nabla_{\theta} \log p(\tau; \theta)]\end{aligned}$$

- ▶ We can estimate this with Monte Carlo sampling.

► Recall,

$$p(\tau; \theta) = \prod_{t \geq 0} p(s_{t+1} | s_t, a_t) \pi_{\theta}(a_t | s_t)$$

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► Thus,

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- When differentiating:

$$\nabla_{\theta} \log p(\tau; \theta) = \sum_{t \geq 0} \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

- It doesn't depend on the transition probabilities!

- Therefore, when sampling a trajectory τ , we can estimate $\nabla_{\theta} \mathcal{J}(\theta)$ as:

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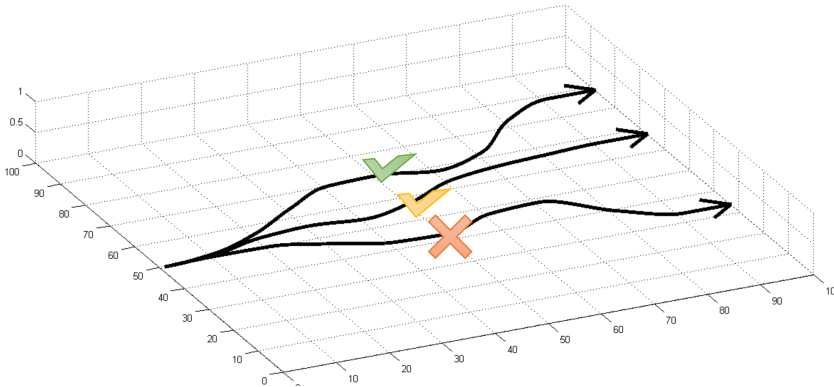
- Interpretation:

- If $r(\tau)$ is high, increase the probabilities of the actions taken.
- If $r(\tau)$ is low, decrease the probabilities of the actions taken.

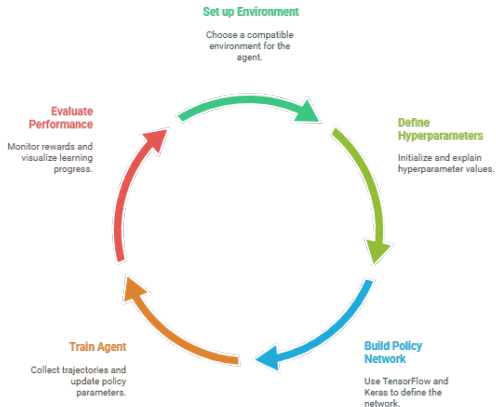
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 - If $r(\tau)$ is high, increase the probabilities of the actions taken.
 - If $r(\tau)$ is low, decrease the probabilities of the actions taken.
- It might seem simplistic to say that if a trajectory is good, then all its actions were good. But in expectation, it averages out!



REINFORCE Algorithm Implementation Cycle

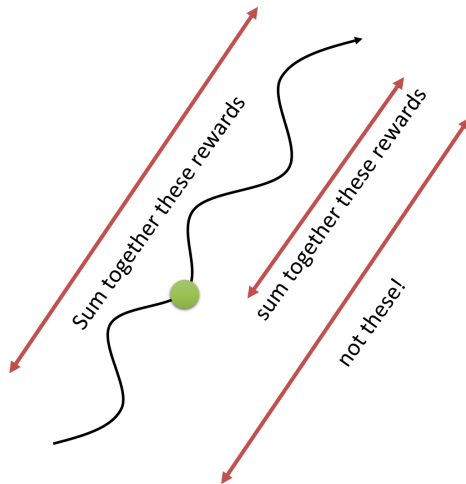


Reinforcement Learning: **Variance Reduction**

- ▶ However, there is a problem.
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- ▶ Can we help the estimator?

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- ▶ This approach suffers from high variance because credit assignment is difficult.
- ▶ Can we help the estimator?
- ▶ **First idea:** Increase the probability of an action only by the cumulative future reward from that state:

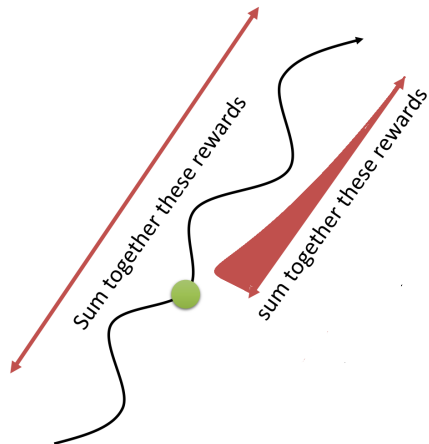
$$\nabla_{\theta} \mathcal{J}(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} r_{t'} \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$



- ▶ But this still doesn't completely solve the credit assignment problem.
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- ▶ It can lead to bias due to delayed rewards.
- ▶ **Second idea:** Use a discount factor γ to reduce the effect of delayed rewards:

$$\nabla_{\theta} \mathcal{J}(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$



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- ▶ **What is important then?** Whether a reward is better or worse than what you expect to get.
- ▶ **Idea:** Introduce a baseline function dependent on the state:

$$\nabla_{\theta} \mathcal{J}(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} - b(s_t) \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

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- ▶ A simple baseline: the moving average of rewards experienced so far from all trajectories.

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- ▶ Essentially, we want to increase the probability of an action from a state if this action was better than the **expected value** from that state.
- ▶ What does this remind you of?
- ▶ **Answer:** Q-function and value function!

- ▶ Intuitively, we are happy with an action a_t in a state s_t if $Q^\pi(s_t, a_t) - V^\pi(s_t)$ is large. In contrast, we are unhappy if it is small.

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- ▶ The term $Q^\pi(s_t, a_t) - V^\pi(s_t)$ is called the **Advantage** and is denoted by $A^\pi(s_t, a_t)$.
- ▶ Using this, we get the estimator:

$$\nabla_\theta \mathcal{J}(\theta) \approx \sum_{t \geq 0} (Q^\pi(s_t, a_t) - V^\pi(s_t)) \nabla_\theta \log \pi_\theta(a_t | s_t)$$

Reinforcement Learning: **Actor-Critic**

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- ▶ We can also incorporate Q -learning techniques, such as experience replay.
- ▶ **Remark:** The advantage function measures how much better an action was compared to the expected value.

Initialize policy parameters θ , critic parameters ϕ

For iteration=1, 2 ... **do**

Sample m trajectories under the current policy

$\Delta\theta \leftarrow 0$

For $i=1, \dots, m$ **do**

For $t=1, \dots, T$ **do**

$$A_t = \sum_{t' \geq t} \gamma^{t'-t} r_t^i - V_\phi(s_t^i)$$

$$\Delta\theta \leftarrow \Delta\theta + A_t \nabla_\theta \log(a_t^i | s_t^i)$$

$$\Delta\phi \leftarrow \sum_i \sum_t \nabla_\phi \|A_t^i\|^2$$

$$\theta \leftarrow \alpha \Delta\theta$$

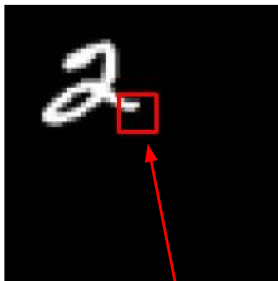
$$\phi \leftarrow \beta \Delta\phi$$

End for

REINFORCE in Action: **Recurrent Attention Model (RAM)**

- ▶ **Objective:** Image classification
- ▶ The model takes a sequence of “glimpses,” selectively focusing on regions of the image to predict the class.
 - Inspired by human perception and eye movements
 - Saves computational resources \Rightarrow improves scalability
 - Can ignore clutter or irrelevant parts of the image
- ▶ **State:** Glimpses observed so far
- ▶ **Action:** (x, y) coordinates (center of the next glimpse) indicating where to look next in the image
- ▶ **Reward:** 1 at the final timestep if the image is correctly classified, 0 otherwise

⁰[Mnih et al., 2014]

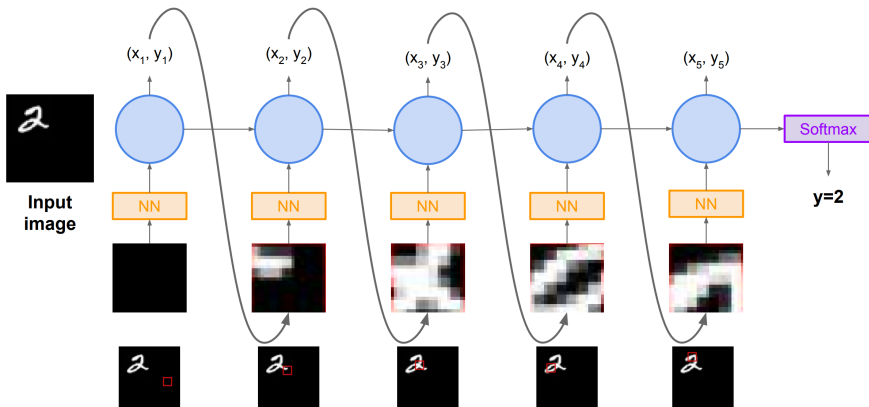


glimpse

- ▶ Glimpsing is a non-differentiable operation.
- ▶ The policy for selecting glimpse locations is learned using REINFORCE.
- ▶ Given the sequence of glimpses observed so far, an RNN models the state and outputs the next action.

⁰[Mnih et al., 2014]

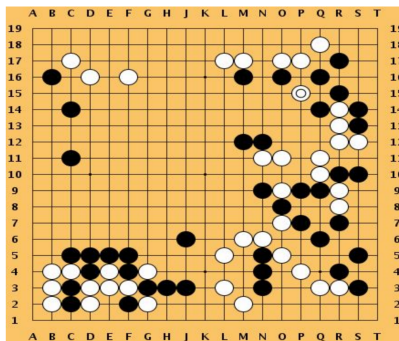
REINFORCE in Action: Recurrent Attention Model (RAM)



⁰[Mnih et al., 2014]

How to Beat the Go World Champion: **AlphaGo**

How to Beat the Go World Champion - AlphaGo



- ▶ Combination of supervised learning and reinforcement learning
- ▶ Integration of traditional methods (Monte Carlo Tree Search) with modern approaches (deep reinforcement learning)

⁰[Silver et al., Nature 2016]

- ▶ Featurize the board (stone color, move legality, biases, etc.)
- ▶ Initialize the policy network with supervised training on professional Go games, then continue training using policy gradients (self-play from random previous iterations, with $+1/-1$ reward for winning/losing)
- ▶ Learn a value network (critic) to estimate the value of board positions
- ▶ Finally, combine the policy and value networks within a Monte Carlo Tree Search algorithm to select actions via lookahead search

⁰[Silver et al., Nature 2016]

Reinforcement Learning: **Value-Based** and **Policy-Based** Methods

► Value-Based Methods

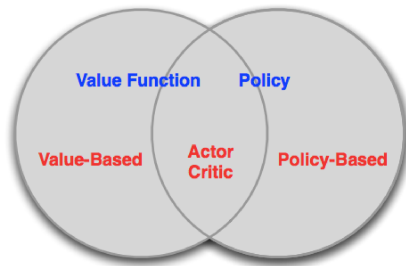
- Learn a value function
- Derive policy implicitly (e.g., ϵ -greedy)

► Policy-Based Methods

- Do not learn a value function
- Learn the policy directly

► Actor-Critic Methods

- Learn both a value function and a policy



Policy Gradient vs. Q-Learning

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- ▶ **Policy Gradient: Pros and Cons**
 - **Pros:** Unbiased estimate of the gradient of expected return.
 - Can handle large action spaces (since only one action needs to be sampled).
 - **Cons:** High variance updates (leads to poor sample efficiency).
 - Does not perform credit assignment effectively.

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 - **Cons:** High variance updates (leads to poor sample efficiency).
 - Does not perform credit assignment effectively.
- ▶ **Q-Learning: Pros and Cons**
 - **Pros:** Lower variance updates, more sample efficient.
 - Performs credit assignment.
 - **Cons:** Biased updates due to function approximation.
 - Difficult to handle large action spaces (since the maximum over actions must be computed).

Reinforcement Learning: **Summary**

Method	Policy Type	Gradient Source	Stability	Efficiency
Q-learning	Deterministic	Value gradients	Medium	High
REINFORCE	Stochastic	Monte Carlo returns	Low	Low
Actor-Critic	Stochastic	TD-based advantage	High	High

► REINFORCE:

- High variance in gradient estimates.
- Slow convergence.

► Actor-Critic:

- Sensitive to hyperparameters.
- Actor and critic updates may interfere.
- Requires careful tuning and exploration strategies.
- Can struggle in sparse-reward environments.

- ▶ **Trust Region Policy Optimization (TRPO):** Improves stability by constraining policy updates.
- ▶ **Proximal Policy Optimization (PPO):** Balances exploration and stability with clipped objective functions.
- ▶ **Soft Actor-Critic (SAC):** Uses entropy regularization for improved robustness and exploration.
- ▶ **Meta-Reinforcement Learning (Meta-RL):** Enables agents to adapt quickly to new tasks.
- ▶ **Multi-agent Actor-Critic:** Facilitates decentralized coordination among multiple agents.

- ▶ It can be hard to learn the exact Q-value for every (state, action) pair in high-dimensional state and action spaces.
- ▶ However, we can just learn a policy that maximizes the reward.
- ▶ We can use gradient ascent on policy parameters.
- ▶ However, this can suffer from high variance. Various strategies exist to tackle this.
- ▶ Actor-Critic methods combine Policy Gradients and Q-learning by training both an actor (the policy) and a critic (the Q-network).
- ▶ The actor decides which action to take, and the critic tells the actor how good its action was and how it should adjust.

Reinforcement Learning: **References**

- [1] Sutton, R. S., & Barto, A. G. (2018).
Reinforcement Learning: An Introduction.
- [2] Williams, R. J. (1992).
Simple Statistical Gradient-Following Algorithms for Connectionist Reinforcement Learning.
Machine Learning Journal.
- [3] Schulman, J., Levine, S., Abbeel, P., Jordan, M., & Moritz, P. (2015).
Trust Region Policy Optimization.
In *ICML*.

- [4] Schulman, J., Wolski, F., Dhariwal, P., Radford, A., & Klimov, O. (2017).
Proximal Policy Optimization Algorithms.
arXiv preprint arXiv:1707.06347.

- [5] Haarnoja, T., Zhou, A., Abbeel, P., & Levine, S. (2018).
Soft Actor-Critic: Off-Policy Maximum Entropy Deep Reinforcement
Learning with a Stochastic Actor.
In *ICML*.

- [6] OpenAI Spinning Up.
<https://spinningup.openai.com>

- [7] Berkeley CS285 Deep RL.
<https://rail.eecs.berkeley.edu/deeprlcourse/>

- [8] Chelsea Finn & Karol Hausman, Stanford CS224R: [Deep Reinforcement Learning](#)
- [9] Fei-Fei Li, Yunzhu Li & Ruohan Gao, Stanford CS231n: [Deep Learning for Computer Vision](#)
- [10] Jimmy Ba & Bo Wang, UofT CSC413/2516: [Neural Networks and Deep Learning](#)
- [11] Sergey Levine, Berkeley CS285: [Deep Reinforcement Learning](#)

Credits

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This project benefited from external collaboration, and we acknowledge their contribution with gratitude.