Infinite integer R.V.

I) Distribution of an I.I.R.V

1.1) Def°

Let $(\Omega, \mathbb{R}, P(\Omega))$ be a proba space

$X : \Omega \to \mathbb{R}$. a random variable.

We say that:

X an I.I.R.V when $X(\Omega) \subset \mathbb{N}$.

1.2) Distribution of X.

$X(\Omega)$

$\forall k \in X(\Omega)$, $P(X = k)$

$$P(\Omega) = 1 \iff \sum_{k \in X(\Omega)} P(X = k) = 1$$

↳ Cvg to 1.

1.3) Calculation of $P(n \in A)$.

Let $A \subset \mathbb{N}$.

For any subset A,

$$P(X \in A) = \sum_{n \in A} P(X = n)$$

1.4) Geometric distribution.

Let $p \in]0;1[$. We consider a random experiment with two outcomes.

$P(S) = p$. and $P(\bar{S}) = 1 - p$.

Run the experiment an infinite number of times, independently.

Let X be a random variable to represent the first success:

$$P(X=1) = p$$

$$P(X=2) = (1-p)p$$

$$\vdots$$

$$P(X=k) = (1-p)^{k-1} p \Rightarrow X \sim \mathcal{G}(p)$$

2) Expected value and variance

2.1) Definition.

If X is an I.I.R.V then

$$\left.\begin{aligned} E(X) &= \sum_{n=0}^{+\infty} n P(X=n) \\ Var(X) &= \sum_{n=0}^{+\infty} (n - E(X))^2 P(X=n) \end{aligned}\right\} \begin{array}{l}\text{Don't exist} \\ \text{if associated series} \\ \text{not cvg.}\end{array}$$

2.2) Properties.

X and Y be two I.I.R.V with an expected value.

and a variance $(a,b) \in \mathbb{R}^2$.

$$\left.\begin{aligned} E(aX+b) &= aE(X)+b \\ E(X+Y) &= E(X)+E(Y) \end{aligned}\right\} E \text{ linear.}$$

$$Var(aX+b) = a^2 Var(X)$$

$$V(X+Y) = V(X) + V(Y) \quad \Delta \text{ if independent.}$$

$$= E(X^2) - E^2(X)$$

3) Generating functions of an I.I.R.V.

3.1) Def°.

Let be X an I.I.R.V.

$$G_X(t) = \sum_{k \in X(\Omega)} P(X = k) t^k$$

$\hookrightarrow R \geq 1$

- continuous over $]-1;1[$
- $G_X(1) = 1$
- class $\mathcal{C}^{+\infty}$ over $]-1;1[$
- $G_{X+Y} = G_X \cdot G_Y$ if X and Y independent.

3.2) Esperance, variance and G_X

Iff $\sum n P(X=n)$ cvg

$$E(X) = G_X'(1)$$

Iff $\sum (n - E(X))^2 P(X=n)$ cvg.

$$Var(X) = G_X''(1) + G_X'(1) - (G_X(1))^2$$