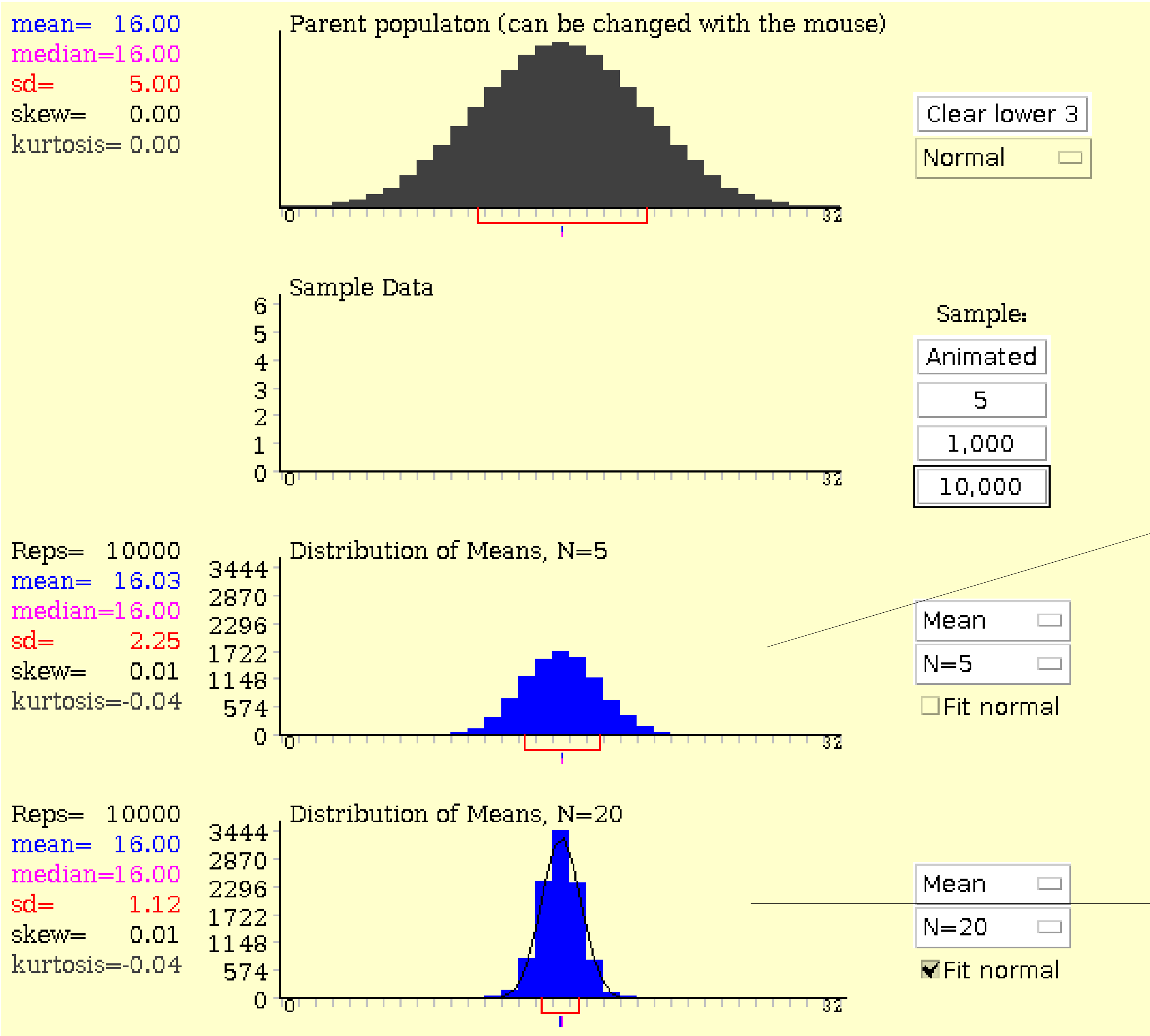
# Central Limit Theorem And Confidence Interval

**The central limit theorem (CLT)** is a statistical theory that states that given a sufficiently large sample size from a population with a finite level of variance, the mean of all samples from the same population will be approximately equal to the mean of the population.

The central limit theorem states that: Given a population with a finite mean μ and a finite non-zero variance σ2, the sampling distribution of the mean approaches a normal distribution with a mean of μ and a variance of σ2/N as N, the sample size, increases.

**Confidence Interval:** A range of values so defined that there is a specified probability that the value of a parameter lies within it.

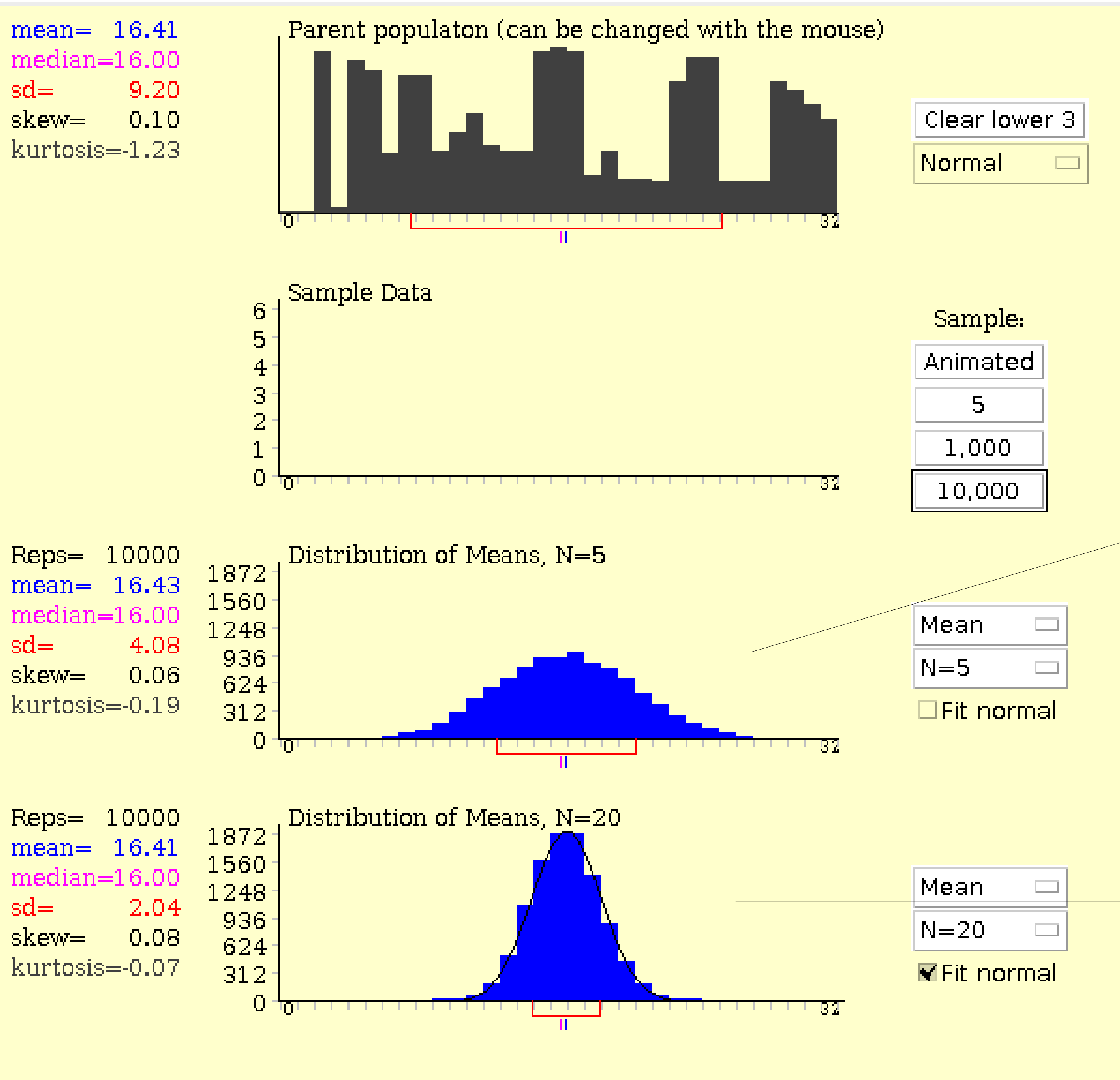
http://onlinestatbook.com/




Population SD = 5
Sample SD = 5/sqrt(5)
= 2.23606
= 2.24
Almost equal to sample SD – 2.25

(vice-versa is not true)

Population SD = 5
Sample SD = 5/sqrt(20)
= 1.118033
= 1.12
Equal to sample SD – 1.12
(vice-versa is not true)




Population SD = 9.20
Standard Error(SD) = 9.20/sqrt(5)
= 4.11436
= 4.11
Almost equal to sample SD – 4.08

(vice-versa is not true)

Population SD = 9.20
Standard Error(SD) = 9.20/sqrt(20)
= 2.05718
= 2.06
Almost equal to sample SD – 2.04
(vice-versa is not true)

**Example:** You sample 36 apples form your farm's harvest of over 2,00,000 apples. The mean weight of the sample is 112 grams (with a 40 gram sample standard deviation). What is the probability that the mean weight of all 2,00,000 apples is within 100 and 124 grams ?

**Useful information:**
The Sample Size(36)
Sample Mean(112 grams)
Standard Deviation(40) – best estimator of population SD

**What to find?**
* Probability(the mean weight of population within 100 and 124 grams)
* Think that we took many samples of size 36 (sampling distribution), the sampling distribution mean is 112 grams.
* From Central Limit Theorem – the sampling distribution mean (112) is same as the population mean.
* Now we have to find P(**Population Mean** is with in **12** of **Sampling Distribution Mean**)
i.e., 112 – 12 = 100
124 + 12 = 124
* From Central Limit Theorem – sample SD = Population SD/Sqrt(sample size)
= 40/Sqrt(36)
= 40/6 = 6.66667
= 6.67
* **12** is **1.8** Standard Deviations away form 112. (124-112/6.6667 = 12/6.6667 = 1.799)

* **Confidence Interval:** Lets find out probability of the ***mean apple wait*** is between **mean** and **1.8 snandard deviations above the mean** by looking at Z-Table. This value need to be doubled to find confidence interval.

**Standard Normal Probabilities**

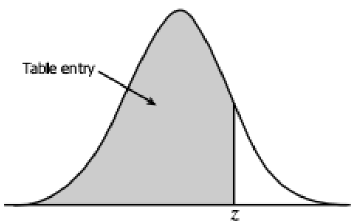
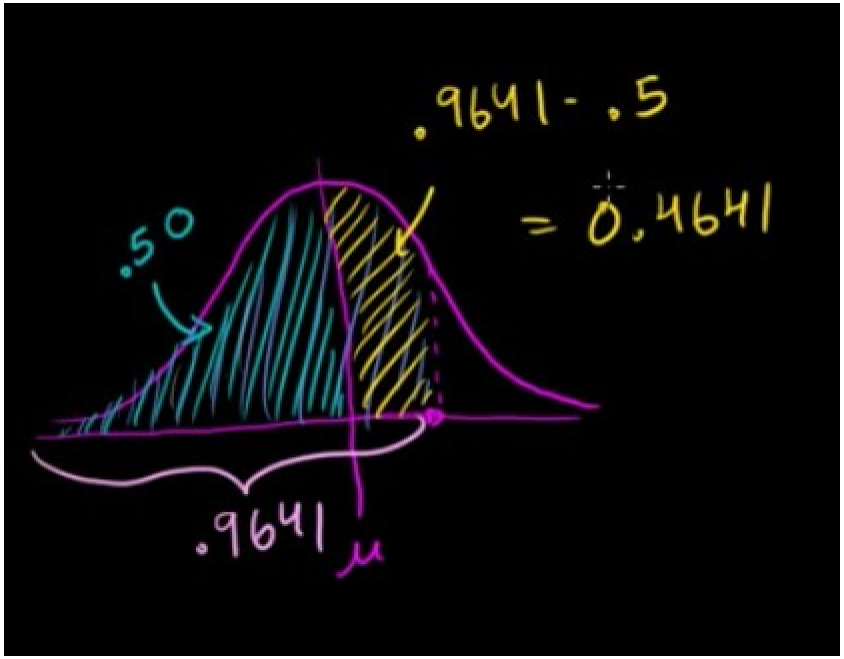



Table entry for $z$ is the area under the standard normal curve to the left of $z$.

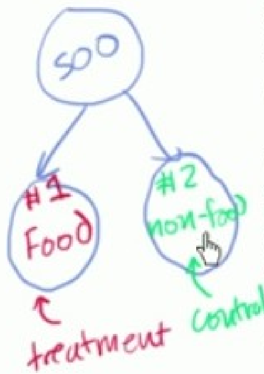
| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |




* The confidence interval is **2 x 0.4642 = 0.9282.**

* We are **92.82% confident that the population mean is between 100 and 124 range.**

# Statistical Significance

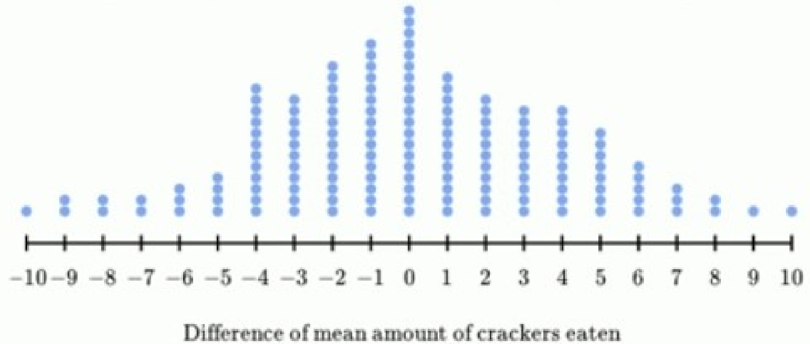



In an experiment aimed at studying the effect of advertising on eating behavior in children, a group of 500 children, 7-11 years old, were randomly assigned to two different groups. After randomization, each child was asked to watch a cartoon in a private room, containing a large bowl of goldfish crackers. The cartoon included 2 commercial breaks.

The first group watched food commercials (mostly snacks), while the second group watched non-food commercials (games and entertainment products). Once the child finished watching the cartoon, the conductors of the experiment weighed the crackers bowl to measure how many grams of crackers the child ate. They found that the mean amount of crackers eaten by the children who watched food commercials is 10 grams greater than the mean amount of crackers eaten by the children who watched non-food commercials.

Using a simulator, they re-randomized the results into two new groups and measured the difference between the means of the new groups. They repeated this simulation 150 times, and plotted the resulting differences, as given below.

**According to the simulations, is the result of the experiment significant?**




**Significance Test:** The probability of some thing **randomly happening** is less than (significance level – alpha ) 5% then a we will consider the experiment as statistically significant.

Here the above experiment states that the kids who watched food commercials eat 10 grams more that the other group. The dot-plot of the group means after re-randomization shows that only 2 times the difference in mean is 10 grams.

This states that the kids who watch food commercials eat 10 grams more that the other group at random is only 1.33%. Which is less than 5%.

Hence the experiment is significant. That is the kids who watched food commercials eat 10 grams more than the other group.

# Hypothesis testing

**Null & Alternative Hypothesis:** The null hypothesis, $H_0$ is the commonly accepted fact; it is the opposite of the alternate hypothesis ($H_1$). Researchers work to reject, nullify or disprove the null hypothesis.

**Why is it Called the “Null”?**
The word “null” in this context means that it’s a commonly accepted fact that researchers work to nullify. It doesn’t mean that the statement is null itself! (Perhaps the term should be called the “**nullifiable hypothesis**” as that might cause less confusion).

Example: A researcher is studying the effects of radical exercise program on knee surgery patients. There is a good chance that the therapy will improve recovery time, but there's also the possibility it will make it worse. Average recovery times for knee surgery is 8.2 weeks.

$H_0$ (Null Hypothesis) : Average recovery times for knee surgery is 8.2 weeks.
$H_1$ (Alternative Hypothesis) : Average recovery times for knee surgery is NOT 8.2 weeks (could be more or less).

**P-Value:** When you perform a hypothesis test in statistics, a p-value helps you determine the significance of your results. The p-value is a number between 0 and 1 and interpreted in the following way: A small p-value (typically ≤ 0.05) indicates strong evidence against the null hypothesis, so you reject the null hypothesis.

A neurologist is testing the effect of a drug on response time by injecting 100 rats with a unit dose of the drug, subjecting each to neurological stimulus, and recording its response time. The neurologist knows that the mean response time for rats not injected with the drug is 1.2 seconds. The mean of the 100 injected rats' response times is 1.05 seconds with a sample standard deviation of 0.5 seconds. Do you think that the drug has an effect on response time?

$H_0$ : Drug has no effect on response time (mean = 1.2 seconds)
$H_1$ : Drug has an effect on response time. (mean != 1.2 seconds)

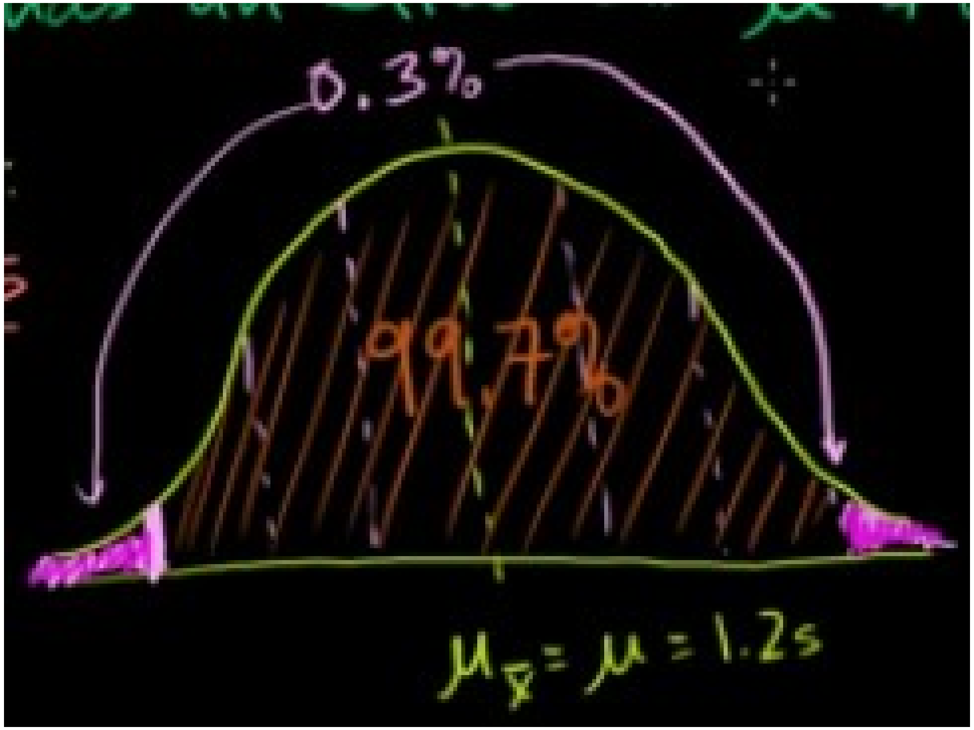
* Assume $H_0$ is true → From central limit theorem the mean of the sampling distribution is 1.2 seconds (population mean).
* Now we have to find how likely(probability) that the response time is 1.05, when drug has no effect (null hypothesis)
* That is how many standard deviations away 1.05 is from 1.2. This gives the area under the curve of a normal distribution.
* Need to find standard deviation of the sampling distribution. The formula for this is = population SD/sqrt(# of samples)
* We don't know the population standard deviation, hence we take best approximation as 0.5 (sample SD) as population SD.

Sampling distribution SD = 0.5/sqrt(100)
= 0.5/10
= 0.05

Z-Score (how may SD away 1.05 from 1.2) = (1.2 – 1.05)/0.05
= 3

* **P-value:** 3 standard deviations away (99.7%). Only 0.3% (P-value is 0.003) chance getting 1.05 seconds response when drug has no effect.

Hence $H_0$ (Null Hypothesis) is rejected.



# One-tailed and Two-tailed Tests

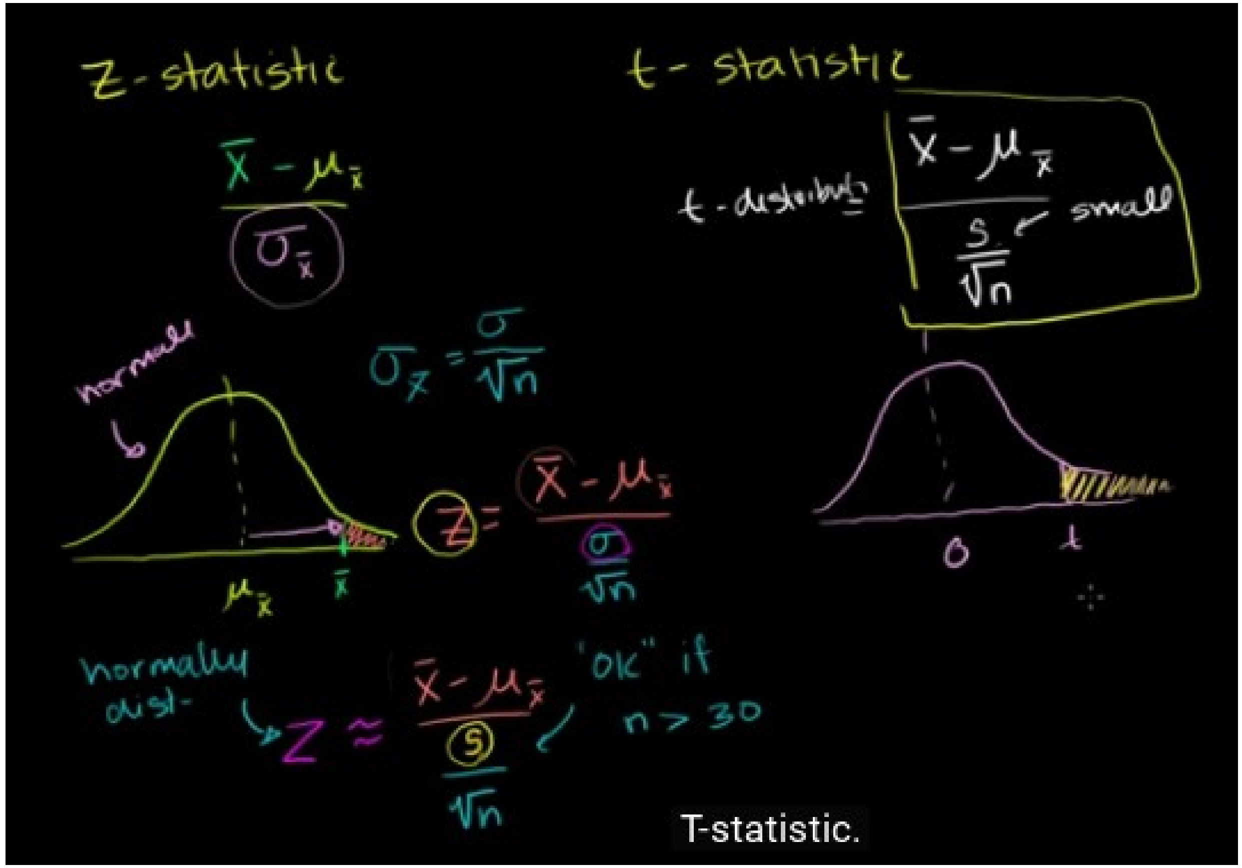
The above example is a Two-tailed test. This depends on the alternative hypothesis we chose. Above example $H_1$ is looking for has an effect on response time, but not for more or less.

$H_0$ : Drug has no effect on response time (mean = 1.2 seconds)
$H_1$ : Drug has an effect on response time. (mean != 1.2 seconds)

One-Tailed test: If the $H_1$ is mentioned as below then it would have been One-Tailed test. The P-Value would have been 0.0015.

$H_1$ : Drug lowers the response time ( < 1.2 seconds)

# Z-Statistic vs T-Statistic




We have to use T-Statistic if the sample size is less than 30.

http://www.sjsu.edu/faculty/gerstman/StatPrimer/t-table.pdf

# Chi-Square Goodness of Fit Test

When an analyst attempts to fit a statistical model to observed data, he or she may wonder how well the model actually reflects the data. How "close" are the observed values to those which would be expected under the fitted model? One statistical test that addresses this issue is the chi-square goodness of fit test.

In general, the chi-square test statistic is of the form

$$X^2 = \sum \frac{(\text{observed - expected})^2}{\text{expected}}$$

**Example:** A new casino game involves rolling 3 dice. The winnings are directly proportional to the total number of sixes rolled. Suppose a gambler plays the game 100 times, with the following observed counts:

| Number of Sixes | Number of Rolls |
|---|---|
| 0 | 48 |
| 1 | 35 |
| 2 | 15 |
| 3 | 3 |

The casino becomes suspicious of the gambler and wishes to determine whether the dice are fair. What do they conclude?

* Probability to rolling a 6 on any given toss – 1/6. Assuming the 3 dice are independent (the roll of one die should not affect the roll of the others), we might assume that the number of sixes in three rolls is distributed Binomial.

* To determine whether the gambler's dice are fair, we may compare his results with the results expected under this distribution. On 3 dies

$$P_p(n \mid N) = \binom{N}{n} p^n q^{N-n} = \frac{N!}{n!\,(N-n)!} p^n (1-p)^{N-n},$$

**Null Hypothesis:**

→ The expected values for 0 sixes P(X=0) = $3^{C}0 \times (1/6)^{0} \times (1-1/6)^{3} = 1 \times 1 \times (5/6)^{3} = 0.578$

→ The expected values for 1 sixes P(X=1) = $3^{C}1 \times (1/6)^{1} \times (1-1/6)^{2} = 3 \times (1/6) \times (5/6)^{2} = 0.347$

→ The expected values for 2 sixes P(X=2) = $3^{C}2 \times (1/6)^{2} \times (1-1/6)^{1} = 3 \times (1/6)^{2} \times (5/6)^{3} = 0.069$

→ The expected values for 3 sixes P(X=3) = $3^{C}3 \times (1/6)^{3} \times (1-1/6)^{0} = 1 \times (1/6)^{3} \times 1 = 0.005$

Since the gambler plays 100 times, the expected counts are the following:

| Number of Sixes | Expected Counts | Observed Counts |
|---|---|---|
| 0 | 58.0 | 48 |
| 1 | 34.5 | 35 |
| 2 | 7.0 | 15 |
| 3 | 0.5 | 3 |

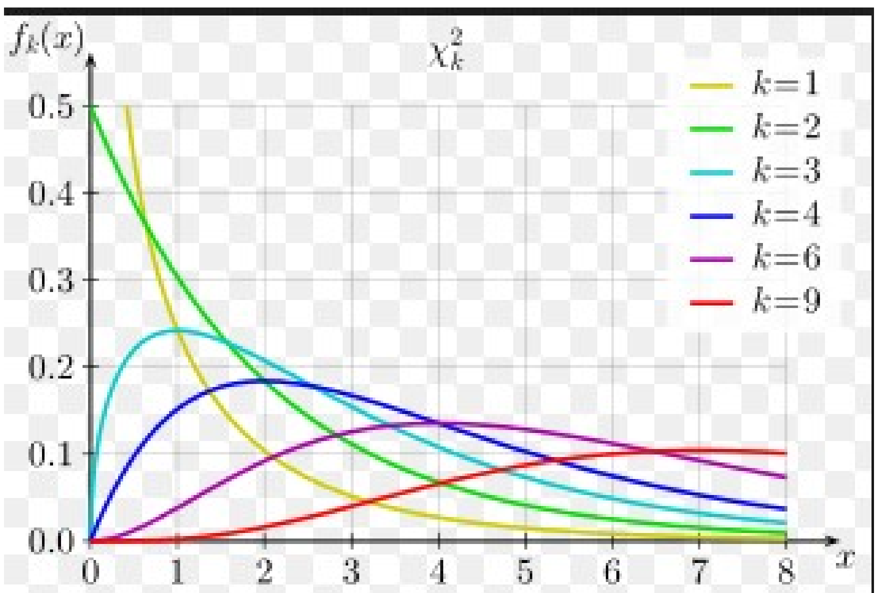
**The chi-square statistic is** = $(48-58)^2/58 + (35-34.5)^2/58 + (15-7)^2/7 + (3-0.5)^2/0.5$

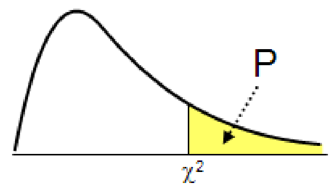
= 1.72 + 0.007 + 9.14 + 12.5

= 23.367.

* We need to look into Chi-Square distribution table and curve. We have four random variables, hence the degrees of freedom is 3.

* If we are interested in a significance level of 0.05 we may reject the null hypothesis (that the dice are fair) if > 7.815, the value corresponding to the 0.05 significance level for the (3) distribution. Since 23.367 is clearly greater than 7.815, we may reject the null hypothesis that the dice are fair at the 0.05 significance level.




| | P | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **DF** | **0.995** | **0.975** | **0.20** | **0.10** | **0.05** | **0.025** | **0.02** | **0.01** | **0.005** | **0.002** | **0.001** |
| **1** | 0.0000393 | 0.000982 | 1.642 | 2.706 | 3.841 | 5.024 | 5.412 | 6.635 | 7.879 | 9.550 | 10.828 |
| **2** | 0.0100 | 0.0506 | 3.219 | 4.605 | 5.991 | 7.378 | 7.824 | 9.210 | 10.597 | 12.429 | 13.816 |
| **3** | 0.0717 | 0.216 | 4.642 | 6.251 | 7.815 | 9.348 | 9.837 | 11.345 | 12.838 | 14.796 | 16.266 |
| **4** | 0.207 | 0.484 | 5.989 | 7.779 | 9.488 | 11.143 | 11.668 | 13.277 | 14.860 | 16.924 | 18.467 |
| **5** | 0.412 | 0.831 | 7.289 | 9.236 | 11.070 | 12.833 | 13.388 | 15.086 | 16.750 | 18.907 | 20.515 |
| **6** | 0.676 | 1.237 | 8.558 | 10.645 | 12.592 | 14.449 | 15.033 | 16.812 | 18.548 | 20.791 | 22.458 |
| **7** | 0.989 | 1.690 | 9.803 | 12.017 | 14.067 | 16.013 | 16.622 | 18.475 | 20.278 | 22.601 | 24.322 |
| **8** | 1.344 | 2.180 | 11.030 | 13.362 | 15.507 | 17.535 | 18.168 | 20.090 | 21.955 | 24.352 | 26.124 |
| **9** | 1.735 | 2.700 | 12.242 | 14.684 | 16.919 | 19.023 | 19.679 | 21.666 | 23.589 | 26.056 | 27.877 |
| **10** | 2.156 | 3.247 | 13.442 | 15.987 | 18.307 | 20.483 | 21.161 | 23.209 | 25.188 | 27.722 | 29.588 |
| **11** | 2.603 | 3.816 | 14.631 | 17.275 | 19.675 | 21.920 | 22.618 | 24.725 | 26.757 | 29.354 | 31.264 |
| **12** | 3.074 | 4.404 | 15.812 | 18.549 | 21.026 | 23.337 | 24.054 | 26.217 | 28.300 | 30.957 | 32.909 |
| **13** | 3.565 | 5.009 | 16.985 | 19.812 | 22.362 | 24.736 | 25.472 | 27.688 | 29.819 | 32.535 | 34.528 |
| **14** | 4.075 | 5.629 | 18.151 | 21.064 | 23.685 | 26.119 | 26.873 | 29.141 | 31.319 | 34.091 | 36.123 |
| **15** | 4.601 | 6.262 | 19.311 | 22.307 | 24.996 | 27.488 | 28.259 | 30.578 | 32.801 | 35.628 | 37.697 |
| **16** | 5.142 | 6.908 | 20.465 | 23.542 | 26.296 | 28.845 | 29.633 | 32.000 | 34.267 | 37.146 | 39.252 |
| **17** | 5.697 | 7.564 | 21.615 | 24.769 | 27.587 | 30.191 | 30.995 | 33.409 | 35.718 | 38.648 | 40.790 |
| **18** | 6.265 | 8.231 | 22.760 | 25.989 | 28.869 | 31.526 | 32.346 | 34.805 | 37.156 | 40.136 | 42.312 |
| **19** | 6.844 | 8.907 | 23.900 | 27.204 | 30.144 | 32.852 | 33.687 | 36.191 | 38.582 | 41.610 | 43.820 |
| **20** | 7.434 | 9.591 | 25.038 | 28.412 | 31.410 | 34.170 | 35.020 | 37.566 | 39.997 | 43.072 | 45.315 |
| **21** | 8.034 | 10.283 | 26.171 | 29.615 | 32.671 | 35.479 | 36.343 | 38.932 | 41.401 | 44.522 | 46.797 |
| **22** | 8.643 | 10.982 | 27.301 | 30.813 | 33.924 | 36.781 | 37.659 | 40.289 | 42.796 | 45.962 | 48.268 |

## F- Statistic (ANOVA)

Figure out how much of the total variance comes from:

- The variance *between* the groups
- The variance *within* the groups

Calculate the ratio:

$$F = \frac{between\ groups}{within\ groups}$$

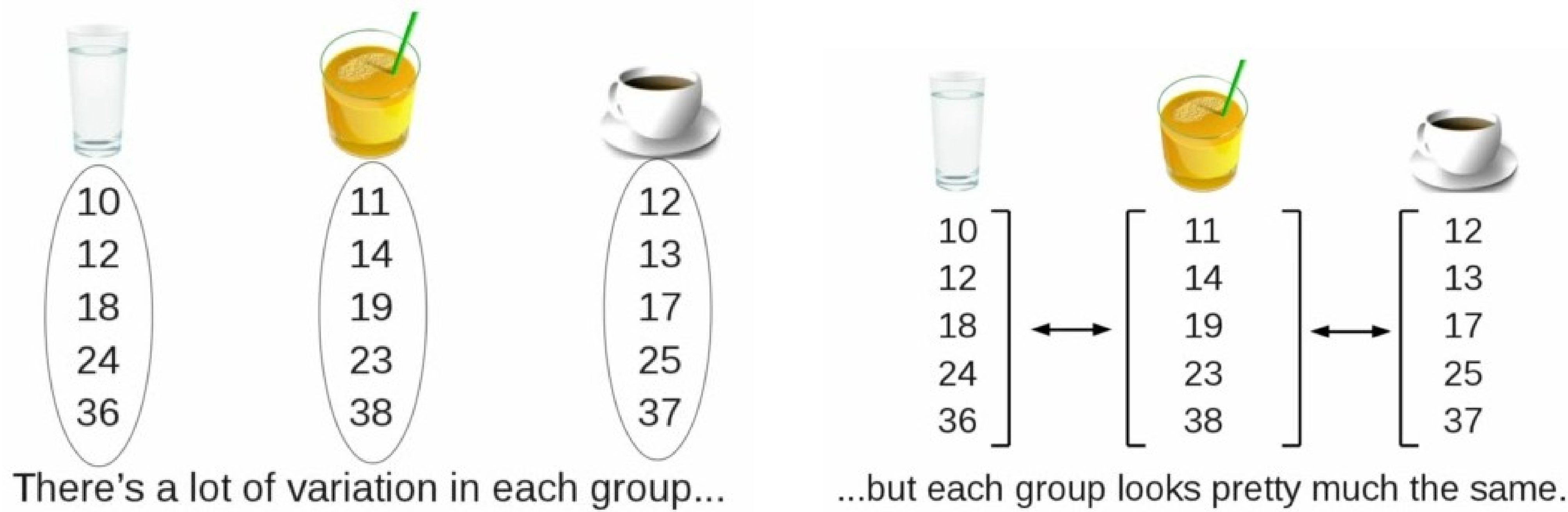
The larger the ratio, the more likely it is that the groups have different means (reject $H_0$).

Suppose that three groups have given water, fruit juice, coffee. Now we want to test the response between three groups. We have to use F-test as we have more than two groups (otherwise we would have used t-test)

**Null Hypothesis:** Drink didn't make much difference

Example 1: Lot of variation with in group, but a little between groups.

This case we will **accept** Null Hypothesis, variation between groups is not much.



Example 2: A little variation with in group, but a lot of variation between groups.

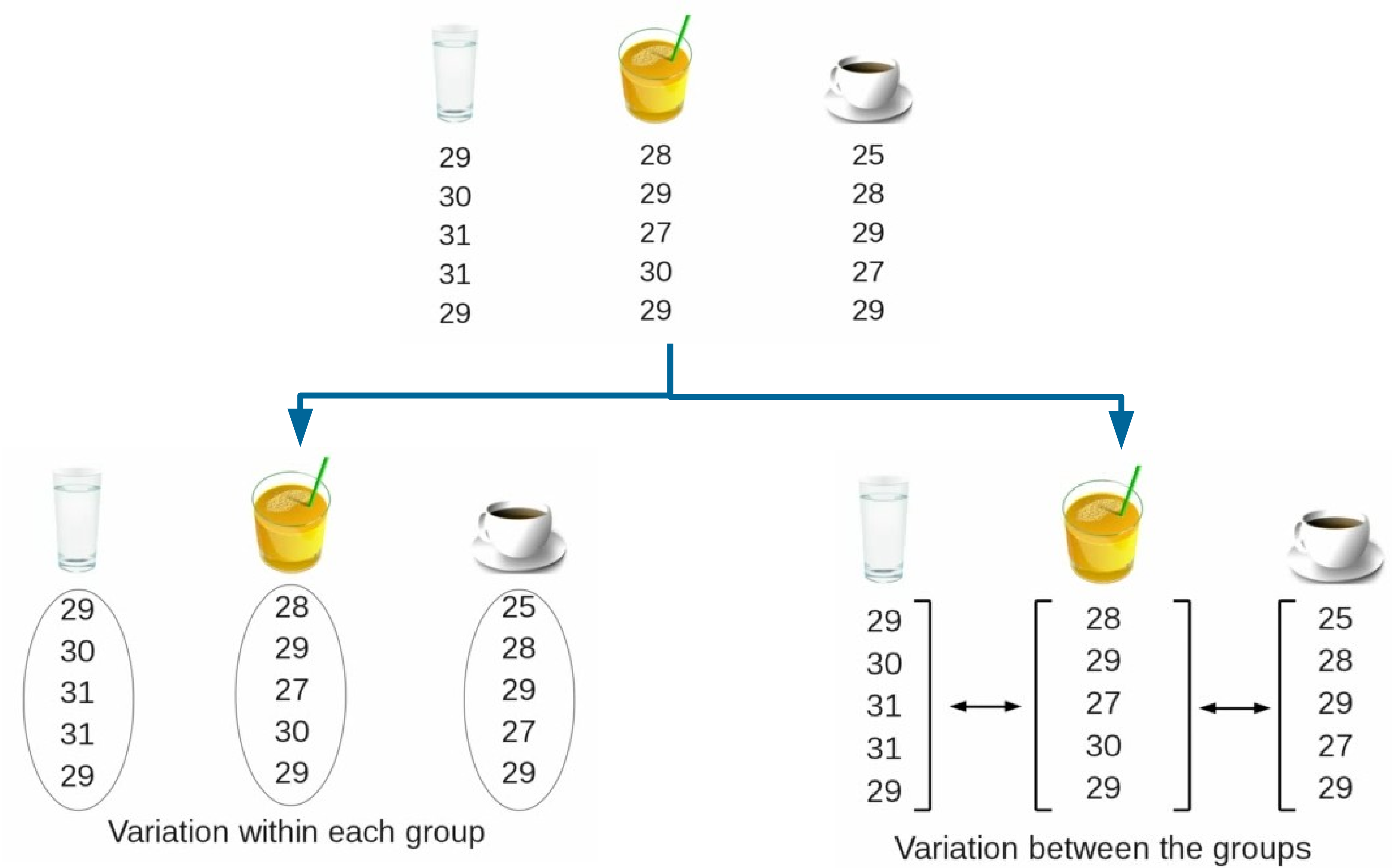
This case we will **reject** the Null Hypothesis

| | | |
|---|---|---|
| 29 | 17 | 10 |
| 29 | 18 | 11 |
| 30 | 19 | 12 |
| 31 | 19 | 12 |
| 31 | 20 | 13 |

Conclusion: it's the drink that makes the difference, not the people.

Example 3: There is not much variation with in OR between groups.

The result of ANOVA calculation is F(2, 12) = 4.7, the p-value = 0.04 (from F-distribution table) . We can **reject** the Null Hypothesis.



Calculate the degrees of freedom as follows:

$$F(b, w)$$

b is the degrees of freedom for variance between groups.

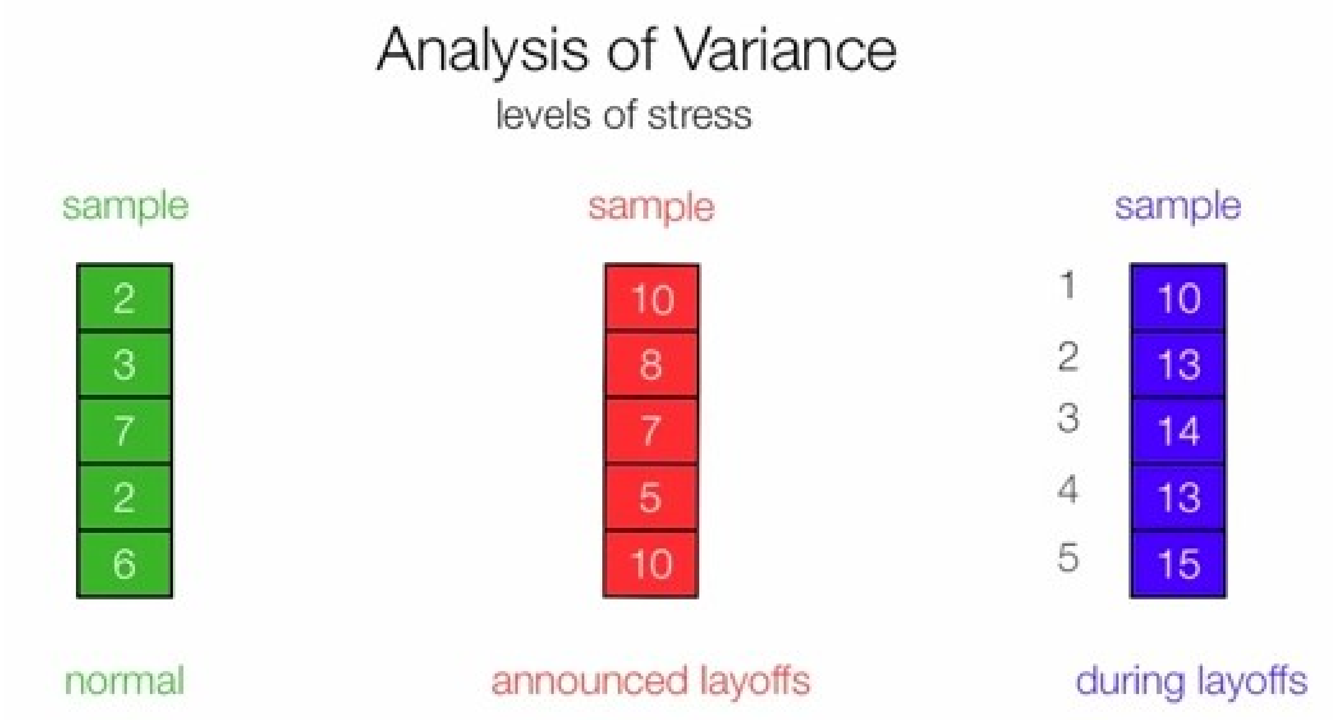
w is the degrees of freedom for variance within groups.

b = number of groups – 1
w = total number of observations – number of groups

# ANOVA Calculation example

Sample 1 : Stress under normal condition
Sample 2 : Stress after announced layoffs
Sample 3 : Stress during layoffs
Need to measure the impact of announced layoffs.

**Null Hypothesis :** No impact of announced layoffs on employee stress



# Analysis of Variance

## Sum of Squares Within Groups

| sample | | | sample | | | sample | | |
|---|---|---|---|---|---|---|---|---|
| 2 | $- 4 = -2^2$ | 4 | 10 | $- 8 = 2^2$ | 4 | 10 | $- 13 = -3^2$ | 9 |
| 3 | $- 4 = -1^2$ | 1 | 8 | $- 8 = 0^2$ | 0 | 13 | $- 13 = 0^2$ | 0 |
| 7 | $- 4 = 3^2$ | 9 | 7 | $- 8 = -1^2$ | 1 | 14 | $- 13 = 1^2$ | 1 |
| 2 | $- 4 = -2^2$ | 4 | 5 | $- 8 = -3^2$ | 9 | 13 | $- 13 = 0^2$ | 0 |
| 6 | $- 4 = 2^2$ | 4 | 10 | $- 8 = 2^2$ | 4 | 15 | $- 13 = 2^2$ | 4 |
| | | 22 | | | 18 | | | 14 |

**S**um of **S**quares **W**ithin Groups = 22 + 18 + 14 = 54

Total Sum of Squares = Sum of Squares Between Groups + Sum of Squares Within Groups

54

| observation | | mean | observation - mean | ( observation - mean ) $^2$ |
|---|---|---|---|---|
| 2 | - | 8.3 | = -6.3 | 40.1 |
| 3 | - | 8.3 | = -5.3 | 28.4 |
| 7 | - | 8.3 | = -1.3 | 1.8 |
| 2 | - | 8.3 | = -6.3 | 40.1 |
| 6 | - | 8.3 | = -2.3 | 5.4 |
| 10 | - | 8.3 | = 1.7 | 2.7 |
| 8 | - | 8.3 | = -0.3 | 0.1 |
| 7 | - | 8.3 | = -1.3 | 1.8 |
| 5 | - | 8.3 | = -3.3 | 11.1 |
| 10 | - | 8.3 | = 1.7 | 2.8 |
| 10 | - | 8.3 | = 1.7 | 2.8 |
| 13 | - | 8.3 | = 4.7 | 21.8 |
| 14 | - | 8.3 | = 5.7 | 32.1 |
| 13 | - | 8.3 | = 4.7 | 21.8 |
| 15 | - | 8.3 | = 6.7 | 44.4 |

**T**otal **S**um of **S**quares

**SST** = 257.3

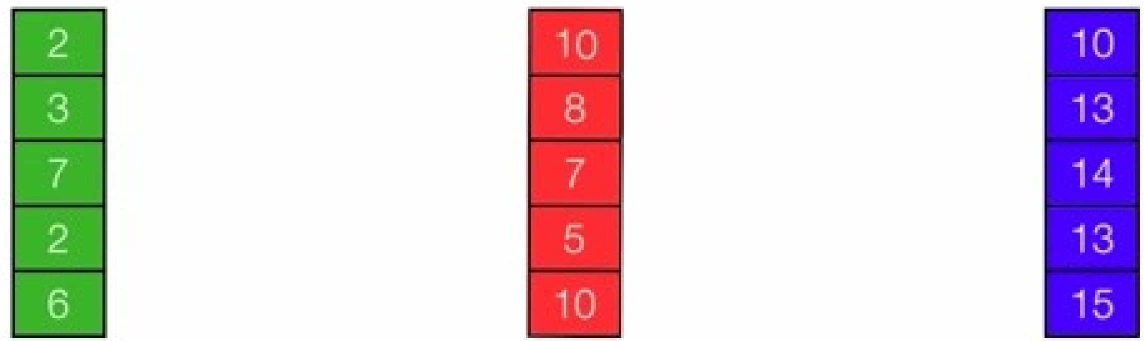
Total Sum of Squares = Sum of Squares Between Groups + Sum of Squares Within Groups

257.3 54

# Analysis of Variance

## Sum of Squares Between Groups


mean




mean　　mean　　mean

1. mean - mean　　mean - mean　　mean - mean

2. $(\text{mean} - \text{mean})^2$　　$(\text{mean} - \text{mean})^2$　　$(\text{mean} - \text{mean})^2$

3. $(\text{mean} - \text{mean})^2 + (\text{mean} - \text{mean})^2 + (\text{mean} - \text{mean})^2$

4. $(\text{mean} - \text{mean})^2 + (\text{mean} - \text{mean})^2 + (\text{mean} - \text{mean})^2 \times 5$

# Analysis of Variance

## Sum of Squares Between Groups

| |
|---|
| 2 |
| 3 |
| 7 |
| 2 |
| 6 |
| 10 |
| 8 |
| 7 |
| 5 |
| 10 |
| 10 |
| 13 |
| 14 |
| 13 |
| 15 |

mean 8.3

| | | |
|---|---|---|
| 2 | 10 | 10 |
| 3 | 8 | 13 |
| 7 | 7 | 14 |
| 2 | 5 | 13 |
| 6 | 10 | 15 |

mean $4 - 8.3 = (-4.3)^2$ $8 - 8.3 = (-.3)^2$ $13 - 8.3 = (4.7)^2$

$18.8 + .1 + 21.8 = 40.7$

$40.7 \times 5 = 203.3$

Total Sum of Squares = Sum of Squares Between Groups + Sum of Squares Within Groups

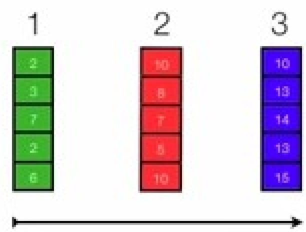
257.3 = 203.3 + 54

# Final Calculations

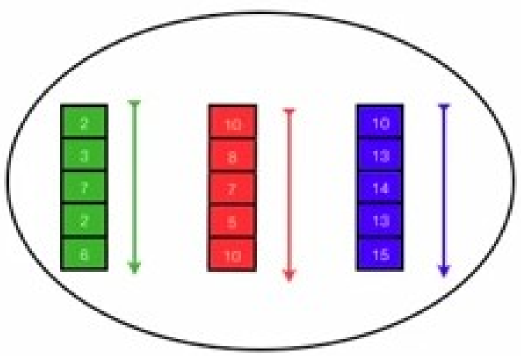
$$\frac{\text{Sum of Squares Between Groups}}{\text{degrees of freedom}} = \frac{203.3}{\phantom{2}}$$

Groups - 1

3 - 1 = 2




$$\frac{\text{Sum of Squares Between Groups}}{\text{degrees of freedom}} = \frac{203.3}{2} = 101.667$$



15 - 3 = 12

Observations - Groups

$$\frac{\text{Sum of Squares Within Groups}}{\text{degrees of freedom}} = \frac{54}{12} = 4.5$$

$$\frac{\text{Sum of Squares Within Groups}}{\text{degrees of freedom}} = \frac{54}{\phantom{12}}$$

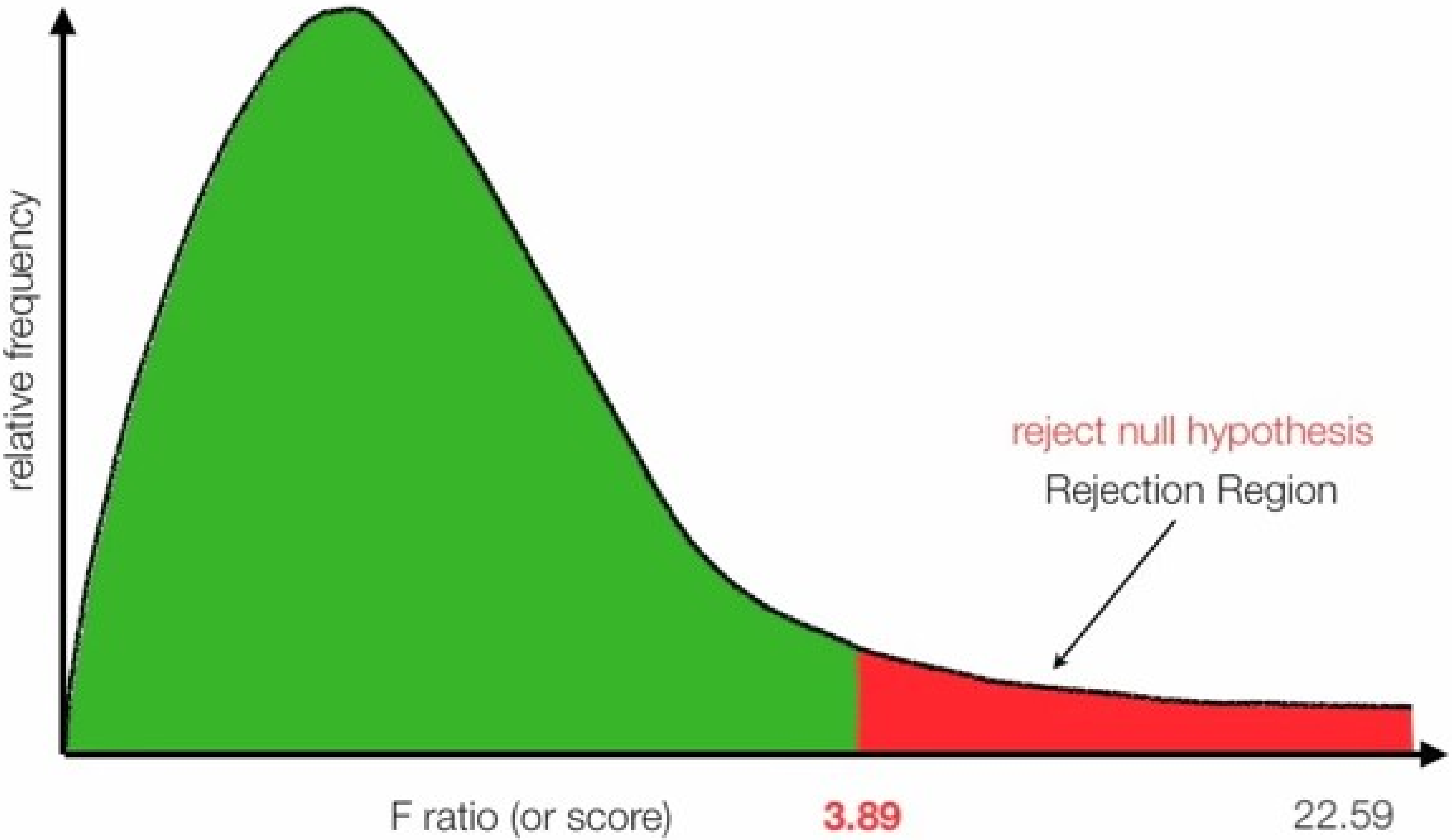
$$\boxed{F = \frac{101.667}{4.5} = 22.59}$$

F Distribution $F(2, 12) = 22.59$, $p < .05$

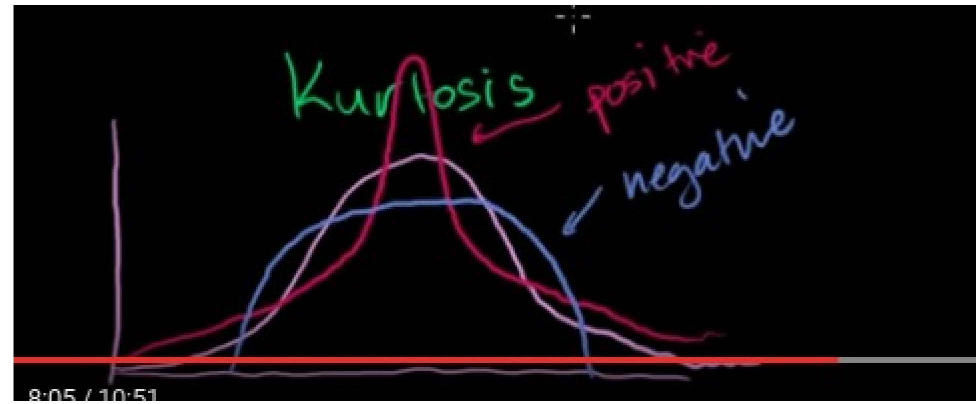
degrees of freedom numerator

degrees of freedom denominator

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 161.5 | 199.5 | 215.7 | 224.6 | 230.2 | 234.0 | 236.8 | 238.9 | 240.5 | 241.9 | 243.9 | 246.0 | 248.0 | 249.1 | 250.1 |
| **2** | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 | 19.43 | 19.45 | 19.45 | 19.46 |
| **3** | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.70 | 8.66 | 8.64 | 8.62 |
| **4** | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 |
| **5** | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.62 | 4.56 | 4.53 | 4.50 |
| | | | | | | | | | | | | | | | |
| **6** | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.94 | 3.87 | 3.84 | 3.81 |
| **7** | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.51 | 3.44 | 3.41 | 3.38 |
| **8** | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.22 | 3.15 | 3.12 | 3.08 |
| **9** | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.01 | 2.94 | 2.90 | 2.86 |
| **10** | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.85 | 2.77 | 2.74 | 2.70 |
| | | | | | | | | | | | | | | | |
| **11** | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.79 | 2.72 | 2.65 | 2.61 | 2.57 |
| **12** | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.62 | 2.54 | 2.51 | 2.47 |
| **13** | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.53 | 2.46 | 2.42 | 2.38 |
| **14** | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.46 | 2.39 | 2.35 | 2.31 |
| **15** | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.40 | 2.33 | 2.29 | 2.25 |


relative frequency
reject null hypothesis
Rejection Region
F ratio (or score)
3.89
22.59


onlinestatbook.com
pos skew
negative


Kurtosis
positive
negative
8:05 / 10:51