TD2

Ex 1

1) We know that:

$$V(r,\theta) = \frac{kQa\cos(\theta)}{r^2} \quad , \quad \vec{E}(r,\theta) = -\text{grad}(V)$$

which can be written as:

$$E \begin{pmatrix} E_r \\ E_\theta \end{pmatrix} \begin{matrix} \vec{u_r} \\ \vec{u_\theta} \end{matrix}$$

So we use $V(r,\theta)$:

$$E_r = -\frac{dV(r,\theta)}{dr}$$
$$= +\frac{2kQa\cos(\theta)}{r^3}$$

$$E_\theta = -\frac{dV(r,\theta)}{d\theta}$$
$$= +\frac{kQa\sin(\theta)}{r^2}$$

2 and 3) Both use reference circle

$\frac{1}{r}$

Ex. 2

Rmq: Spherical coordinates (r, φ, θ) if ignored $\Rightarrow$ constant.

$$V(r) = kq \frac{1}{r} e^{-\frac{r}{a_0}}$$

1) $E = -\text{grad}(V(r))$ so:

$$E(M) = -\frac{dV(r)}{dr}$$

$$V(r) = \underbrace{\boxed{-kq}}_{cst} \boxed{\frac{1}{r}} \boxed{e^{-\frac{r}{a_0}}}$$

Both depend on r $\Rightarrow$ uv.

$$E(M) = -kq\left[-\frac{1}{r^2}e^{-\frac{r}{a_0}} + \left(-\frac{1}{a_0}\right)e^{-\frac{r}{a_0}}\left(\frac{1}{r}\right)\right]$$

facto. by $-\frac{1}{r}$

$$= +\frac{kq}{r}\left(\frac{1}{r}e^{-\frac{r}{a_0}} + \frac{1}{a_0}e^{-\frac{r}{a_0}}\right)$$

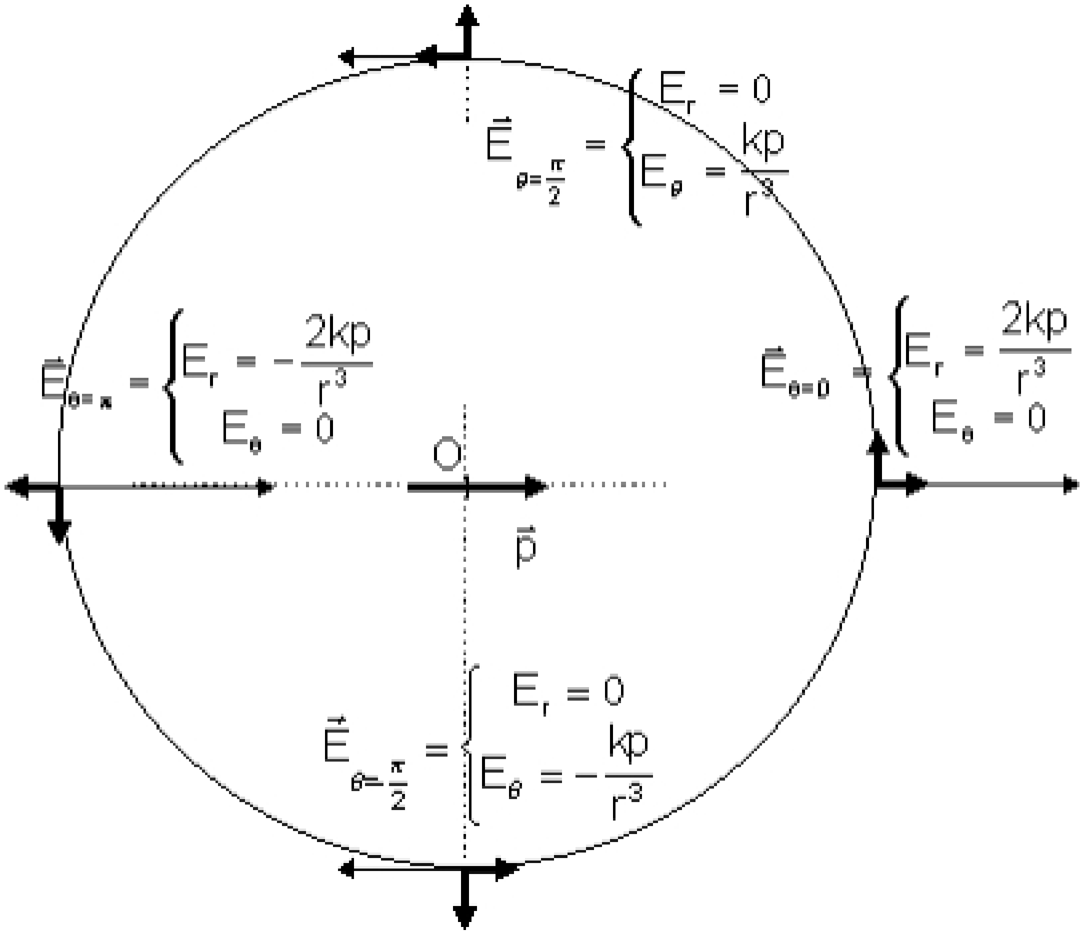
facto by $e^{-\frac{r}{a_0}}$

$$= \frac{kq}{r}e^{-\frac{r}{a_0}}\left(\frac{a_0+r}{ra_0}\right)$$

2) Because $V(r)$ only depends on r so does E.

Therefore, we have:

$$\vec{E}(M)\begin{pmatrix} E(M) \\ 0 \\ 0 \end{pmatrix}\begin{matrix} \vec{u_r} \\ \vec{u_\varphi} \\ \vec{u_\theta} \end{matrix}$$



$\vec{E}_{\theta=\frac{\pi}{2}} = \begin{cases} E_r = 0 \\ E_\theta = \dfrac{kp}{r^3} \end{cases}$
$\vec{E}_{\theta=\pi} = \begin{cases} E_r = -\dfrac{2kp}{r^3} \\ E_\theta = 0 \end{cases}$
$\vec{E}_{\theta=0} = \begin{cases} E_r = \dfrac{2kp}{r^3} \\ E_\theta = 0 \end{cases}$
O
$\vec{p}$
$\vec{E}_{\theta=\frac{\pi}{2}} = \begin{cases} E_r = 0 \\ E_\theta = -\dfrac{kp}{r^3} \end{cases}$