Mathematics for AI

Problem Set: Calculus

November 27, 2022

Some of the following problems are chosen from Dennis G. Zill, Advanced Engineering Mathematics, 6th ed., Jones & Bartlett Learning.

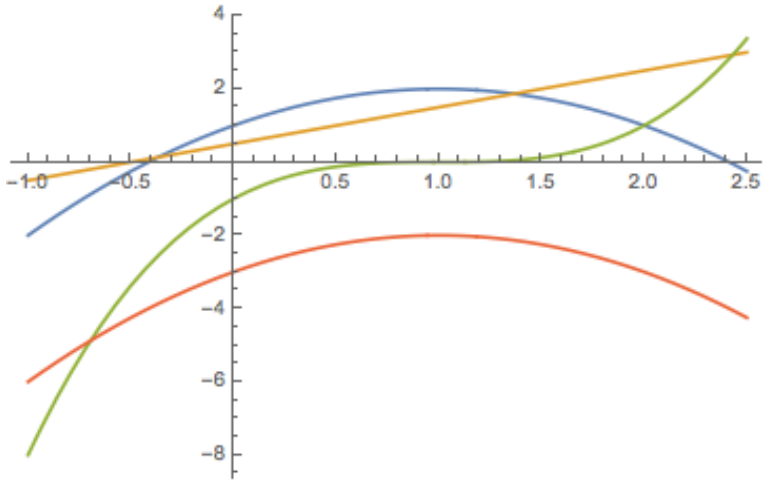
1. Find all the values of x where the tangent lines to $y = x^3$ and $y = x^4$ are parallel.
2. Find the equation of the tangent line of $y = e^{x+2}$ at $x = -1$.
3. The kth order polynomial approximation of f at point $(a, f(a))$ is

$$p_k(x) = \sum_{i=0}^{k} \frac{f^{(i)}(a)}{i!}(x-a)^i.$$

   Find the Taylor polynomial of the kth order, p_k, of $\ln x$ at $x = 1$. Draw the graphs of $\ln x, p_1$ and p_2, and note that, in a neighborhood of $x = 1$, the graph of p_2 is closer to the graph of $\ln x$ than that of p_1.
4. Use the chain rule to find the derivative of $f \circ g$ for $f(u) = \sin u$ and $g(x) = 2x + 1$.
5. What is the least squares best fitting line $y = \hat{C} + \hat{D}t$ through

$$(-1, -1) \qquad (0, -3) \qquad (1, 1)?$$

   Minimize the sum of the squares of the errors at $t = -1, 0, 1$.
6. Match the functions in the graphs of the first figure with their derivatives in the second figure: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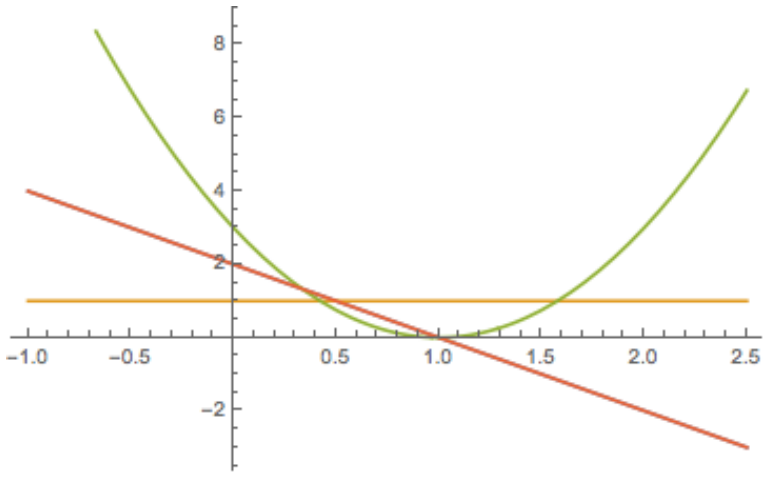
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