

Let us assume  $P(z) = N(0, 1)$

approx posterior

$$q(z|x) \approx p(z|x)$$

gaussian distribution

$$q(z|x) = \underbrace{N(\mu, \sigma^2)}_{\text{Variational bayes}}$$

Variational bayes



$$\approx N(0, 1)$$

Optimization can be given by

Evidence lower bound (ELBO)

$$\mathcal{L}(x) = \underbrace{\mathbb{E}_{q(z|x)} [\log_p(x|z)]}_{\text{Data consistency}} - \underbrace{KL(q(z|x) || P(z))}_{\text{Regularization term}}$$

• Data consistency: we want to make sure that our generated samples look like the original data.

Exp 11

## EXPERIMENTS USING VARIATIONAL AUTOENCODER

AIM

To implement a Variational Autoencoder (VAE) for the MNIST dataset learn probabilistic latent representation and evaluate.

OBJECTIVES

- \* To understand the architecture and concept of Variational Autoencoders (VAEs)
- \* To train a VAE on the MNIST dataset for unsupervised image generation and compression
- \* To compute reconstruction metrics such as MSE, PSNR, SSIM
- \* To visualize the training loss trend (Reconstruction + KL Divergence) and latentspace.

PSEUDOCODE:

- ↳ Import necessary libraries
- ↳ Load MNIST Dataset
- ↳ Define Variational Autoencoder (VAE) model.

Encoder:

Input Image  $\rightarrow$  Flatten  $\rightarrow$  Dense  $\rightarrow \mu(\text{mean}), \log \sigma^2 (\log \text{variance})$

Sampling:

~~Reparameterise  $z = \mu + \sigma \cdot \epsilon$   $\epsilon \sim N(0, 1)$~~

Decoder:

$z \rightarrow$  Dense  $\rightarrow$  Reconstruct image

- ↳ Define loss function

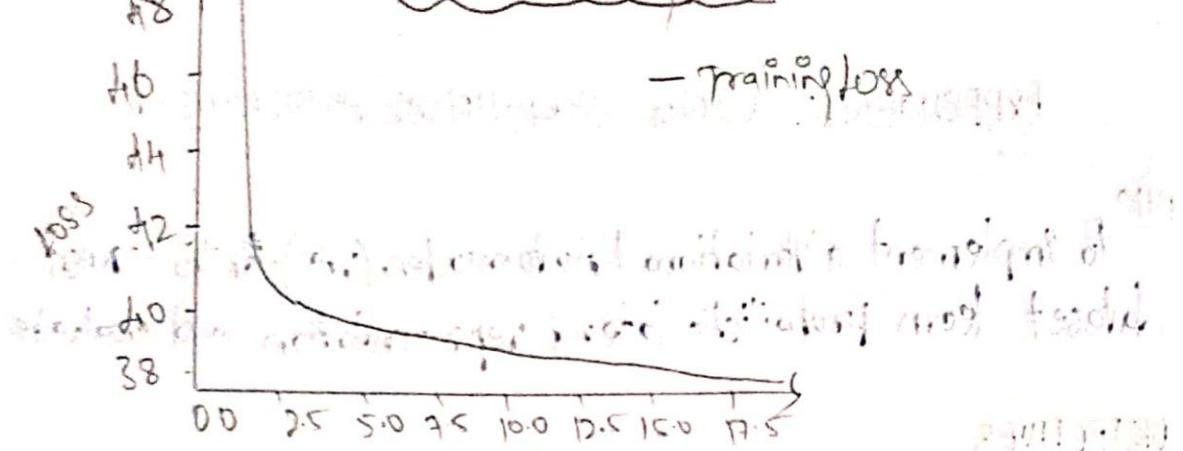
Reconstruction Loss =  $\text{MSE}(x, x_{\text{reconstructed}})$

KL Divergence =  $0.5 * \sum (\mu^2 + \sigma^2 - \log \sigma^2 - 1)$

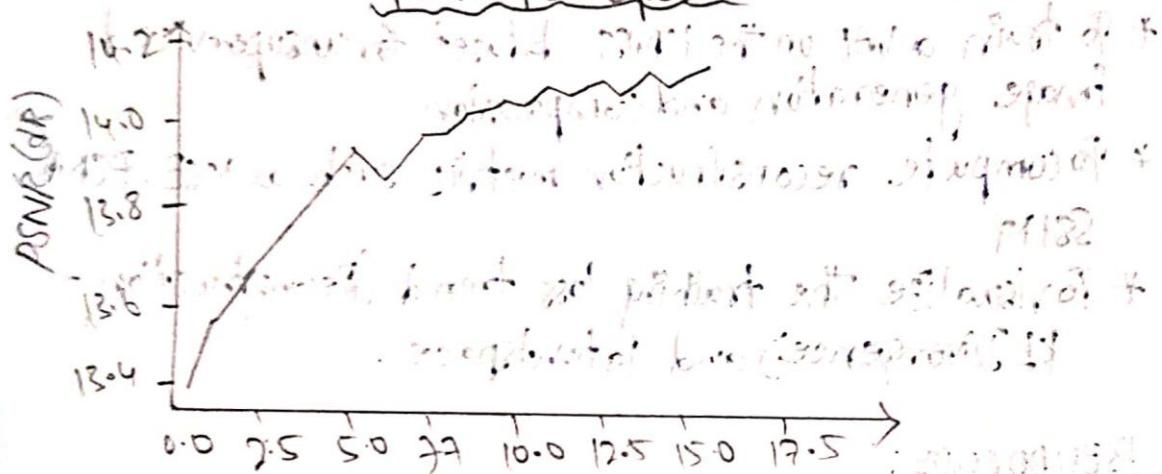
Total Loss = Reconstruction +  $\beta * \text{KL}$  ( $\beta=1$ )

- ↳ Train for N epochs:

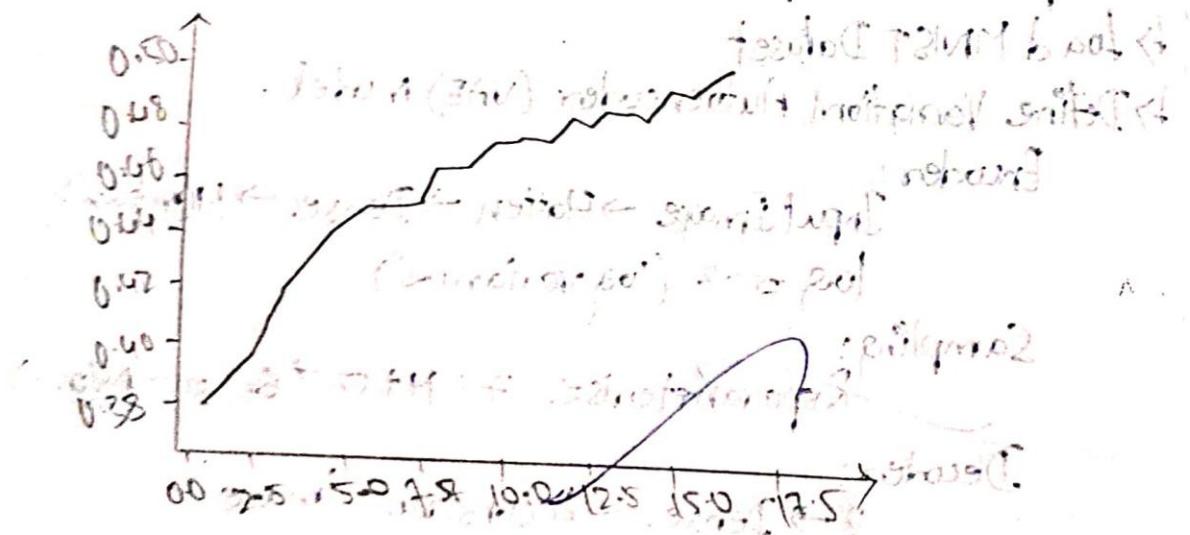
## VAE Training Loss



Ergebnis: Epochslänge 1, mit backtracking PSNR pro Epoch sinkt linear



SSIM: Bereich für SSIM liegt zwischen 0 und 1



Wert auf 0.65 erholt

Ergebnis: SSIM ist ein Maß für die Ähnlichkeit zweier Bilder

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Ergebnis: SSIM ist ein Maß für die Ähnlichkeit zweier Bilder

→ Evaluate performance:

\* Compute average MSE, PSNR, SSIM on test data.

→ Visualize:

(CONCLUSIONS) OBSERVATIONS:

Epoch [1/20] | Loss: 49.2570 | PSNR: 13.347 | SSIM: 0.381

Epoch [2/20] | Loss: 42.5184 | PSNR: 13.565 | SSIM: 0.403

Epoch [3/20] | Loss: 41.4425 | PSNR: 13.672 | SSIM: 0.422

Epoch [4/20] | Loss: 40.7862 | PSNR: 13.971 | SSIM: 0.441

Epoch [5/20] | Loss: 40.3012 | PSNR: 13.864 | SSIM: 0.449

⋮ ⌋ (0.0000 - 0.0000)

Epoch [18/20] | Loss: 38.0021 | PSNR: 14.197 | SSIM: 0.493

Epoch [20/20] | Loss: 37.9765 | PSNR: 14.172 | SSIM: 0.499

Final Evaluation: (0.0000 - 0.0000) + 0.0000

Average Training Loss: 39.9563

Final PSNR: 14.172

Final SSIM: 0.499

In the models Latent Space Representative (2D) the class '9' is more scattered so we got more number of '9' in reconstruction. \*\*

Q&A

RESULTS:

We have implemented a Variational Autoencoder for MNIST with 0.499 SSIM successfully

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LAB\_11

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Lab 11: Variational Autoencoder on MNIST

```

import torch
import torch.nn as nn
import torch.optim as optim
from torchvision import datasets, transforms
from torch.utils.data import DataLoader
import matplotlib.pyplot as plt
import numpy as np
from skimage.metrics import peak_signal_noise_ratio as psnr, structural_similarity as ssim

# Device
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")

```

```

import kagglehub
path = kagglehub.dataset_download("arnavsharma45/mnist-dataset")
print("Path to dataset files:", path)

Using Colab cache for faster access to the 'mnist-dataset' dataset.
Path to dataset files: /kaggle/input/mnist-dataset

```

Dataset Preparation

```

transform = transforms.Compose([
    transforms.ToTensor(),
])

```

Variables Terminal

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<https://colab.research.google.com/drive/1qSCyuvB0OYd1uuqlonVoA0ckzS6bb0Z?authuser=2#scrollTo=zVetPElg2p9k>

LAB\_11

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Using Colab cache for faster access to the 'mnist-dataset' dataset.
Path to dataset files: /kaggle/input/mnist-dataset

Dataset Preparation

```

transform = transforms.Compose([
    transforms.ToTensor(),
])

train_data = datasets.MNIST(root='./data', train=True, download=True, transform=transform)
test_data = datasets.MNIST(root='./data', train=False, download=True, transform=transform)

train_loader = DataLoader(train_data, batch_size=128, shuffle=True)
test_loader = DataLoader(test_data, batch_size=128, shuffle=False)

```

Define VAE Model

```

class VAE(nn.Module):
    def __init__(self, latent_dim=2):
        super(VAE, self).__init__()
        # Encoder
        self.fc1 = nn.Linear(28*28, 400)
        self.fc21 = nn.Linear(400, latent_dim)  # Mean ( $\mu$ )
        self.fc22 = nn.Linear(400, latent_dim)  # Log variance ( $\log \sigma^2$ )
        # Decoder
        self.fc3 = nn.Linear(latent_dim, 400)
        self.fc4 = nn.Linear(400, 28*28)

```

Variables Terminal

Define VAE Model

```

class VAE(nn.Module):
    def __init__(self, latent_dim=2):
        super(VAE, self).__init__()
        # Encoder
        self.fc1 = nn.Linear(28*28, 400)
        self.fc21 = nn.Linear(400, latent_dim)  # Mean ( $\mu$ )
        self.fc22 = nn.Linear(400, latent_dim)  # log variance ( $\log \sigma^2$ )
        # Decoder
        self.fc3 = nn.Linear(latent_dim, 400)
        self.fc4 = nn.Linear(400, 28*28)
        self.relu = nn.ReLU()
        self.sigmoid = nn.Sigmoid()

    def encode(self, x):
        h1 = self.relu(self.fc1(x))
        return self.fc21(h1), self.fc22(h1)  #  $\mu$  and  $\log \sigma^2$ 

    def reparameterize(self, mu, logvar):
        std = torch.exp(0.5 * logvar)
        eps = torch.randn_like(std)
        return mu + eps * std  #  $z = \mu + \sigma * \epsilon$ 

    def decode(self, z):
        h3 = self.relu(self.fc3(z))
        return self.sigmoid(self.fc4(h3))

```

Variables Terminal

Loss Function

```

def vae_loss(recon_x, x, mu, logvar):
    MSE = nn.functional.mse_loss(recon_x, x.view(-1, 28*28), reduction='sum')
    KLD = -0.5 * torch.sum(1 + logvar - mu.pow(2) - logvar.exp())
    return MSE + KLD

```

Initialize Model & Optimizer

```

model = VAE(latent_dim=2).to(device)
optimizer = optim.Adam(model.parameters(), lr=1e-3)

epochs = 20
train_losses, psnr_history, ssim_history = [], [], []

```

Training & Metric Evaluation per Epoch

```

for epoch in range(epochs):
    model.train()
    train_loss = 0

    for data, _ in train_loader:
        data = data.to(device)
        optimizer.zero_grad()
        recon_batch, mu, logvar = model(data)
        loss = vae_loss(recon_batch, data, mu, logvar)

```

Variables Terminal

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Training & Metric Evaluation per Epoch

```

for epoch in range(epochs):
    model.train()
    train_loss = 0

    for data, _ in train_loader:
        data = data.to(device)
        optimizer.zero_grad()
        recon_batch, mu, logvar = model(data)
        loss = vae.loss(recon_batch, data, mu, logvar)
        loss.backward()
        train_loss += loss.item()
        optimizer.step()

    avg_train_loss = train_loss / len(train_loader.dataset)
    train_losses.append(avg_train_loss)

# ---- Validation ----
model.eval()
mse_list, psnr_list, ssim_list = [], [], []
with torch.no_grad():
    for data, _ in test_loader:
        data = data.to(device)
        recon_batch, mu, logvar = model(data)
        x_orig = data.cpu().numpy()
        x_recon = recon_batch.view(-1, 1, 28, 28).cpu().numpy()

    for i in range(len(x_orig)):
        orig = x_orig[i, 0]
        rec = x_recon[i, 0]
        mse_val = np.mean((orig - rec) ** 2)
        mse_list.append(mse_val)
        psnr_list.append(psnr(orig, rec, data_range=1))
        ssim_list.append(ssim(orig, rec, data_range=1))

psnr_history.append(np.mean(psnr_list))
ssim_history.append(np.mean(ssim_list))

print(f"Epoch [{epoch+1}/{epochs}] | Loss: {avg_train_loss:.4f} | "
      f"PSNR: {np.mean(psnr_list):.3f} | SSIM: {np.mean(ssim_list):.3f}")

```

Variables Terminal

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# ---- Validation ----
model.eval()
mse\_list, psnr\_list, ssim\_list = [], [], []
with torch.no\_grad():
 for data, \_ in test\_loader:
 data = data.to(device)
 recon\_batch, mu, logvar = model(data)
 x\_orig = data.cpu().numpy()
 x\_recon = recon\_batch.view(-1, 1, 28, 28).cpu().numpy()

 for i in range(len(x\_orig)):
 orig = x\_orig[i, 0]
 rec = x\_recon[i, 0]
 mse\_val = np.mean((orig - rec) \*\* 2)
 mse\_list.append(mse\_val)
 psnr\_list.append(psnr(orig, rec, data\_range=1))
 ssim\_list.append(ssim(orig, rec, data\_range=1))

psnr\_history.append(np.mean(psnr\_list))
ssim\_history.append(np.mean(ssim\_list))

print(f"Epoch [{epoch+1}/{epochs}] | Loss: {avg\_train\_loss:.4f} | "
 f"PSNR: {np.mean(psnr\_list):.3f} | SSIM: {np.mean(ssim\_list):.3f}")

Epoch [1/20] | Loss: 49.2570 | PSNR: 13.347 | SSIM: 0.381
Epoch [2/20] | Loss: 42.5184 | PSNR: 13.565 | SSIM: 0.403
Epoch [3/20] | Loss: 41.4425 | PSNR: 13.672 | SSIM: 0.422
Epoch [4/20] | Loss: 46.7862 | PSNR: 13.771 | SSIM: 0.441
Epoch [5/20] | Loss: 46.3012 | PSNR: 13.864 | SSIM: 0.449
Epoch [6/20] | Loss: 39.9422 | PSNR: 13.836 | SSIM: 0.451
Epoch [7/20] | Loss: 39.9422 | PSNR: 13.836 | SSIM: 0.451
Epoch [8/20] | Loss: 39.9422 | PSNR: 13.836 | SSIM: 0.451
Epoch [9/20] | Loss: 39.9422 | PSNR: 13.836 | SSIM: 0.451
Epoch [10/20] | Loss: 39.9422 | PSNR: 13.836 | SSIM: 0.451
Epoch [11/20] | Loss: 39.9422 | PSNR: 13.836 | SSIM: 0.451
Epoch [12/20] | Loss: 39.9422 | PSNR: 13.836 | SSIM: 0.451
Epoch [13/20] | Loss: 39.9422 | PSNR: 13.836 | SSIM: 0.451
Epoch [14/20] | Loss: 39.9422 | PSNR: 13.836 | SSIM: 0.451
Epoch [15/20] | Loss: 39.9422 | PSNR: 13.836 | SSIM: 0.451
Epoch [16/20] | Loss: 39.9422 | PSNR: 13.836 | SSIM: 0.451
Epoch [17/20] | Loss: 39.9422 | PSNR: 13.836 | SSIM: 0.451
Epoch [18/20] | Loss: 39.9422 | PSNR: 13.836 | SSIM: 0.451
Epoch [19/20] | Loss: 39.9422 | PSNR: 13.836 | SSIM: 0.451
Epoch [20/20] | Loss: 39.9422 | PSNR: 13.836 | SSIM: 0.451

Variables Terminal

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```

print(f"Epoch [{epoch+1}/{epochs}] | Loss: {avg_train_loss:.4f} | "
      f"PSNR: {np.mean(psnr_list):.3f} | SSIM: {np.mean(ssim_list):.3f}")

Epoch [1/20] | Loss: 49.2570 | PSNR: 13.347 | SSIM: 0.381
Epoch [2/20] | Loss: 42.5184 | PSNR: 13.565 | SSIM: 0.483
Epoch [3/20] | Loss: 41.4425 | PSNR: 13.672 | SSIM: 0.422
Epoch [4/20] | Loss: 46.7862 | PSNR: 13.371 | SSIM: 0.441
Epoch [5/20] | Loss: 39.0551 | PSNR: 13.864 | SSIM: 0.450
Epoch [6/20] | Loss: 39.9422 | PSNR: 13.836 | SSIM: 0.451
Epoch [7/20] | Loss: 39.6417 | PSNR: 13.925 | SSIM: 0.465
Epoch [8/20] | Loss: 39.4002 | PSNR: 13.942 | SSIM: 0.465
Epoch [9/20] | Loss: 39.1871 | PSNR: 13.978 | SSIM: 0.473
Epoch [10/20] | Loss: 39.0199 | PSNR: 14.015 | SSIM: 0.474
Epoch [11/20] | Loss: 38.8723 | PSNR: 14.014 | SSIM: 0.479
Epoch [12/20] | Loss: 38.7733 | PSNR: 14.031 | SSIM: 0.477
Epoch [13/20] | Loss: 38.6962 | PSNR: 14.093 | SSIM: 0.482
Epoch [14/20] | Loss: 38.4940 | PSNR: 14.084 | SSIM: 0.485
Epoch [15/20] | Loss: 38.3884 | PSNR: 14.108 | SSIM: 0.489
Epoch [16/20] | Loss: 38.2775 | PSNR: 14.130 | SSIM: 0.490
Epoch [17/20] | Loss: 38.2031 | PSNR: 14.103 | SSIM: 0.489
Epoch [18/20] | Loss: 38.0947 | PSNR: 14.181 | SSIM: 0.497
Epoch [19/20] | Loss: 38.0021 | PSNR: 14.147 | SSIM: 0.493
Epoch [20/20] | Loss: 37.9765 | PSNR: 14.172 | SSIM: 0.499

Evaluation

```

```

print("\n▣ Final Evaluation:")
print("Average Training Loss: {np.mean(train_losses):.4f}")
print(Final PSNR: {psnr_history[-1]:.3f})
print(Final SSIM: {ssim_history[-1]:.3f})

```

Variables Terminal

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```

print("\n▣ Final Evaluation:")
print("Average Training Loss: {np.mean(train_losses):.4f}")
print(Final PSNR: {psnr_history[-1]:.3f})
print(Final SSIM: {ssim_history[-1]:.3f})

▣ Final Evaluation:
Average Training Loss: 39.7563
Final PSNR: 14.172
Final SSIM: 0.499

```

Evaluation

Visualizations

```

# (a) Loss Curve
plt.figure(figsize=(6,4))
plt.plot(train_losses, label="Training Loss", color='tab:red', linewidth=2)
plt.xlabel("Epochs"); plt.ylabel("Loss")
plt.title("VAE Training Loss")
plt.legend(); plt.grid(True)
plt.show()

# (b) PSNR Curve
plt.figure(figsize=(6,4))
plt.plot(psnr_history, label="PSNR", color='tab:blue', marker='o')
plt.xlabel("Epochs"); plt.ylabel("PSNR (dB)")

```

Variables Terminal

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Final PSNR: 19.122  
Final SSIM: 0.499

### Visualizations

```

# (a) Loss curve
plt.figure(figsize=(6,4))
plt.plot(train_losses, label="Training Loss", color='tab:red', linewidth=2)
plt.xlabel("Epochs"); plt.ylabel("Loss")
plt.title("VAE Training Loss")
plt.legend(); plt.grid(True)
plt.show()

# (b) PSNR Curve
plt.figure(figsize=(6,4))
plt.plot(PSNR_history, label="PSNR", color='tab:blue', marker='o')
plt.xlabel("Epochs"); plt.ylabel("PSNR (dB)")
plt.title("PSNR per Epoch")
plt.legend(); plt.grid(True)
plt.show()

# (c) SSIM Curve
plt.figure(figsize=(6,4))
plt.plot(ssim_history, label="SSIM", color='tab:green', marker='s')
plt.xlabel("Epochs"); plt.ylabel("SSIM")
plt.title("SSIM per Epoch")
plt.legend(); plt.grid(True)
plt.show()

```

### Variables Terminal

```

# (d) Reconstruction Visualization
test_iter = iter(test_loader)
test_imgs, _ = next(test_iter)
test_imgs = test_imgs.to(device)
reconstructed, _ = model(test_imgs)
reconstructed = reconstructed.view(-1, 1, 28, 28).cpu().detach()

fig, axes = plt.subplots(2, 10, figsize=(12, 3))
for i in range(10):
    axes[0, i].imshow(test_imgs[i, 0].cpu(), cmap='gray')
    axes[0, i].axis('off')
    axes[1, i].imshow(reconstructed[i, 0], cmap='gray')
    axes[1, i].axis('off')
    axes[0, 0].set_title("Original")
    axes[1, 0].set_title("Reconstructed")
    plt.suptitle("VAE Reconstruction Results")
plt.show()

# (e) Latent Space Visualization
with torch.no_grad():
    mu_vals, labels = [], []

```

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```
# (e) Latent Space Visualization
with torch.no_grad():
    mu_vals, labels = [], []
    for data, target in test_loader:
        data = data.to(device)
        mu, _ = model.encode(data.view(-1, 28*28))
        mu_vals.append(mu.cpu())
        labels.append(target)
    mu_vals = torch.cat(mu_vals)
    labels = torch.cat(labels)

    plt.figure(figsize=(7,6))
    plt.scatter(mu_vals[:, 0], mu_vals[:, 1], c=labels, cmap='tab10', s=10)
    plt.colorbar()
    plt.title("Latent Space Representation ( $\mu$ )")
    plt.xlabel("Latent Dim 1")
    plt.ylabel("Latent Dim 2")
    plt.show()
```

VAE Training Loss

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```
plt.show()
```

VAE Training Loss

PSNR per Epoch

Variables Terminal

