Reinforcement Learning
Policy Search – TRPO & PPO

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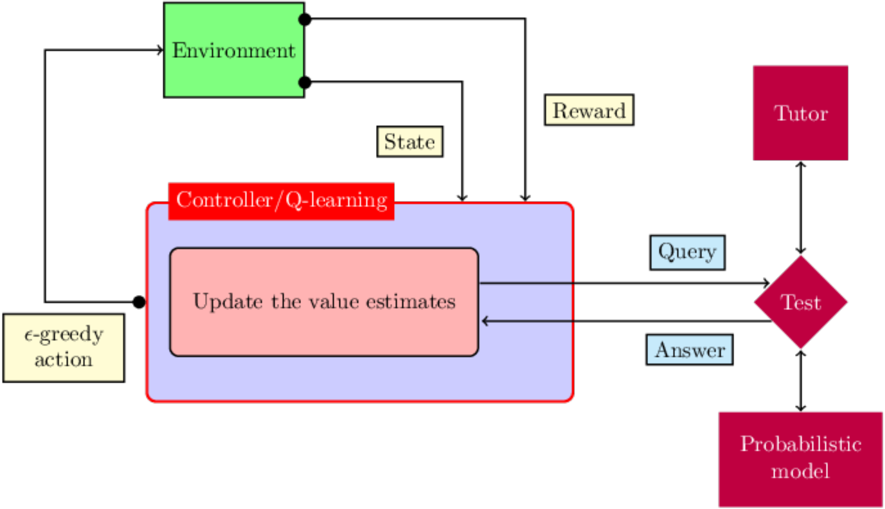
Environment
State
Reward
Tutor
Controller/Q-learning
Query
Update the value estimates
Test
ϵ-greedy action
Answer
Probabilistic model

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Policy Gradient: **Recap**

Recap

- Learning the exact Q-value for every (state, action) pair is challenging in high-dimensional spaces.
- Instead, we can directly learn a policy that maximizes expected reward.
- Policy parameters can be optimized using gradient ascent.
- However, policy gradients can suffer from high variance; various strategies exist to address this.
- Actor-Critic methods combine policy gradients and value-based methods by training both an actor (the policy) and a critic (the value function).
- The actor selects actions, while the critic evaluates the actions and guides the actor's learning.

Learning Outcomes

- Understand the challenges of vanilla policy gradients.
- Explain the motivation and theory behind Trust Region Policy Optimization (TRPO).
- Describe and implement Proximal Policy Optimization (PPO).
- Identify the trade-offs between stability and performance in modern policy optimization algorithms.
- Recognize applications and evaluate limitations of TRPO and PPO.

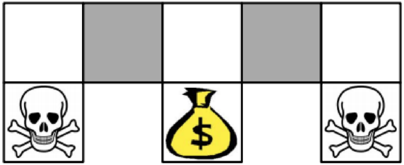
Motivation

- **Vanilla policy gradient methods** (e.g., REINFORCE, A2C) suffer from:
  - High variance in gradient estimates
  - Unstable policy updates
  - Requirement for small learning rates
  - Large policy updates may collapse performance
- **Goal:** Achieve stable and efficient policy updates while ensuring policy improvement

Policy Search: **Types of Policies**

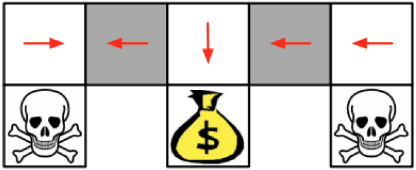
Types of Policies

- Policy π determines how the agent chooses actions
- Deterministic Policy:
$$\pi(s) = a$$
- Stochastic Policy:
$$\pi(a|s) = Pr(a_t = a|s_t = s)$$
- So far have focused on deterministic policies or ϵ-greedy policies
- ϵ-greedy policies are also near deterministic as we decrease the value of epsilon with training
- Is deterministic policy optimal?



- Consider a Grid World as in the image
- The agent can move in four direction (N, E, W, S) if valid
- The agent **cannot** differentiate the grey states

Example: Aliased Grid world



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  - move W in both grey states (shown by red arrows)
  - move E in both grey states

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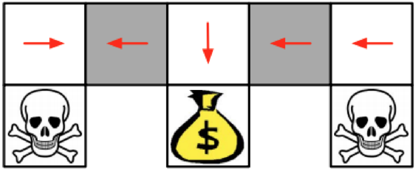

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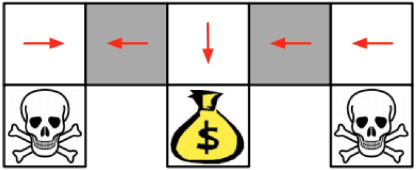

- Under aliasing, an optimal deterministic policy will either
  - move W in both grey states (shown by red arrows)
  - move E in both grey states
- Either way, it can get stuck and never reach the money
- Similarly, Value-based RL learns a near-deterministic policy (e.g., ϵ-greedy)
- As a result, it will traverse the corridor for a long time depending on the value of Epsilon

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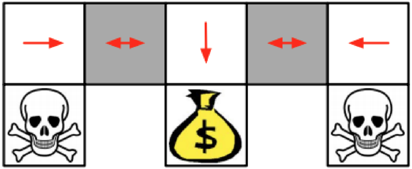

- An optimal stochastic policy will randomly move E or W in grey states
  - π_θ(wall to N and S, move E) $= 0.5$
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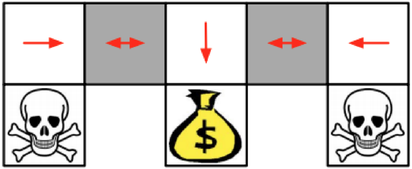

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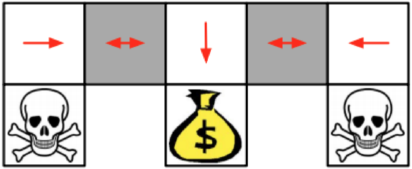

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- It will reach the goal state in a few steps with high probability
- Policy-based RL can learn the optimal stochastic policy

Policy Search: **Limitations of policy gradients**

Limitations of policy gradients

- Sample efficiency is poor
  - We throw out each batch of data immediately after just one gradient step
  - Why? PG is an on-policy expectation.

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$$\Pi = \left\{ \pi : \pi \in \mathbb{R}^{|S| \times |A|}, \pi_{sa} \geq 0 \right\}$$

  - Policy gradients take steps in parameter space
  - Step size is hard to get right as a result

Policy Search: **Importance Sampling**

Importance Sampling

- Importance sampling is a technique for estimating expectations using samples drawn from a different distribution.

$$
\begin{aligned}
E_{x\sim P}[f(x)] &= \int P(x)f(x)dx \\
&= \int P(x)\frac{Q(x)}{Q(x)}f(x)dx \\
&= \int Q(x)\frac{P(x)}{Q(x)}f(x)dx \\
&= E_{x\sim Q}\left[\frac{P(x)}{Q(x)}f(x)\right]
\end{aligned}
$$

Importance Sampling

$$\therefore \quad E_{x\sim P}[f(x)] = E_{x\sim Q}\left[\frac{P(x)}{Q(x)}f(x)\right] \approx \frac{1}{|D|}\sum_{x\in D}\frac{P(x)}{Q(x)}f(x)$$

- The ratio P(x)/Q(x) is the importance sampling weight for x

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- The ratio P(x)/Q(x) is the importance sampling weight for x
- What is the variance of an importance sampling estimator?

$$\begin{aligned} var(\hat{\mu}_Q) &= \frac{1}{N} var\left(\frac{P(x)}{Q(x)}f(x)\right) \\ &= \frac{1}{N}\left(E_{x\sim Q}\left[\left(\frac{P(x)}{Q(x)}f(x)\right)^2\right] - E_{x\sim Q}\left[\frac{P(x)}{Q(x)}f(x)\right]^2\right) \\ &= \frac{1}{N}\left(E_{x\sim P}\left[\frac{P(x)}{Q(x)}f(x)^2\right] - E_{x\sim P}[f(x)]^2\right) \end{aligned}$$

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- The term in red is problematic - if $\frac{P(x)}{Q(x)}$ is large in the wrong places, the variance of the estimator explodes.

- Now, let's put this in policy gradient. $\pi_{\theta'}$ represents new policy.

$$\nabla_{\theta'} \mathcal{J}(\theta') = E_{\tau \sim \pi_{\theta'}} \left[\sum_{t \geq 0} \gamma^t \nabla_{\theta'} \log \pi_{\theta'}(a_t|s_t) A^{\pi_{\theta'}}(s_t, a_t) \right]$$

$$= E_{\tau \sim \pi_{\theta}} \left[\sum_{t \geq 0} \frac{P(\tau_t|\pi_{\theta'})}{P(\tau_t|\pi_{\theta})} \gamma^t \nabla_{\theta'} \log \pi_{\theta'}(a_t|s_t) A^{\pi_{\theta'}}(s_t, a_t) \right]$$

Importance Sampling for Policy Gradients

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- Looks useful - what's the issue?
- Exploding or vanishing importance sampling weights. Even for policies only slightly different from each other, many small differences multiply to become a big difference.

- Solution?

Importance Sampling for Policy Gradients

- Solution?
- Stay close to the previous policy!
- We can use KL divergence for that.
- What is KL-divergence between policies?

$$D_{KL}(\pi'||\pi)[s] = \sum_{a \in A} \pi'(a|s) \log \frac{\pi'(a|s)}{\pi(a|s)}$$

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- Now, we have

$$\nabla_{\theta'} \mathcal{J}(\theta') \text{ s.t. } D_{KL}(\pi'||\pi) \leq \epsilon$$

Relative Policy Performance Identity

- But, recall that for $\nabla_{\theta'} \mathcal{J}(\theta')$ we will still have to compute $\log \pi_{\theta'}(a_t|s_t) A^{\pi_{\theta'}}(s_t, a_t)$ based on current policy.
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- This is not desirable.
- So, we make use of Relative Policy Performance Identity. This states that for two policies, $\pi_{\theta'}$ and π_θ

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- Using importance sampling, we get

$$\mathcal{J}(\pi_{\theta'}) - \mathcal{J}(\pi_\theta) = E_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T} \frac{\pi_{\theta'}(s_t, a_t)}{\pi_\theta(s_t, a_t)} \gamma^t A^{\pi_\theta}(s_t, a_t) \right]$$

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- Therefore, we can use this as our loss function

$$\mathcal{L}_{\theta'}(\pi_{\theta'}) = \mathcal{J}(\pi_{\theta'}) - \mathcal{J}(\pi_{\theta})$$

Choosing a Step Size for Policy Gradients

- But, the problem is more than step size
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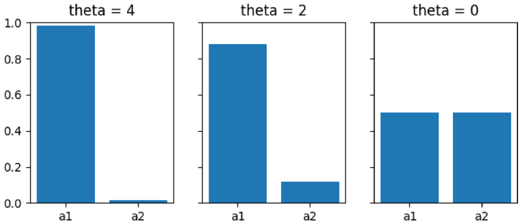

Policy Search: **Trust Region Policy Optimization (TRPO)**

Trust Region Policy Optimization (TRPO)

- TRPO updates policies by taking the largest step possible to improve performance, while satisfying a special constraint on how close the new and old policies are allowed to be.
- The constraint is expressed in terms of KL-Divergence
- This is different from normal policy gradient, which keeps new and old policies close in parameter space
- TRPO nicely avoids this kind of collapse, and tends to quickly and monotonically improve performance
- TRPO uses conjugate gradients for computing the hessian matrix for KL divergence derivative

[0] https://spinningup.openai.com/en/latest/algorithms/trpo.html

- TRPO uses backtracking line search with exponential decay (decay coeff $\alpha \in (0, 1)$, budget L) to make appropriate step sizes

Algorithm 2 Line Search for TRPO

Compute proposed policy step $\Delta_k = \sqrt{\frac{2\delta}{\hat{g}_k^T \hat{H}_k^{-1} \hat{g}_k}} \hat{H}_k^{-1} \hat{g}_k$
for $j = 0, 1, 2, ..., L$ **do**
  Compute proposed update $\theta = \theta_k + \alpha^j \Delta_k$
  if $\mathcal{L}_{\theta_k}(\theta) \geq 0$ and $\bar{D}_{KL}(\theta || \theta_k) \leq \delta$ **then**
    accept the update and set $\theta_{k+1} = \theta_k + \alpha^j \Delta_k$
    break
  end if
end for

Trust Region Policy Optimization (TRPO)

Algorithm 3 Trust Region Policy Optimization

Input: initial policy parameters θ_0
for $k = 0, 1, 2, \ldots$ **do**
  Collect set of trajectories $\mathcal{D}_k$ on policy $\pi_k = \pi(\theta_k)$
  Estimate advantages $\hat{A}_t^{\pi_k}$ using any advantage estimation algorithm
  Form sample estimates for

- policy gradient $\hat{g}_k$ (using advantage estimates)
- and KL-divergence Hessian-vector product function $f(v) = \hat{H}_k v$

  Use CG with n_{cg} iterations to obtain $x_k \approx \hat{H}_k^{-1} \hat{g}_k$
  Estimate proposed step $\Delta_k \approx \sqrt{\frac{2\delta}{x_k^T \hat{H}_k x_k}} x_k$
  Perform backtracking line search with exponential decay to obtain final update

$$\theta_{k+1} = \theta_k + \alpha^j \Delta_k$$

end for

Policy Search: **Proximal Policy Optimization (PPO)**

Proximal Policy Optimization (PPO)

- PPO is motivated by the same question as TRPO: how can we take the biggest possible improvement step on a policy using the data we currently have, without stepping so far that we accidentally cause performance collapse?
- Where TRPO tries to solve this problem with a complex second-order method, PPO is a family of first-order methods that use a few other tricks to keep new policies close to old.
- It approximately enforce KL constraint without computing natural gradients.
- PPO methods are significantly simpler to implement, and empirically seem to perform at least as well as TRPO.
- There are two primary variants of PPO: PPO-Penalty and PPO-Clip.

[0] https://spinningup.openai.com/en/latest/algorithms/ppo.html

Proximal Policy Optimization (PPO)

- Adaptive KL Penalty
  - Policy update solves unconstrained optimization problem

$$\theta k + 1 = \arg\max_{\theta} \mathcal{L}_{\theta_k}(\theta) - \beta_k \overline{D}_{KL}(\theta || \theta_k)$$

  - Penalty coefficient β_k changes between iterations to approximately enforce KL-divergence constraint

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$$\theta k + 1 = \arg\max_{\theta} \mathcal{L}_{\theta_k}(\theta) - \beta_k \overline{D}_{KL}(\theta || \theta_k)$$

  - Penalty coefficient β_k changes between iterations to approximately enforce KL-divergence constraint
- Clipped Objective
  - New objective function: let $r_t(\theta) = \frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_k}(a_t|s_t)}$. Then,

$$\mathcal{L}_{\theta_k}^{CLIP} = E_{\tau \sim \pi_k}\left[\sum_{t=0}^{T}\left[r_t(\theta)\hat{A}_t^{\pi_k}, clip(r_t(\theta), 1-\epsilon, 1+\epsilon)\hat{A}_t^{\pi_k}\right]\right]$$

  - ϵ is a hyperparameter (e.g., $\epsilon = 0.2$)
  - Policy update is

$$\theta_{k+1} = \arg\max_{\theta} \mathcal{L}_{\theta_k}^{CLIP}$$

Proximal Policy Optimization (PPO)

Algorithm 4 PPO with Adaptive KL Penalty

Input: initial policy parameters θ_0, initial KL penalty β_0, target KL-divergence δ
for $k = 0, 1, 2, \ldots$ **do**
  Collect set of partial trajectories $\mathcal{D}_k$ on policy $\pi_k = \pi(\theta_k)$
  Estimate advantages $\hat{A}_t^{\pi_k}$ using any advantage estimation algorithm
  Compute policy update

$$\theta_{k+1} = \arg\max_{\theta} \mathcal{L}_{\theta_k}(\theta) - \beta_k \bar{D}_{KL}(\theta || \theta_k)$$

  by taking K steps of minibatch SGD (via Adam)
  if $\bar{D}_{KL}(\theta_{k+1} || \theta_k) \geq 1.5\delta$ **then**
    $\beta_{k+1} = 2\beta_k$
  else if $\bar{D}_{KL}(\theta_{k+1} || \theta_k) \leq \delta / 1.5$ **then**
    $\beta_{k+1} = \beta_k / 2$
  end if
end for

- PPO clip is more widely used as it seems to work at least as well as PPO with KL penalty, but is simpler to implement

Algorithm 5 PPO with Clipped Objective

Input: initial policy parameters θ_0, clipping threshold ϵ
for $k = 0, 1, 2, \ldots$ **do**
  Collect set of partial trajectories $\mathcal{D}_k$ on policy $\pi_k = \pi(\theta_k)$
  Estimate advantages $\hat{A}_t^{\pi_k}$ using any advantage estimation algorithm
  Compute policy update

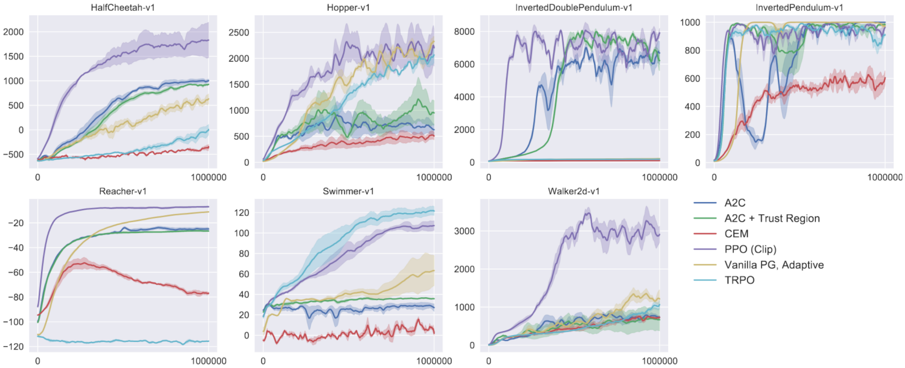
$$\theta_{k+1} = \arg\max_{\theta} \mathcal{L}_{\theta_k}^{CLIP}(\theta)$$

  by taking K steps of minibatch SGD (via Adam), where

$$\mathcal{L}_{\theta_k}^{CLIP}(\theta) = \mathop{\mathrm{E}}_{\tau\sim\pi_k}\left[\sum_{t=0}^{T}\left[\min(r_t(\theta)\hat{A}_t^{\pi_k}, \mathrm{clip}\left(r_t(\theta), 1-\epsilon, 1+\epsilon\right)\hat{A}_t^{\pi_k})\right]\right]$$

end for

Empircal Performance of PPO



[0] Schulman, Wolski, Dhariwal, Radford, Klimov, 2017

Policy Search: **Summary**

PPO vs TRPO

Feature	**TRPO**	**PPO**
Optimization	Constrained (KL)	Unconstrained (clipped)
Implementation	Complex	Simple
Performance	Strong	Comparable
Speed	Slower	Faster (SGD-friendly)
Used in	Robotics, theory	Industry, games

Limitations of TRPO & PPO

- Both methods remain sensitive to reward scaling, exploration strategies, and advantage estimation quality.
- PPO's clipping mechanism is heuristic and may under- or over-constrain policy updates.
- Both can struggle in environments with very sparse rewards.
- Stability and convergence are not guaranteed in general MDPs.

Summary

- In some cases, learning a stochastic policy is preferable to a deterministic policy.
- Policy gradient methods often suffer from poor sample efficiency.
- Importance sampling can help improve sample efficiency.
- However, it is important to ensure that the current policy is not too different from the policy used to collect trajectories.
- Small changes in policy parameters can sometimes lead to large, unexpected changes in the policy.
- TRPO uses importance sampling to take multiple gradient steps and constrains the optimization objective in policy space.
- PPO achieves similar goals by approximately enforcing a KL-divergence constraint without computing natural gradients.

Policy Search: **References**

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