Homework 2
Rational expressions

Version du September 20, 2020

This homework has to be returned tomorrow, on Thursday, at the beginning of the tutorial.

Exercise 1 (The keypad strikes again)**.** *Here is another keypad similar to the one described in the previous assignment. It features ten numeric keys numbered from "*`0`*" to "*`9`*" as well as a key "*`E`*" used to confirm the input.*

The keypad has already been configured, but the passcode has been lost. Unlike the previous assignment, your goal now is to find the passcode by analysing the program. The keypad may accept more than one passcode!

Here is the embedded program:

```
/* As an example, tab[9][2] == 5. */
int tab[10][10] =
{        /* 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 */
  /* 0 */ { 0, 7, 0, 0, 0, 0, 0, 0, 0, 0 },
  /* 1 */ { 0, 7, 0, 0, 0, 0, 0, 0, 0, 0 },
  /* 2 */ { 1, 7, 0, 0, 0, 0, 0, 0, 0, 0 },
  /* 3 */ { 0, 8, 0, 0, 0, 0, 0, 0, 0, 0 },
  /* 4 */ { 0, 8, 0, 0, 0, 0, 0, 0, 0, 0 },
  /* 5 */ { 0, 7, 1, 0, 0, 0, 0, 0, 0, 0 },
  /* 6 */ { 0, 7, 0, 0, 1, 0, 0, 0, 0, 0 },
  /* 7 */ { 6, 9, 5, 3, 2, 0, 0, 0, 0, 0 },
  /* 8 */ { 6, 9, 5, 3, 2, 0, 0, 0, 0, 0 },
  /* 9 */ { 6, 9, 5, 4, 2, 0, 0, 0, 0, 0 }
};

int pos = 0;
for (;;) /* infinite loop */
  {
    int key = get_key();
    if (key == -1) /* key 'E' */
      {
        if (pos == 1 || pos == 4 || pos == 8)
          {
            open_door();
          }
        pos = 0;
      }
    else  /* 0 <= key <= 9 */
      {
        pos = tab[pos][key];
      }
  }
```

1. *What can we learn from the columns 5-9 of this table?*
2. *Write two sequences of keys that open the door.*
3. *Justify that the set L of sequences that open the door is infinite.*

4. *Write a rational expression for the language L. Note: use the alphabet $\{0,1,2,3,4,5,6,7,8,9\}$ and ignore the key "$\boldsymbol{E}$" whose sole purpose is to mark the end of the input sequence sent to the keypad.*

Exercise 2. *For any alphabet Σ, for any letter $a \in \Sigma$, for all languages A, L and M on Σ:*

1. *Justify that $\{a\}.L = \{a\}.M \implies L = M$.*
2. *Prove that $AL = AM \not\implies L = M$.*
3. *Prove that $L^\star = M^\star \not\implies L = M$.*
4. *Prove that $\forall n > 1$, $L^n \neq \{u^n \mid u \in L\}$.*
5. *Prove that $\forall n > 1$, $L^n = M^n \not\implies L = M$.*

Exercise 3 (Rational expressions)**.** *Compute an equivalent rational expression for the following languages:*

1. *The words in $\{a,b\}^\star$ with an even number of a.*
2. *The words in $\{a,b,c\}^\star$ with exactly 2 or 3 occurences of the letter c.*
3. *The words in $\{a,b,c\}^\star$ such that the letter c never appears immediately before the letter b.*