Differential: scalar

$$f'(x) = \frac{df}{dx}$$

$$df = f'(x).dx$$

With 3 variables x,y and z:

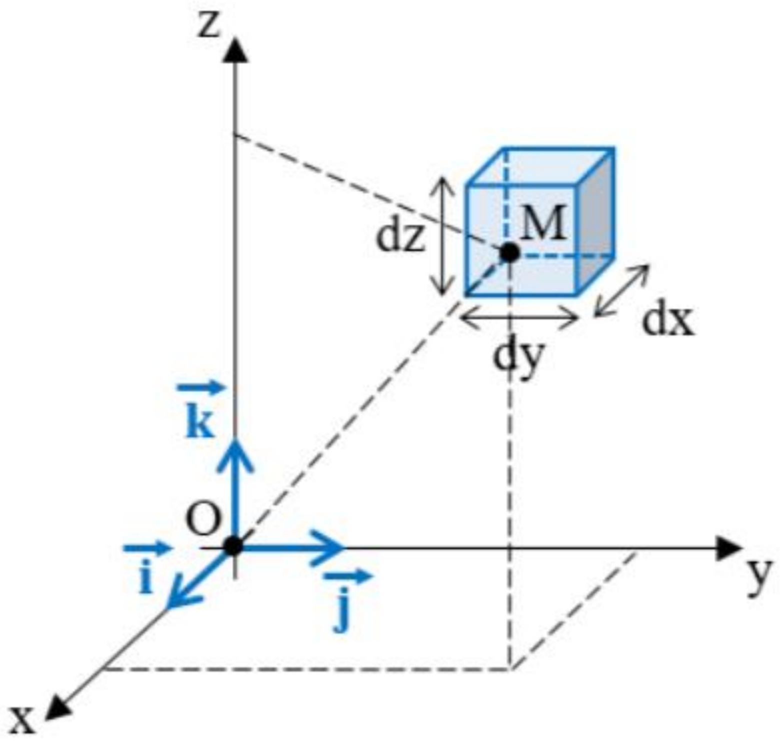
$$df(x,y,z) = \frac{\partial f}{\partial x}.dx + \frac{\partial f}{\partial y}.dy + \frac{\partial f}{\partial z}.dz$$

Gradiant: vector

Partial derivative of f with respect to x (y and z are supposed constant here in the partial derivative calculation).

In 3D cartesian frame: $\overrightarrow{grad}(f) = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix} = \frac{\partial f}{\partial x}.\vec{u}_x + \frac{\partial f}{\partial y}.\vec{u}_y + \frac{\partial f}{\partial z}.\vec{u}_z$

Cartesian coordinates



$\left(O,\vec{i},\vec{j},\vec{k}\right)$ $\left(O,\vec{u}_x,\vec{u}_y,\vec{u}_z\right)$

$\overrightarrow{OM} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$

$\vec{dl} = dx\,\vec{i} + dy\,\vec{j} + dz\,\vec{k}$

$dA = dx\,dy$ or $dA = dy\,dz$ or $dA = dx\,dz$

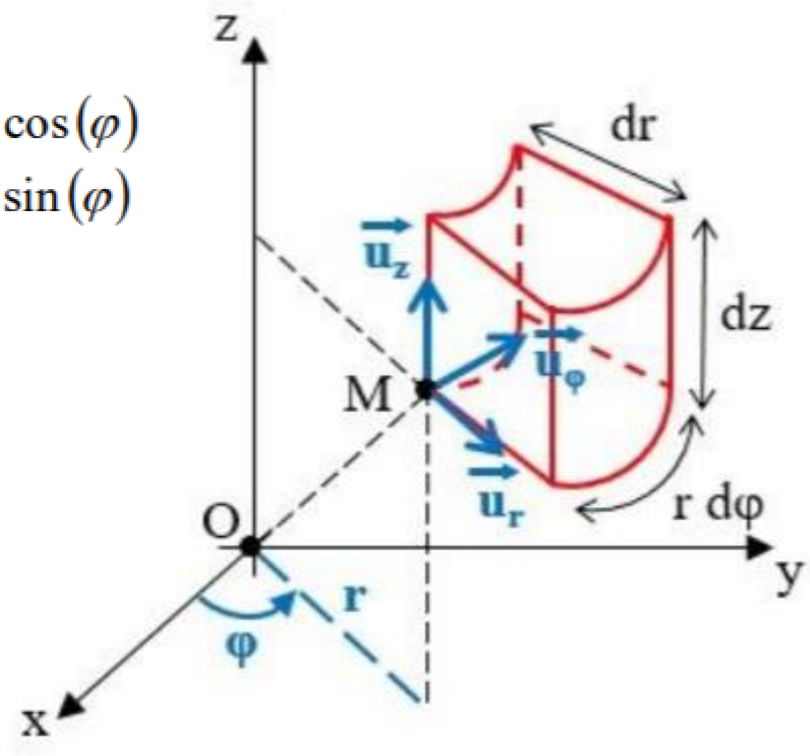
$d\tau = dx\,dy\,dz$

V(x,y,z) Gradient:

$$\vec{\nabla}V = \frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}$$

Cylindrical coordinates

$x = r\cos(\varphi)$
$y = r\sin(\varphi)$
$z = z$



$$\left(O, \vec{u}_r, \vec{u}_\varphi, \vec{u}_z\right)$$

$$\overrightarrow{OM} = r\,\vec{u}_r + z\,\vec{u}_z$$

$$\overrightarrow{dl} = dr\,\vec{u}_r + r\,d\varphi\,\vec{u}_\varphi + dz\,\vec{u}_z$$

$dA = r\,d\varphi\,dz$ or $dA = r\,dr\,d\varphi$

or $dA = dr\,dz$

$d\tau = r\,dr\,d\varphi\,dz$

V(r,φ,z) Gradient in cylindrical frame:

$$\vec{\nabla}V = \frac{\partial V}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial V}{\partial \varphi}\vec{u}_\varphi + \frac{\partial V}{\partial z}\vec{u}_z$$

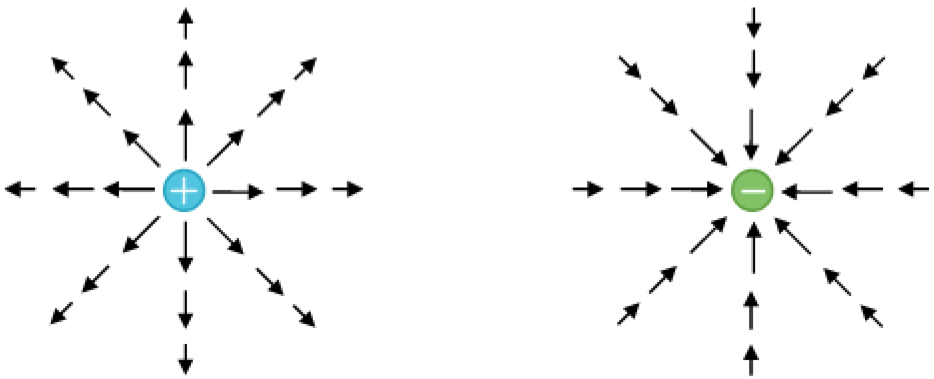
$$x = r\cos(\varphi)$$
$$y = r\sin(\varphi)$$
$$z = z$$

Coordinates	Cartesian	Polar	Cylindrical	Spherical
Variables	(x, y, z)	(r, θ)	(r, θ, z)	(r, θ, φ)
$\overrightarrow{grad}$	$\frac{\partial}{\partial x}$ $\frac{\partial}{\partial y}$ $\frac{\partial}{\partial z}$	$\frac{\partial}{\partial r}$ $\frac{1}{r}\frac{\partial}{\partial \theta}$	$\frac{\partial}{\partial r}$ $\frac{1}{r}\frac{\partial}{\partial \theta}$ $\frac{\partial}{\partial z}$	$\frac{\partial}{\partial r}$ $\frac{1}{r}\frac{\partial}{\partial \theta}$ $\frac{1}{r\sin(\theta)}\frac{\partial}{\partial \varphi}$

Electric field

Coulomb's Law: $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q\, q_i}{r^2}\, \hat{r}_i$ newtons

Electric field: $\vec{E} = \frac{\vec{F}}{q} = \frac{1}{4\pi\epsilon_0} \frac{q_i}{r^2}\, \hat{r}_i$ newtons/coulomb



$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_i \frac{\mathrm{q}_i}{r^2}\, \hat{r}_i$$

Electric potential Energy

$$U_r = \frac{qQ}{4\pi\epsilon_0}\frac{1}{r}$$

U_r represent the *electric potential energy* stored in charge q when it is distance r away from Q. The change in energy going from A to B can be written as,

$$\text{electric potential energy difference}_{AB} = U_B - U_A$$

electric potential

$$\text{electric potential } = \frac{U_r}{q}$$

$$\mathrm{V}_r = \frac{Q}{4\pi\epsilon_0}\frac{1}{r}$$