TD 5
Stability of Rational Languages

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Exercice 1 – Complementing rational expressions

Let $\Sigma = \{a, b\}$. Let L be the language matched to the rational expression $a^\star(ba^\star ba^\star ba^\star)^\star$. Our goal is to match a rational expression to the complement $\overline{L} = \Sigma^\star \setminus L$.

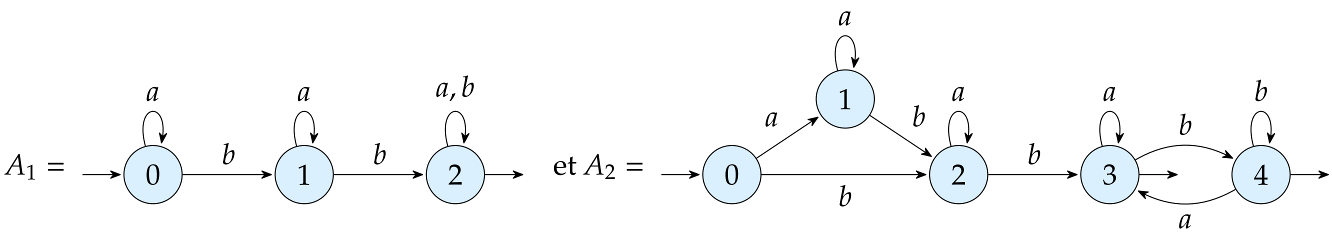
1. Is $\overline{L}$ rational? Why?
2. Write a finite automaton A_L accepting L.
3. Write $\overline{A_L}$, the complement of the automaton A_L.
4. Use Brzozowski and McCluskey's algorithm to compute the rational expression matched to the automaton $\overline{A_L}$.
5. Is the complement automaton still valid if $\Sigma = \{a, b, c\}$? If this isn't the case, what should be changed in order to make it correct?

Exercice 2 – Relations between rational languages

1. Let L_1 and L_2 be two rational languages such that $L_2 \subset L_1$. Is $L_1 \setminus L_2$ rational?
2. Let L_1 and L_2 be two languages such that $L_2 \subset L_1$. Suppose that L_2 is rational; is L_1 then rational? Why?

Exercice 3 – Intersection rational languages

We consider the two following automata:



Our goal is to show that these two automata are equivalent by computing $\overline{L(A_1)} \cap L(A_2)$ et $L(A_1) \cap \overline{L(A_2)}$.

1. Suppose the two automata are equivalent. What should then be the values of $\overline{L(A_1)} \cap L(A_2)$ and $L(A_1) \cap \overline{L(A_2)}$?
2. Compute and trim $\overline{A_1}$ and $\overline{A_2}$.
3. Given two non-deterministic automata $A = (\Sigma, Q, Q_0, F, \delta)$ et $A' = (\Sigma, Q', Q'_0, F', \delta')$, the synchronized product $A \,\&\, A'$ is the automaton $(\Sigma, Q^\&, Q_0^\&, F^\&, \delta^\&)$ such that:
   — $Q^\& = Q \times Q'$,
   — $Q_0^\& = Q_0 \times Q'_0$,
   — $F^\& = F \times F'$,
   — $\delta^\& = \{((s, s'), l, (d, d')) \in Q^\& \times \Sigma \times Q^\& \mid (s, l, d) \in \delta \text{ and } (s', l, d') \in \delta'\}$.

It is rather easy to prove that the words accepted by $A \,\&\, A'$ are both accepted by A and by A'. Indeed, $L(A \,\&\, A') = L(A) \cap L(A')$.

Using this definition, compute the automata $A_1 \,\&\, \overline{A_2}$ et $A_2 \,\&\, \overline{A_1}$.

4. Are A_1 and A_2 equivalent? Why?
5. Use the minimization algorithm shown in class to reduce the automaton A_2. Detail the state partitions of the automaton at each iteration step of the algorithm.

Exercice 4 – A difficult language to express

1. Let $\Sigma = \{a, b\}$. Let L be a rational language over Σ. Using only rational expressions and set operators on languages, how would you define the language L' of words with **exactly one** factor in the language L?

   As an example, if $L = \{ab, ba\}$, then $a\underline{ab}b \in L'$, $bbb\underline{ba} \in L'$, but $a\underline{ab}b\underline{ba} \notin L'$.

   (A clue: use the set difference operator.)
2. Is this language rational?