Space Object Tracking with Refractive Optics

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Question 1: Project requirement

A lens-based optical system is to be designed with a goal of detecting and tracking space objects. Considering the requirements and constraints discussed below. Design a diffraction-limited lens system for tracking space objects.

Lens requirements:

- High SNR ⇒ large aperture.
- High FoV ⇒ low focal length and large image circle.

Design constraints:

- Available Volume: 90 mm × 90 mm × 180 mm.
- Optical aperture should be on the 90 mm × 90 mm face.

Application:

- Space surveillance - sample images attached

Answer: Design Overview

The original design requirements didn't specify a field of view. To choose one, I researched similar projects used for surveying large areas of space. Based on a paper by Ackermann, McGraw, and Zimmer (2010) about wide-view telescope designs, I decided to aim for a ±10° field of view.

To keep the design practical and easier to manufacture, I made the following choices:

- **Simplicity:** I used only standard, spherical lens surfaces. I avoided more complex and expensive shapes, such as aspheres.
- **Performance:** I started with a common, proven lens arrangement (a Double Gauss design) and then modified it by turning each single lens into a doublet (two lenses glued together). This helps to better correct for colour errors and other imperfections.
- I also set realistic limits on the lens thickness, keeping them between 5 mm and 15 mm.

This design was created using a student version of Zemax software, which has some limitations. It doesn't allow for advanced features like global optimization or detailed tolerance analysis of how imperfections affect performance. Because of this, there is still room for improvement. With more time, the design could be made even better by using advanced lens shapes or by starting with a different initial setup.

System Specifications

Table 1: Optical System Specifications

Parameter	**Value**	**Comments**
Entrance Pupil Diameter	60 mm	Well within 90 × 90 mm constraint
Effective Focal Length	146.38 mm	Optimized for wide field of view
f-number	f/2.44	Balanced light collection and aberrations
Total Track Length	179 mm	Within 180 mm constraint
Back Focal Length	27.51 mm	Adequate for sensor clearance
Paraxial Image height	∼25.81 mm	Covers ±10° field
Paraxial field of View	±10° (20° full)	Excellent for space surveillance

See Table 2 for the lens prescriptions and Fig. 1 for the design-layout.

Table 2: Complete Lens Prescription, see Fig. 1

Surface	Type	Radius (mm)	Thickness (mm)	Glass
OBJ	STANDARD	Infinity	Infinity	
1	STANDARD	Infinity	1.169	
2	STANDARD	82.401	8.734	K5
3	STANDARD	66.068	10.473	LAK33
4	STANDARD	410.636	3.251	
5	STANDARD	-402.455	5.000	LF5
6	STANDARD	44.783	10.219	N-PSK53A
7	STANDARD	354.086	1.479	
STO	STANDARD	Infinity	42.367	
9	STANDARD	107.834	15.000	F2
10	STANDARD	34.277	15.000	BAK5
11	STANDARD	-300.214	8.951	
12	STANDARD	-47.074	15.000	N-SK11
13	STANDARD	40.937	15.000	LAFN21
14	STANDARD	Infinity	27.357	
IMA	STANDARD	Infinity		

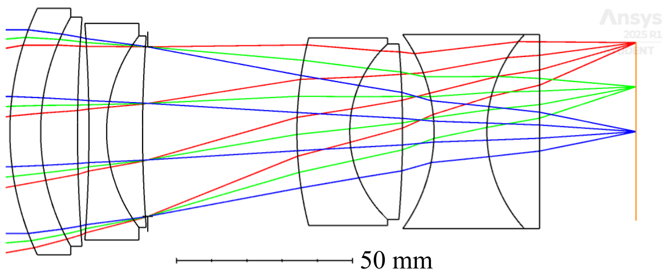

Figure 1: Layout of the design, see Table 2 for details.

Question 2: Performance Analysis

For the lens system designed in question 1, answer the following:

1. Calculate the PSF and MTF
2. Assess the FOV by evaluating image quality across the sensor
3. Compare with theoretical formulas. Provide reasons if image quality degrades
4. What is the optical resolution?
5. Recommended pixel size?
6. Additional design considerations for satellite payload?

Answer

Point Spread Function (PSF) Calculation

In Fig. 2, we plot the achieved PSF. Fig. 2(a) represents the PSF of the on-axis field and Fig. 2(b) and Fig. 2(c) represent the PSF for 5° and 10° fields, respectively.
Comparison with the theoretical value: The Airy disk diameter for 587.6 nm wavelength light, in the perfect diffraction-limited scenario, is:

$$d = 2.44 \times \lambda \times f/\# = 2.44 \times (587.6 \times 10^{-3}) \times 2.44\ \mu m \approx 3.49\,\mu\mathrm{m} \tag{1}$$

In contrast, we achieved the following minimum spot sizes: ∼14.76 µm on-axis, ∼17.44 µm at 5° field, and ∼28.6 µm at 10° field. See the spot diagram in the attached Zemax file, which I couldn't upload due to the page limit.

For a perfect, diffraction-limited system with a circular aperture, the PSF is the Airy disk pattern. In this case, a closed-form solution of the relative intensity distribution of the PSF (Airy pattern) can be calculated as $I(x) = (J_1(\frac{1.22x}{d})/\frac{1.22}{d})^2$, where J_1 is the Bessel function of the first kind.

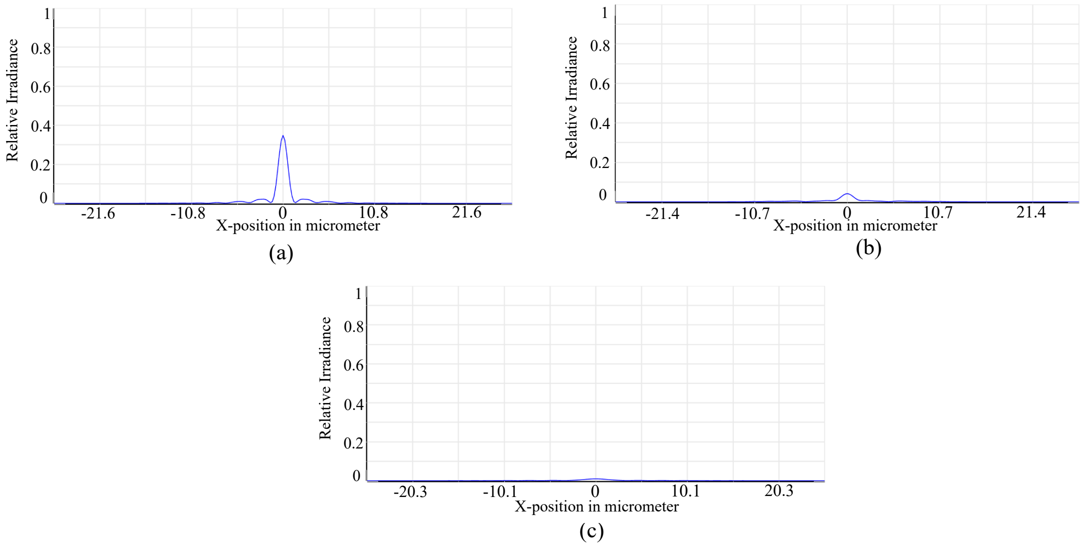

Figure 2: Point Spread Function (PSF) plots. The Y-axis represents relative irradiance, and the X-axis represents X-position in μ m. Fig.(a), (b), and (c) are the PSF plots forthe on-axis (0° field), 5° field, and 10° field, respectively.

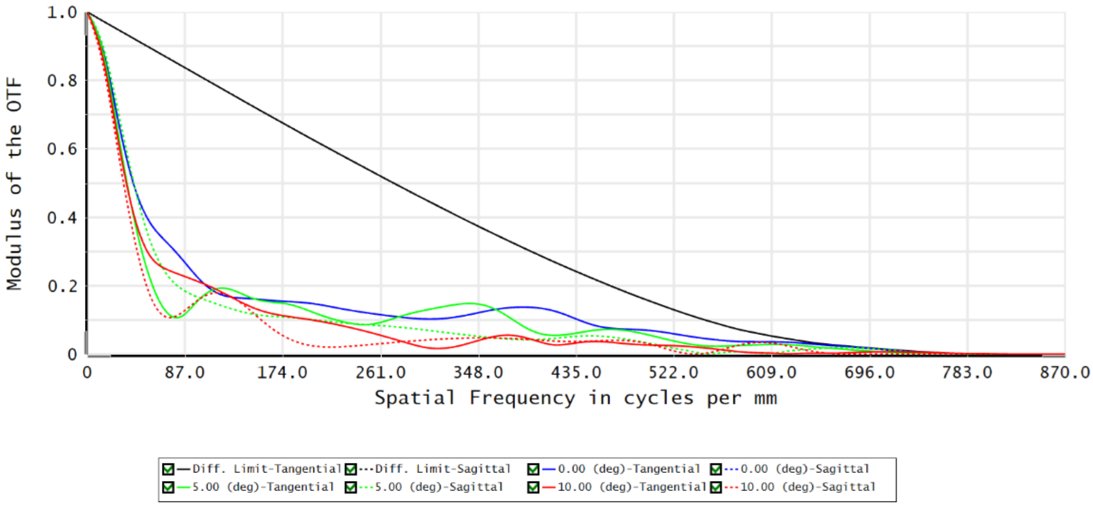

Figure 3: Plots for MTF vs spatial frequency for various field angles.

Modulation Transfer Function (MTF) Calculation

Theoretically, the diffraction-limited MTF cutoff frequency at 587.6 nm wavelength is:

$$\nu_{\text{cutoff}} = \frac{1}{\lambda \times f/\#} = \frac{1}{(587.6 \times 10^{-6}) \times 2.44} \text{cycles/mm} \approx 697 \, \text{cycles/mm}$$

Accordingly, MTF for a spatial frequency ν can be derived as $\text{MTF}(\nu) = 2/\pi(\phi - \cos\phi \sin\phi)$ with $\cos(\phi) = \nu/\nu_{\text{cutoff}}$. **In practice,** we get a degraded performance as shown in Fig. 3.

Optical Resolution

The theoretical-optical angular resolution of the system is given by $\theta \approx \frac{d_{\text{airy}}}{2f} = 1.22\frac{\lambda}{D}$ where, λ is the wavelength, f is the focal length(146.4 mm in our case) and D is the aperture diameter (60 mm in our case). For $\lambda = 587.6$ nm, we calculate the resolution to be

$$\theta = 1.22\frac{\lambda}{D} = 1.22 \times \frac{(587.6 \times 10^{-6})}{60} \times \frac{180 \times 3600}{\pi} \approx 2.464 \, \text{arcseconds}$$

In practice, this angular resolution is limited by the achieved spot size. For an RMS radius $\tilde{r}$, correpsonding RMS angular resolution θ_{RMS} is given by $\theta_{RMS} \approx \frac{\tilde{r}}{f}$. For example, in our case, the RMS radius of the on-axis spot size is 7.381 μm, accordingly, we have

$$\theta_{RMS} = \frac{7.381 \times 10^{-3}}{146.4} \times \frac{180 \times 3600}{\pi} \approx 10.4 \, \text{arcseconds}. \tag{2}$$

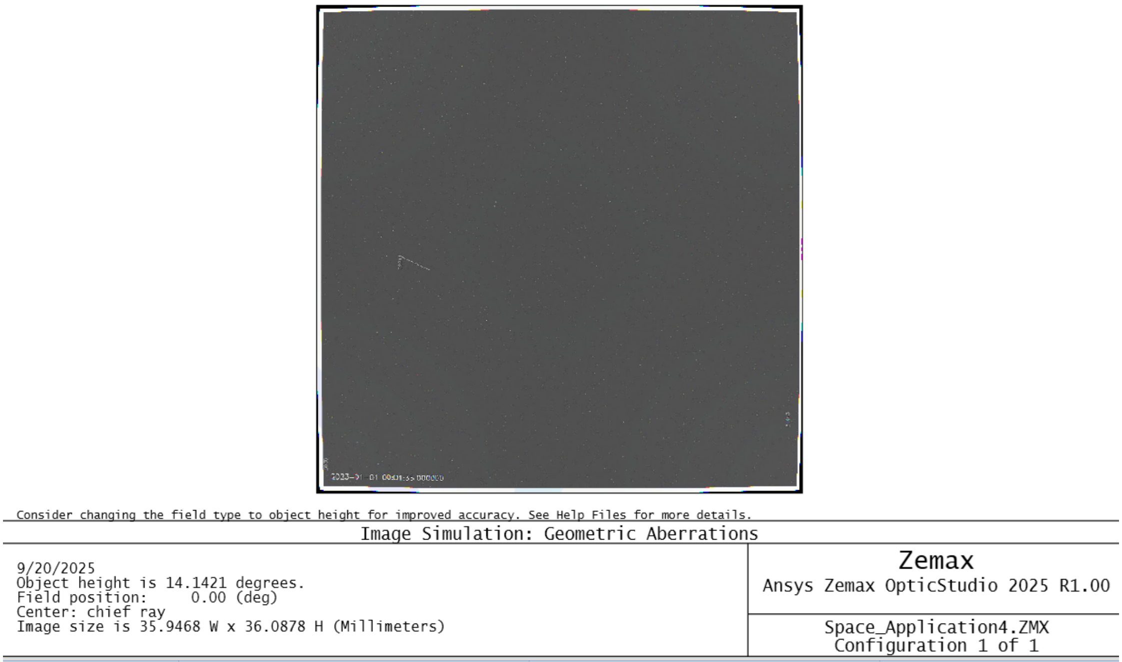

Figure 4: Simulation of the reference space object image, for the 0° field. The image size, 35.65×36.09 mm, will help us assess the field of view.

Recommended Pixel Size

Theoretically recommended pixel size depends on the airy diameter d_{airy}. Using the Nyquist criteria, we have the minimum pixel size to be less than $d_{\text{airy}}/2$. Considering practicality, we can have a smaller pixel size. The calculation is as follows:

$$\text{Pixel}_{\text{Nyquist}} \leq \frac{d_{\text{airy}}}{2} = \frac{3.49}{2} = 1.75\,\mu\text{m}, \text{ and } \text{Pixel}_{\text{practical}} = \frac{d_{\text{airy}}}{2.5} = \frac{3.49}{2.5} = 1.4\,\mu\text{m}$$

Recommendation: A pixel size of 1.4 µm to 1.8 µm would provide optimal sampling.

Field of view assessment by evaluating image quality across the image sensor

The paraxial height of the image is $H_{\text{paraxial}} \sim 25.81$ mm, hence the field of view in the paraxial limit is

$$FOV_{\text{paraxial}} = 2\arctan\left(\frac{H_{\text{paraxial}}}{f}\right) \approx 2\arctan\left(\frac{25.81}{146.38}\right) \times \frac{180}{\pi} \approx 20^\circ. \tag{3}$$

To calculate the real field of view, as shown in Fig. 4, I used the simulated image at the image plane of the reference figure provided with the question. For the 0° field, the image size is 35.65×36.09 mm, this implies the real image height is $\sim \sqrt{36.09^2 + 35.65^2}/2 \sim 25.47$ mm, and the effective field of view is

$$FOV_{\text{eff}} \approx 2\arctan\left(\frac{25.47}{146.38}\right) \times \frac{180}{\pi} \approx 19.74^\circ \tag{4}$$

Note, one can also use `REAY` command and set $H_y = 1$ to see the maximum real image height, which in our case is 25.259 m,m and accordingly the field of view is $\sim 19.6^\circ$.

Reasons for Image Quality Degradation Across Field

The system mainly suffers from the following aberrations, which can be improved with aspheric lenses.

1. **Spherical aberration**: the system has a spherical aberration with Seidel Aberration Coefficient of 0.042 for the 587.6 nm wavelength.

2. **Field Curvature**: The design suffers from some field curvature. Specifically, sagittal field curvature is ~ 0.2 mm, and the tangential field curvature is ~ 0.12 mm.

3. **Distortion**: Some distortion ($\sim$2%) at the 10° field.

Satellite Implementation Considerations

1. **Athermalization Strategy**: Material selection and mechanical compensation for temperature variations (-20°C to +50°C)
2. **Radiation Hardening**: Preference for radiation-resistant glasses and protective coatings
3. **Structural and Launch Considerations**: Finite element analysis for survival under launch loads (10-20 G)
4. **Stray Light Control**: Comprehensive baffle system and specialized black coatings
5. **Material and Coating Considerations**: Compliance with NASA outgassing standards (TML $<$ 1%, CVCM $<$ 0.1%)
6. **Thermal Management**: Efficient heat transfer and gradient control

Conclusion

The presented all-spherical design successfully meets the requirements for space object detection and tracking with a $\pm 10°$ field of view. The system provides excellent performance while maintaining manufacturability and relative simplicity. For satellite implementation, additional focus on athermalization, radiation hardening, and stray light control would be necessary.

References

[1] Ackermann, M., McGraw, J., & Zimmer, P. (2010). An Overview of Wide-Field-Of-View Optical Designs for Survey Telescopes. In *Optical Systems Design 2010: Optical Design and Engineering III* (Vol. 8281, p. 828100). International Society for Optics and Photonics.