



ADVANCED STATISTICS

EFFECTS ON LEARNING OF SMALL CLASS SIZES

An Analytical Study

INTRODUCTION

EFFECT OF CLASS SIZE

Whether small classes really help with learning.

DEMOGRAPHIC FACTORS

Which sex/race benefits the most from the implemented schemes.

RESOURCE ALLOCATION

Cost of implementation of these schemes in general schools.

ANALYSIS

Visual and statistical means to figure out the effect of various factors on learning.

THE DATASET

FOR EACH STUDENT

DEMOGRAPHIC

These fields described the sex and race of the students in the study.

ADVANCED STATISTICS

CLASS TYPES

Description of the class the student is in. They are either regular, regular with aid, or smaller classes.

TEACHER EXPERIENCE

The dataset had teachers with experiences ranging from 1 to 27 years.

SCORES

Scaled scores for Math and Reading were provided for each student. These were the metrics for performance.

FREE LUNCH SCHEME

Some students, from lower financial backgrounds availed the free lunch scheme at their school.

EXPLORATORY DATA ANALYSIS

RACE DISTRIBUTION

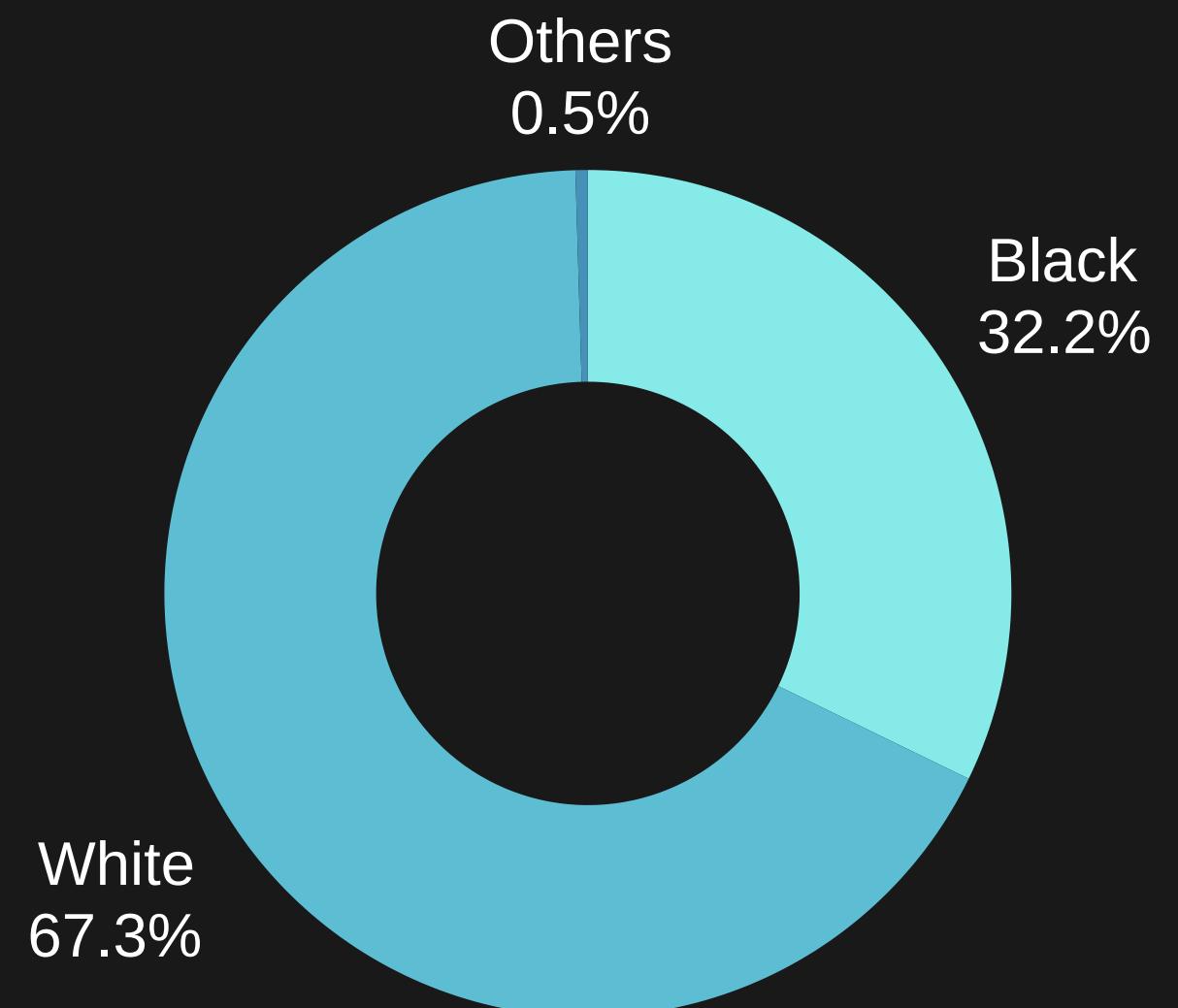
CLASS SIZE EFFECT ON
PERFORMANCE

EXPERIENCE EFFECT ON
PERFORMANCE

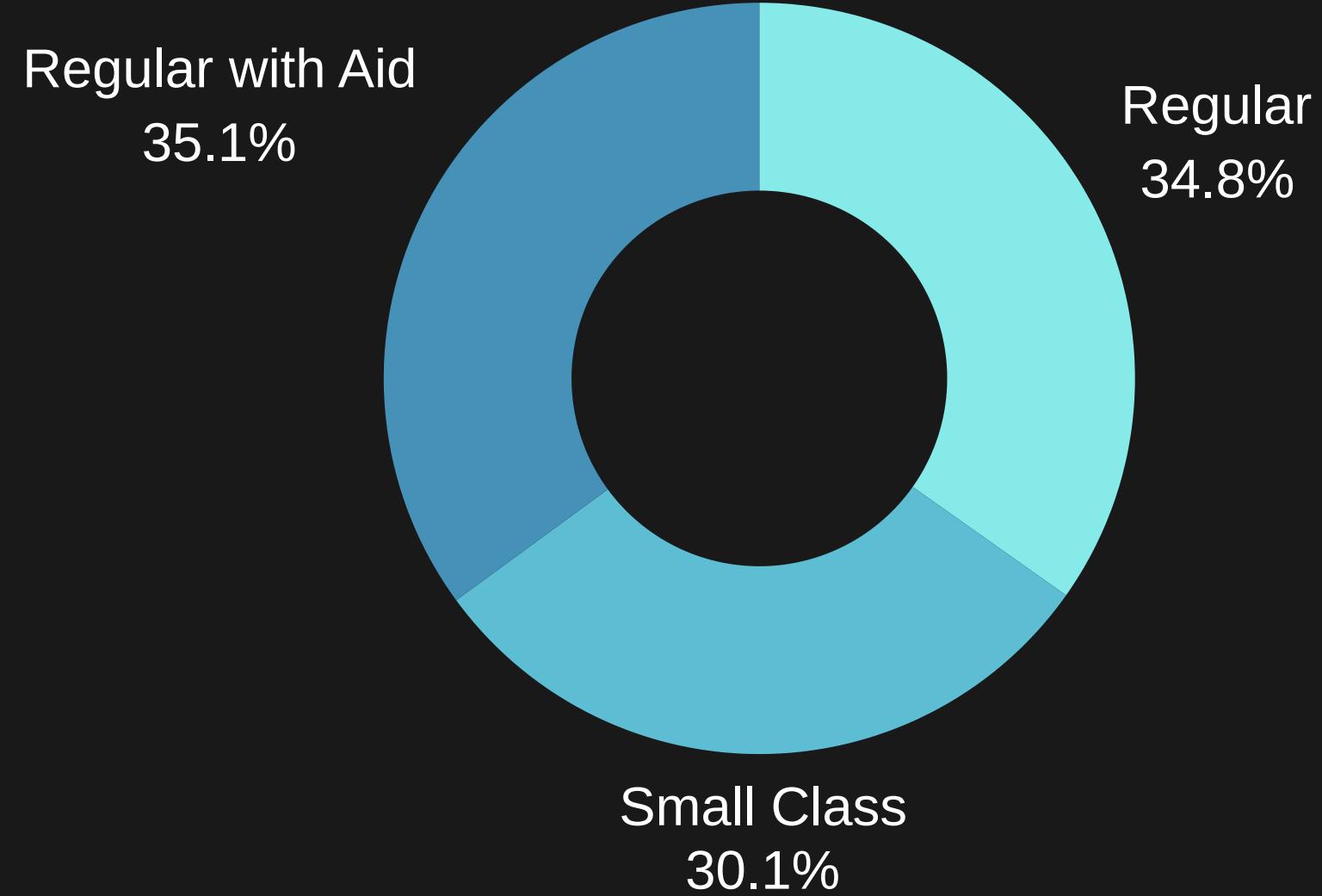
CLASS SIZE
DISTRIBUTION

RACE EFFECT ON
PERFORMANCE

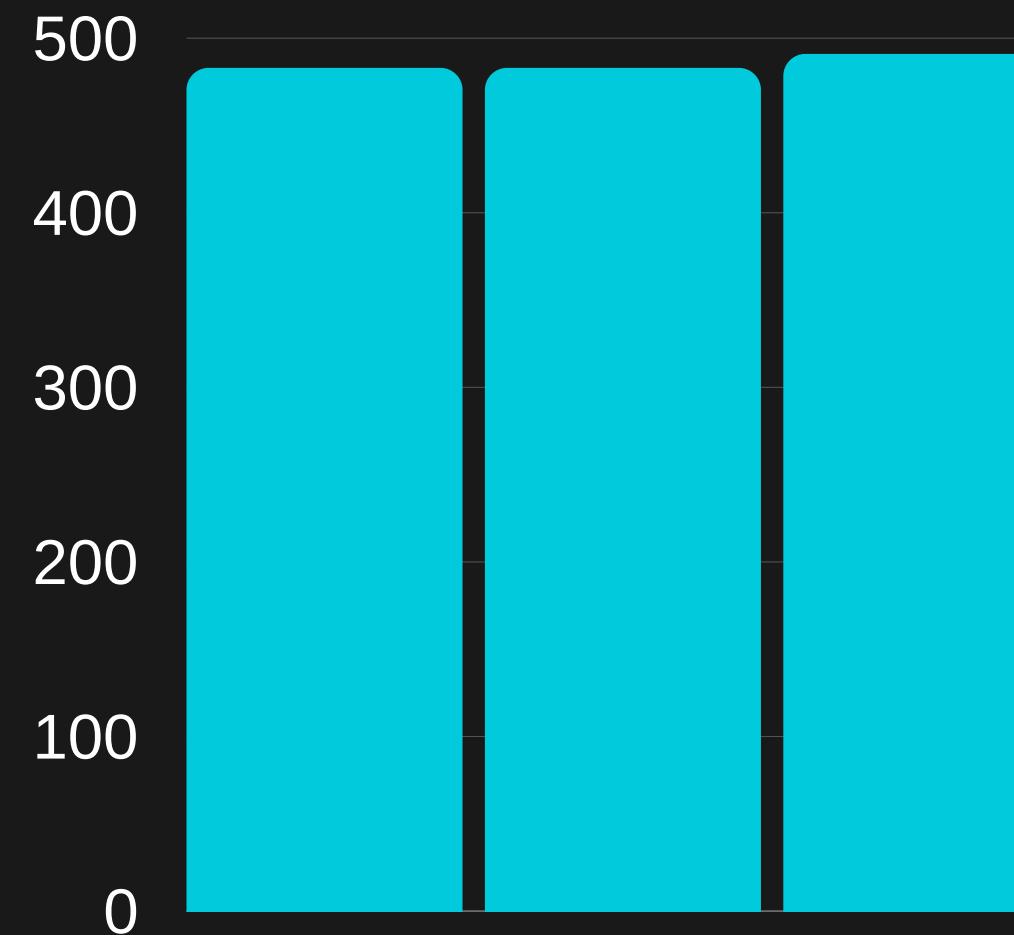
FINANCE EFFECT ON
PERFORMANCE



RACE DISTRIBUTION



CLASS DISTRIBUTION

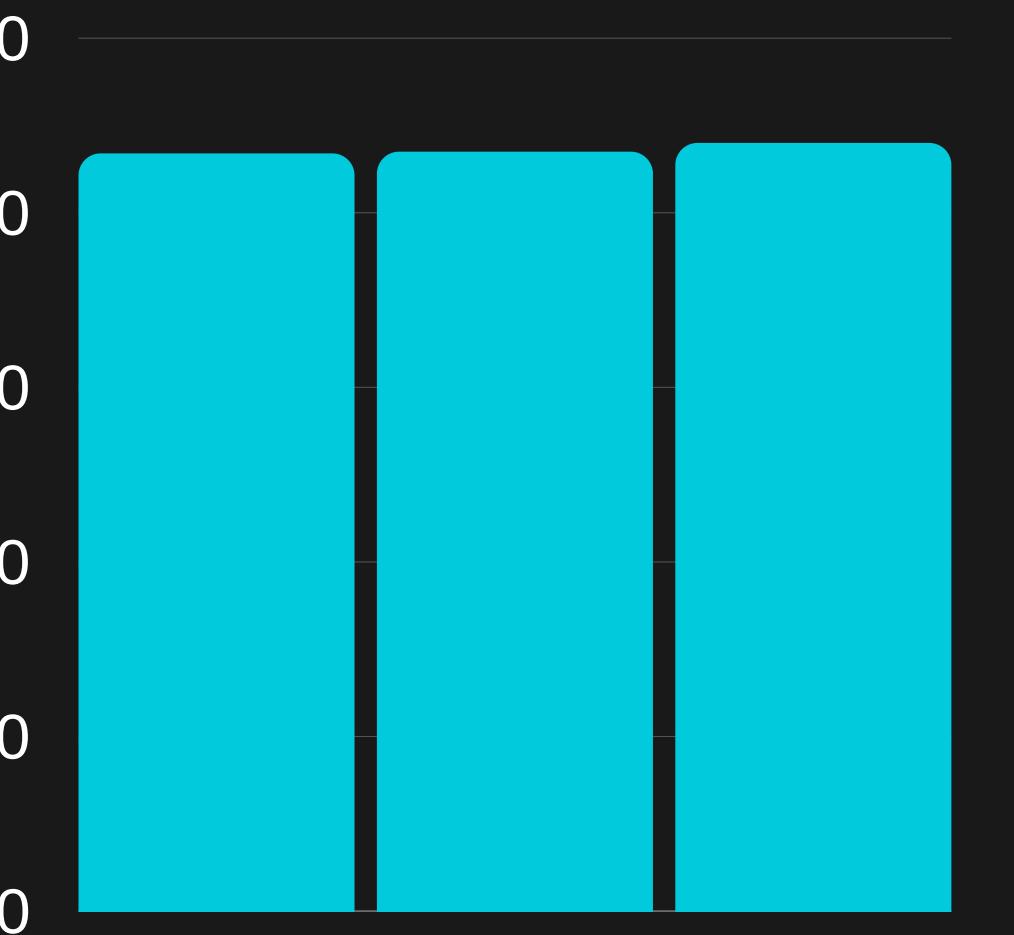


A bar chart titled "MATH SCORES" comparing three classroom types. The y-axis represents scores from 0 to 500. The "Regular" group has a score of approximately 480. The "Regular with Aid" group has a score of approximately 485. The "Small Class" group has a score of approximately 490.

Classroom Type	Math Score
Regular	480
Regular with Aid	485
Small Class	490

Regular
Regular with Aid
Small Class

MATH SCORES



A bar chart titled "READING SCORES" comparing three classroom types. The y-axis represents scores from 0 to 500. The "Regular" group has a score of approximately 430. The "Regular with Aid" group has a score of approximately 435. The "Small Class" group has a score of approximately 440.

Classroom Type	Reading Score
Regular	430
Regular with Aid	435
Small Class	440

Regular
Regular with Aid
Small Class

READING SCORES

AVAILING FREE LUNCH SCHEME

500

400

300

200

100

0

Yes

No

MATH SCORES

500

400

300

200

100

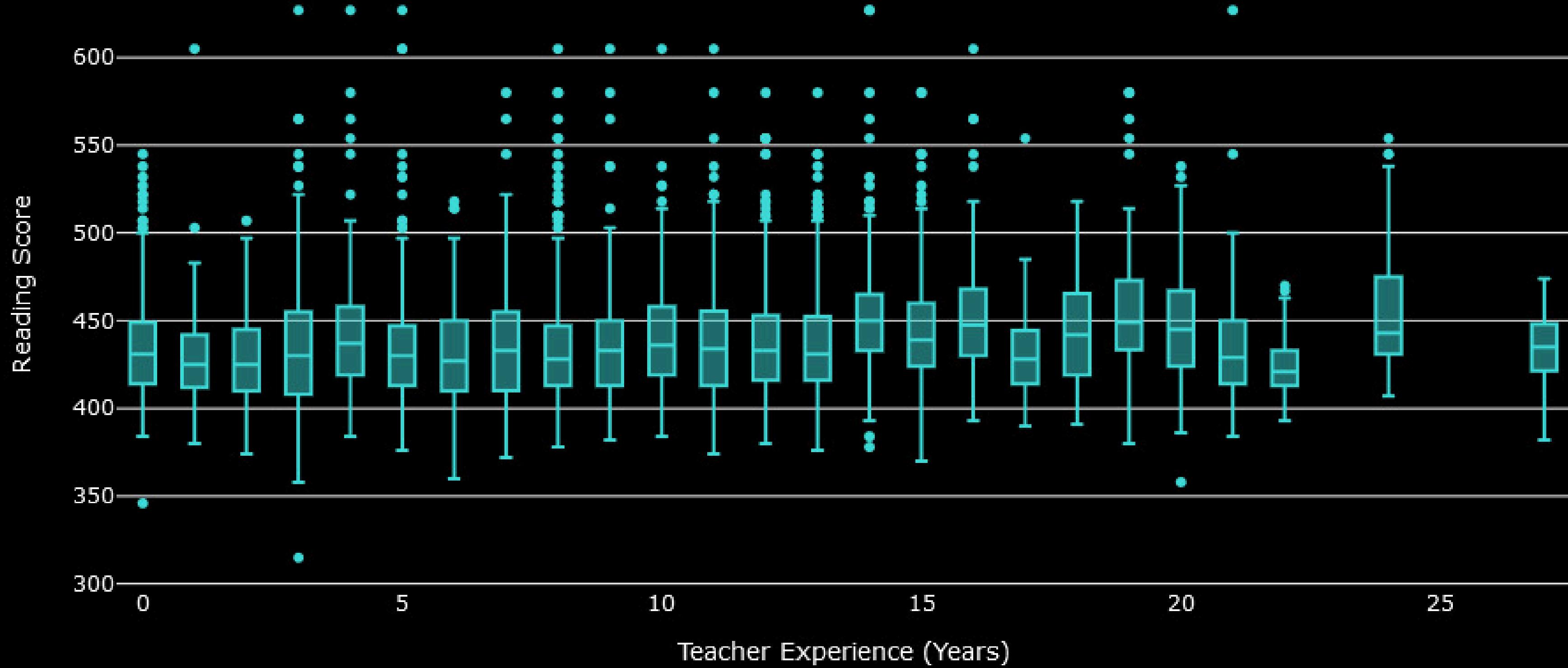
0

Yes

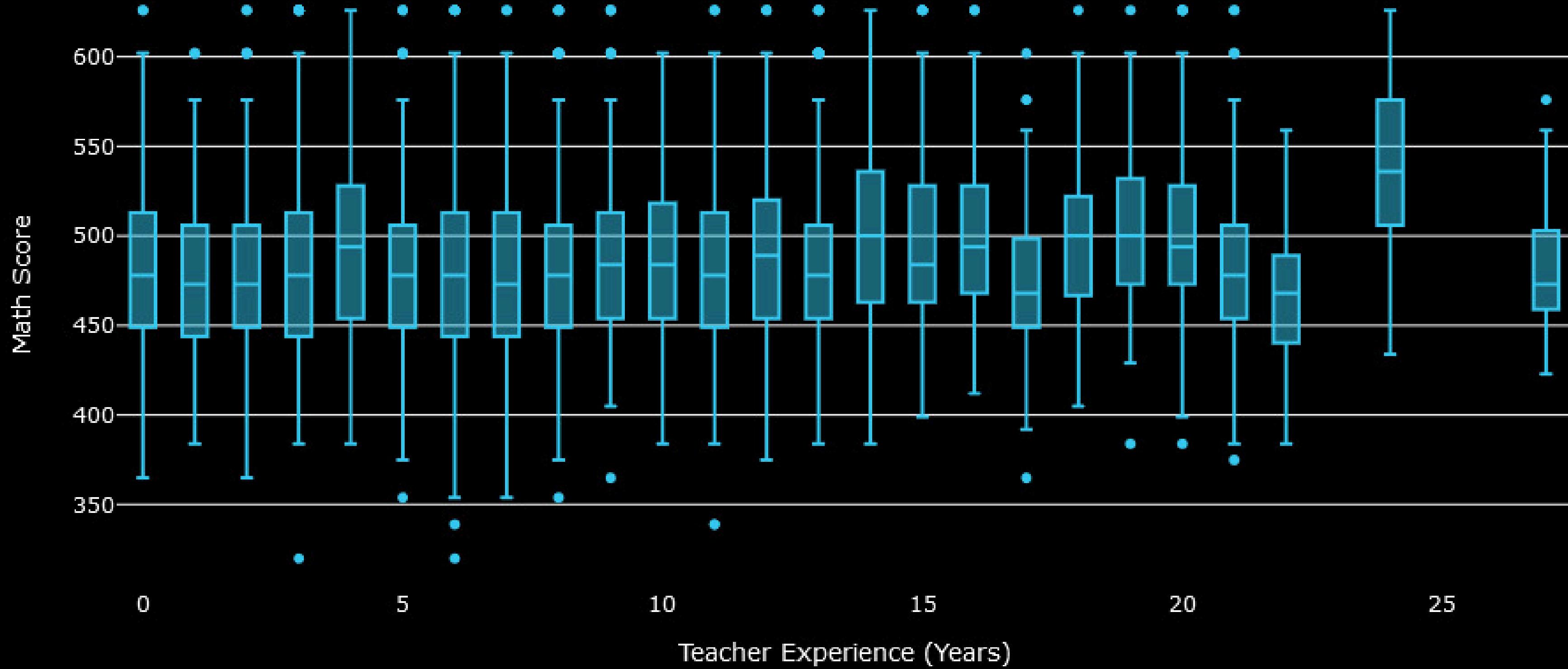
No

READING SCORES

Reading Scores vs. Teacher Experience



Math Scores vs. Teacher Experience



HYPOTHESIS TESTING

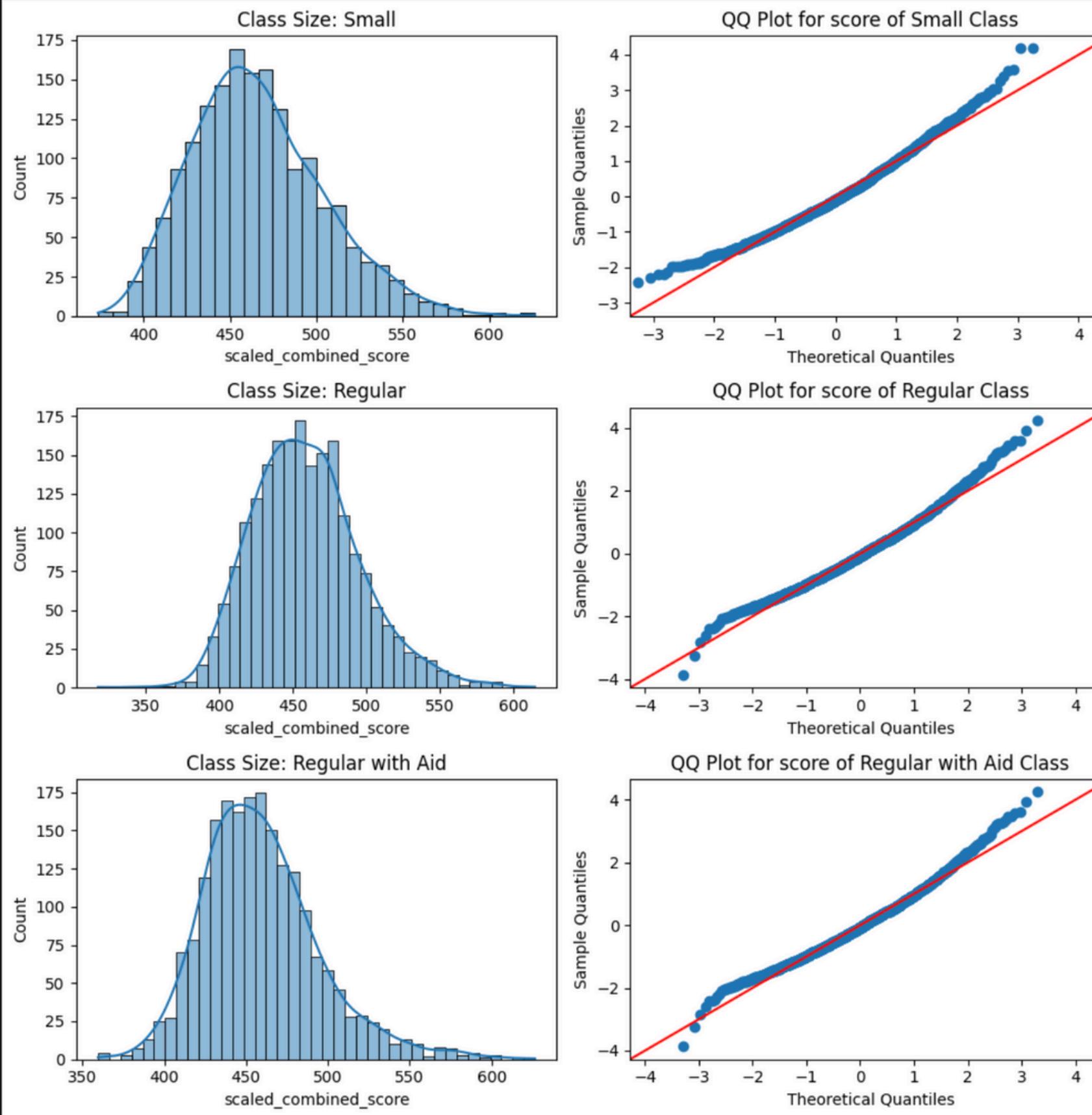
NULL HYPOTHESIS

THERE IS NO STATISTICALLY SIGNIFICANT DIFFERENCE IN ACADEMIC PERFORMANCE ATTRIBUTABLE TO VARIATIONS IN CLASS SIZES

	df	sum_sq	mean_sq	F	PR(>F)
class_type	2.0	5.796135e+04	28980.676432	21.390791	5.552856e-10
Residual	5745.0	7.783442e+06	1354.820215	NaN	NaN

Multiple Comparison of Means – Tukey HSD, FWER=0.05							
group1	group2	meandiff	p-adj	lower	upper	reject	
regular	regular.with.aide	0.2776	0.969	-2.446	3.0013	False	
regular	small.class	7.0544	0.0	4.2225	9.8862	True	
regular.with.aide	small.class	6.7768	0.0	3.9498	9.6037	True	

- ACADEMIC PERFORMANCE IS THE COMBINED MATH AND READING SCORE.
- THERE IS STATISTICAL DIFFERENCE BETWEEN THE ACADEMIC PERFORMANCE BETWEEN THE THREE CLASS SIZES.
- THE SMALL CLASS SIZES PERFORM BETTER THAN THE OTHER TWO.



```

from scipy.stats import levene

# Perform Levene's test
statistic, p_value = levene(df[df['class_type'] == 'small.class']['scaled_combined_score'],
                             df[df['class_type'] == 'regular']['scaled_combined_score'],
                             df[df['class_type'] == 'regular.with.aide']['scaled_combined_score'])

# Display the results
print(f"Statistic: {statistic}")
print(f"P-value: {p_value}")

# The null hypothesis of Levene's test is that the variances are equal across all groups.
# A small p-value suggests that there is evidence to reject the null hypothesis, indicating
# that at least one group has a different variance.

```

✓ 0.0s

Statistic: 7.266342070923406
P-value: 0.000705102773581642

DATA IS NORMALLY DISTRIBUTED, INDEPENDENT OF ONE ANOTHER, BUT THE VARIANCES ARE NOT EQUAL.

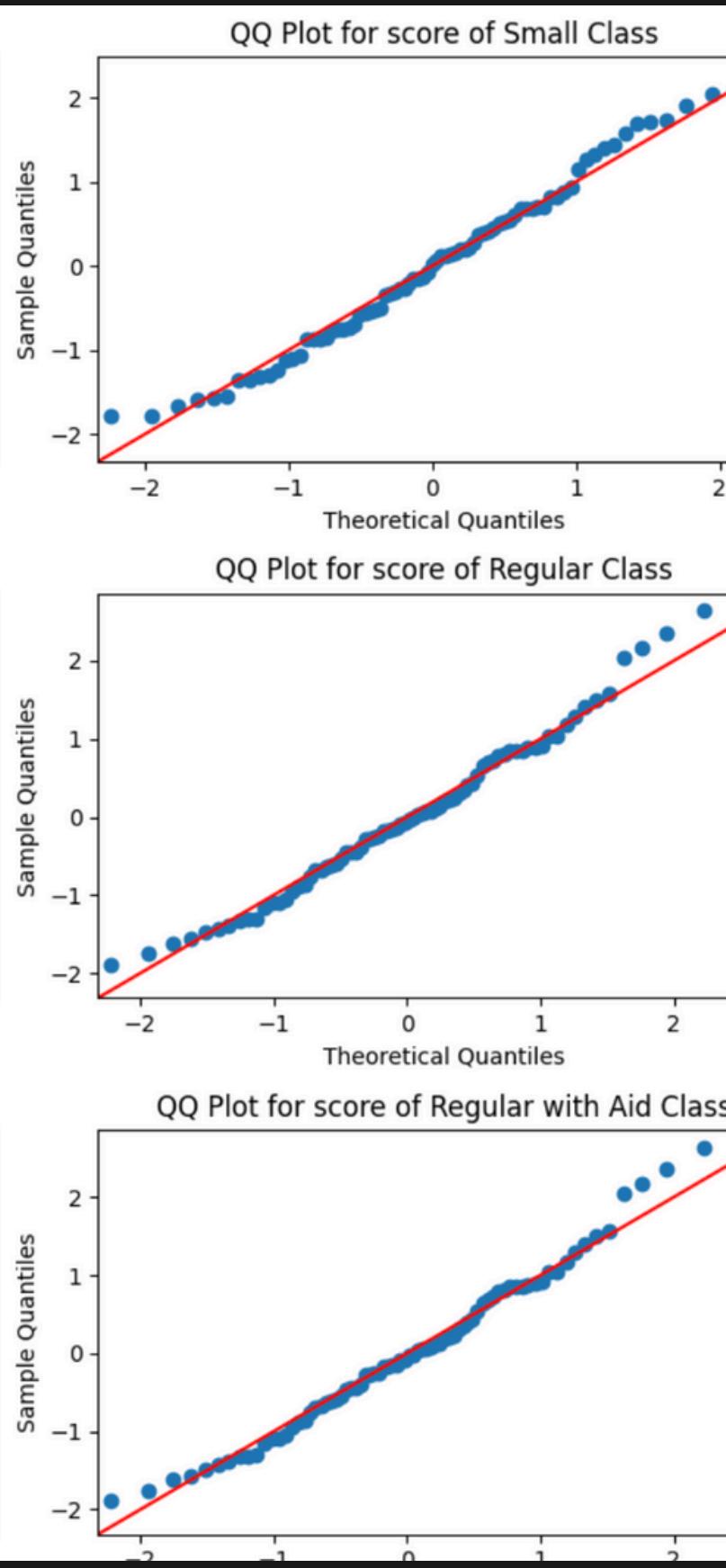
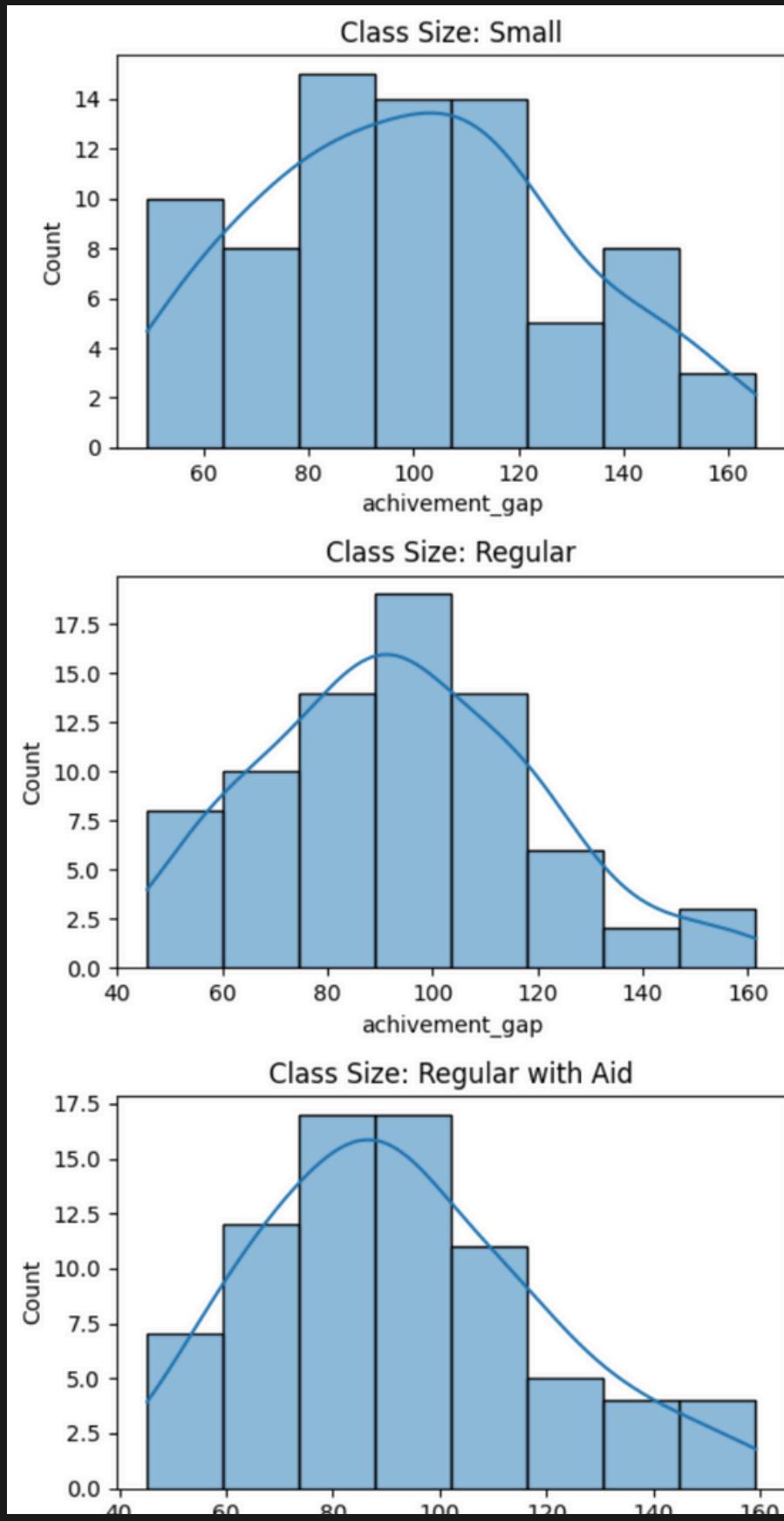
NULL HYPOTHESIS

**THERE IS NO STATISTICALLY SIGNIFICANT DIFFERENCE IN ACHIEVEMENT
GAP ATTRIBUTABLE TO VARIATIONS IN CLASS SIZES**

	df	sum_sq	mean_sq	F	PR(>F)
class_type	2.0	2422.410083	1211.205042	1.664353	0.191615
Residual	227.0	165195.480796	727.733396	NaN	NaN

Multiple Comparison of Means – Tukey HSD, FWER=0.05							
group1	group2	meandiff	p-adj	lower	upper	reject	
regular	regular.with.aide	-1.2441	0.9562	-11.5348	9.0465	False	
regular	small.class	6.1663	0.3355	-4.1244	16.4569	False	
regular.with.aide	small.class	7.4104	0.2057	-2.8466	17.6674	False	

- ACHIEVEMENT GAP IS THE DIFFERENCE IN THE MEAN COMBINED SCORES OF THE TOP 10 PERFORMERS AND THE BOTTOM 10 PERFORMERS.
- THERE IS NO STATISTICAL DIFFERENCE BETWEEN THE ACHIEVEMENT GAP BETWEEN THE THREE CLASS SIZES.
- THE TOP PERFORMERS IN EACH CLASS SIZE ARE EQUALLY AHEAD OF THE BOTTOM PERFORMERS IN EACH CLASS.



```

from scipy.stats import levene

# Perform Levene's test
statistic, p_value = levene(achievement_gap_df[achievement_gap_df['class_type'] == 'Small'],
                             achievement_gap_df[achievement_gap_df['class_type'] == 'Regular'],
                             achievement_gap_df[achievement_gap_df['class_type'] == 'Regular with Aid'])

# Display the results
print(f"Statistic: {statistic}")
print(f"P-value: {p_value}")

# The null hypothesis of Levene's test is that the variances are equal across groups.
# A small p-value suggests that there is evidence to reject the null hypothesis,
# that at least one group has a different variance.

```

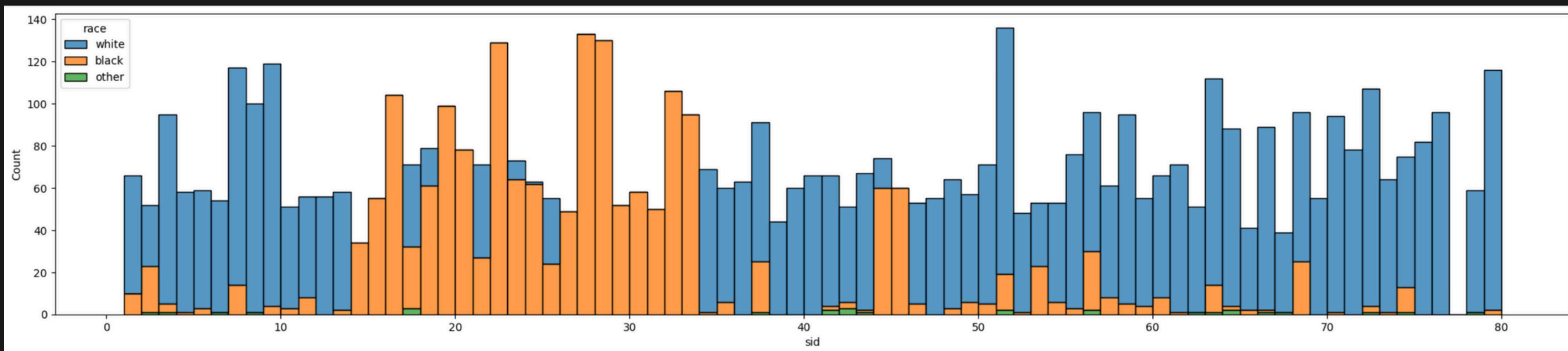
✓ 0.0s

Statistic: 0.9033267965949597
P-value: 0.4066708894820553

DATA IS NORMALLY DISTRIBUTED, INDEPENDENT OF ONE ANOTHER, AND THE VARIANCES ARE EQUAL.

NULL HYPOTHESIS

THERE IS NO STATISTICALLY SIGNIFICANT DIFFERENCE IN ACADEMIC PERFORMANCE ATTRIBUTABLE TO VARIATIONS IN MAJORITARIAN RACE



	df	sum_sq	mean_sq	F	PR(>F)
dominant_race	1.0	1.367009e+05	136700.908922	101.948572	8.987480e-24
Residual	5746.0	7.704703e+06	1340.881061	NaN	NaN

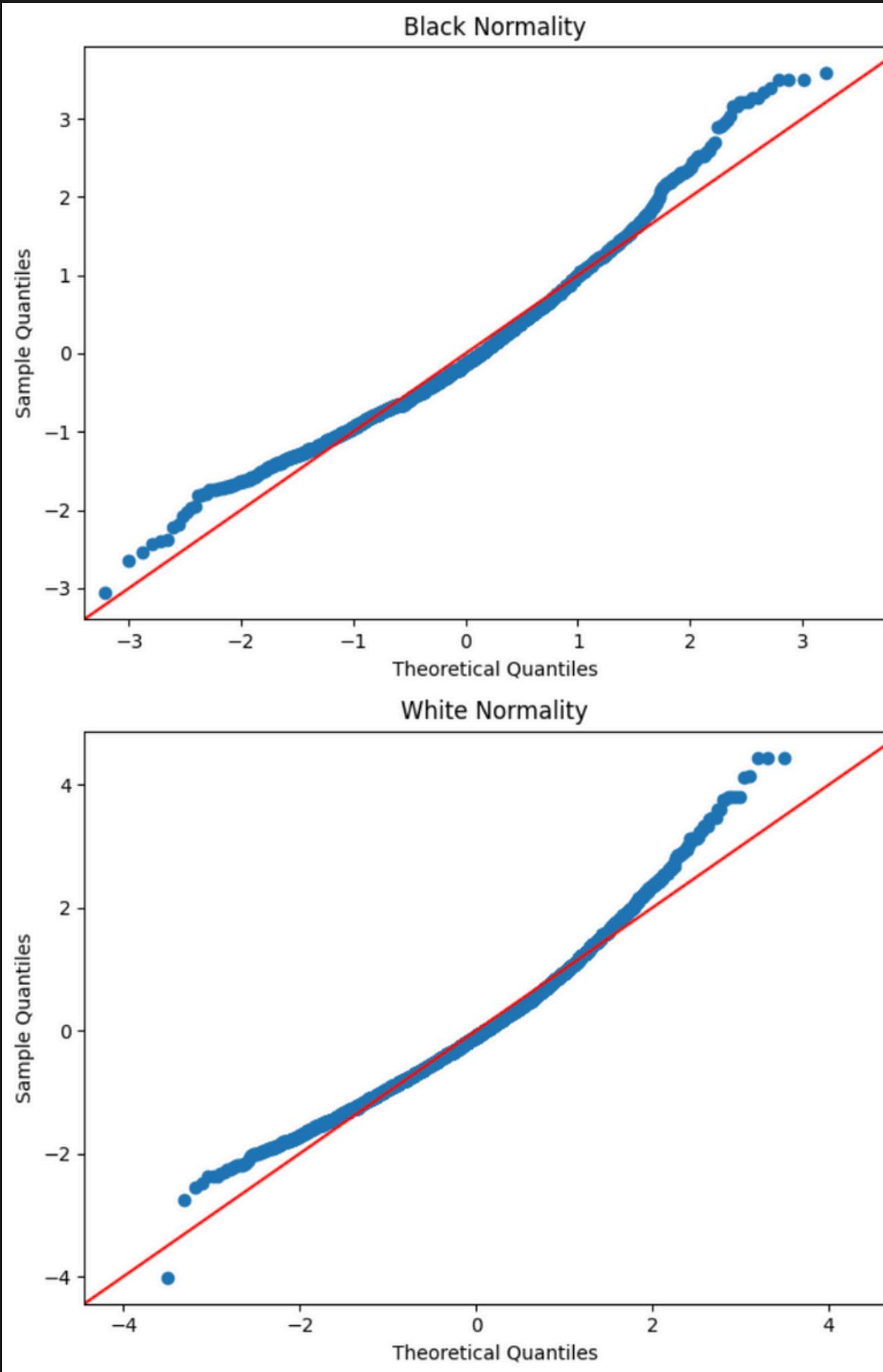
T-statistic: -10.096958533319205

P-value: 8.987479506901872e-24

The difference between the two groups is statistically significant.

The mean score of students from black dominant is 453.0654174884944 and white dominant is 464.12053465815

- THERE IS STATISTICALLY SIGNIFICANT DIFFERENCE BETWEEN THE ACADEMIC PERFORMANCE BETWEEN THE SCHOOLS THAT ARE WHITE DOMINATED AND THE ONES THAT ARE BLACK DOMINATED.
- WHITE DOMINATED SCHOOLS GENERALLY PERFORM BETTER.



```
from scipy.stats import levene

# Perform Levene's test
statistic, p_value = levene(df[df['dominant_race'] == 'white']['scaled_combined_score'], df[df['dominant_race'] == 'black']['scaled_combined_score'])

# Display the results
print(f"Statistic: {statistic}")
print(f"P-value: {p_value}")

# The null hypothesis of Levene's test is that the variances are equal across all groups.
# A small p-value suggests that there is evidence to reject the null hypothesis, indicating
# that at least one group has a different variance.

✓ 0.0s
Statistic: 0.691365464461825
P-value: 0.40573487467891167
```

- DATA IS RIGHT SKEWED NORMALLY DISTRIBUTED.
- VARIANCE IN TOTAL SCORE OF WHITE DOMINANT AND BLACK DOMINANT SCHOOLS ARE SIMILAR.

NULL HYPOTHESIS

THERE IS NO STATISTICALLY SIGNIFICANT INTERACTION BETWEEN THE EFFECT OF CLASS SIZE ON VARIOUS DEMOGRAPHICS LIKE RACE, SEX, AND FINANCIAL BACKGROUND

TWO WAY ANOVA

- EFFECT ON COMBINED SCORE BY CLASS TYPE AND RACE
- EFFECT ON COMBINED SCORE BY CLASS TYPE AND SEX
- EFFECT ON COMBINED SCORE BY CLASS TYPE AND AVAILING OF FREE LUNCH SCHEME

NO INTERACTION BETWEEN CLASS TYPE AND RACE

	sum_sq	df	F	PR(>F)
C(class_type)	5.463163e+04	2.0	20.864192	9.366554e-10
C(race)	2.679669e+05	2.0	102.338382	2.134023e-44
C(class_type):C(race)	1.861682e+03	4.0	0.355494	8.403538e-01
Residual	7.513614e+06	5739.0	NaN	NaN

NO INTERACTION BETWEEN CLASS TYPE AND AVAILING FREE LUNCH

	sum_sq	df	F	PR(>F)
C(class_type)	5.397023e+04	2.0	21.485562	5.054530e-10
C(free_lunch)	5.715789e+05	1.0	455.091364	3.072137e-97
C(class_type):C(free_lunch)	1.118483e+02	2.0	0.044527	9.564502e-01
Residual	7.211751e+06	5742.0	NaN	NaN

STATISTICALLY SIGNIFICANT INTERACTION BETWEEN CLASS TYPE AND SEX

	sum_sq	df	F	PR(>F)
C(class_type)	5.799640e+04	2.0	21.632525	4.368492e-10
C(sex)	6.874993e+04	1.0	51.287141	8.981350e-13
C(class_type):C(sex)	1.759474e+04	2.0	6.562798	1.422544e-03
Residual	7.697097e+06	5742.0	NaN	NaN

GIRLS AND BOYS PERFORM SIMILARLY IN SMALL CLASS SIZES

```
small_class_girl_score = df[(df["class_type"] == "small.class") & (df['sex'] == 'girl')]["scaled_combined_score"]
small_class_boy_score = df[(df["class_type"] == "small.class") & (df['sex'] == 'boy')]["scaled_combined_score"]

t_statistic, p_value = ttest_ind(small_class_girl_score, small_class_boy_score)

print(f'T-statistic: {t_statistic}')
print(f'P-value: {p_value}')

# Interpret the results
alpha = 0.01 # significance level
if p_value < alpha:
    print("The difference between the two groups is statistically significant.")
else:
    print("There is no statistically significant difference between the two groups.")

print(f"\nThe mean score for girls in small class {small_class_girl_score.mean()} and for boys in small class is {small_class_boy_score.mean()}")
```

✓ 0.0s

Python

```
T-statistic: 1.8597357003421364
P-value: 0.06309246068571991
There is no statistically significant difference between the two groups.
```

The mean score for girls in small class 467.78028503562945 and for boys in small class is 464.36700336700335

GIRLS PERFORM BETTER IN REGULAR CLASS SIZES

```
regular_class_girl_score = df[(df["class_type"] == "regular") & (df['sex'] == 'girl')]["scaled_combined_score"]
regular_class_boy_score = df[(df["class_type"] == "regular") & (df['sex'] == 'boy')]["scaled_combined_score"]

t_statistic, p_value = ttest_ind(regular_class_girl_score, regular_class_boy_score)

print(f'T-statistic: {t_statistic}')
print(f'P-value: {p_value}')

# Interpret the results
alpha = 0.01 # significance level
if p_value < alpha:
    print("The difference between the two groups is statistically significant.")
else:
    print("There is no statistically significant difference between the two groups.")

print(f"\nThe mean score for girls in regular class {regular_class_girl_score.mean()} and for boys in regular class is {regular_class_boy_score.mean()}")
```

✓ 0.0s

Python

T-statistic: 7.176771428293762

P-value: 1.0017674896636543e-12

The difference between the two groups is statistically significant.

The mean score for girls in regular class 464.9326131687243 and for boys in regular class is 453.3341439688716

GIRLS PERFORM BETTER IN REGULAR CLASS SIZES WITH AID

```
regular_with_aid_class_girl_score = df[(df["class_type"] == "regular.with.aide") & (df['sex'] == 'girl')]["scaled_combined_score"]
regular_with_aid_class_boy_score = df[(df["class_type"] == "regular.with.aide") & (df['sex'] == 'boy')]["scaled_combined_score"]

t_statistic, p_value = ttest_ind(regular_with_aid_class_girl_score, regular_with_aid_class_boy_score)

print(f'T-statistic: {t_statistic}')
print(f'P-value: {p_value}')

# Interpret the results
alpha = 0.01 # significance level
if p_value < alpha:
    print("The difference between the two groups is statistically significant.")
else:
    print("There is no statistically significant difference between the two groups.")

print(f"\nThe mean score for girls in regular class with aid {regular_with_aid_class_girl_score.mean()} and for boys in regular class with aid is {regular_with_aid_class_boy_score.mean()}")
```

✓ 0.0s

Python

T-statistic: 3.3250605820745864

P-value: 0.0008998637697378911

The difference between the two groups is statistically significant.

The mean score for girls in regular class with aid 461.96632653061226 and for boys in regular class with aid is 456.6753623188406



IN INDIA

- CLASS PERFORMANCE IN SCIENCES BEGINS TO DETERIORATE AFTER THE CLASS SIZE OF 41.
- THIS NUMBER IS AS HIGH AS 51 IN NON-SCIENCE CLASSES.
- US\$ 3.6 BILLION INCURRED BY INDIA (2017-18) IN REDUCING PTR.
- REDUCING CLASS SIZE TO 30 OR BELOW IN SECONDARY EDUCATION MAY LOWER LEARNING AND INCREASE COSTS.
- PROPOSED INCREASING THE PTR TO 40 PUPILS PER TEACHER IN BOTH ELEMENTARY AND SECONDARY SCHOOLS, ESTIMATING A POTENTIAL ANNUAL SAVING OF USD 19.4 BILLION IN TEACHER SALARY EXPENSES WITHOUT COMPROMISING LEARNING OUTCOMES.

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https://riseprogramme.org/sites/default/files/2021-01/RISE_WP-0059_Datta_Kingdon.pdf

THANK YOU