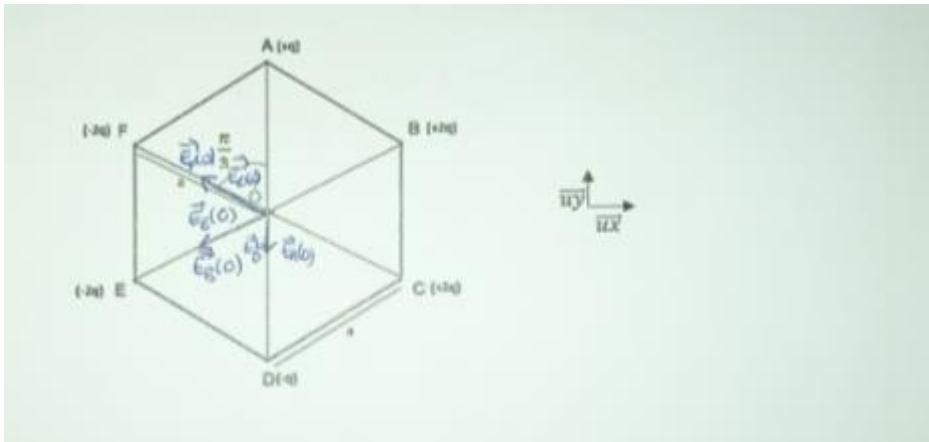


Test 1 Correction – Physics S3

Exercise 1



$$\|\vec{E}_A(O)\| = \frac{q}{4\pi\epsilon_0 a^2}$$

$$\|\vec{E}_B(O)\| = \frac{2q}{4\pi\epsilon_0 a^2}$$

$$\|\vec{E}_D(O)\| = \frac{q}{4\pi\epsilon_0 a^2}$$

$$\|\vec{E}_C(O)\| = \frac{2q}{4\pi\epsilon_0 a^2}$$

$$\|\vec{E}_E(O)\| = \frac{2q}{4\pi\epsilon_0 a^2}$$

$$\|\vec{E}_F(O)\| = \frac{2q}{4\pi\epsilon_0 a^2}$$

$$\vec{E}_A(O) = \|\vec{E}_A(O)\| \cdot (-\hat{w}_j) = -\frac{q}{4\pi\epsilon_0 a^2} \hat{w}_j = \vec{E}_B(O)$$

$$\begin{aligned}\vec{E}_F(O) &= \|\vec{E}_F(O)\| \cdot \left(-\cos(\pi/6) \hat{u}_x + \sin(\pi/6) \hat{u}_y \right) \\ &= -\frac{2q \cos(\pi/6)}{4\pi\epsilon_0 a^2} \hat{u}_x + \frac{2q \sin(\pi/6)}{4\pi\epsilon_0 a^2} \hat{u}_y = \vec{E}_C(O)\end{aligned}$$

$$\begin{aligned}\vec{E}_E(O) &= \|\vec{E}_E(O)\| \cdot \left(-\cos(\pi/6) \hat{u}_x - \sin(\pi/6) \hat{u}_y \right) \\ &= -\frac{2q \cos(\pi/6)}{4\pi\epsilon_0 a^2} \hat{u}_x - \frac{2q \sin(\pi/6)}{4\pi\epsilon_0 a^2} \hat{u}_y = \vec{E}_B(O)\end{aligned}$$

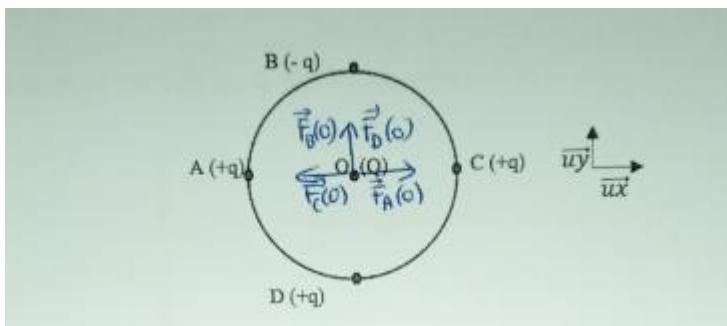
$$\vec{E}(0) = \vec{E}_A(0) + \vec{E}_B(0) + \vec{E}_C(0) + \vec{E}_D(0) + \vec{E}_E(0) + \vec{E}_F(0)$$

$$\vec{E}(0) = \left[-\frac{2q \cos(\pi/6)}{4\pi\epsilon_0 a^2} \times 2 - \frac{2q \cos(\pi/6)}{4\pi\epsilon_0 a^2} \times 2 \right] \vec{u}_x + \left[-\frac{q}{4\pi\epsilon_0 a^2} \times 2 \right. \\ \left. + \frac{2q \sin(\pi/6)}{4\pi\epsilon_0 a^2} \times 2 - \frac{2q \sin(\pi/6)}{4\pi\epsilon_0 a^2} \times 2 \right] \vec{u}_y.$$

$$\vec{E}(0) = -\frac{8q \cos(\pi/6)}{4\pi\epsilon_0 a^2} \vec{u}_x - \frac{q\sqrt{2}}{4\pi\epsilon_0 a^2} \vec{u}_y$$

$$\|\vec{E}(0)\| = \sqrt{\left(\frac{8q \cos(\pi/6)}{4\pi\epsilon_0 a^2}\right)^2 + \left(\frac{q\sqrt{2}}{4\pi\epsilon_0 a^2}\right)^2} \stackrel{A \cdot N}{=} \sqrt{\left(\frac{40\mu B}{8\pi\epsilon_0 \cdot 10^3 \cdot 10^{-4}}\right)^2 + \left(\frac{10\mu}{4\pi\epsilon_0 \cdot 10^3 \cdot 10^{-4}}\right)^2} \\ = \sqrt{(276 \frac{\mu}{\epsilon_0})^2 + (79,6 \frac{\mu}{\epsilon_0})^2}$$

Bonus



$$\|\vec{F}_A(0)\| = k \frac{qQ}{R^2} \quad \vec{F}_A(0) = k \frac{qQ}{R^2} \vec{u}_x; \quad \vec{F}_C(0) = -k \frac{qQ}{R^2} \vec{u}_x$$

$$\|\vec{F}_C(0)\| = \|\vec{F}_D(0)\| \quad \vec{F}_D(0) = k \frac{qQ}{R^2} \vec{u}_y; \quad \vec{F}_B(0) = +k \frac{qQ}{R^2} \vec{u}_y$$

$$= \|\vec{F}_B(0)\| \quad \vec{F}(0) = \vec{F}_A(0) + \vec{F}_B(0) + \vec{F}_C(0) + \vec{F}_D(0) = 2k \frac{qQ}{R^2} \vec{u}_y$$

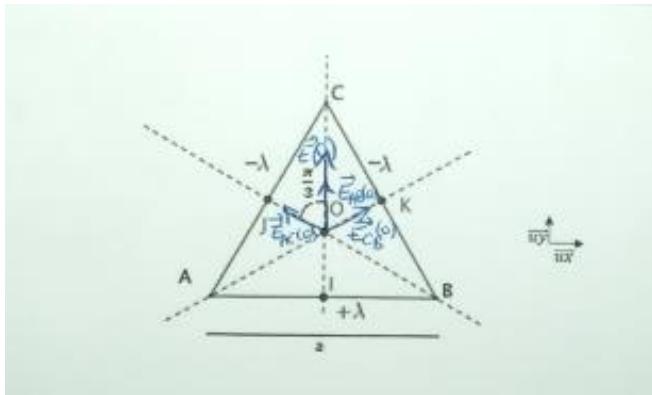
$$\|\vec{F}(0)\| = \sqrt{\left(\frac{2qQ}{R^2}\right)^2} = 2 \frac{qQ}{R^2} k.$$

$$V_A(0) = V_B(0) = V_C(0) = \frac{kq}{R} \quad V_B(0) = -\frac{kq}{R}$$

$$V(0) = V_A(0) + V_B(0) + V_C(0) + V_D(0) = \frac{3kq}{R} - \frac{kq}{R}$$

$$V(0) = 2 \frac{kq}{R}.$$

Exercise 2



2)

$$\|\vec{E}_{AB}(0)\| = \frac{k\lambda 2\sqrt{3}}{a} \int_{-\pi/3}^{\pi/3} \cos x \, dx \quad (\tan(\pi/3) = \sqrt{3} = \frac{a/2}{xc})$$

$$\rightarrow x = \frac{a}{2\sqrt{3}}$$

$$\|\vec{E}_{AB}(0)\| = \frac{k\lambda 2\sqrt{3}}{a} (\sin(\pi/3) - \sin(-\pi/3)) = \frac{k\lambda 2\sqrt{3}}{a} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{k\lambda 6}{a}$$

$$= \frac{k\lambda 2\sqrt{3}}{a} \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right)$$

$$= \frac{k\lambda 2\sqrt{3}}{a} \cdot \sqrt{3} = \frac{k\lambda 6}{a}$$

$$\|\vec{E}_{AB}(0)\| = \|\vec{E}_{CB}(0)\| = \|\vec{E}_{AC}(0)\| = \frac{k\lambda 6}{a}$$

3) $\|\vec{E}_{AB}\| = E$

$$\vec{E}_{AB} = \|\vec{E}_{AB}\| \vec{w}_y = \frac{k\lambda 6}{a} \vec{w}_y$$

$$\vec{E}_{CB} = \|\vec{E}_{CB}\| \cdot (\cos(\pi/3) \vec{w}_y + \sin(\pi/3) \vec{w}_x)$$

$$\vec{E}_{CB} = \frac{3k\lambda}{a} \vec{w}_y + \frac{3k\lambda\sqrt{3}}{a} \vec{w}_x$$

$$\vec{E}_{AC} = \|\vec{E}_{AC}\| \cdot (\cos(\pi/3) \vec{w}_y - \sin(\pi/3) \vec{w}_x)$$

$$\vec{E}_{AC} = \frac{k\lambda 2\sqrt{3}}{a} \cdot \frac{1}{2} \vec{w}_y - \frac{3k\lambda 2\sqrt{3}}{a} \frac{\sqrt{3}}{2} \vec{w}_x$$

$$\vec{E}_{AC} = \frac{3k\lambda}{a} \vec{w}_y - \frac{3}{2} \frac{k\lambda\sqrt{3}}{a} \vec{w}_x$$

$$\vec{E}(0) = \vec{E}_{AB} + \vec{E}_{AC} + \vec{E}_{CB} = 0 \vec{w}_x + \frac{12k\lambda}{a} \vec{w}_y$$

$$\|\vec{E}(0)\| = \sqrt{\left(\frac{12k\lambda}{a}\right)^2} = \frac{12k\lambda}{a} = 2 \|\vec{E}_{AB}\|$$

Exercise 3

$$\vec{E} = -\vec{\text{grad}} V = \underbrace{-\frac{\partial V}{\partial r} \vec{u}_r}_{E_r} - \underbrace{\frac{1}{r} \frac{\partial V}{\partial \theta} \vec{u}_\theta}_{E_\theta} - \underbrace{\frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \vec{u}_\phi}_{E_\phi}$$

$$E_r = \frac{C_1 \sin \theta}{r^2} e^{-C_2 \phi} \quad \vec{E}_r$$

$$E_\theta = -\frac{C_1 \cos \theta}{r^2} e^{-C_2 \phi}$$

$$E_\phi = \frac{C_1 \sin \theta}{r^2 \sin \theta} C_2 e^{-C_2 \phi} = \frac{C_1 C_2}{r^2} e^{-C_2 \phi}$$

$$E_r = 10 \quad E_\theta = 0 \quad E_\phi = 10.$$

$$\|\vec{E}\| = \sqrt{10^2 + 10^2} = \sqrt{200} = 14.14.$$