Math

Series :

- $\sum u_n$ CV $\Rightarrow \lim\limits_{n \to +\infty} u_n = 0$
- u_n CV $\Leftrightarrow \sum (u_n - u_{n-1})$ CV
- u_n and v_n constant sign
  - $0 \leq u_n \leq v_n$
    - if v_n CV then u_n CV
    - if u_n DV then v_n DV
  - if $u_n = O(v_n)$ if $\sum v_n$ CV $\Rightarrow \sum u_n$ CV
  - if $u_n = o(v_n)$ if $\sum u_n$ DV $\Rightarrow \sum v_n$ DV
  - if $u_n \sim v_n$: $\sum u_n$, $\sum v_n$ same nature
  - Riemann
    - $\left(\frac{1}{n^\alpha}, \alpha \in \mathbb{R}\right)$, $\sum \frac{1}{n^\alpha}$ if $\alpha > 1 \Rightarrow \sum \frac{1}{n^\alpha}$ CV
  - Reimann's Rule
    - if $\exists \alpha > 1$ such that $\lim\limits_{n \to +\infty} n^\alpha u_n \to 0$ $\Rightarrow \sum u_n$ CV
  - D'Alembert
    - if $\lim\limits_{n \to +\infty} \frac{u_{n+1}}{u_n} \to \ell$, $\ell \in \mathbb{R}^+ \cup \{+\infty\}$
      - $\ell < 1 \Rightarrow \sum u_n$ CV

→ $\ell > 1 \Rightarrow \sum u_n$ DV

→ **Cauchy**

→ if $\lim_{n \to +\infty} \sqrt[n]{u_n} \to \ell$, $\ell \in \mathbb{R}^+ \cup \{+\infty\}$

→ $\ell < 1 \Rightarrow \sum u_n$ CV

→ $\ell > 1 \Rightarrow \sum u_n$ DV

→ **Leibniz**

$$\left.\begin{array}{l} u_n \text{ alternative} \\ |u_n| \searrow \\ \lim |u_n| = 0 \end{array}\right\} \sum u_n \text{ CV}$$

→ if $\sum u_n$ absolutly CV $\Rightarrow \sum u_n$ CV

→ if $\sum u_n$ CV et $\sum u_n$ absolutly DV

$\Rightarrow \sum u_n$ semi - CV

Taylor Expansions :

→ $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + o(x^3)$

→ $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$

→ $(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)x^2}{2!} + o(x^2)$

→ $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + o(x^4)$

→ $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^5)$

→ $\frac{1}{1-x} = 1 + x + x^2 + x^3 + o(x^3)$

Generating Functions :

- $G_X(t) = \sum_{k=0}^{n} P(X=k) \times t^k$
- $G_X(1) = 1$
- $E(X) = G_X'(1)$
- $Var(X) = G_X''(1) + G_X'(1) - (G_X'(1))^2$