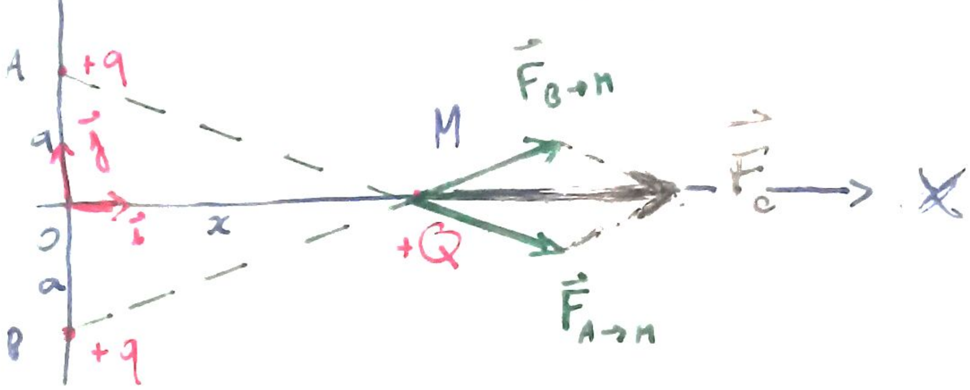
Exercise 2

$$\vec{F}_e = \vec{F}_{A\to M} + \vec{F}_{B\to M}$$



2/ $$\vec{F}_{A\to M} = \frac{qQ}{4\pi\varepsilon_0}\,\frac{\vec{AM}}{AM^3} = \frac{qQ}{4\pi\varepsilon_0}\cdot\frac{\vec{AO}+\vec{OM}}{\left(\sqrt{AO^2+OM^2}\right)^3}$$

$$\|\vec{i}\| = \|\vec{j}\| = 1$$

Electrical force cretated by A and applied on M :

$$\vec{F}_{A\to M} = \frac{qQ}{4\pi\varepsilon_0}\cdot\frac{-a\vec{j}+x\vec{i}}{(a^2+x^2)^{3/2}}$$

$$\vec{F}_{B\to M} = \frac{qQ}{4\pi\varepsilon_0}\cdot\frac{\vec{BM}}{BM^3} = \frac{qQ}{4\pi\varepsilon_0}\cdot\frac{\vec{BO}+\vec{OM}}{(a^2+x^2)^{3/2}}$$

Electrical force cretated by B and applied on M :

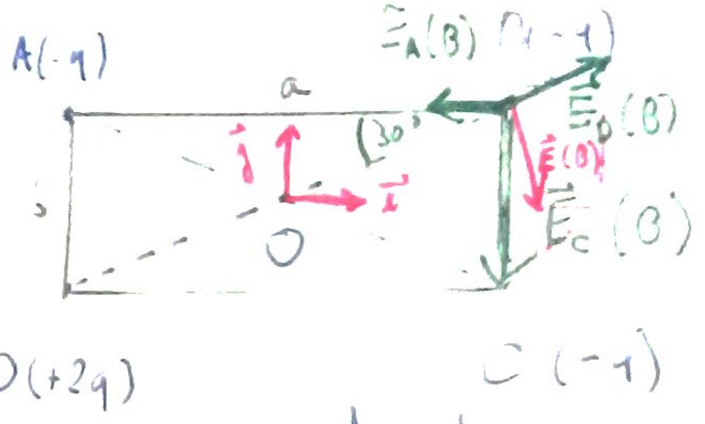
$$\vec{F}_{B\to M} = \frac{qQ}{4\pi\varepsilon_0}\cdot\frac{a\vec{j}+x\vec{i}}{(a^2+x^2)^{3/2}}$$

According to the superposition principle: $\vec{F}_e = \vec{F}_{A\to M} + \vec{F}_{B\to M}$

$$\vec{F}_e = \frac{qQ}{4\pi\varepsilon_0}\,\frac{2x\vec{i}}{(a^2+x^2)^{3/2}} \Rightarrow \boxed{\vec{F}_e = \frac{qQ}{2\pi\varepsilon_0}\,\frac{x}{(a^2+x^2)^{3/2}}\cdot\vec{i}}$$

l'intensité de $\vec{F}_e$ est $F_e = \|\vec{F}_e\| = \dfrac{qQ}{2\pi\varepsilon_0}\,\dfrac{x}{(a^2+x^2)^{3/2}}$

Exercice 2 :



1./ $\bullet\ \|\vec{E}_A(B)\| = \dfrac{+q}{4\pi\varepsilon_0 a^2}$

=> Electric field created by A and applied on B

$\bullet\ \boxed{\|\vec{E}_D(B)\| = \dfrac{2q}{4\pi\varepsilon_0 (a^2+b^2)}}$

$k = \dfrac{1}{4\pi\varepsilon_0}$

$\boxed{\|\vec{E}_C(B)\| = \dfrac{q}{4\pi\varepsilon_0 b^2}}$

2./

According to the superposition principle:

$$\vec{E}(B) = \vec{E}_A(B) + \vec{E}_C(B) + \vec{E}_D(B)$$

$$\vec{E}(B) = -k\cdot\frac{q}{a^2}\vec{i} - k\,\frac{q}{b^2}\vec{j} + k\frac{2q}{(a^2+b^2)}\left(\cos 30°\cdot\vec{i} + \sin(30°)\,\vec{j}\right)$$

$$\vec{E}(B) = k\cdot q\left[\left(-\frac{1}{a^2} + \frac{\sqrt{3}}{(a^2+b^2)}\right)\vec{i} + \left(-\frac{1}{b^2} + \frac{1}{a^2+b^2}\right)\vec{j}\right]$$

$$\boxed{\|\vec{E}(B)\| = E(B) = kq\cdot\sqrt{\left(\frac{\sqrt{3}}{a^2+b^2} - \frac{1}{a^2}\right)^2 + \left(\frac{1}{a^2+b^2} - \frac{1}{b^2}\right)^2}}$$

3./ $\vec{E}(O) = \vec{E}_A(O) + \vec{E}_B(O) + \vec{E}_C(O) + \vec{E}_D(O)$

$$\vec{E}_A(O) = k\cdot\frac{q}{\frac{a^2+b^2}{4}}\left(-\cos 30°\,\vec{i} + \sin 30°\,\vec{j}\right) = \frac{4\,k\cdot q}{a^2+b^2}\left(-\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}\right)$$

$$\vec{E}_A(O) = \frac{2\,k\cdot q}{a^2+b^2}\left(-\sqrt{3}\,\vec{i} + \vec{j}\right)$$

$$\vec{E}_B(O) = k \cdot \frac{q}{\frac{a^2+b^2}{4}} \cdot \left(\cos 30^\circ \vec{i} + \sin 30^\circ \vec{j}\right)$$

$$\vec{E}_B(O) = \frac{2k \cdot q}{a^2+b^2}\left(\sqrt{3} \cdot \vec{i} + \vec{j}\right)$$

$$\vec{E}_C(O) = k \cdot \frac{q}{\frac{a^2+b^2}{4}}\left(\cos 30^\circ \vec{i} - \sin 30^\circ \vec{j}\right)$$

$$\vec{E}_C(O) = \frac{2kq}{a^2+b^2}\left(\sqrt{3}\,\vec{i} - \vec{j}\right)$$

$$\vec{E}_D(O) = k \cdot \frac{2q}{\frac{a^2+b^2}{4}}\left(\cos 30^\circ \vec{i} + \sin 30^\circ \vec{j}\right)$$

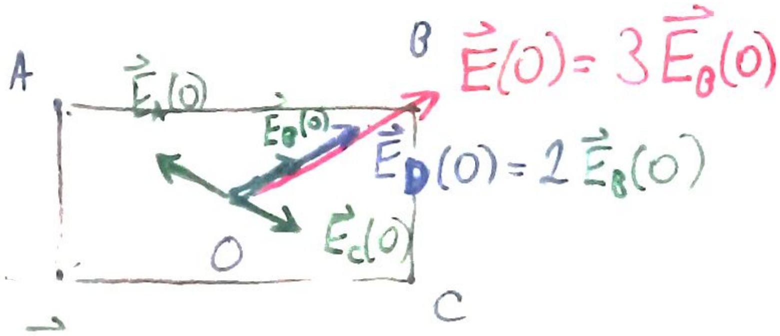
$$\vec{E}_D(O) = \frac{4kq}{a^2+b^2}\left(\sqrt{3}\,\vec{i} + \vec{j}\right)$$

$$\vec{E}(O) = \frac{2kq}{a^2+b^2}\left(3\sqrt{3}\,\vec{i} + 3\vec{j}\right)$$

$$\vec{E}(O) = \frac{6kq}{a^2+b^2}\left(\sqrt{3}\,\vec{i} + \vec{j}\right)$$

$$E(O) = \|\vec{E}(O)\| = \frac{6kq}{a^2+b^2}\left(\sqrt{3}^2 + 1^2\right)^{1/2} = \frac{6kq}{a^2+b^2} \times 2$$

$$\boxed{E(O) = \frac{12kq}{a^2+b^2}}$$



$$\vec{E}_A(O) + \vec{E}_C(O) = \vec{0}$$

4./

Electrostatic potential on O V(O):

$$V(O) = V_A(O) + V_B(O) + V_C(O) + V_D(O)$$

$$V_A(O) = k \cdot \frac{(-q)}{OA} = \frac{-kq}{\frac{\sqrt{a^2+b^2}}{2}} \; ; \; \underline{V_A(O) = \frac{-2kq}{\sqrt{a^2+b^2}}}$$

$$\underline{V_B(O) = V_C(O) = V_A(O)} \qquad OB = OC = OA$$

$$V_D(O) = k \cdot \frac{(2q)}{OD} = \frac{2kq}{\frac{\sqrt{a^2+b^2}}{2}} \; ; \; \underline{V_D(O) = \frac{4kq}{\sqrt{a^2+b^2}}}$$

$$\boxed{V(O) = \frac{-2kq}{\sqrt{a^2+b^2}}}$$