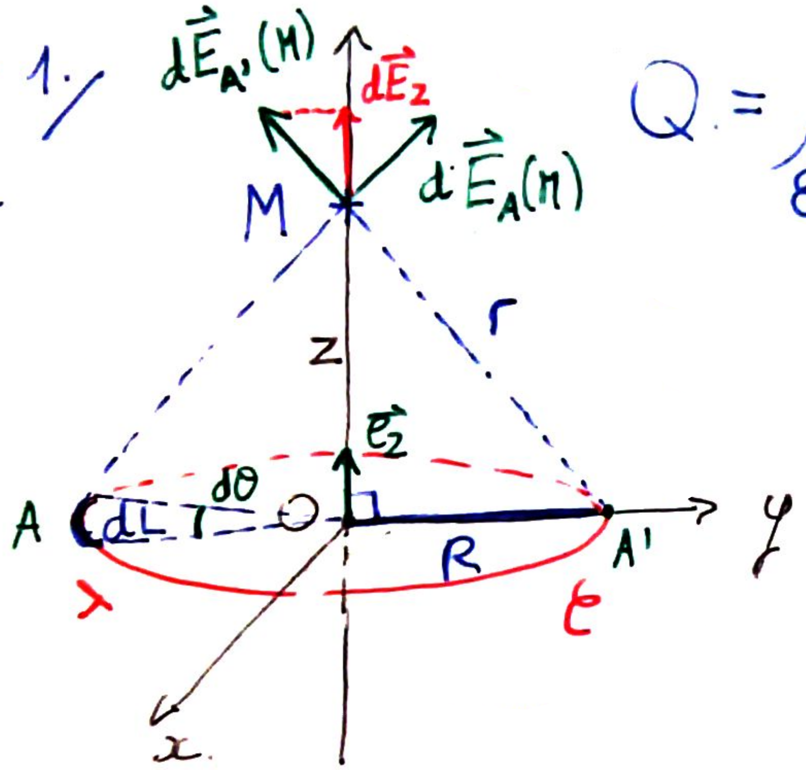
Exercise 2

1./

$dQ = \lambda dL$



$Q = \oint_{\mathcal{E}} \lambda dL = 2\pi R\lambda$

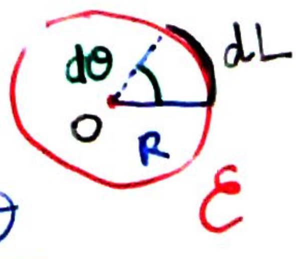
$\vec{E}(M) = E(z)\vec{e_z}$

$\vec{E}(O) = \vec{0}$

$$d\vec{E}_A(M) = k\cdot\frac{\lambda dL}{r^3}\vec{r} = \frac{k\lambda dL}{(\sqrt{R^2+z^2})^3}(\vec{AO}+\vec{OM})$$

$$d\vec{E}_A(M) = \frac{k\lambda dL}{(R^2+z^2)^{3/2}}(-R\vec{e_\rho}+z\vec{e_z})$$

$$d\vec{E}_z(M) = \frac{k\lambda dL}{(R^2+z^2)^{3/2}}\cdot z\,\vec{e_z}$$



$dL = R\,d\theta$

$$\boxed{d\vec{E}_z(M) = \frac{kR\lambda z\,d\theta}{(R^2+z^2)^{3/2}}\vec{e_z}}$$

$$\vec{E}(M) = \int_{\mathcal{C}} d\vec{E}_z(M)$$

$$\vec{E}(M) = \frac{kR\lambda z}{(R^2+z^2)^{3/2}} \int_0^{2\pi} d\theta \cdot \vec{e_z}$$

$$\boxed{\vec{E}(M) = \frac{2\pi R k \lambda z}{(R^2+z^2)^{3/2}} \cdot \vec{e_z}}$$

2/ $\vec{E}(M) = -\vec{grad}\, V(M)$,

$$\vec{grad}\, V(M) = \vec{\nabla} V(M) = \frac{\partial V}{\partial \rho}\vec{e_\rho} + \frac{1}{\rho}\frac{\partial V}{\partial \theta}\vec{e_\theta} + \frac{\partial V}{\partial z}\vec{e_z}$$

$$\vec{E}(M) = E(\cancel{\rho}; \cancel{\theta}; z)\vec{e_z} \Rightarrow V(M) = V(\cancel{\rho}, \cancel{\theta}, z)$$

$$\cdot\vec{e_z} \quad E(z)\vec{e_z} = \cancel{-\frac{\partial V}{\partial \rho}\vec{e_\rho}} \cancel{- \frac{1}{\rho}\frac{\partial V}{\partial \theta}\vec{e_\theta}} - \frac{\partial V}{\partial z}\vec{e_z}$$

$$E(z) = -\frac{\partial V}{\partial z} \Rightarrow V(z) = -\int E\, dz$$

$$I = \int \frac{z\, dz}{(R^2+z^2)^{3/2}} = \int \underbrace{z}_{\frac{u'}{2}} \cdot \underbrace{(R^2+z^2)^{-3/2}}_{u^{n-1}} dz$$

$(u^n)' = n u' u^{n-1}$

$n = -1/2$

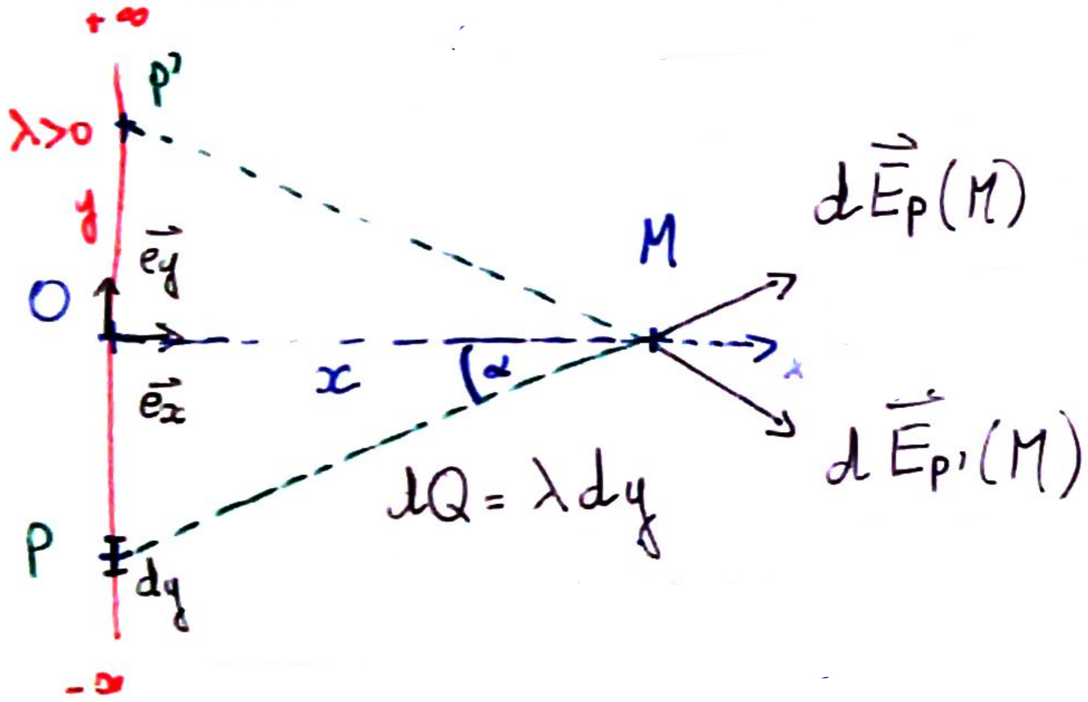
$$; \quad (u^{-1/2})' = -\frac{1}{2} u' u^{-3/2} \Rightarrow -\cancel{2}(u^{-1/2})' = \frac{u'}{2} u^{-3/2}$$

$$I = -\frac{1}{\sqrt{R^2+z^2}} \; ; \; \boxed{V(z) = \frac{2\pi R k \lambda}{\sqrt{R^2+z^2}}}$$

$$dV(M) = \frac{k\,dQ}{r} = \frac{k\lambda\,dL}{\sqrt{R^2+z^2}} = \frac{k\lambda R\,d\theta}{\sqrt{R^2+z^2}}.$$

$$V(M) = \int_C dV(M) = \int_0^{2\pi} \frac{k\lambda R\,d\theta}{\sqrt{R^2+z^2}} = \frac{2\pi R k\lambda}{\sqrt{R^2+z^2}}.$$

Exercise 1



$$\boxed{dE_x(M) = \frac{k\lambda \cos\alpha \, d\alpha}{x}}$$

Every single point P on the wire has a symetrical point P'.
By semtrical analysis, the resultant elmectric field is towards Ox axis.
Only the projection on Ox is then taken into account in the calculations.

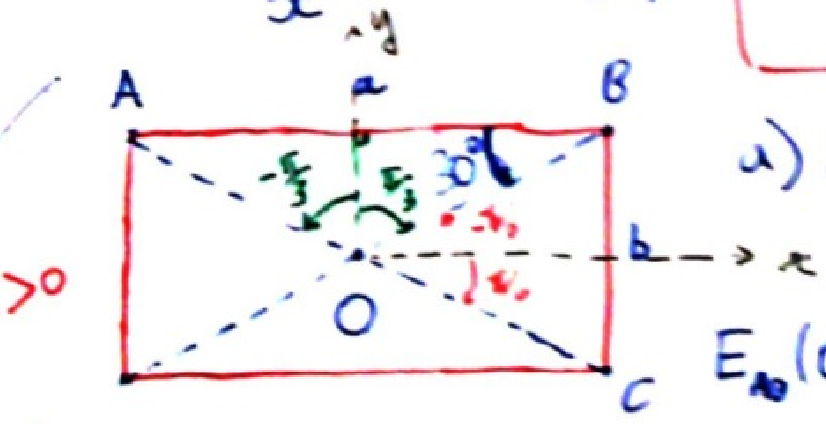
1./

$$E(M) = \int_{\text{fil } \infty} dE_x(M) = \frac{k\lambda}{x} \int_{-\pi/2}^{\pi/2} \cos\alpha \, d\alpha$$

$$E(M) = \frac{k\lambda}{x} [\sin\alpha]_{-\pi/2}^{\pi/2} \; ; \quad \boxed{\vec{E}(M) = \frac{2k\lambda}{x} \vec{e_x}}$$

2./

$\lambda > 0$



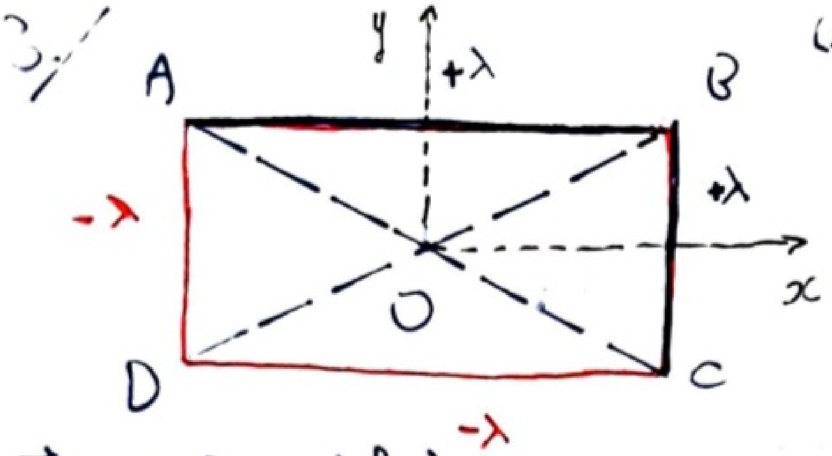
a) $$dE_{AB}(O) = \frac{k\lambda}{b/2} \cos\alpha \, d\alpha$$

$$E_{AB}(O) = \frac{2k\lambda}{b} \int_{-\pi/3}^{\pi/3} \cos\alpha \, d\alpha$$

$$\boxed{E_{AB}(O) = \frac{2k\lambda}{b} \cdot \sqrt{3}} = E_{DC}(O) \; ; \quad E_{BC}(O) = \frac{k\lambda}{a/2} \int_{-\pi/6}^{\pi/6} \cos\alpha \, d\alpha$$

$$\boxed{E_{BC}(O) = \frac{2k\lambda}{a}} = E_{AD}(O) \qquad \boxed{\vec{E}(O) = \vec{0}}$$

3/ $(\lambda > 0)$



$$\vec{E}_{AB}(O) = \frac{2k\lambda}{b} \cdot \sqrt{3}\ \vec{e_y}$$

$$\vec{E}_{BC}(O) = \frac{2k\lambda}{a}\ \vec{e_x}$$

$$\vec{E}_{DC}(O) = \frac{2k\lambda}{b} \sqrt{3} \cdot \vec{e_y}$$

$$\vec{E}_{AD}(O) = \frac{2k\lambda}{a}\ \vec{e_x}$$

$$\vec{E}(O) = \vec{E}_{AB}(O) + \vec{E}_{BC}(O) + \vec{E}_{DC}(O) + \vec{E}_{AD}(O)$$

$$\vec{E}(O) = \frac{4k\lambda}{a} \vec{e_x} + \frac{4k\lambda\sqrt{3}}{b} \vec{e_y}$$

$$\|\vec{E}(O)\| = E(O) = \sqrt{\left(\frac{4k\lambda}{a}\right)^2 + \left(\frac{4k\lambda\sqrt{3}}{b}\right)^2}$$

$$E(O) = \sqrt{\frac{16k^2\lambda^2}{a^2} + \frac{48k^2\lambda^2}{b^2}}$$

$$\boxed{E(O) = 4k\lambda\sqrt{\frac{1}{a^2} + \frac{3}{b^2}}}$$