Assignment 3

April 5, 2018

1 Introduction:

It is required to design a hand-written numeral recognition system that being trained using given data. Features should be extracted from that data using different methods, train the model with different algorithms and then test the model and compare between these methods accuraces.

1.1 Features generation:

- Centroid features.
- AutoEncoder.

1.2 Classification algorithms:

- k-means clustering.
- GMM.
- SVM.

In [49]:
```
#Initializing needed libaries
import scipy as sp
import numpy as np
import pandas as pd
import matplotlib as plt
from scipy.fftpack import dct as dct
from scipy import io as spio
from scipy import ndimage as img
from random import randint
from sklearn.decomposition import PCA
from sklearn.cluster import KMeans
from sklearn.mixture import GaussianMixture
from sklearn.metrics import accuracy_score
from sklearn.preprocessing import normalize
from sklearn.preprocessing import StandardScaler
from sklearn import svm
import time
from sklearn.metrics import confusion_matrix
from scipy import linalg as la
from numpy.linalg import inv
```

```
from keras.layers import Input, Dense
from keras.models import Model
from keras import backend as K
from scipy.ndimage.measurements import center_of_mass
import math
import warnings
warnings.filterwarnings('ignore')
%matplotlib inline
```

2 Core Functions:

2.0.1 -Extract Data

This function handles the input data, first it reads the input features and labels, then it reshapes features into an image pixels format while labeles transform it into class number instead of hot one.

2.0.2 -Standarize

To make data in standard form with mean=0 and variance of 1, this function takes the input data and standarize them.

The standarization process decreases the the values of input data making information dense in smaller values while keeping all information, due to smaller values computational process becomes faster which is in favour of the algorithm.

2.0.3 -Unroll

Unrolling of input into a single vector for other functions.

2.0.4 -dct_2D

DCT is a powerful transformation for features which decreases number of features dramatically while keeping most of information and variations in data.

first 2D DCT transformation is applied on the data then Zigzag reading of DCT coefficient to make the most of the transformation.

2.0.5 -pca_fit & pca_trans

PCA is another powerful transformation where it reduces input features into smaller number of feature, it reduces the number of dimensions of the input data while keeping high variations of the data.

pca_fit forms the model on the training data, while pca_trans transforms the test features into the same model of the training data.

2.0.6 -KMeans

K-Means is one of the most known clusrting algorithms, in order to use it with our classification problem it was applied on each class training data in order to produce the means of the each class data, then at test time nearest class is assigned to the input vector with the nearest distance from cluster of the assigned class.

2.0.7 -GMM

Guassian Mixture Model is another important algorithm of clustering, it takes longer time for training however it helps a lot in increasing accuracy when compared to kmeans using same number of clusters

2.0.8 -predict_acc

In this function it estimates which class should be assigned to the test example, and then calculates the accuracy of the predictions according to the true labels.

2.0.9 -accuracy

In this function it calculates the correctly predicted classes ratio to the whole test set and returns the accuracy.

2.0.10 -find_class

This function estimates the class that should be assigned to the currrebt test vector of features, this function is uded by predict_acc

2.0.11 -Centroids

Slicing the input image into 9 slices that have the same size and returns an array containing these slices.

2.0.12 -Get_Centroid_Features

Taking these slices that (centroids) function returns, and then compute the distance between the center of mass of each slice and center of mass of the image and finally returns these distances as the desired features.

2.0.13 -features_diagonalization

We can diagonalize the covariance matrix of the new features using this function that returns a diagonal matrix containing eignvalues of the features.

```
In [50]: #Functions definations
         #extract data from Mat format and normalize the image
         def extract(Data):
             features=[]
             output=[]
             for i in range (Data.shape[0]):
                     features.append(Data[i,0])
                     output.append(Data[i,1])
             features=np.array(features)
```

```
    output=np.array(output)
    features=np.reshape(features,[features.shape[0]*features.shape[1],28,28])/1.0
    output=np.reshape(output,[output.shape[0]*output.shape[1],output.shape[2]])
    output=[np.where(r==1)[0][0] for r in output]
    return features, np.array(output)
def standarize(x):
    stnd=[]
    for i in range(x.shape[0]):
        scaler = StandardScaler()
        scaler.fit(x[i])
        temp=scaler.transform(x[i])
        stnd.append(temp)
    return np.array(stnd)
def unroll(a):
    a=a.reshape(a.shape[0],a.shape[1]*a.shape[2])
    return a
#2D dct
def dct_2D(x):
    a=[]
    for i in range (x.shape[0]):
        x_dct=dct(dct(x[i],norm='ortho').T,norm='ortho').T;
        a.append([x_dct[0,0], x_dct[0,1], x_dct[1,0], x_dct[2,0], x_dct[1,1],\
                  x_dct[0,2], x_dct[0,3], x_dct[1,2], x_dct[2,1], x_dct[3,0],\
                  x_dct[4,0], x_dct[3,1], x_dct[2,2], x_dct[1,3], x_dct[0,4],\
                  x_dct[0,5], x_dct[1,4], x_dct[2,3], x_dct[3,2], x_dct[4,1]])
    return np.array(a)
#PCA
def pca_fit(x,n):
    #x = StandardScaler().fit_transform(x)
    pca = PCA(n_components=n)
    pca.fit(x)
    pca_comp=pca.transform(x)
    var=sum(pca.explained_variance_ratio_)*100
    return pca_comp,var,pca
def pca_trans(x,pca):
    #x = StandardScaler().fit_transform(x)
    pca_comp=pca.transform(x)
    var=sum(pca.explained_variance_ratio_)*100.0
    return pca_comp,var
#K-Means
def kmeans(clusters,classes_n,Features_Train,class_margin):
    kmeans_=[]
    for i in range (classes_n):
        kmeans_temp=KMeans(n_clusters=clusters,n_init=10,max_iter=5000,algorithm='full'
        fit(Features_Train[i*class_margin:i*class_margin+class_margin-1])
        kmeans_.append(kmeans_temp.cluster_centers_)
    kmeans_=np.array(kmeans_)
    return kmeans_
```

```
#GMM
def GMM(Mixtures,classes_n,Features_Train,class_margin):
    G=[]
    for i in range (classes_n):
        G_temp=GaussianMixture(n_components=Mixtures,n_init=10,max_iter=5000,covariance
        fit(Features_Train[i*class_margin:i*class_margin+class_margin-1])
        G.append(G_temp.means_)
    G=np.array(G)
    return G
#Predict
def predict_acc(test_features,label_set,model):
    Y_predict=np.zeros_like(label_set)
    for i in range (Y_predict.shape[0]):
        Y_predict[i]=find_class(test_features[i],model)
    acc1=accuracy(label_set,Y_predict)
    return acc1,Y_predict
#accuracy calc
def accuracy(original,predicted):
    acc=original-predicted
    acc[acc != 0] = 1
    acc=(np.count_nonzero(acc == 0)*1.0/original.shape[0])*100.0
    return acc
#class decision
def find_class(x,y):
    min_d=np.ones(y.shape[0])*100000000.0
    for i in range(y.shape[0]):
        for j in range(y.shape[1]):
            temp=np.linalg.norm(x-y[i][j])
            if temp<min_d[i]:
                min_d[i]=temp
    min_class_idx=np.argmin(min_d)
    return min_class_idx
#Grid slicing
def Centroids(x):
    Slice_1=[]
    Slice_2=[]
    Slice_3=[]
    Slice_4=[]
    Slice_5=[]
    Slice_6=[]
    Slice_7=[]
    Slice_8=[]

    for i in range (x.shape[0]):
        Slice_1.append(np.hstack(x[i][0:9,0:9]))
        Slice_2.append(np.hstack(x[i][0:9,9:18]))
        Slice_3.append(np.hstack(x[i][0:9,18:27]))
```

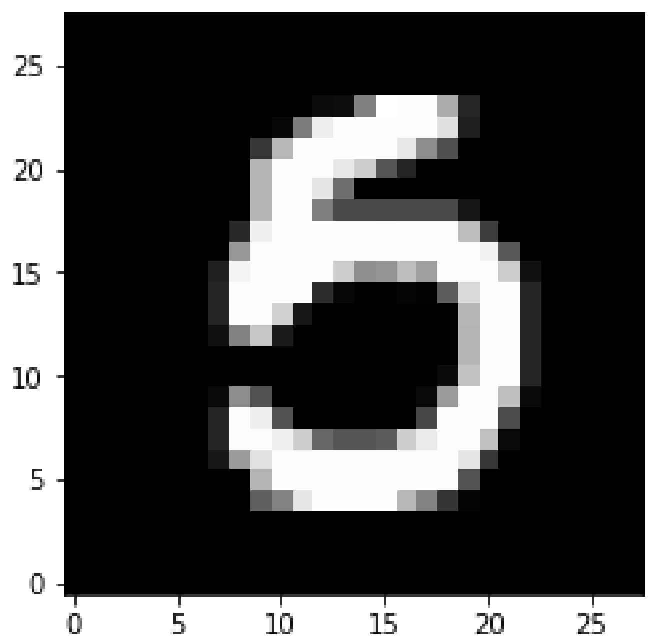
```
        Slice_4.append(np.hstack(x[i][9:18,0:9]))
        Slice_5.append(np.hstack(x[i][9:18,18:27]))

        Slice_6.append(np.hstack(x[i][18:27,0:9]))
        Slice_7.append(np.hstack(x[i][18:27,9:18]))
        Slice_8.append(np.hstack(x[i][18:27,18:27]))

    Slices = { "S1": np.array(Slice_1).reshape(x.shape[0],9,9),
               "S2": np.array(Slice_2).reshape(x.shape[0],9,9),
               "S3": np.array(Slice_3).reshape(x.shape[0],9,9),
               "S4": np.array(Slice_4).reshape(x.shape[0],9,9),
               "S5": np.array(Slice_5).reshape(x.shape[0],9,9),
               "S6": np.array(Slice_6).reshape(x.shape[0],9,9),
               "S7": np.array(Slice_7).reshape(x.shape[0],9,9),
               "S8": np.array(Slice_8).reshape(x.shape[0],9,9)
             }
    return Slices
def Get_Centroid_Features(x_slices, x_train):
    Features=[]
    for i in range (x_train.shape[0]):
        stack=[]
        for j in range (1,9):
            a=np.array(center_of_mass(x_slices["S"+str(j)][i])).reshape(1,2)
            b=np.array(center_of_mass(x_train[i])).reshape(1,2)

            if math.isnan(a[0,0]):
                a=np.zeros((1,2))

            stack.append(np.linalg.norm(a-b))
        temp1=np.array(stack)
        temp2=temp1.reshape(1,8)
        Features.append(temp2)
    return np.array(Features).reshape((x_train.shape[0],8))
#feature diagnolization
def features_diagonalization(x):
    m = x.shape[1]
    covariance_matrix = (1/m) * np.dot(np.transpose(x),x)
    # covariance_matrix.shape
    e_vals, e_vecs = la.eig(covariance_matrix)
    diagonal_eignvalues = np.dot(np.dot(inv(e_vecs),covariance_matrix),e_vecs)
    return diagonal_eignvalues
```

2.1 Step 1:

- Read Data
- Extract the features and labels
- Standarize the features
- Unroll the Features

- Show an example of handwritten image

```
In [69]: #Read Data
         R_MNIST=spio.loadmat('./ReducedMNIST.mat')
         R_MINST_Train=R_MNIST['SmallTrainData']
         R_MINST_Test=R_MNIST['SmallTestData']
         #extract features and labels
         X_Train,Y_Train= extract(R_MINST_Train)
         X_Train_std=standarize(X_Train)
         X_Test,Y_Test= extract(R_MINST_Test)
         X_Test_std=standarize(X_Test)
         #unroll images
         X_Train_unroll=unroll(X_Train_std)
         X_Train_unroll_norm=unroll(X_Train)/255.0
         X_Test_unroll=unroll(X_Test_std)
         X_Test_unroll_norm=unroll(X_Test)/255.0
         #show a random picture example
         img_num=randint(0,X_Train.shape[0])
         plt.pyplot.imshow(img.rotate(X_Train[img_num],90),origin='lower')
         plt.pyplot.gray()
         plt.pyplot.show()
         print("Label = "+ str(Y_Train[img_num])+" image = "+ str(img_num))
```



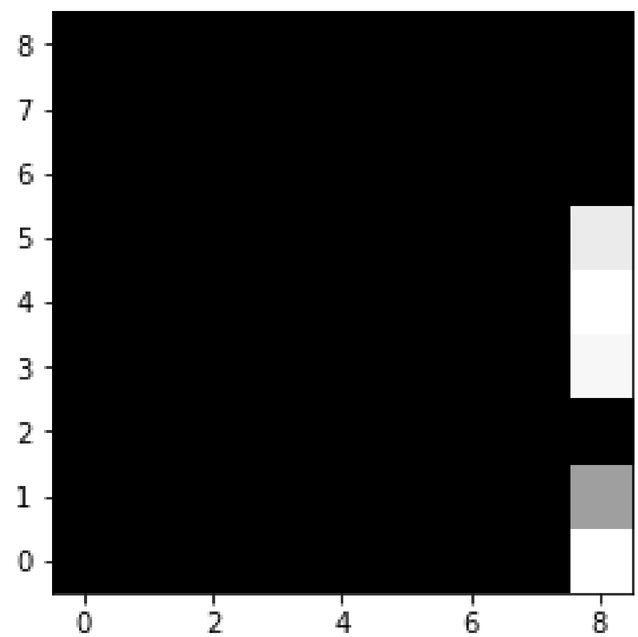

```
Label = 5 image = 5782
```

2.2 Step 2:

Extracting the centroid features from training and test data by calling (Centroids) and (Get_Centroid_Features) functions.

```
In [52]: #extracting centeroids features for Train and Test Data
         Slices_Train=Centroids(X_Train)
         Centroid_Features_Train= Get_Centroid_Features(Slices_Train,X_Train)
         Slices_Test=Centroids(X_Test)
         Centroid_Features_Test= Get_Centroid_Features(Slices_Test, X_Test)
```

```
In [53]: #plot a slice example
         img_num=randint(0,X_Train.shape[0])
         plt.pyplot.imshow(img.rotate(Slices_Train["S2"][img_num],90),origin='lower')
         plt.pyplot.gray()
         plt.pyplot.show()
```



2.3 Step 3:

Creating Autoencoder model that extracts 10 features and training it using the given data.

2.4 Autoencoding:

"Autoencoding" is a data compression algorithm where the compression and decompression functions are 1) data-specific, 2) lossy, and 3) learned automatically from examples rather than engineered by a human. Additionally, in almost all contexts where the term "autoencoder" is used, the compression and decompression functions are implemented with neural networks.

To build an autoencoder, you need three things: an encoding function, a decoding function, and a distance function between the amount of information loss between the compressed representation of your data and the decompressed representation (i.e. a "loss" function). The encoder and decoder will be chosen to be parametric functions (typically neural networks), and to be differentiable with respect to the distance function, so the parameters of the encoding/decoding functions can be optimize to minimize the reconstruction loss, using Stochastic Gradient Descent.

One reason why they have attracted so much research and attention is because they have long been thought to be a potential avenue for solving the problem of unsupervised learning, i.e. the learning of useful representations without the need for labels. Then again, autoencoders are not a true unsupervised learning technique (which would imply a different learning process altogether), they are a self-supervised technique, a specific instance of supervised learning where the targets are generated from the input data. In order to get self-supervised models to learn interesting features, you have to come up with an interesting synthetic target and loss function, and that's where problems arise: merely learning to reconstruct your input in minute detail might not be the right choice here. At this point there is significant evidence that focusing on the reconstruction of a picture at the pixel level, for instance, is not conductive to learning interesting, abstract features of the kind that label-supervized learning induces (where targets are fairly abstract concepts "invented" by humans such as "dog", "car"...). In fact, one may argue that the best features in this regard are those that are the worst at exact input reconstruction while achieving high performance on the main task that you are interested in (classification, localization, etc).

```
In [55]: encoding_dim = 10  #size of output of encoder
         Compression_ratio=784/encoding_dim
         input_img = Input(shape=(784,)) #input placeholder
         encoded = Dense(encoding_dim, activation='relu')(input_img) #encoding layer output
         decoded = Dense(784, activation='sigmoid')(encoded) #decoding layer output
         autoencoder = Model(input_img, decoded) #autoencoder model
         encoder = Model(input_img, encoded) #encoder model
         encoded_input = Input(shape=(encoding_dim,)) #input placeholder for decoder input
         decoder_layer = autoencoder.layers[-1] # retrieve the last layer of the autoencoder mod
         decoder = Model(encoded_input, decoder_layer(encoded_input)) #decoder model
         autoencoder.compile(optimizer='adam', loss='binary_crossentropy')#compile autoencoder m
         print(round(Compression_ratio,3))
         print("Compression ratio =" , round(Compression_ratio,4))
```

```
78.4
Compression ratio = 78.4
```

```
In [56]: #Train the Autoencoder
         autoencoder.fit(X_Train_unroll_norm, X_Train_unroll_norm,
                         epochs=70,
                         batch_size=256,
                         shuffle=True,
                         validation_data=(X_Test_unroll_norm, X_Test_unroll_norm))
```

```
Train on 10000 samples, validate on 1000 samples
Epoch 1/70
10000/10000 [==============================] - 2s 224us/step - loss: 0.5927 - val_loss: 0.3894
```

```
Epoch 2/70
10000/10000 [==============================] - 2s 177us/step - loss: 0.3180 - val_loss: 0.2868
Epoch 3/70
10000/10000 [==============================] - 2s 159us/step - loss: 0.2826 - val_loss: 0.2762
Epoch 4/70
10000/10000 [==============================] - 2s 174us/step - loss: 0.2737 - val_loss: 0.2678
Epoch 5/70
10000/10000 [==============================] - 2s 204us/step - loss: 0.2646 - val_loss: 0.2571
Epoch 6/70
10000/10000 [==============================] - 2s 181us/step - loss: 0.2532 - val_loss: 0.2448
Epoch 7/70
10000/10000 [==============================] - 2s 178us/step - loss: 0.2415 - val_loss: 0.2333
Epoch 8/70
10000/10000 [==============================] - 2s 203us/step - loss: 0.2309 - val_loss: 0.2237
Epoch 9/70
10000/10000 [==============================] - 2s 206us/step - loss: 0.2219 - val_loss: 0.2157
Epoch 10/70
10000/10000 [==============================] - 2s 178us/step - loss: 0.2143 - val_loss: 0.2088
Epoch 11/70
10000/10000 [==============================] - 2s 238us/step - loss: 0.2080 - val_loss: 0.2033
Epoch 12/70
10000/10000 [==============================] - 3s 333us/step - loss: 0.2026 - val_loss: 0.1986
Epoch 13/70
10000/10000 [==============================] - 3s 284us/step - loss: 0.1980 - val_loss: 0.1946
Epoch 14/70
10000/10000 [==============================] - 3s 262us/step - loss: 0.1942 - val_loss: 0.1911
Epoch 15/70
10000/10000 [==============================] - 4s 351us/step - loss: 0.1911 - val_loss: 0.1884
Epoch 16/70
10000/10000 [==============================] - 2s 190us/step - loss: 0.1884 - val_loss: 0.1860
Epoch 17/70
10000/10000 [==============================] - 4s 361us/step - loss: 0.1861 - val_loss: 0.1840
Epoch 18/70
10000/10000 [==============================] - 2s 219us/step - loss: 0.1841 - val_loss: 0.1821
Epoch 19/70
10000/10000 [==============================] - 3s 253us/step - loss: 0.1824 - val_loss: 0.1806
Epoch 20/70
10000/10000 [==============================] - 3s 332us/step - loss: 0.1807 - val_loss: 0.1791
Epoch 21/70
10000/10000 [==============================] - 2s 207us/step - loss: 0.1792 - val_loss: 0.1777
Epoch 22/70
10000/10000 [==============================] - 4s 409us/step - loss: 0.1778 - val_loss: 0.1764
Epoch 23/70
10000/10000 [==============================] - 2s 221us/step - loss: 0.1766 - val_loss: 0.1752
Epoch 24/70
10000/10000 [==============================] - 4s 397us/step - loss: 0.1755 - val_loss: 0.1742
Epoch 25/70
10000/10000 [==============================] - 2s 215us/step - loss: 0.1745 - val_loss: 0.1734
```

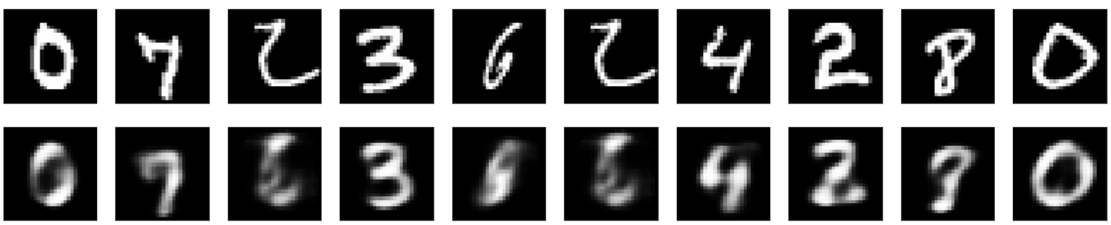
```
Epoch 26/70
10000/10000 [==============================] - 3s 278us/step - loss: 0.1736 - val_loss: 0.1725
Epoch 27/70
10000/10000 [==============================] - 3s 288us/step - loss: 0.1729 - val_loss: 0.1719
Epoch 28/70
10000/10000 [==============================] - 2s 213us/step - loss: 0.1721 - val_loss: 0.1713
Epoch 29/70
10000/10000 [==============================] - 4s 367us/step - loss: 0.1714 - val_loss: 0.1707
Epoch 30/70
10000/10000 [==============================] - 2s 204us/step - loss: 0.1708 - val_loss: 0.1701
Epoch 31/70
10000/10000 [==============================] - 4s 416us/step - loss: 0.1703 - val_loss: 0.1695
Epoch 32/70
10000/10000 [==============================] - 2s 197us/step - loss: 0.1698 - val_loss: 0.1692
Epoch 33/70
10000/10000 [==============================] - 3s 280us/step - loss: 0.1693 - val_loss: 0.1687
Epoch 34/70
10000/10000 [==============================] - 3s 313us/step - loss: 0.1689 - val_loss: 0.1684
Epoch 35/70
10000/10000 [==============================] - 2s 223us/step - loss: 0.1685 - val_loss: 0.1681
Epoch 36/70
10000/10000 [==============================] - 4s 412us/step - loss: 0.1681 - val_loss: 0.1676
Epoch 37/70
10000/10000 [==============================] - 2s 207us/step - loss: 0.1678 - val_loss: 0.1673
Epoch 38/70
10000/10000 [==============================] - 4s 433us/step - loss: 0.1675 - val_loss: 0.1670
Epoch 39/70
10000/10000 [==============================] - 2s 175us/step - loss: 0.1671 - val_loss: 0.1667
Epoch 40/70
10000/10000 [==============================] - 3s 340us/step - loss: 0.1667 - val_loss: 0.1664
Epoch 41/70
10000/10000 [==============================] - 3s 271us/step - loss: 0.1665 - val_loss: 0.1661
Epoch 42/70
10000/10000 [==============================] - 3s 324us/step - loss: 0.1662 - val_loss: 0.1660
Epoch 43/70
10000/10000 [==============================] - 3s 295us/step - loss: 0.1659 - val_loss: 0.1657
Epoch 44/70
10000/10000 [==============================] - 2s 202us/step - loss: 0.1656 - val_loss: 0.1654
Epoch 45/70
10000/10000 [==============================] - 4s 428us/step - loss: 0.1654 - val_loss: 0.1652
Epoch 46/70
10000/10000 [==============================] - 4s 371us/step - loss: 0.1652 - val_loss: 0.1651
Epoch 47/70
10000/10000 [==============================] - 2s 153us/step - loss: 0.1649 - val_loss: 0.1646
Epoch 48/70
10000/10000 [==============================] - 2s 204us/step - loss: 0.1647 - val_loss: 0.1647
Epoch 49/70
10000/10000 [==============================] - 2s 214us/step - loss: 0.1645 - val_loss: 0.1643
```

```
Epoch 50/70
10000/10000 [==============================] - 2s 201us/step - loss: 0.1642 - val_loss: 0.1641
Epoch 51/70
10000/10000 [==============================] - 3s 332us/step - loss: 0.1640 - val_loss: 0.1640
Epoch 52/70
10000/10000 [==============================] - 2s 245us/step - loss: 0.1638 - val_loss: 0.1639
Epoch 53/70
10000/10000 [==============================] - 2s 210us/step - loss: 0.1636 - val_loss: 0.1636
Epoch 54/70
10000/10000 [==============================] - 2s 229us/step - loss: 0.1634 - val_loss: 0.1634
Epoch 55/70
10000/10000 [==============================] - 4s 360us/step - loss: 0.1633 - val_loss: 0.1633
Epoch 56/70
10000/10000 [==============================] - 2s 216us/step - loss: 0.1631 - val_loss: 0.1632
Epoch 57/70
10000/10000 [==============================] - 2s 218us/step - loss: 0.1629 - val_loss: 0.1629
Epoch 58/70
10000/10000 [==============================] - 4s 363us/step - loss: 0.1627 - val_loss: 0.1627
Epoch 59/70
10000/10000 [==============================] - 3s 335us/step - loss: 0.1625 - val_loss: 0.1626
Epoch 60/70
10000/10000 [==============================] - 4s 374us/step - loss: 0.1623 - val_loss: 0.1625
Epoch 61/70
10000/10000 [==============================] - 3s 287us/step - loss: 0.1622 - val_loss: 0.1622
Epoch 62/70
10000/10000 [==============================] - 2s 193us/step - loss: 0.1620 - val_loss: 0.1621
Epoch 63/70
10000/10000 [==============================] - 2s 163us/step - loss: 0.1619 - val_loss: 0.1619
Epoch 64/70
10000/10000 [==============================] - 2s 227us/step - loss: 0.1617 - val_loss: 0.1618
Epoch 65/70
10000/10000 [==============================] - 4s 390us/step - loss: 0.1616 - val_loss: 0.1617
Epoch 66/70
10000/10000 [==============================] - 2s 210us/step - loss: 0.1614 - val_loss: 0.1615
Epoch 67/70
10000/10000 [==============================] - 4s 408us/step - loss: 0.1613 - val_loss: 0.1614
Epoch 68/70
10000/10000 [==============================] - 2s 183us/step - loss: 0.1611 - val_loss: 0.1612
Epoch 69/70
10000/10000 [==============================] - 3s 261us/step - loss: 0.1610 - val_loss: 0.1612
Epoch 70/70
10000/10000 [==============================] - 3s 312us/step - loss: 0.1608 - val_loss: 0.1611
```

Out[56]: <keras.callbacks.History at 0x7f5192b03390>

In [57]:
```
#encode and decode Test Images
encoded_imgs = encoder.predict(X_Test_unroll_norm)
decoded_imgs = decoder.predict(encoded_imgs)
```

```
In [11]: #display Original test images and decoded Images examples
         n = 10  # how many digits we will display
         plt.pyplot.figure(figsize=(20, 4))
         for i in range(n):
             # display original
             ax = plt.pyplot.subplot(2, n, i + 1)
             img_num=randint(0,X_Test.shape[0])
             plt.pyplot.imshow(img.rotate(X_Test[img_num].reshape(28, 28),90),origin='lower')
             plt.pyplot.gray()
             ax.get_xaxis().set_visible(False)
             ax.get_yaxis().set_visible(False)
             # display reconstruction
             ax = plt.pyplot.subplot(2, n, i + 1 + n)
             plt.pyplot.imshow(img.rotate(decoded_imgs[img_num].reshape(28, 28),90),origin='lowe
             plt.pyplot.gray()
             ax.get_xaxis().set_visible(False)
             ax.get_yaxis().set_visible(False)
         plt.pyplot.show()
```



```
In [58]: #Create Encoded data for train and test datasets
         encoded_Train = encoder.predict(X_Train_unroll_norm)
         encoded_Test = encoder.predict(X_Test_unroll_norm)
```

2.5 Step 4:

Using output features from the Autoencoder (10 features), train the model using K-means clustering algorithm with different number of clusters and comparing output accuracy when classifing test images.

```
In [13]: #KMeans 1 cluster AutoEncoder
         tic = time.time()
         kmeans_encode1=kmeans(1,10,encoded_Train,1000)
         toc = time.time()
         print("elapsed time =",round(toc-tic,4),"sec")

elapsed time = 0.2005 sec
```

```
In [14]: #Encoder KMeans 1 cluster Prediction
         tic = time.time()
         acc_kmeans_encoded1,Y_encoded_KMeans1=predict_acc(encoded_Test,Y_Test,kmeans_encode1)
         toc = time.time()
         print("accuracy =",round(acc_kmeans_encoded1,2),"%")
         print("elapsed time =",round(toc-tic,4),"sec\n")
         print("Confusion Matrix:")
         confusion_encoded_KMeans1 = pd.crosstab(Y_Test, Y_encoded_KMeans1,rownames=['Actual'],
         display(confusion_encoded_KMeans1)
```

```
accuracy = 68.5 %
elapsed time = 0.2063 sec

Confusion Matrix:
```

Predicted	0	1	2	3	4	5	6	7	8	9	All
Actual											
0	79	4	1	0	1	12	0	0	3	0	100
1	0	90	1	0	0	0	0	0	9	0	100
2	2	6	74	2	1	0	5	2	8	0	100
3	1	8	2	71	0	7	0	2	8	1	100
4	0	4	0	0	64	2	1	0	1	28	100
5	6	15	1	7	1	40	5	0	17	8	100
6	3	7	2	0	2	3	83	0	0	0	100
7	0	10	1	1	4	1	0	67	5	11	100
8	3	4	3	8	1	9	0	0	63	9	100
9	0	3	0	0	29	3	0	2	9	54	100
All	94	151	85	89	103	77	94	73	123	111	1000

```
In [15]: #KMeans 2 cluster AutoEncoder
         tic = time.time()
         kmeans_encode2=kmeans(2,10,encoded_Train,1000)
         toc = time.time()
         print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 0.5895 sec
```

```
In [16]: #Encoder KMeans 2 cluster Prediction
         tic = time.time()
         acc_kmeans_encoded2,Y_encoded_KMeans2=predict_acc(encoded_Test,Y_Test,kmeans_encode2)
         toc = time.time()
         print("accuracy =",round(acc_kmeans_encoded2,2),"%")
         print("elapsed time =",round(toc-tic,4),"sec\n")
         print("Confusion Matrix:")
         confusion_encoded_KMeans2 = pd.crosstab(Y_Test, Y_encoded_KMeans2,rownames=['Actual'],
         display(confusion_encoded_KMeans2)
```

```
accuracy = 69.5 %
elapsed time = 0.3687 sec

Confusion Matrix:
```

Predicted	0	1	2	3	4	5	6	7	8	9	All
Actual											
0	81	0	1	0	0	17	0	0	1	0	100
1	0	92	1	2	0	0	0	0	5	0	100
2	1	2	73	3	2	1	8	2	8	0	100
3	1	2	3	75	1	6	0	3	9	0	100
4	0	3	0	0	55	7	2	0	1	32	100
5	8	6	5	7	1	50	5	1	12	5	100
6	9	2	5	0	3	3	77	0	1	0	100
7	0	8	4	0	2	1	0	69	3	13	100
8	2	2	0	7	2	13	2	0	61	11	100
9	0	3	0	0	25	2	0	2	6	62	100
All	102	120	92	94	91	100	94	77	107	123	1000

In [60]:
```python
#KMeans 4 cluster AutoEncoder
tic = time.time()
kmeans_encode4=kmeans(4,10,encoded_Train,1000)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 0.9261 sec
```

In [64]:
```python
#Encoder KMeans 4 cluster Prediction
tic = time.time()
acc_kmeans_encoded4,Y_encoded_KMeans4=predict_acc(encoded_Test,Y_Test,kmeans_encode4)
toc = time.time()
print("accuracy =",round(acc_kmeans_encoded4,2),"%")
print("elapsed time =",round(toc-tic,4),"sec\n")
for t in range(5):
    print("\n")
print("Confusion Matrix:")
confusion_encoded_KMeans4 = pd.crosstab(Y_Test, Y_encoded_KMeans4,rownames=['Actual'],
display(confusion_encoded_KMeans4)
```

```
accuracy = 77.5 %
elapsed time = 0.7188 sec
```

```
Confusion Matrix:


Predicted   0    1   2   3   4   5   6   7    8    9   All
Actual
0          82    0   0   1   0  11   3   0    3    0   100
1           0   98   0   0   0   0   1   0    1    0   100
2           2    2  76   4   1   1   2   3    9    0   100
3           0    1   4  79   0   5   0   1    7    3   100
4           0    1   0   1  71   4   5   1    3   14   100
5           7    0   1   4   3  59   4   0   14    8   100
6           6    0   5   0   3   2  83   0    1    0   100
7           0    7   2   0   1   1   0  80    1    8   100
8           1    0   2   6   2   6   1   0   69   13   100
9           0    1   2   0  12   1   0   1    5   78   100
All        98  110  92  95  93  90  99  86  113  124  1000
```

```
In [65]: #KMeans 8 cluster AutoEncoder
         tic = time.time()
         kmeans_encode8=kmeans(8,10,encoded_Train,1000)
         toc = time.time()
         print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 1.3526 sec
```

```
In [66]: #Encoder KMeans 8 cluster Prediction
         tic = time.time()
         acc_kmeans_encoded8,Y_encoded_KMeans8=predict_acc(encoded_Test,Y_Test,kmeans_encode8)
         toc = time.time()
         print("accuracy =",round(acc_kmeans_encoded8,2),"%")
         print("elapsed time =",round(toc-tic,4),"sec\n")
         for t in range(7):
             print("\n")
         print("Confusion Matrix:")
         confusion_encoded_KMeans8 = pd.crosstab(Y_Test, Y_encoded_KMeans8,rownames=['Actual'],
         display(confusion_encoded_KMeans8)
```

```
accuracy = 81.8 %
elapsed time = 1.2506 sec
```

Confusion Matrix:

Predicted Actual	0	1	2	3	4	5	6	7	8	9	All
0	86	0	0	0	0	11	2	0	1	0	100
1	0	99	0	0	0	0	1	0	0	0	100
2	3	2	83	2	1	0	2	2	5	0	100
3	1	1	5	79	0	2	0	3	8	1	100
4	0	2	1	2	74	1	4	0	0	16	100
5	4	1	1	3	3	72	4	0	10	2	100
6	5	0	2	0	3	0	89	0	1	0	100
7	0	5	0	0	1	1	0	81	1	11	100
8	1	0	0	4	2	10	0	0	75	8	100
9	0	2	0	1	10	1	0	4	2	80	100
All	100	112	92	91	94	98	102	90	103	118	1000

In [21]:
```
#KMeans 16 cluster AutoEncoder
tic = time.time()
kmeans_encode16=kmeans(16,10,encoded_Train,1000)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 1.9621 sec
```

In [22]:
```
#Encoder KMeans 16 cluster Prediction
tic = time.time()
acc_kmeans_encoded16,Y_encoded_KMeans16=predict_acc(encoded_Test,Y_Test,kmeans_encode16
toc = time.time()
print("accuracy =",round(acc_kmeans_encoded16,2),"%")
print("elapsed time =",round(toc-tic,4),"sec\n")
print("Confusion Matrix:")
confusion_encoded_KMeans16 = pd.crosstab(Y_Test, Y_encoded_KMeans16,rownames=['Actual']
display(confusion_encoded_KMeans16)
```

```
accuracy = 81.8 %
elapsed time = 2.6752 sec

Confusion Matrix:
```

Predicted	0	1	2	3	4	5	6	7	8	9	All
Actual											
0	89	0	0	0	0	9	2	0	0	0	100
1	0	99	0	0	0	0	1	0	0	0	100
2	3	3	81	1	1	1	2	2	4	2	100
3	0	2	2	82	0	4	0	2	6	2	100
4	0	1	1	1	78	2	2	0	0	15	100
5	2	0	0	3	1	75	3	1	11	4	100
6	2	0	1	0	3	1	91	0	2	0	100
7	0	4	1	0	1	0	0	83	3	8	100
8	2	2	2	8	4	7	0	1	69	5	100
9	0	1	0	2	17	1	1	5	2	71	100
All	98	112	88	97	105	100	102	94	97	107	1000

2.6 Step 5:

Using the same features from autoencoder, but with GMM algorithm with different number of GMM.

```
In [23]: #encoded GMM 1
         tic = time.time()
         G_encoded1=GMM(1,10,encoded_Train,1000)
         toc = time.time()
         print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 0.8652 sec
```

```
In [24]: #encoded GMM 1 Predictions
         tic = time.time()
         acc_GMM_encoded1,Y_encoded_GMM1=predict_acc(encoded_Test,Y_Test,G_encoded1)
         toc = time.time()
         print("accuracy =",round(acc_GMM_encoded1,2),"%")
         print("elapsed time =",round(toc-tic,4),"sec")
         print("\nConfusion Matrix:")
         confusion_encoded_GMM1 = pd.crosstab(Y_Test, Y_encoded_GMM1,rownames=['Actual'], colnam
         display(confusion_encoded_GMM1)
```

```
accuracy = 68.5 %
elapsed time = 0.1993 sec

Confusion Matrix:
```

Predicted	0	1	2	3	4	5	6	7	8	9	All
Actual											
0	79	4	1	0	1	12	0	0	3	0	100
1	0	90	1	0	0	0	0	0	9	0	100
2	2	6	74	2	1	0	5	2	8	0	100
3	1	8	2	71	0	7	0	2	8	1	100
4	0	4	0	0	64	2	1	0	1	28	100
5	6	15	1	7	1	40	5	0	17	8	100
6	3	7	2	0	2	3	83	0	0	0	100
7	0	10	1	1	4	1	0	67	5	11	100
8	3	4	3	8	1	9	0	0	63	9	100
9	0	3	0	0	29	3	0	2	9	54	100
All	94	151	85	89	103	77	94	73	123	111	1000

In [67]:
```
#encoded GMM 2
tic = time.time()
G_encoded2=GMM(2,10,encoded_Train,1000)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 2.8205 sec
```

In [68]:
```
#encoded GMM 2 Predictions
tic = time.time()
acc_GMM_encoded2,Y_encoded_GMM2=predict_acc(encoded_Test,Y_Test,G_encoded2)
toc = time.time()
print("accuracy =",round(acc_GMM_encoded2,2),"%")
print("elapsed time =",round(toc-tic,4),"sec")
for t in range(9):
    print("\n")
print("\nConfusion Matrix:")
confusion_encoded_GMM2 = pd.crosstab(Y_Test, Y_encoded_GMM2,rownames=['Actual'], colnam
display(confusion_encoded_GMM2)
```

```
accuracy = 71.0 %
elapsed time = 0.3324 sec
```

Confusion Matrix:

Predicted	0	1	2	3	4	5	6	7	8	9	All
Actual											
0	77	0	2	0	1	15	1	0	4	0	100
1	0	90	0	0	0	0	0	0	10	0	100
2	3	4	71	2	2	3	4	2	9	0	100
3	2	6	4	69	0	4	0	2	13	0	100
4	1	4	0	0	63	7	3	0	0	22	100
5	10	1	0	9	3	49	7	0	15	6	100
6	4	2	1	1	6	8	78	0	0	0	100
7	0	9	0	0	3	1	0	73	3	11	100
8	3	2	3	4	3	7	1	0	67	10	100
9	0	5	0	0	15	2	0	1	4	73	100
All	100	123	81	85	96	96	94	78	125	122	1000

In [27]:
```
#encoded GMM 4
tic = time.time()
G_encoded4=GMM(4,10,encoded_Train,1000)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

elapsed time = 9.8912 sec

In [28]:
```
#encoded GMM 4 Predictions
tic = time.time()
acc_GMM_encoded4,Y_encoded_GMM4=predict_acc(encoded_Test,Y_Test,G_encoded4)
toc = time.time()
print("accuracy =",round(acc_GMM_encoded4,2),"%")
print("elapsed time =",round(toc-tic,4),"sec")
print("\nConfusion Matrix:")
confusion_encoded_GMM4 = pd.crosstab(Y_Test, Y_encoded_GMM4,rownames=['Actual'], colnam
display(confusion_encoded_GMM4)
```

accuracy = 73.8 %
elapsed time = 0.6614 sec

Confusion Matrix:

```
Predicted    0    1   2   3   4   5   6   7   8    9   All
Actual
0           82    1   2   0   1  13   1   0   0    0   100
1            1   95   0   3   0   0   0   0   1    0   100
2            5    3  73   3   3   1   4   2   6    0   100
3            3    5   2  75   1   3   0   2   8    1   100
4            0    4   2   0  65   1   3   0   1   24   100
5           15    1   2   8   1  60   3   0   6    4   100
6            6    2   3   0   3   3  82   0   1    0   100
7            0    8   0   0   3   1   0  70   3   15   100
8            5    2   1   8   2  10   1   0  64    7   100
9            1    4   0   0  17   3   0   0   3   72   100
All        118  125  85  97  96  95  94  74  93  123  1000
```

2.7 Step 6:

Same autoencoder 10 features with SVM algorithm with linear and nonlinear kernals.

```
In [70]: #encoded Linear SVM
         tic = time.time()
         svm_encoded_lin = svm.SVC(kernel='linear')
         svm_encoded_lin.fit(encoded_Train,Y_Train)
         toc = time.time()
         print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 3.8738 sec
```

```
In [71]: #encoded Linear SVM Prediction
         tic = time.time()
         acc_encoded_svm=accuracy_score(Y_Test,svm_encoded_lin.predict(encoded_Test))*100
         toc = time.time()
         print('accuracy = ',round(acc_encoded_svm,2),"%")
         print("elapsed time =",round(toc-tic,4),"sec")
         for t in range(4):
             print("\n")
         print("\nConfusion Matrix:")
         confusion_svm_encoded_lin = pd.crosstab(Y_Test, svm_encoded_lin.predict(encoded_Test),\
                                        rownames=['Actual'], colnames=['Predicted'], margin
         display(confusion_svm_encoded_lin)
```

```
accuracy =  82.7 %
elapsed time = 0.0676 sec
```

```
Confusion Matrix:


Predicted    0   1    2    3    4   5    6   7   8   9   All
Actual
0           90   0    0    0    0   8    1   0   1   0   100
1            0  91    2    2    0   1    0   0   4   0   100
2            1   1   81    0    2   1    8   2   4   0   100
3            2   0    3   84    0   4    0   1   5   1   100
4            0   1    0    1   87   0    3   0   0   8   100
5            7   0    3    5    4  71    3   1   5   1   100
6            0   1    6    0    1   1   90   0   1   0   100
7            0   1    3    1    0   2    0  83   3   7   100
8            4   0    5    7    3   3    0   2  74   2   100
9            1   2    2    2   11   2    0   2   2  76   100
All        105  97  105  102  108  93  105  91  99  95  1000
```

```
In [72]: #encoded non-Linear SVM
         tic = time.time()
         svm_encoded_nlin = svm.SVC(kernel='rbf')
         svm_encoded_nlin.fit(encoded_Train,Y_Train)
         toc = time.time()
         print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 9.9671 sec
```

```
In [73]: #encoded non-Linear SVM Prediction
         tic = time.time()
         acc_encoded_nsvm=accuracy_score(Y_Test,svm_encoded_nlin.predict(encoded_Test))*100
         toc = time.time()
         print('accuracy = ',round(acc_encoded_nsvm,2),"%")
         print("elapsed time =",round(toc-tic,4),"sec")
         for t in range(6):
             print("\n")
         print("\nConfusion Matrix:")
         confusion_svm_encoded_nlin = pd.crosstab(Y_Test,svm_encoded_nlin.predict(encoded_Test),
                                                  rownames=['Actual'], colnames=['Predicted'], margin
         display(confusion_svm_encoded_nlin)
```

```
accuracy =  77.5 %
elapsed time = 0.3825 sec
```

Confusion Matrix:

Predicted	0	1	2	3	4	5	6	7	8	9	All
Actual											
0	89	0	10	0	0	1	0	0	0	0	100
1	0	89	9	1	0	0	0	0	0	1	100
2	2	0	96	1	0	0	0	1	0	0	100
3	0	0	17	78	0	1	0	1	3	0	100
4	0	0	17	0	74	1	1	0	0	7	100
5	8	0	28	2	0	61	1	0	0	0	100
6	4	0	21	0	2	0	73	0	0	0	100
7	0	0	22	0	0	0	0	71	1	6	100
8	1	0	28	2	0	2	0	0	62	5	100
9	0	1	9	1	6	0	0	1	0	82	100
All	104	90	257	85	82	66	75	74	66	101	1000

2.8 Step 7:

Creating autoencoder model that gives 20 features instead of 10 and training it using training data.

In [74]:
```
K.clear_session()#clear old model
```

In [75]:
```
encoding_dim = 20  #size of output of encoder
Compression_ratio=784/encoding_dim
input_img = Input(shape=(784,)) #input placeholder
encoded = Dense(encoding_dim, activation='relu')(input_img) #encoding layer output
decoded = Dense(784, activation='sigmoid')(encoded) #decoding layer output
autoencoder = Model(input_img, decoded) #autoencoder model
encoder = Model(input_img, encoded) #encoder model
encoded_input = Input(shape=(encoding_dim,)) #input placeholder for decoder input
decoder_layer = autoencoder.layers[-1] # retrieve the last layer of the autoencoder mod
decoder = Model(encoded_input, decoder_layer(encoded_input)) #decoder model
autoencoder.compile(optimizer='adam', loss='binary_crossentropy')#compile autoencoder m
print("Compression ratio =" , round(Compression_ratio,3))
```

```
Compression ratio = 39.2
```

In [76]:
```
# Train the Autoencoder
autoencoder.fit(X_Train_unroll_norm, X_Train_unroll_norm,
                epochs=70,
                batch_size=256,
                shuffle=True,
                validation_data=(X_Test_unroll_norm, X_Test_unroll_norm))
```

```
Train on 10000 samples, validate on 1000 samples
Epoch 1/70
10000/10000 [==============================] - 2s 193us/step - loss: 0.5432 - val_loss: 0.3245
Epoch 2/70
10000/10000 [==============================] - 2s 197us/step - loss: 0.2912 - val_loss: 0.2742
Epoch 3/70
10000/10000 [==============================] - 2s 165us/step - loss: 0.2666 - val_loss: 0.2545
Epoch 4/70
10000/10000 [==============================] - 2s 189us/step - loss: 0.2461 - val_loss: 0.2353
Epoch 5/70
10000/10000 [==============================] - 2s 178us/step - loss: 0.2288 - val_loss: 0.2204
Epoch 6/70
10000/10000 [==============================] - 1s 147us/step - loss: 0.2155 - val_loss: 0.2087
Epoch 7/70
10000/10000 [==============================] - 2s 189us/step - loss: 0.2051 - val_loss: 0.1995
Epoch 8/70
10000/10000 [==============================] - 2s 186us/step - loss: 0.1967 - val_loss: 0.1921
Epoch 9/70
10000/10000 [==============================] - 2s 224us/step - loss: 0.1899 - val_loss: 0.1861
Epoch 10/70
10000/10000 [==============================] - 2s 198us/step - loss: 0.1843 - val_loss: 0.1810
Epoch 11/70
10000/10000 [==============================] - 2s 229us/step - loss: 0.1795 - val_loss: 0.1768
Epoch 12/70
10000/10000 [==============================] - 4s 391us/step - loss: 0.1751 - val_loss: 0.1728
Epoch 13/70
10000/10000 [==============================] - 2s 235us/step - loss: 0.1711 - val_loss: 0.1690
Epoch 14/70
10000/10000 [==============================] - 3s 348us/step - loss: 0.1675 - val_loss: 0.1657
Epoch 15/70
10000/10000 [==============================] - 3s 275us/step - loss: 0.1642 - val_loss: 0.1625
Epoch 16/70
10000/10000 [==============================] - 3s 336us/step - loss: 0.1610 - val_loss: 0.1598
Epoch 17/70
10000/10000 [==============================] - 3s 266us/step - loss: 0.1581 - val_loss: 0.1572
Epoch 18/70
10000/10000 [==============================] - 2s 241us/step - loss: 0.1554 - val_loss: 0.1547
Epoch 19/70
10000/10000 [==============================] - 4s 377us/step - loss: 0.1529 - val_loss: 0.1520
Epoch 20/70
10000/10000 [==============================] - 2s 192us/step - loss: 0.1504 - val_loss: 0.1497
```

```
Epoch 21/70
10000/10000 [==============================] - 3s 328us/step - loss: 0.1482 - val_loss: 0.1479
Epoch 22/70
10000/10000 [==============================] - 2s 238us/step - loss: 0.1461 - val_loss: 0.1459
Epoch 23/70
10000/10000 [==============================] - 2s 192us/step - loss: 0.1442 - val_loss: 0.1440
Epoch 24/70
10000/10000 [==============================] - 4s 363us/step - loss: 0.1426 - val_loss: 0.1428
Epoch 25/70
10000/10000 [==============================] - 2s 195us/step - loss: 0.1410 - val_loss: 0.1412
Epoch 26/70
10000/10000 [==============================] - 4s 385us/step - loss: 0.1397 - val_loss: 0.1399
Epoch 27/70
10000/10000 [==============================] - 2s 219us/step - loss: 0.1385 - val_loss: 0.1388
Epoch 28/70
10000/10000 [==============================] - 3s 286us/step - loss: 0.1374 - val_loss: 0.1380
Epoch 29/70
10000/10000 [==============================] - 3s 341us/step - loss: 0.1365 - val_loss: 0.1369
Epoch 30/70
10000/10000 [==============================] - 2s 212us/step - loss: 0.1357 - val_loss: 0.1361
Epoch 31/70
10000/10000 [==============================] - 4s 394us/step - loss: 0.1349 - val_loss: 0.1354
Epoch 32/70
10000/10000 [==============================] - 2s 180us/step - loss: 0.1342 - val_loss: 0.1350
Epoch 33/70
10000/10000 [==============================] - 4s 350us/step - loss: 0.1335 - val_loss: 0.1343
Epoch 34/70
10000/10000 [==============================] - 3s 256us/step - loss: 0.1329 - val_loss: 0.1337
Epoch 35/70
10000/10000 [==============================] - 3s 320us/step - loss: 0.1323 - val_loss: 0.1332
Epoch 36/70
10000/10000 [==============================] - 4s 390us/step - loss: 0.1318 - val_loss: 0.1328
Epoch 37/70
10000/10000 [==============================] - 3s 261us/step - loss: 0.1312 - val_loss: 0.1323
Epoch 38/70
10000/10000 [==============================] - 1s 143us/step - loss: 0.1308 - val_loss: 0.1317
Epoch 39/70
10000/10000 [==============================] - 2s 201us/step - loss: 0.1303 - val_loss: 0.1312
Epoch 40/70
10000/10000 [==============================] - 2s 202us/step - loss: 0.1299 - val_loss: 0.1309
Epoch 41/70
10000/10000 [==============================] - 2s 219us/step - loss: 0.1294 - val_loss: 0.1305
Epoch 42/70
10000/10000 [==============================] - 2s 222us/step - loss: 0.1290 - val_loss: 0.1301
Epoch 43/70
10000/10000 [==============================] - 3s 349us/step - loss: 0.1286 - val_loss: 0.1297
Epoch 44/70
10000/10000 [==============================] - 3s 330us/step - loss: 0.1283 - val_loss: 0.1294
```

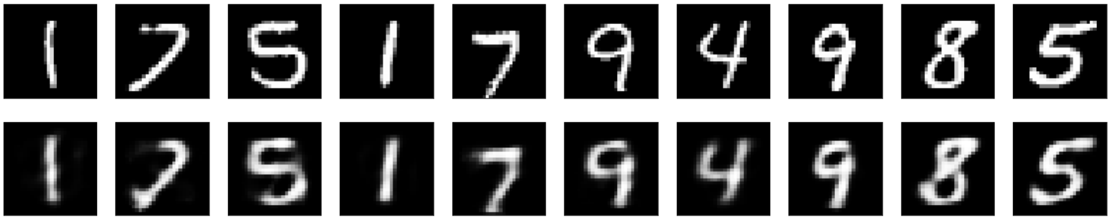
```
Epoch 45/70
10000/10000 [==============================] - 2s 168us/step - loss: 0.1280 - val_loss: 0.1290
Epoch 46/70
10000/10000 [==============================] - 2s 197us/step - loss: 0.1275 - val_loss: 0.1287
Epoch 47/70
10000/10000 [==============================] - 2s 220us/step - loss: 0.1272 - val_loss: 0.1283
Epoch 48/70
10000/10000 [==============================] - 2s 244us/step - loss: 0.1269 - val_loss: 0.1280
Epoch 49/70
10000/10000 [==============================] - 4s 381us/step - loss: 0.1266 - val_loss: 0.1277
Epoch 50/70
10000/10000 [==============================] - 2s 195us/step - loss: 0.1262 - val_loss: 0.1275
Epoch 51/70
10000/10000 [==============================] - 3s 265us/step - loss: 0.1259 - val_loss: 0.1270
Epoch 52/70
10000/10000 [==============================] - 4s 372us/step - loss: 0.1256 - val_loss: 0.1267
Epoch 53/70
10000/10000 [==============================] - 2s 234us/step - loss: 0.1253 - val_loss: 0.1264
Epoch 54/70
10000/10000 [==============================] - 2s 224us/step - loss: 0.1250 - val_loss: 0.1262
Epoch 55/70
10000/10000 [==============================] - 2s 225us/step - loss: 0.1247 - val_loss: 0.1259
Epoch 56/70
10000/10000 [==============================] - 2s 239us/step - loss: 0.1243 - val_loss: 0.1258
Epoch 57/70
10000/10000 [==============================] - 3s 262us/step - loss: 0.1241 - val_loss: 0.1253
Epoch 58/70
10000/10000 [==============================] - 4s 406us/step - loss: 0.1238 - val_loss: 0.1248
Epoch 59/70
10000/10000 [==============================] - 3s 343us/step - loss: 0.1235 - val_loss: 0.1244
Epoch 60/70
10000/10000 [==============================] - 2s 245us/step - loss: 0.1232 - val_loss: 0.1241
Epoch 61/70
10000/10000 [==============================] - 3s 276us/step - loss: 0.1230 - val_loss: 0.1238
Epoch 62/70
10000/10000 [==============================] - 4s 352us/step - loss: 0.1226 - val_loss: 0.1235
Epoch 63/70
10000/10000 [==============================] - 2s 211us/step - loss: 0.1224 - val_loss: 0.1233
Epoch 64/70
10000/10000 [==============================] - 3s 289us/step - loss: 0.1221 - val_loss: 0.1229
Epoch 65/70
10000/10000 [==============================] - 3s 280us/step - loss: 0.1218 - val_loss: 0.1225
Epoch 66/70
10000/10000 [==============================] - 2s 238us/step - loss: 0.1215 - val_loss: 0.1224
Epoch 67/70
10000/10000 [==============================] - 2s 211us/step - loss: 0.1213 - val_loss: 0.1220
Epoch 68/70
10000/10000 [==============================] - 2s 225us/step - loss: 0.1210 - val_loss: 0.1218
```

```
Epoch 69/70
10000/10000 [==============================] - 2s 239us/step - loss: 0.1208 - val_loss: 0.1214
Epoch 70/70
10000/10000 [==============================] - 3s 278us/step - loss: 0.1206 - val_loss: 0.1212
```

```
Out[76]: <keras.callbacks.History at 0x7f51925b9f28>
```

```
In [77]: #encode and decode Test Images
         encoded_imgs = encoder.predict(X_Test_unroll_norm)
         decoded_imgs = decoder.predict(encoded_imgs)
```

```
In [37]: #display Original test images and decoded Images examples
         n = 10  # how many digits we will display
         plt.pyplot.figure(figsize=(20, 4))
         for i in range(n):
             # display original
             ax = plt.pyplot.subplot(2, n, i + 1)
             img_num=randint(0,X_Test.shape[0])
             plt.pyplot.imshow(img.rotate(X_Test[img_num].reshape(28, 28),90),origin='lower')
             plt.pyplot.gray()
             ax.get_xaxis().set_visible(False)
             ax.get_yaxis().set_visible(False)
             # display reconstruction
             ax = plt.pyplot.subplot(2, n, i + 1 + n)
             plt.pyplot.imshow(img.rotate(decoded_imgs[img_num].reshape(28, 28),90),origin='lowe
             plt.pyplot.gray()
             ax.get_xaxis().set_visible(False)
             ax.get_yaxis().set_visible(False)
         plt.pyplot.show()
```



```
In [20]: #Create Encoded data for train and test datasets
         encoded_Train = encoder.predict(X_Train_unroll_norm)
         encoded_Test = encoder.predict(X_Test_unroll_norm)
```

2.9 Step 8:

Autoencoder 20 features - K-means clustering algorithm

In [39]:
```
#KMeans 1 cluster AutoEncoder
tic = time.time()
kmeans_encode1=kmeans(1,10,encoded_Train,1000)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 0.2773 sec
```

In [40]:
```
#Encoder KMeans 1 cluster Prediction
tic = time.time()
acc_kmeans_encoded1,Y_encoded_KMeans1=predict_acc(encoded_Test,Y_Test,kmeans_encode1)
toc = time.time()
print("accuracy =",round(acc_kmeans_encoded1,2),"%")
print("elapsed time =",round(toc-tic,4),"sec\n")
print("Confusion Matrix:")
confusion_encoded_KMeans1 = pd.crosstab(Y_Test, Y_encoded_KMeans1,rownames=['Actual'],
display(confusion_encoded_KMeans1)
```

```
accuracy = 70.8 %
elapsed time = 0.2781 sec

Confusion Matrix:
```

Predicted	0	1	2	3	4	5	6	7	8	9	All
Actual											
0	76	3	1	0	2	14	1	0	3	0	100
1	0	91	1	0	0	1	0	0	7	0	100
2	2	6	72	2	2	1	5	2	8	0	100
3	1	3	3	64	1	18	3	2	4	1	100
4	1	7	0	1	73	1	1	0	4	12	100
5	2	14	0	12	1	44	9	0	16	2	100
6	6	10	2	0	0	2	80	0	0	0	100
7	1	10	1	0	1	0	0	70	6	11	100
8	3	7	2	9	1	5	0	0	70	3	100
9	1	5	1	0	17	0	0	5	3	68	100
All	93	156	83	88	98	86	99	79	121	97	1000

In [41]:
```
#KMeans 2 cluster AutoEncoder
tic = time.time()
kmeans_encode2=kmeans(2,10,encoded_Train,1000)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 0.7324 sec
```

```
In [42]: #Encoder KMeans 2 cluster Prediction
         tic = time.time()
         acc_kmeans_encoded2,Y_encoded_KMeans2=predict_acc(encoded_Test,Y_Test,kmeans_encode2)
         toc = time.time()
         print("accuracy =",round(acc_kmeans_encoded2,2),"%")
         print("elapsed time =",round(toc-tic,4),"sec\n")
         print("Confusion Matrix:")
         confusion_encoded_KMeans2 = pd.crosstab(Y_Test, Y_encoded_KMeans2,rownames=['Actual'],
         display(confusion_encoded_KMeans2)
```

```
accuracy = 74.9 %
elapsed time = 0.4994 sec

Confusion Matrix:
```

Predicted	0	1	2	3	4	5	6	7	8	9	All
Actual											
0	82	0	3	2	0	8	2	0	3	0	100
1	0	91	1	0	0	1	2	0	5	0	100
2	1	3	72	2	3	1	6	2	10	0	100
3	1	1	3	79	0	4	2	2	8	0	100
4	0	3	0	0	65	4	2	0	1	25	100
5	3	0	0	12	4	67	7	0	3	4	100
6	4	0	4	1	2	3	86	0	0	0	100
7	0	7	1	0	3	3	0	74	3	9	100
8	3	0	2	8	2	8	0	1	65	11	100
9	0	3	1	0	17	2	0	7	2	68	100
All	94	108	87	104	96	101	107	86	100	117	1000

```
In [43]: #KMeans 4 cluster AutoEncoder
         tic = time.time()
         kmeans_encode4=kmeans(4,10,encoded_Train,1000)
         toc = time.time()
         print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 1.1783 sec
```

```
In [44]: #Encoder KMeans 4 cluster Prediction
         tic = time.time()
         acc_kmeans_encoded4,Y_encoded_KMeans4=predict_acc(encoded_Test,Y_Test,kmeans_encode4)
         toc = time.time()
         print("accuracy =",round(acc_kmeans_encoded4,2),"%")
         print("elapsed time =",round(toc-tic,4),"sec\n")
         print("Confusion Matrix:")
         confusion_encoded_KMeans4 = pd.crosstab(Y_Test, Y_encoded_KMeans4,rownames=['Actual'],
         display(confusion_encoded_KMeans4)
```

```
accuracy = 79.2 %
elapsed time = 0.7612 sec

Confusion Matrix:


Predicted    0    1    2    3    4    5    6    7    8    9   All
Actual
0           89    0    0    2    0    5    2    0    2    0   100
1            0   94    1    0    0    0    1    0    4    0   100
2            2    1   78    4    3    0    3    2    7    0   100
3            0    1    3   83    0    3    1    1    7    1   100
4            0    4    1    0   69    0    2    2    2   20   100
5            4    2    1    6    1   66    7    0    9    4   100
6            3    0    8    0    2    2   83    0    2    0   100
7            0    5    1    1    0    0    0   76    3   14   100
8            2    0    0    5    2    6    1    1   77    6   100
9            0    3    0    0   13    0    0    6    1   77   100
All        100  110   93  101   90   82  100   88  114  122  1000
```

In [78]:
```
#KMeans 8 cluster AutoEncoder
tic = time.time()
kmeans_encode8=kmeans(8,10,encoded_Train,1000)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 3.0829 sec
```

In [79]:
```
#Encoder KMeans 8 cluster Prediction
tic = time.time()
acc_kmeans_encoded8,Y_encoded_KMeans8=predict_acc(encoded_Test,Y_Test,kmeans_encode8)
toc = time.time()
print("accuracy =",round(acc_kmeans_encoded8,2),"%")
print("elapsed time =",round(toc-tic,4),"sec\n")
for t in range(3):
    print("\n")
print("Confusion Matrix:")
confusion_encoded_KMeans8 = pd.crosstab(Y_Test, Y_encoded_KMeans8,rownames=['Actual'],
display(confusion_encoded_KMeans8)
```

```
accuracy = 81.8 %
elapsed time = 1.21 sec
```

```
Confusion Matrix:


Predicted    0    1   2   3   4   5    6   7    8    9   All
Actual
0           86    0   0   0   0  11    2   0    1    0   100
1            0   99   0   0   0   0    1   0    0    0   100
2            3    2  83   2   1   0    2   2    5    0   100
3            1    1   5  79   0   2    0   3    8    1   100
4            0    2   1   2  74   1    4   0    0   16   100
5            4    1   1   3   3  72    4   0   10    2   100
6            5    0   2   0   3   0   89   0    1    0   100
7            0    5   0   0   1   1    0  81    1   11   100
8            1    0   0   4   2  10    0   0   75    8   100
9            0    2   0   1  10   1    0   4    2   80   100
All        100  112  92  91  94  98  102  90  103  118  1000
```

In [80]:
```
#KMeans 16 cluster AutoEncoder
tic = time.time()
kmeans_encode16=kmeans(16,10,encoded_Train,1000)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 1.9765 sec
```

In [81]:
```
#Encoder KMeans 16 cluster Prediction
tic = time.time()
acc_kmeans_encoded16,Y_encoded_KMeans16=predict_acc(encoded_Test,Y_Test,kmeans_encode16
toc = time.time()
print("accuracy =",round(acc_kmeans_encoded16,2),"%")
print("elapsed time =",round(toc-tic,4),"sec\n")
for t in range(9):
    print("\n")
print("Confusion Matrix:")
confusion_encoded_KMeans16 = pd.crosstab(Y_Test, Y_encoded_KMeans16,rownames=['Actual']
display(confusion_encoded_KMeans16)
```

```
accuracy = 82.7 %
elapsed time = 2.1724 sec
```

```
Confusion Matrix:


Predicted   0    1   2   3    4    5    6   7    8    9   All
Actual
0          85    0   0   0    0   11    4   0    0    0   100
1           0   99   0   0    0    0    1   0    0    0   100
2           2    3  79   3    1    1    1   2    6    2   100
3           0    3   2  83    1    3    0   1    6    1   100
4           0    1   1   0   83    2    3   0    0   10   100
5           4    0   0   2    2   75    3   0   10    4   100
6           1    1   1   0    1    2   94   0    0    0   100
7           0    2   0   0    1    0    0  82    2   13   100
8           2    0   3   7    3    5    1   0   72    7   100
9           0    1   0   0   15    1    1   3    4   75   100
All        94  110  86  95  107  100  108  88  100  112  1000
```

2.10 Step 9:

Autoencoder 20 features - GMM algorithm

```
In [49]: #encoded GMM 1
         tic = time.time()
         G_encoded1=GMM(1,10,encoded_Train,1000)
         toc = time.time()
         print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 0.5118 sec
```

```
In [50]: #encoded GMM 1 Predictions
         tic = time.time()
         acc_GMM_encoded1,Y_encoded_GMM1=predict_acc(encoded_Test,Y_Test,G_encoded1)
         toc = time.time()
         print("accuracy =",round(acc_GMM_encoded1,2),"%")
```

```
        print("elapsed time =",round(toc-tic,4),"sec")
        print("\nConfusion Matrix:")
        confusion_encoded_GMM1 = pd.crosstab(Y_Test, Y_encoded_GMM1,rownames=['Actual'], colnam
        display(confusion_encoded_GMM1)
```

```
accuracy = 70.8 %
elapsed time = 0.2528 sec

Confusion Matrix:
```

Predicted	0	1	2	3	4	5	6	7	8	9	All
Actual											
0	76	3	1	0	2	14	1	0	3	0	100
1	0	91	1	0	0	1	0	0	7	0	100
2	2	6	72	2	2	1	5	2	8	0	100
3	1	3	3	64	1	18	3	2	4	1	100
4	1	7	0	1	73	1	1	0	4	12	100
5	2	14	0	12	1	44	9	0	16	2	100
6	6	10	2	0	0	2	80	0	0	0	100
7	1	10	1	0	1	0	0	70	6	11	100
8	3	7	2	9	1	5	0	0	70	3	100
9	1	5	1	0	17	0	0	5	3	68	100
All	93	156	83	88	98	86	99	79	121	97	1000

```
In [51]: #encoded GMM 2
         tic = time.time()
         G_encoded2=GMM(2,10,encoded_Train,1000)
         toc = time.time()
         print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 5.0127 sec
```

```
In [52]: #encoded GMM 2 Predictions
         tic = time.time()
         acc_GMM_encoded2,Y_encoded_GMM2=predict_acc(encoded_Test,Y_Test,G_encoded2)
         toc = time.time()
         print("accuracy =",round(acc_GMM_encoded2,2),"%")
         print("elapsed time =",round(toc-tic,4),"sec")
         print("\nConfusion Matrix:")
         confusion_encoded_GMM2 = pd.crosstab(Y_Test, Y_encoded_GMM2,rownames=['Actual'], colnam
         display(confusion_encoded_GMM2)
```

```
accuracy = 74.5 %
elapsed time = 0.622 sec

Confusion Matrix:
```

```
Predicted   0    1   2   3    4   5   6   7    8    9   All
Actual
0          79    1   2   1    1  13   0   0    2    1   100
1           0   92   1   0    0   0   1   0    6    0   100
2           4    4  75   0    2   2   2   2    9    0   100
3           0    4   3  72    3   3   0   1   14    0   100
4           1    3   0   0   67   4   2   0    1   22   100
5           3    5   0   6    1  62   8   0   12    3   100
6           3    7   3   1    3   1  82   0    0    0   100
7           0    8   0   1    5   1   0  70    5   10   100
8           2    1   2   6    3   7   0   0   76    3   100
9           1    5   0   0   19   1   0   2    2   70   100
All        93  130  86  87  104  94  95  75  127  109  1000
```

In [82]:
```
#encoded GMM 4
tic = time.time()
G_encoded4=GMM(4,10,encoded_Train,1000)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 5.7405 sec
```

In [83]:
```
#encoded GMM 4 Predictions
tic = time.time()
acc_GMM_encoded4,Y_encoded_GMM4=predict_acc(encoded_Test,Y_Test,G_encoded4)
toc = time.time()
print("accuracy =",round(acc_GMM_encoded4,2),"%")
print("elapsed time =",round(toc-tic,4),"sec")
for t in range(10):
    print("\n")
print("\nConfusion Matrix:")
confusion_encoded_GMM4 = pd.crosstab(Y_Test, Y_encoded_GMM4,rownames=['Actual'], colnam
display(confusion_encoded_GMM4)
```

```
accuracy = 75.2 %
elapsed time = 0.7746 sec
```

```
Confusion Matrix:


Predicted   0    1   2   3   4    5   6   7    8    9   All
Actual
0          79    0   1   0   0   16   2   0    2    0   100
1           0   94   0   0   0    0   1   0    5    0   100
2           3    3  76   1   2    2   4   2    7    0   100
3           0    1   3  71   2   10   0   3    9    1   100
4           1    2   1   0  66    6   2   0    2   20   100
5           4    0   0   6   5   64   6   0    8    7   100
6           4    0   5   0   4    4  83   0    0    0   100
7           0    5   3   0   2    3   0  74    2   11   100
8           2    2   3   3   2    6   0   1   70   11   100
9           0    2   3   0  14    1   0   1    4   75   100
All        93  109  95  81  97  112  98  81  109  125  1000
```

2.11 Step 10:

Autoencoder 20 features - SVM algorithm

In [55]:
```
#encoded Linear SVM
tic = time.time()
svm_encoded_lin = svm.SVC(kernel='linear')
svm_encoded_lin.fit(encoded_Train,Y_Train)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 6.7535 sec
```

In [56]:
```
#encoded Linear SVM Prediction
tic = time.time()
acc_encoded_svm=accuracy_score(Y_Test,svm_encoded_lin.predict(encoded_Test))*100
toc = time.time()
print('accuracy = ',round(acc_encoded_svm,2),"%")
print("elapsed time =",round(toc-tic,4),"sec")
print("\nConfusion Matrix:")
```

```
confusion_svm_encoded_lin = pd.crosstab(Y_Test, svm_encoded_lin.predict(encoded_Test),\
                                        rownames=['Actual'], colnames=['Predicted'], margin
display(confusion_svm_encoded_lin)
```

```
accuracy =  88.4 %
elapsed time = 0.0661 sec

Confusion Matrix:
```

Predicted	0	1	2	3	4	5	6	7	8	9	All
Actual											
0	95	0	1	0	0	4	0	0	0	0	100
1	0	92	1	0	0	1	1	1	4	0	100
2	1	3	82	0	2	1	5	2	3	1	100
3	0	0	1	90	0	5	0	1	3	0	100
4	0	1	2	0	89	1	1	0	0	6	100
5	1	1	3	3	1	86	1	0	4	0	100
6	0	0	3	0	0	2	95	0	0	0	100
7	0	0	0	2	3	0	0	90	1	4	100
8	0	0	4	4	0	7	2	3	78	2	100
9	0	1	1	0	9	1	0	0	1	87	100
All	97	98	98	99	104	108	105	97	94	100	1000

In [21]:
```
#encoded non-Linear SVM
tic = time.time()
svm_encoded_nlin = svm.SVC(kernel='rbf')
svm_encoded_nlin.fit(encoded_Train,Y_Train)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 21.9054 sec
```

In [22]:
```
#encoded non-Linear SVM Prediction
tic = time.time()
acc_encoded_nsvm=accuracy_score(Y_Test,svm_encoded_nlin.predict(encoded_Test))*100
toc = time.time()
print('accuracy = ',round(acc_encoded_nsvm,2),"%")
print("elapsed time =",round(toc-tic,4),"sec")
print("\nConfusion Matrix:")
confusion_svm_encoded_nlin = pd.crosstab(Y_Test,svm_encoded_nlin.predict(encoded_Test),
                                        rownames=['Actual'], colnames=['Predicted'], margin
display(confusion_svm_encoded_nlin)
```

```
accuracy =  65.9 %
elapsed time = 0.5429 sec
```

```
Confusion Matrix:


Predicted   0   1    2   3   4   5   6   7   8   9   All
Actual
0          72   0   28   0   0   0   0   0   0   0   100
1           0  86   13   0   0   0   0   0   0   1   100
2           0   0  100   0   0   0   0   0   0   0   100
3           0   0   33  66   0   0   0   1   0   0   100
4           0   0   39   0  57   0   0   0   0   4   100
5           0   0   56   1   0  43   0   0   0   0   100
6           0   0   38   0   0   0  62   0   0   0   100
7           0   0   40   0   0   0   0  57   0   3   100
8           0   0   50   0   0   0   0   0  49   1   100
9           0   0   30   0   3   0   0   0   0  67   100
All        72  86  427  67  60  43  62  58  49  76  1000
```

2.12 Step 11:

centroid features - KMeans clustering classifing algorithm

```
In [84]: #KMeans 1 cluster centroid features
         tic = time.time()
         kmeans_centroid1=kmeans(1,10,Centroid_Features_Train,1000)
         toc = time.time()
         print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 0.1766 sec
```

```
In [85]: #centroid features KMeans 1 cluster Prediction
         tic = time.time()
         acc_kmeans_centroid1,Y_centroid_KMeans1=predict_acc(Centroid_Features_Test,Y_Test,kmean
         toc = time.time()
         print("accuracy =",round(acc_kmeans_centroid1,2),"%")
         print("elapsed time =",round(toc-tic,4),"sec\n")
         for t in range(2):
             print("\n")
         print("Confusion Matrix:")
         confusion_centroid_KMeans1 = pd.crosstab(Y_Test, Y_centroid_KMeans1,rownames=['Actual']
         display(confusion_centroid_KMeans1)
```

```
accuracy = 39.1 %
elapsed time = 0.1646 sec
```

```
Confusion Matrix:


Predicted    0    1   2    3   4   5   6    7  8   9   All
Actual
0           61    4  16   11   1   6   0    1  0   0   100
1            2   97   0    1   0   0   0    0  0   0   100
2           37    7  20   17   0   6   1   12  0   0   100
3           10   21  12   31   0  11   0   15  0   0   100
4           11    5   2    0  30   5  10   27  0  10   100
5           18   10   1   21   4  33   4    5  2   2   100
6           24   19  15    2   0   0  31    7  1   1   100
7            0   10   2   10   1   7   3   54  0  13   100
8           13   29   0   15   3   6   1   26  0   7   100
9            6   18   4    1   2   8   6   21  0  34   100
All        182  220  72  109  41  82  56  168  3  67  1000
```

In [86]:
```python
#KMeans 2 cluster centroid features
tic = time.time()
kmeans_centroid2=kmeans(2,10,Centroid_Features_Train,1000)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 0.2772 sec
```

In [87]:
```python
#centroid features KMeans 2 cluster Prediction
tic = time.time()
acc_kmeans_centroid2,Y_centroid_KMeans2=predict_acc(Centroid_Features_Test,Y_Test,kmean
toc = time.time()
print("accuracy =",round(acc_kmeans_centroid2,2),"%")
print("elapsed time =",round(toc-tic,4),"sec\n")
for t in range(9):
    print("\n")
print("Confusion Matrix:")
confusion_centroid_KMeans2 = pd.crosstab(Y_Test, Y_centroid_KMeans2,rownames=['Actual']
display(confusion_centroid_KMeans2)
```

```
accuracy = 47.4 %
elapsed time = 0.2732 sec
```

```
Confusion Matrix:


Predicted    0    1   2   3    4   5   6   7  8    9   All
Actual
0           94    1   1   0    1   0   3   0  0    0   100
1            2   95   0   1    0   0   2   0  0    0   100
2           28    7  40   9    0   2   2   8  0    4   100
3           26    7   5  43    0   2   2  10  0    5   100
4           16    2   4   0   50   1   9   0  0   18   100
5           38    9   5   9    6  22   7   1  0    3   100
6           27    0  11   0    7   0  50   0  0    5   100
7            4   11   1   9   12   2   1  39  2   19   100
8           19   24   3   6   10   5   6  13  1   13   100
9            9   19   7   0   19   1   3   1  1   40   100
All        263  175  77  77  105  35  85  72  4  107  1000
```

In [63]:
```
#KMeans 4 cluster centroid features
tic = time.time()
kmeans_centroid4=kmeans(4,10,Centroid_Features_Train,1000)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 0.4591 sec
```

In [64]:
```
#centroid features KMeans 4 cluster Prediction
tic = time.time()
acc_kmeans_centroid4,Y_centroid_KMeans4=predict_acc(Centroid_Features_Test,Y_Test,kmean
toc = time.time()
print("accuracy =",round(acc_kmeans_centroid4,2),"%")
print("elapsed time =",round(toc-tic,4),"sec\n")
print("Confusion Matrix:")
confusion_centroid_KMeans4 = pd.crosstab(Y_Test, Y_centroid_KMeans4,rownames=['Actual']
display(confusion_centroid_KMeans4)
```

```
accuracy = 56.4 %
elapsed time = 0.5595 sec

Confusion Matrix:
```

Predicted	0	1	2	3	4	5	6	7	8	9	All
Actual											
0	82	0	4	6	4	2	2	0	0	0	100
1	0	95	1	0	0	0	2	0	0	2	100
2	10	0	49	17	6	1	6	6	5	0	100
3	8	0	14	46	1	3	7	10	6	5	100
4	7	2	2	4	45	7	7	0	4	22	100
5	18	7	0	11	6	36	3	8	9	2	100
6	16	0	1	1	2	0	78	0	0	2	100
7	3	2	1	11	6	1	1	53	7	15	100
8	8	1	2	9	13	3	2	27	25	10	100
9	1	8	7	10	5	1	4	5	4	55	100
All	153	115	81	115	88	54	112	109	60	113	1000

In [88]:
```
#KMeans 8 cluster centroid features
tic = time.time()
kmeans_centroid8=kmeans(8,10,Centroid_Features_Train,1000)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 0.6708 sec
```

In [89]:
```
#centroid features KMeans 8 cluster Prediction
tic = time.time()
acc_kmeans_centroid8,Y_centroid_KMeans8=predict_acc(Centroid_Features_Test,Y_Test,kmean
toc = time.time()
print("accuracy =",round(acc_kmeans_centroid8,2),"%")
print("elapsed time =",round(toc-tic,4),"sec\n")
for t in range(2):
    print("\n")
print("Confusion Matrix:")
confusion_centroid_KMeans8 = pd.crosstab(Y_Test, Y_centroid_KMeans8,rownames=['Actual']
display(confusion_centroid_KMeans8)
```

```
accuracy = 64.0 %
elapsed time = 0.8203 sec
```

Confusion Matrix:

Predicted	0	1	2	3	4	5	6	7	8	9	All
Actual											
0	78	1	1	7	1	5	3	1	2	1	100
1	0	97	0	0	0	0	1	0	0	2	100
2	9	0	55	10	2	0	6	0	15	3	100
3	4	0	6	59	3	12	2	7	5	2	100
4	6	2	3	1	53	8	7	2	2	16	100
5	12	0	0	8	5	54	3	5	10	3	100
6	19	1	1	0	2	1	74	1	1	0	100
7	1	1	4	13	3	1	2	59	5	11	100
8	7	1	3	4	0	11	0	13	58	3	100
9	1	5	3	3	5	4	3	16	7	53	100
All	137	108	76	105	74	96	101	104	105	94	1000

In [90]:
```
#KMeans 16 cluster centroid features
tic = time.time()
kmeans_centroid16=kmeans(16,10,Centroid_Features_Train,1000)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

elapsed time = 1.2157 sec

In [91]:
```
#centroid features KMeans 16 cluster Prediction
tic = time.time()
acc_kmeans_centroid16,Y_centroid_KMeans16=predict_acc(Centroid_Features_Test,Y_Test,kme
toc = time.time()
print("accuracy =",round(acc_kmeans_centroid16,2),"%")
print("elapsed time =",round(toc-tic,4),"sec\n")
for t in range(7):
    print("\n")
print("Confusion Matrix:")
confusion_centroid_KMeans16 = pd.crosstab(Y_Test, Y_centroid_KMeans16,rownames=['Actual
display(confusion_centroid_KMeans16)
```

accuracy = 70.5 %
elapsed time = 1.6371 sec

```
Confusion Matrix:


Predicted    0   1   2   3   4    5    6   7    8    9   All
Actual
0           83   0   1   7   0    4    1   1    2    1   100
1            0  92   1   0   0    0    4   0    1    2   100
2            5   0  68   3   1    2    5   0   13    3   100
3            7   0   4  57   0   13    2   6   10    1   100
4            4   2   2   0  65    6    4   3    2   12   100
5           14   0   1   5   3   63    3   2    9    0   100
6            7   0   1   0   3    2   85   0    2    0   100
7            0   0   3   3   5    1    1  67    8   12   100
8            8   1   5   7   1    9    0  10   55    4   100
9            2   2   2   0   6    3    2  10    3   70   100
All        130  97  88  82  84  103  107  99  105  105  1000
```

2.13 Step 12:

centroid features - GMM classifing algorithm

```
In [92]: #centroid features GMM 1
         tic = time.time()
         G_centroid1=GMM(1,10,Centroid_Features_Train,1000)
         toc = time.time()
         print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 0.306 sec
```

```
In [94]: #Centroid Features GMM 1 Predictions
         tic = time.time()
         acc_GMM_centroid1,Y_centroid_GMM1=predict_acc(Centroid_Features_Test,Y_Test,G_centroid1
         toc = time.time()
         print("accuracy =",round(acc_GMM_centroid1,2),"%")
         print("elapsed time =",round(toc-tic,4),"sec")
         print("\nConfusion Matrix:")
         confusion_centroid_GMM1 = pd.crosstab(Y_Test, Y_centroid_GMM1,rownames=['Actual'], coln
         display(confusion_centroid_GMM1)
```

```
accuracy = 39.1 %
elapsed time = 0.1784 sec

Confusion Matrix:
```

Predicted	0	1	2	3	4	5	6	7	8	9	All
Actual											
0	61	4	16	11	1	6	0	1	0	0	100
1	2	97	0	1	0	0	0	0	0	0	100
2	37	7	20	17	0	6	1	12	0	0	100
3	10	21	12	31	0	11	0	15	0	0	100
4	11	5	2	0	30	5	10	27	0	10	100
5	18	10	1	21	4	33	4	5	2	2	100
6	24	19	15	2	0	0	31	7	1	1	100
7	0	10	2	10	1	7	3	54	0	13	100
8	13	29	0	15	3	6	1	26	0	7	100
9	6	18	4	1	2	8	6	21	0	34	100
All	182	220	72	109	41	82	56	168	3	67	1000

In [95]:
```
#centroid features GMM 2
tic = time.time()
G_centroid2=GMM(2,10,Centroid_Features_Train,1000)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 0.5598 sec
```

In [96]:
```
#Centroid Features GMM 2 Predictions
tic = time.time()
acc_GMM_centroid2,Y_centroid_GMM2=predict_acc(Centroid_Features_Test,Y_Test,G_centroid2
toc = time.time()
print("accuracy =",round(acc_GMM_centroid2,2),"%")
print("elapsed time =",round(toc-tic,4),"sec")
for t in range(3):
    print("\n")
print("\nConfusion Matrix:")
confusion_centroid_GMM2 = pd.crosstab(Y_Test, Y_centroid_GMM2,rownames=['Actual'], coln
display(confusion_centroid_GMM2)
```

```
accuracy = 45.8 %
elapsed time = 0.2467 sec
```

Confusion Matrix:

```
Predicted    0    1   2   3   4   5   6   7  8   9   All
Actual
0           94    4   1   0   1   0   0   0  0   0   100
1            2   96   0   1   0   0   1   0  0   0   100
2           28    7  40   9   0   2   2   8  0   4   100
3           24   10   2  42   0   2   5  10  0   5   100
4           16    5   4   0  49   1  10   0  0  15   100
5           38   13   6   8   6  22   3   1  0   3   100
6           31   19   5   0   1   0  40   1  0   3   100
7            4   11   1   9  10   3   5  39  2  16   100
8           19   30   3   5  10   5   1  13  1  13   100
9            9   20   8   0  19   1   6   1  1  35   100
All        265  215  70  74  96  36  73  73  4  94  1000
```

In [97]:
```python
#centroid features GMM 4
tic = time.time()
G_centroid4=GMM(4,10,Centroid_Features_Train,1000)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 2.1262 sec
```

In [98]:
```python
#Centroid Features GMM 4 Predictions
tic = time.time()
acc_GMM_centroid4,Y_centroid_GMM4=predict_acc(Centroid_Features_Test,Y_Test,G_centroid4
toc = time.time()
print("accuracy =",round(acc_GMM_centroid4,2),"%")
print("elapsed time =",round(toc-tic,4),"sec")
for t in range(8):
    print("\n")
print("\nConfusion Matrix:")
confusion_centroid_GMM4 = pd.crosstab(Y_Test, Y_centroid_GMM4,rownames=['Actual'], coln
display(confusion_centroid_GMM4)
```

```
accuracy = 53.4 %
elapsed time = 0.5312 sec
```

```
Confusion Matrix:


Predicted    0    1   2    3   4   5   6    7   8    9   All
Actual
0           78    0   4    9   3   2   1    0   3    0   100
1            0   95   1    0   0   0   2    0   2    0   100
2            7    1  50   15   6   0   3   11   6    1   100
3            6    0  12   46   2   4   7   12   5    6   100
4            8    1   2    3  45   7   6    0   5   23   100
5           13    1   0    9   7  33   3   10  14   10   100
6           28    0   1    2   4   0  63    0   0    2   100
7            4    3   1   11   7   0   1   53   6   14   100
8            7    4   3    8  13   4   0   27  25    9   100
9            4   11   4   10   8   1   4    5   7   46   100
All        155  116  78  113  95  51  90  118  73  111  1000
```

2.14 Step 13:

centroid features - SVM classifing algorithm

```
In [75]: #centroid feature Linear SVM
         tic = time.time()
         svm_centroid_lin = svm.SVC(kernel='linear')
         svm_centroid_lin.fit(Centroid_Features_Train,Y_Train)
         toc = time.time()
         print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 4.788 sec
```

```
In [76]: #centroid feature Linear SVM Prediction
         tic = time.time()
         acc_centroid_svm=accuracy_score(Y_Test,svm_centroid_lin.predict(Centroid_Features_Test)
         toc = time.time()
         print('accuracy = ',round(acc_centroid_svm,2),"%")
         print("elapsed time =",round(toc-tic,4),"sec")
```

```
print("\nConfusion Matrix:")
confusion_svm_centroid_lin = pd.crosstab(Y_Test, svm_centroid_lin.predict(Centroid_Feat
                                        rownames=['Actual'], colnames=['Predicted'], margin
display(confusion_svm_centroid_lin)
```

```
accuracy =  62.0 %
elapsed time = 0.1362 sec

Confusion Matrix:
```

Predicted	0	1	2	3	4	5	6	7	8	9	All
Actual											
0	84	0	3	4	0	4	2	1	2	0	100
1	0	95	0	0	0	0	2	0	1	2	100
2	9	1	73	0	1	1	8	1	6	0	100
3	8	0	2	49	4	18	3	7	9	0	100
4	7	1	4	3	53	1	4	3	3	21	100
5	18	0	2	15	9	31	6	5	11	3	100
6	6	0	1	2	2	1	88	0	0	0	100
7	0	2	6	7	11	4	1	45	9	15	100
8	7	2	3	4	11	9	3	13	42	6	100
9	2	3	10	4	3	3	2	8	5	60	100
All	141	104	104	88	94	72	119	83	88	107	1000

In [77]:
```
#centroid feature non-Linear SVM
tic = time.time()
svm_centroid_nlin = svm.SVC(kernel='rbf')
svm_centroid_nlin.fit(Centroid_Features_Train,Y_Train)
toc = time.time()
print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 2.6397 sec
```

In [78]:
```
#centroid non-Linear SVM Prediction
tic = time.time()
acc_centroid_nsvm=accuracy_score(Y_Test,svm_centroid_nlin.predict(Centroid_Features_Tes
toc = time.time()
print('accuracy = ',round(acc_centroid_nsvm,2),"%")
print("elapsed time =",round(toc-tic,4),"sec")
print("\nConfusion Matrix:")
confusion_svm_centroid_nlin = pd.crosstab(Y_Test,svm_centroid_nlin.predict(Centroid_Fea
                                        rownames=['Actual'], colnames=['Predicted'], margin
display(confusion_svm_centroid_nlin)
```

```
accuracy =  81.1 %
elapsed time = 0.2521 sec
```

```
Confusion Matrix:


Predicted    0   1    2   3   4   5    6   7   8    9   All
Actual
0           93   0    1   1   0   1    1   0   3    0   100
1            1  95    0   0   0   0    2   1   0    1   100
2            2   0   86   3   1   0    2   0   5    1   100
3            2   0    2  75   1  11    1   2   6    0   100
4            2   2    1   0  76   0    2   0   0   17   100
5           11   0    0   5   3  72    3   0   6    0   100
6            3   0    1   0   1   2   92   0   1    0   100
7            0   0    1   2   3   1    0  73   5   15   100
8            4   1    3   6   1   6    0   7  69    3   100
9            0   1    6   1   5   1    0   5   1   80   100
All        118  99  101  93  91  94  103  88  96  117  1000
```

```
In [99]: X_Train_conc=np.concatenate((encoded_Train,Centroid_Features_Train),axis=1)
         X_Test_conc=np.concatenate((encoded_Test,Centroid_Features_Test),axis=1)
```

```
In [102]: #concatenated K-Means
          tic = time.time()
          kmeans_conc=kmeans(16,10,X_Train_conc,1000)
          toc = time.time()
          print("elapsed time =",round(toc-tic,4),"sec")
```

```
elapsed time = 1.637 sec
```

```
In [104]: #concatenated kmeans Predictions
          tic = time.time()
          acc_kmeans_conc,Y_conc=predict_acc(X_Test_conc,Y_Test,kmeans_conc)
          toc = time.time()
          print("accuracy =",round(acc_kmeans_conc,2),"%")
          print("elapsed time =",round(toc-tic,4),"sec")
          for t in range(3):
              print("\n")
          print("\nConfusion Matrix:")
          confusion_kmeans_conc = pd.crosstab(Y_Test,Y_conc,rownames=['Actual'], colnames=['Pred
          display(confusion_kmeans_conc)
```

```
accuracy = 81.9 %
elapsed time = 1.6228 sec
```

Confusion Matrix:

Predicted	0	1	2	3	4	5	6	7	8	9	All
Actual											
0	90	0	0	1	0	7	2	0	0	0	100
1	0	98	1	0	0	0	1	0	0	0	100
2	2	0	82	3	0	1	2	2	8	0	100
3	0	2	5	77	0	7	0	3	5	1	100
4	0	2	3	0	79	1	1	0	1	13	100
5	5	1	1	8	2	70	4	1	7	1	100
6	2	0	1	0	1	1	93	0	1	1	100
7	0	1	1	0	1	0	0	85	2	10	100
8	0	0	3	8	3	9	0	0	67	10	100
9	0	2	2	1	10	0	1	2	4	78	100
All	99	106	99	98	96	96	104	93	95	114	1000

Aftar that, PCA analysis is used to diagonalize the covariance of the new feature using the formule (S=U^-1 * sigma* U) sigma : covariance matrix of the PCA output U is the matrix of Eigenvectors S is a diagonal matrix containing the Eigenvalues

In [48]:
```
# Concatenated feature matrix can be passed to (features_diagonalization) function and
# features is returned
import pandas as pd
from pandas import DataFrame

diagonalized_covariance = features_diagonalization(X_Train_conc)
df= pd.DataFrame(diagonalized_covariance)
display(df)
```

	0	1	2	3	...	24	
0	1.241785e+06	-2.910383e-11	1.136868e-10	-2.364686e-11	...	-1.591616e-11	-2.091838
1	6.048140e-11	3.985821e+04	1.509193e-11	1.145395e-11	...	-4.021672e-12	-1.276135
2	1.656559e-10	8.753887e-12	1.854361e+04	4.391154e-12	...	5.029532e-12	6.690648
3	-5.860556e-11	-6.528467e-11	8.943957e-12	1.656175e+04	...	2.507328e-12	-3.213430
..	...	...	...	...	...	...	
24	-1.672085e-11	-2.004263e-11	-3.134271e-12	2.534306e-12	...	2.968486e+03	-6.057377
25	-8.793322e-11	-3.424816e-12	-1.432410e-12	-3.299583e-12	...	4.329592e-12	3.429456
26	1.234568e-11	3.099743e-12	5.562883e-12	1.261213e-13	...	5.062617e-14	-2.643219
27	3.525713e-11	-5.385914e-12	9.485746e-13	2.763123e-12	...	4.023448e-13	-3.780087

[28 rows x 28 columns]

- The highest values are across the digonal of the matrix.

3 Comparison of Models

- Different Models can be compared with each other according to accuracy,time of training and time of prediction as shown in the following table:

Model type	Features generation	Time of training	Time of prediction	Accuracy
K-means 1 cluster	Autoencoder 10 features	0.2149 sec	0.2204 sec	68.1 %
	Autoencoder 20 features	0.243 sec	0.2417 sec	73.5 %
	Centroid features	0.1048 sec	0.1505 sec	39.1 %
	dct	0.1867 sec	0.203 sec	59.0 %
	pca	0.2049 sec	0.4456 sec	74.6 %
K-means 2 cluster	Autoencoder 10 features	0.6302 sec	0.4122 sec	72.4 %
	Autoencoder 20 features	0.6786 sec	0.4124 sec	78.1 %
	Centroid features	0.211 sec	0.2994 sec	47.4 %
	dct	0.7688 sec	0.2694 sec	63.8 %
	pca	0.9964 sec	0.2717 sec	80.0 %
K-means 4 cluster	Autoencoder 10 features	1.1509 sec	0.7499 sec	77.7 %
	Autoencoder 20 features	1.669 sec	0.8415 sec	80.8 %
	Centroid features	0.4457 sec	0.5447 sec	56.4 %
	dct	1.7464 sec	0.5254 sec	70.3 %
	pca	2.0397 sec	0.4357 sec	83.6 %
K-means 8 cluster	Autoencoder 10 features	1.3082 sec	1.4513 sec	79.3 %
	Autoencoder 20 features	1.7365 sec	1.522 sec	85.4 %
	Centroid features	0.7509 sec	1.0431 sec	64.0 %
	dct	1.7679 sec	0.8271 sec	75.0 %
	pca	2.3743 sec	0.8552 sec	89.6 %
K-means 16 cluster	Autoencoder 10 features	1.9183 sec	2.7627 sec	80.4 %
	Autoencoder 20 features	1.8665 sec	2.804 sec	88.1 %
	Centroid features	1.5426 sec	1.976 sec	70.5 %
	Concatenating centroid and encoder features	1.9795 sec	1.7749 sec	82.0 %
	dct	2.2373 sec	1.6846 sec	78.2 %
	pca	2.6695 sec	1.8107 sec	91.7 %
	Concatenated dct and pca features	3.395 sec	1.6362 sec	91.2 %

Model type	Features generation	Time of training	Time of prediction	Accuracy
1 GMM	Autoencoder 10 features	0.7902 sec	0.2434 sec	70.1 %
	Autoencoder 20 features	0.5094 sec	0.2465 sec	73.5 %
	Centroid features	0.3852 sec	0.1796 sec	39.1 %
	dct	0.4935 sec	0.1967 sec	59.0 %
	pca	0.8466 sec	0.1948 sec	74.6 %
2 GMM	Autoencoder 10 features	3.0359 sec	0.3434 sec	69.6 %
	Autoencoder 20 features	3.5069 sec	0.373 sec	76.2 %
	Centroid features	0.7093 sec	0.3169 sec	45.8 %
	dct	1.8323 sec	0.2372 sec	65.1 %
	pca	6.0392 sec	0.279 sec	79.9 %
4 GMM	Autoencoder 10 features	8.2123 sec	0.7396 sec	76.2 %
	Autoencoder 20 features	9.5174 sec	0.7264 sec	77.9 %
	Centroid features	1.8107 sec	0.4924 sec	53.4 %
	dct	5.58 sec	0.6094 sec	70.9 %
	pca	7.2452 sec	0.4895 sec	83.3 %
SVM - linear kernals	Autoencoder 10 features	4.4774 sec	0.0679 sec	82.7 %
	Autoencoder 20 features	5.7885 sec	0.0626 sec	89.1 %
	Centroid features	4.4105 sec	0.193 sec	62.0 %
	dct	2.2755 sec	0.1475 sec	82.0 %
	pca	4.4507 sec	0.2876 sec	90.3 %
SVM - non linear kernals	Autoencoder 10 features	10.2875 sec	0.3847 sec	77.0 %
	Autoencoder 20 features	11.4055 sec	0.5152 sec	68.7 %
	Centroid features	2.4326 sec	0.265 sec	81.1 %
	dct	2.5146 sec	0.0797 sec	92.1 %
	pca	5.3565 sec	0.5247 sec	97.3 %

3.1 Conclusion:

Classification of Handwriiten numbers is an important part of day to day services and industries, using machines can increase efficiency of this process, descision of which algorithm to use depends on accuracy of this algorithm and processing time of it in our problem next points were noticed:

- As the number of clusters increases the accuracy increases.
- As the number of clusters increases computational time increases.

- GMM gives better accuracy for the same number of clusters over the KMeans, with more computational time in dct and pca features.
- Concatenation of the PCA and DCT didn't increase the accuracy dramatically, it almost remained the same as using PCA only.
- Centroid features was the worst among the feature reduction algorithms in most clustering algorithms however it did a better job with non linear svm than autoencoder.
- Using Autoencoder has the upper hand regarding its accuracy since it tailors the output features on the type of the input it was trained on.
- As the number of output nodes in the encoder increases accuracy increases.
- Autoencoder has relatively slower timing due to time it takes to be trained on dataset rather than being a ready mathematical transformation.
- Non linear classification can hurt the algorithm rather than benefit from it.
- SVM with non-linear kernel was the most powerful algorithm of classification among the algorithms for this problem regarding accuracy using pca features.
- Autoencoder can benefit from deeping the layers but that will cause the model to be computationally to be more expensive in terms of time and memory usage.
- As the number of epochs loops increases loss function decreases and gets slower as learning algorithm reaches the minimum point.