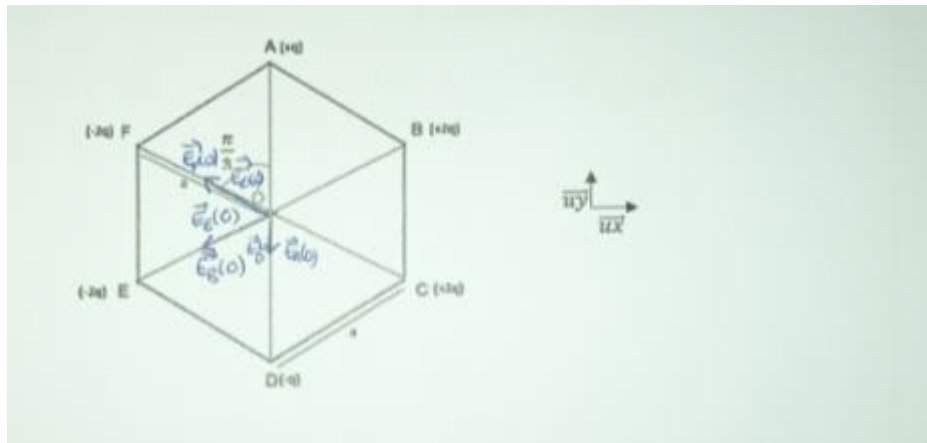


Test 1 Correction – Physics S3

Exercise 1



$$\begin{aligned} \|\vec{E}_A(O)\| &= \frac{q}{4\pi\epsilon_0 a^2} & \|\vec{E}_B(O)\| &= \frac{2q}{4\pi\epsilon_0 a^2} \\ \|\vec{E}_D(O)\| &= \frac{q}{4\pi\epsilon_0 a^2} & \|\vec{E}_C(O)\| &= \frac{2q}{4\pi\epsilon_0 a^2} \\ \|\vec{E}_E(O)\| &= \frac{2q}{4\pi\epsilon_0 a^2} & \|\vec{E}_F(O)\| &= \frac{2q}{4\pi\epsilon_0 a^2} \end{aligned}$$

$$\begin{aligned} \vec{E}_A(O) &= \|\vec{E}_A(O)\| \cdot (-\vec{u}_y) = -\frac{q}{4\pi\epsilon_0 a^2} \vec{u}_y = \vec{E}_D(O) \\ \vec{E}_F(O) &= \|\vec{E}_F(O)\| \cdot (-\cos(\pi/6)\vec{u}_x + \sin(\pi/6)\vec{u}_y) \\ &= -\frac{2q}{4\pi\epsilon_0 a^2} \cos(\pi/6)\vec{u}_x + \frac{2q}{4\pi\epsilon_0 a^2} \sin(\pi/6)\vec{u}_y = \vec{E}_C(O) \\ \vec{E}_E(O) &= \|\vec{E}_E(O)\| \cdot (-\cos(\pi/6)\vec{u}_x - \sin(\pi/6)\vec{u}_y) \\ &= -\frac{2q}{4\pi\epsilon_0 a^2} \cos(\pi/6)\vec{u}_x - \frac{2q}{4\pi\epsilon_0 a^2} \sin(\pi/6)\vec{u}_y = \vec{E}_B(O) \end{aligned}$$

$$\vec{E}(0) = \vec{E}_A(0) + \vec{E}_B(0) + \vec{E}_C(0) + \vec{E}_D(0) + \vec{E}_E(0) + \vec{E}_F(0)$$

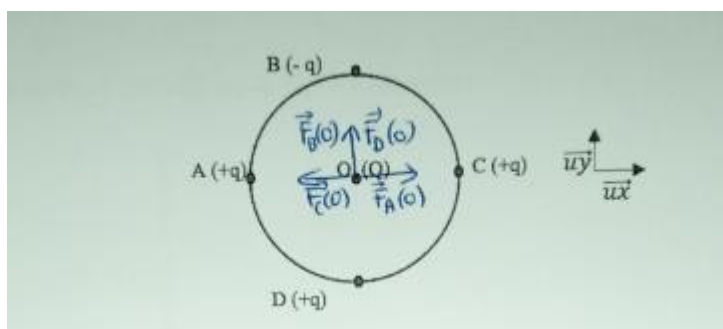
$$\vec{E}(0) = \left[-\frac{2q \cos(\pi/6)}{4\pi\epsilon_0 a^2} \times 2 - \frac{2q \cos(\pi/6)}{4\pi\epsilon_0 a^2} \times 2 \right] \vec{u}_x + \left[-\frac{q}{4\pi\epsilon_0 a^2} \times 2 + \frac{2q \sin(\pi/6)}{4\pi\epsilon_0 a^2} \times 2 - \frac{2q \sin(\pi/6)}{4\pi\epsilon_0 a^2} \times 2 \right] \vec{u}_y$$

$$\vec{E}(0) = -\frac{8q \cos(\pi/6)}{4\pi\epsilon_0 a^2} \vec{u}_x - \frac{q \times 2}{4\pi\epsilon_0 a^2} \vec{u}_y$$

$$\|\vec{E}(0)\| = \sqrt{\left(\frac{8q \cos(\pi/6)}{4\pi\epsilon_0 a^2}\right)^2 + \left(\frac{2q}{4\pi\epsilon_0 a^2}\right)^2} \stackrel{A=N}{=} \sqrt{\left(\frac{40\mu\sqrt{3}}{8\pi\epsilon_0 \times 10^{-2} \times 10^{-9}}\right)^2 + \left(\frac{10\mu}{4\pi\epsilon_0 \times 10^{-2} \times 10^{-9}}\right)^2}$$

$$= \sqrt{\left(276 \frac{\mu}{\epsilon_0}\right)^2 + \left(79,6 \frac{\mu}{\epsilon_0}\right)^2}$$

Bonus



$$\|\vec{F}_A(0)\| = k \frac{qQ}{R^2} \quad \vec{F}_A(0) = \frac{kqQ}{R^2} \vec{u}_x; \quad \vec{F}_C(0) = -\frac{kqQ}{R^2} \vec{u}_x$$

$$\|\vec{F}_C(0)\| = \|\vec{F}_D(0)\| \quad \vec{F}_D(0) = +\frac{kqQ}{R^2} \vec{u}_y; \quad \vec{F}_B(0) = +\frac{kqQ}{R^2} \vec{u}_y$$

$$= \|\vec{F}_B(0)\|$$

$$\vec{F}(0) = \vec{F}_A(0) + \vec{F}_B(0) + \vec{F}_C(0) + \vec{F}_D(0) = \frac{2kqQ}{R^2} \vec{u}_y$$

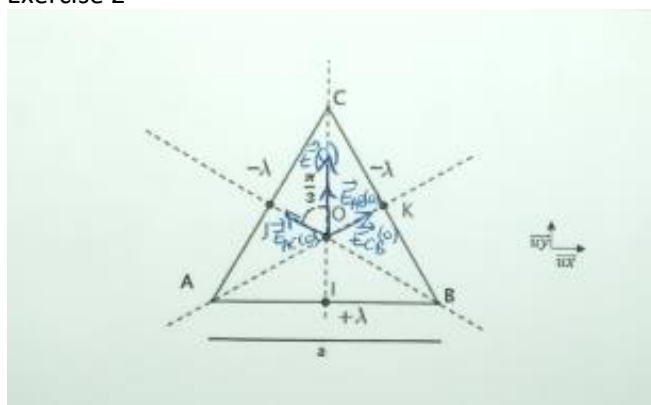
$$\|\vec{F}(0)\| = \sqrt{\left(\frac{2qQk}{R^2}\right)^2} = \frac{2qQk}{R^2}$$

$$V_A(0) = V_D(0) = V_C(0) = \frac{kq}{R} \quad V_B(0) = -\frac{kq}{R}$$

$$V(0) = V_A(0) + V_B(0) + V_C(0) + V_D(0) = \frac{3kq}{R} - \frac{kq}{R}$$

$$V(0) = 2 \frac{kq}{R}$$

Exercise 2



2)

$$\|\vec{E}_{AB}(0)\| = \frac{k\lambda 2\sqrt{3}}{a} \int_{-\pi/3}^{\pi/3} \cos \alpha \, d\alpha \quad (\tan(\pi/3) = \sqrt{3} = \frac{(a/2)}{x})$$

$$\|\vec{E}_{AB}(0)\| = \frac{k\lambda 2\sqrt{3}}{a} (\sin(\pi/3) - (\sin(-\pi/3))) = \frac{k\lambda 2\sqrt{3}}{a} \cdot \frac{x\sqrt{3}}{x}$$

$$= \frac{k\lambda 6}{a}$$

$$= \frac{k\lambda 2\sqrt{3}}{a} \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right)$$

$$= \frac{k\lambda 2\sqrt{3}}{a} \cdot \sqrt{3} = \frac{k\lambda 6}{a}$$

$$\|\vec{E}_{AB}(0)\| = \|\vec{E}_{CB}(0)\| = \|\vec{E}_{AC}(0)\| = \frac{k\lambda 6}{a}$$

3) $\|\vec{E}_{AB}\| = E$

$$\vec{E}_{AB} = \|\vec{E}_{AB}\| \vec{u}_y = \frac{k\lambda 6}{a} \vec{u}_y$$

$$\vec{E}_{CB} = \|\vec{E}_{CB}\| \cdot (\cos(\pi/3) \vec{u}_y + \sin(\pi/3) \cdot \vec{u}_x)$$

$$\vec{E}_{CB} = \frac{3k\lambda}{a} \vec{u}_y + \frac{3k\lambda\sqrt{3}}{a} \vec{u}_x$$

$$\vec{E}_{AC} = \|\vec{E}_{AC}\| \cdot (\cos(\pi/3) \vec{u}_y - \sin(\pi/3) \vec{u}_x)$$

$$\vec{E}_{AC} = \frac{k\lambda 2 \times 3}{a} \cdot \frac{1}{2} \vec{u}_y - \frac{3k\lambda 2\sqrt{3}}{a} \frac{\sqrt{3}}{2} \vec{u}_x$$

$$\vec{E}_{AC} = \frac{3k\lambda}{a} \vec{u}_y - 3 \frac{k\lambda\sqrt{3}}{a} \vec{u}_x$$

$$\vec{E}(0) = \vec{E}_{AB} + \vec{E}_{AC} + \vec{E}_{CB} = 0 \vec{u}_x + \frac{12k\lambda}{a} \vec{u}_y$$

$$\|\vec{E}(0)\| = \sqrt{\left(\frac{12k\lambda}{a}\right)^2} = \frac{12k\lambda}{a} = 2 \|\vec{E}_{AB}\|$$

Exercise 3

$$\vec{E} = -\vec{\text{grad}} V = \underbrace{-\frac{\partial V}{\partial r}}_{E_r} \vec{u}_r - \underbrace{\frac{1}{r} \frac{\partial V}{\partial \theta}}_{E_\theta} \vec{u}_\theta - \underbrace{\frac{1}{r \sin \theta} \frac{\partial V}{\partial \varphi}}_{E_\varphi} \vec{u}_\varphi$$

$$E_r = \frac{C_1 \sin \theta}{r^2} e^{-C_2 \varphi}$$

$$E_\theta = -\frac{C_1 \cos(\theta)}{r^2} e^{-C_2 \varphi}$$

$$E_\varphi = \frac{C_1 \sin \theta}{r^2 \sin \theta} C_2 e^{-C_2 \varphi} = \frac{C_1 C_2}{r^2} e^{-C_2 \varphi}$$

$$E_r = 10 \quad E_\theta = 0 \quad E_\varphi = 10.$$

$$\|\vec{E}\| = \sqrt{10^2 + 10^2} = \sqrt{200} = 14,14.$$