Exercise sheet n° 2

Ex 1

$$V(r,\theta) = \frac{kQa\cos(\theta)}{r^2} \qquad M(r\vec{v_r}, \theta\vec{u_\theta})$$

Scrap it

Ex 2

$$OM = r, \quad V(r) = kq\,\frac{1}{r}\,e^{-\frac{r}{a_0}}$$

1) $\vec{E}(M) = E(r).\vec{e_r}, \quad \vec{E} = -\vec{grad}(V)$

2) $\vec{E}(r) = -grad(V(r))$ $\quad \theta, \varphi$ constant

$$= -\frac{d\,V(r)}{dr}.\vec{e_r}$$

$$= -\frac{q}{4\pi\varepsilon_0}\,\frac{d}{dr}\left(\frac{1}{r}e^{-\frac{r}{a_0}}\right)$$

$$= -\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r^2}e^{-\frac{r}{a_0}} + \frac{1}{r}\left(-\frac{1}{a_0}\right)e^{-r/a_0}\right]\vec{e_r}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r^2} + \frac{1}{ra_0}\right)e^{-\frac{r}{a_0}}\,\vec{v_r}$$

Exercise sheet 3.

Every charge dq at a random point P on the wire has a symmetrical point P' (with respect to O) ⇒

⇒ Every $d\vec{E}_P$ has a symmetrical $d\vec{E}_{P'}$ * See Figure 1

→ The resultant electric field $\vec{E}$ is towards On axis. Therefore the only projection involved in the electrical field calculation is $d\vec{E}_n$

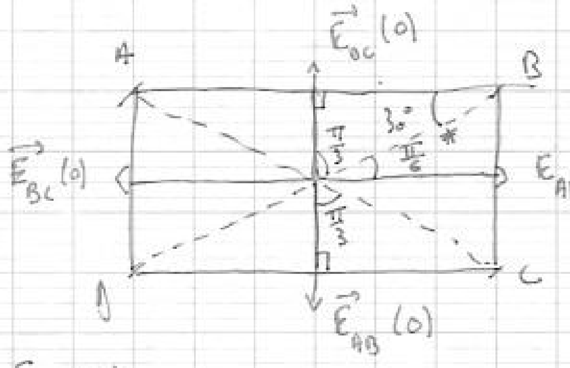
So $$E_x(M) = \int_{-\frac{\pi}{2}}^{\pi/2} \frac{k\lambda}{x} \cos(\alpha)\, d\alpha$$

$$= \frac{k\lambda}{x} \left[\sin(\alpha)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{2k\lambda}{x}$$

$$\boxed{\vec{E}(M) = \frac{2k\lambda}{x} \vec{u}_x}$$

1.2) We consider each point as a finite wire with angles like:



So $$E_{AB}(0) = \frac{k\lambda}{\frac{b}{2}} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos(\alpha)\, d\alpha$$

So $$E_{AB}(0) = \frac{2k\lambda}{b}\sqrt{3}$$

* 

So is $E_{DC}(0)$

$$E_{AD}(0) = \frac{k\lambda}{\frac{a}{2}} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos(\alpha)\, d\alpha = \frac{2k\lambda}{a}\left(\frac{1}{2} - \left(-\frac{1}{2}\right)\right)$$

Continuous distribution.

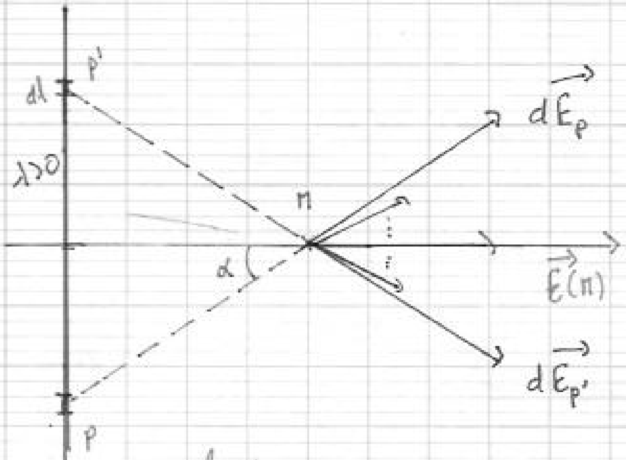


Figure 1

$dl = dy$

$dQ = \lambda dl$

$= \lambda dy$

$$d\vec{E_p}(M) = \frac{k\lambda dy}{PM^2}\vec{u_{PM}}$$

$$d\vec{E} = \frac{k\,dq}{r^2}\cdot\vec{u_r} \quad , \quad dq = \lambda dl = \lambda dy$$

$$\vec{u_{PM}} = \frac{\vec{PM}}{PM}$$

$$d\vec{E_p}(M) = \frac{k\lambda dy}{PM^3}\vec{PM} \qquad \vec{PM}\begin{pmatrix} PM\cos(\alpha) \\ PM\sin(\alpha) \end{pmatrix}\begin{matrix} \vec{u_x} \\ \vec{u_y} \end{matrix}$$

$$= \frac{k\lambda dy}{PM^3}\begin{pmatrix} PM\cos(\alpha) \\ PM\sin(\alpha) \end{pmatrix}$$

, $\vec{E}(M)$ is only projected on $\vec{u_x}$ so we ignore $\vec{u_y}$.

$$= \frac{k\lambda dy}{PM^3} PM\cos(\alpha)$$

We want to integrate according to another variable ($d\alpha$) so:

* $\tan'(\alpha) = \frac{1}{\cos^2(\alpha)}$

$dy \xrightarrow{?} d\alpha$ Well, $\tan(\alpha) = \frac{y}{x} \Rightarrow y = x\tan(\alpha)$

$$\frac{dy}{d\alpha} = \frac{x}{\cos^2(\alpha)}^{*} \longrightarrow dy = \frac{x}{\cos^2(\alpha)}d\alpha$$

$$dE_n(M) = \frac{h\lambda x}{PM^2 \cos(\alpha)} d\alpha$$

but $\cos(\alpha) = \frac{x}{PM} \Rightarrow PM = \frac{x}{\cos(\alpha)}$

$$PM^2 = \frac{x^2}{\cos^2(\alpha)}$$

$$dE_n(M) = \frac{h\lambda \cancel{x}}{\cancel{\cos(\alpha)}} \cdot \frac{\cos^{\cancel{2}}(\alpha)}{x^{\cancel{2}}} d\alpha = \frac{h\lambda \cos(\alpha)}{x} d\alpha.$$