Chapter 5

ELECTROSTATICS

FORCE, FIELD,

POTENTIAL AND ELECTROSTATIC ENERGIES

CONTENTS

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I – Goals

- Study of the electrostatic interactions between charged objects.
- Establish the fundamental law of electrostatics:
    - Electric force $\vec{F}_e$ / Electric potential V
    - Electric field $\vec{E}$ / Electric potential energy: E_{pe}

- Fields of application of electrostatics:
    - Biophysics (study of molecules)
    - Nuclear physics (charged particles accelerator, LHC,...)
    - Plasma, electric arc, lightning,…

II – Charges and electrostatic phenomenons : the constituents of matter

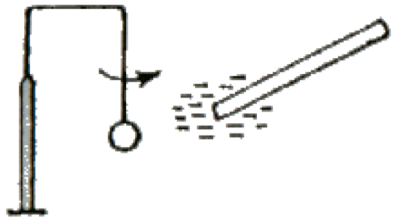
II -1. The Atom

An atom is build out of a nucleus around which turn electrons.
An atome is electrically neutral when there are as protons as electrons.
An object is electrised (+) or (-) if it leaves or gets back electrons.

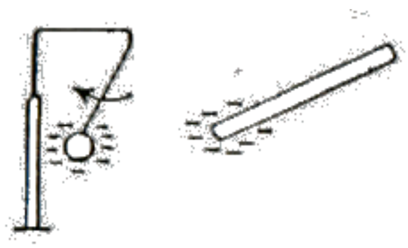
Electrons' charge : $q_{e-} = -e = -1,6.10^{-19}$C

Nucleus: formed of
- Neutrons : no charge
- Protons' charge $+e = 1,6.10^{-19}$ C

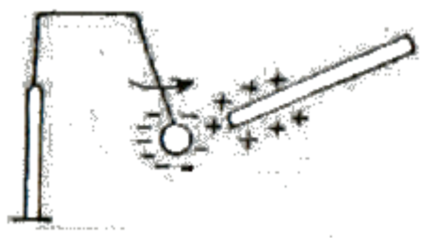
II – 2. Electrostatic phenomenons : electrisation by contact



When an ebonite rod is rubbed it gains what we know to be electrons that are negative charges. It is then able to attract a small ball of elederberry (sureau in french).
An object having an excess of electrons will try to expell them.



During the contact with the ball, some of the electrons leave the rod to get on the ball. Thus the ball is negatively charged.
And the ball is repelled from the rod. This repulsion is all the more important that the distance between the two objects is small.



The ball being negatively charged, let's approach a glass rod which will be positively charged. Unlike the previous case, the ball is attracted by this positive object.

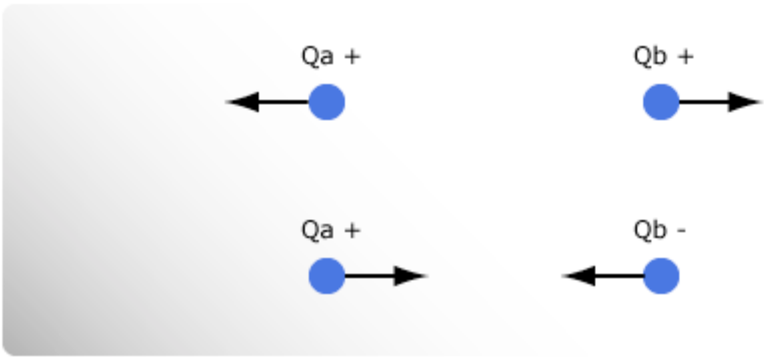
III – <u>Fundamental laws of electrostatics</u>

1. ***Electrostatic force*** **:** $\vec{F}_e$

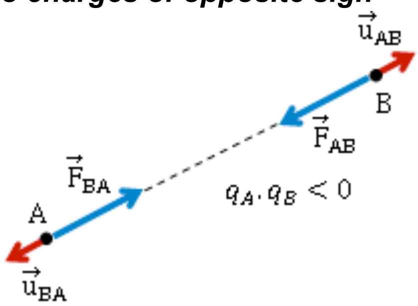
The electrostatic force $\vec{F}_e$ is acting between two charged bodies q_1 and q_2 - two point charges or two volumic objects – **<u>its expression is given by Coulomb's law</u>**.

$$F_e = k\frac{|q_1|.|q_2|}{r^2}$$

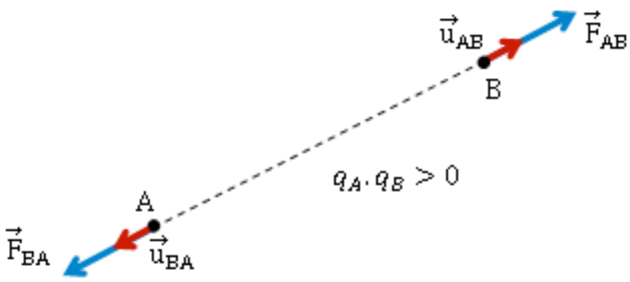
k = 9.10^9 SI Coulomb's constant and r is the distance between the two charges.



Two charges of opposite sign => Attraction



Two charges of identical sign => Repulsion



Analogy with the gravitation

$\vec{F}_G$	$\vec{F}_e$
Attractive	Attractive or repulsive
Independant of charges	Independant of masses
Inversely propotional to the square of the distance	
Negligible at atomic level Predominent at macroscopic level or astronomic	Predominent at the microscopic level (atomic ≈ 10^{-10} m)
$F_G = G\frac{m_1 . m_2}{r^2}$ **G = 6,67.10^{-11} SI gravitational constant**	$F_e = k\frac{\lvert q_1\rvert . \lvert q_2\rvert}{r^2}$ **k = 9.10^{9} SI Coulomb's constant**

2. ***Electrostatic field:*** $\vec{E}(M)$

The expression of the electrostatic field is given by analogy with the gravitational field,

a) Field created by a point charge

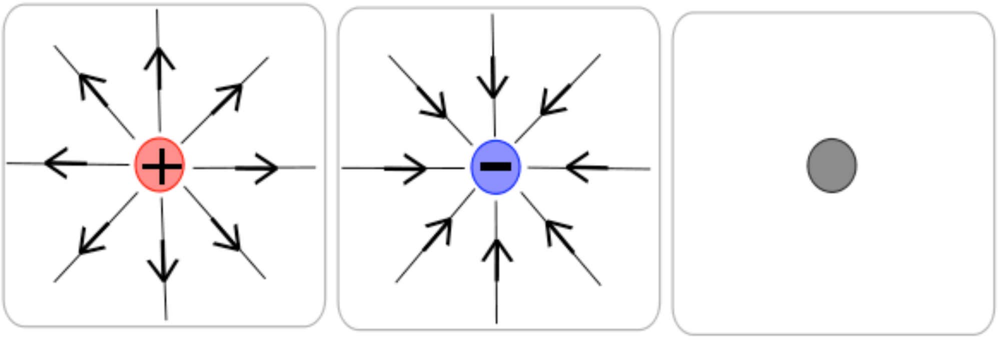
Caracteristic of the electric field at point M when produced by a charge located at point O:

- Direction: the line (OM).
- Ways: divergent if q is positive and convergent if q is negative.
- Intensity: $E(M) = k.\frac{\lvert q\rvert}{(OM)^2}$ E(M) is not defined at point O (r = 0) and tends to zero at infinity

b) Field lines

It is the geometric locus where the electric vector field is everywhere tangential.

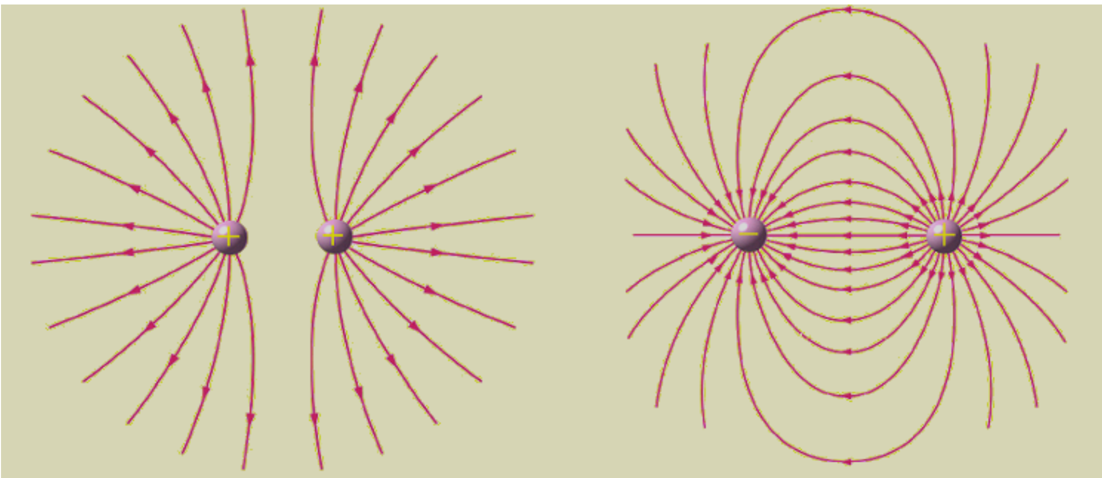
Example 1 : Field lines of a point charge



Radially divergent electric field lines	Radially convergent electric field lines	Neutral particle, no electric field lines

Example 2 : Field lines of an electric doublet

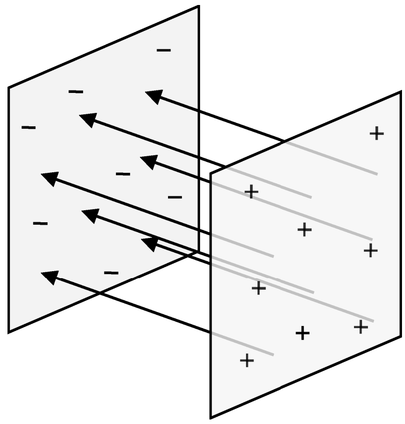
a) 2 positive charges
b) Electric dipole



(a) (b)

Example 3 : Electric field lines in a plane capacitor

The field lines are perpendicular to the plates and directed from the positive to the negative plate.



c) Electric field $\vec{E}$ created by a discrete distribution: Superposition Principle

Discrete distribution of charges = Finite number (N) of point charges.

Let's have q_1 , q_2 , q_1 , … , q_N respectively placed at points O_1 , O_2 , O_1 , … , O_N and a point M which is placed outside the distribution.

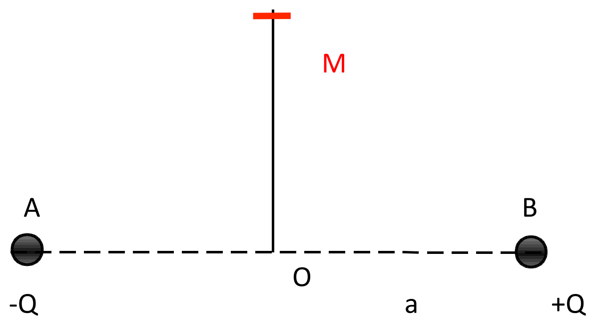
Exercise - application 1: study of an electric dipole

A dipole is a couple or doublet of two opposed charges separated by a distance *a.*

a is called the size of the dipole ≈ 10^{-10}m (atomic scale).

1- Express and represent the field at point M created by the dipole of center O.

M belongs to the perpendicular bisector of the dipole such that $AMO = \alpha = 30°$.

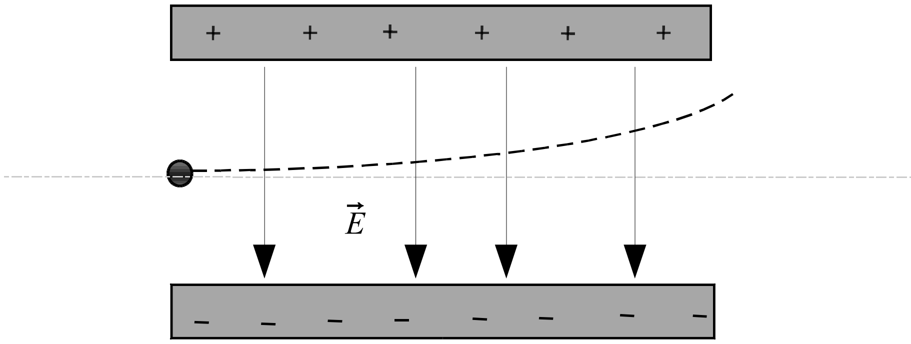


3 – Relationship between the force $\vec{F}_e$ and the electric field $\vec{E}$ at a given point

Let's consider a discrete distribution of N point charges q_1, q_2, q_3, … , q_N (N finite) and an observation point M which is external to the distribution of charges.

Exercise 2

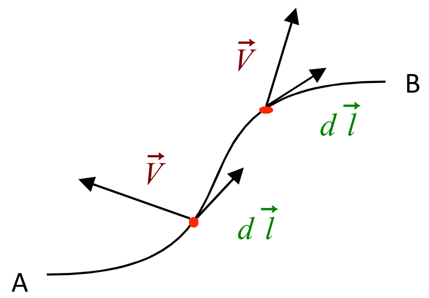
An {e^- } is thrown between two plates of a capacitor in which the electrostatic field is uniformed.



4- Electrostatic potential V(M)

a) Mathematical notion of the circulation of a vector.

For any vector $\vec{V}$ the circulation of $\vec{V}$ along the path AB is: $C(\vec{V})_{A\rightarrow B}=\int_A^B \vec{V}.d\vec{l}$



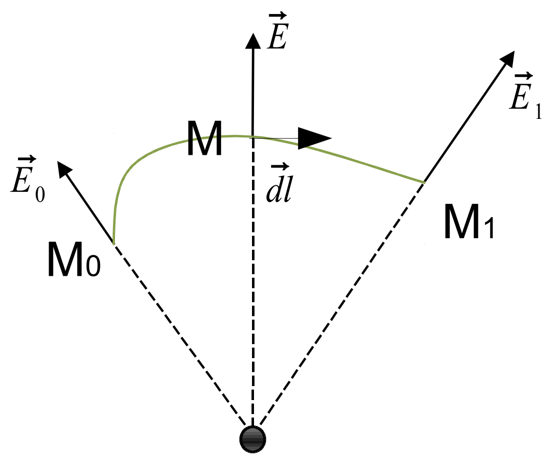
b - Circulation of a field $\vec{V}$ created by a charge q.

A charge q > 0 is positionned in O, it creates at any point M of the space, the field:

$$E=k\frac{|q|}{r^2}$$

Consider two points M_0 and M_1 it is shown that the circulation of the vector field $\vec{E}$ from M_0 toward M_1 is:

$$C(\vec{E})=k\frac{q}{r_0}-k\frac{q}{r_1}=V(M_0)-V(M_1)$$



c – Potential created by N charges (N finite)

$$V(M)=\sum_{i=1}^{N}\frac{kq_i}{r_i}$$

Notice that:

- the unit of V(M) is Volts : V
- V(M) is a scalar function (not a vectorial one)
- The charges q_i are algebraic values.

5 – *Relationship between the electric field $\vec{E}$ and the potential V(M)*

The *E*- field is related to the scalar potential V(M) by the relation: $\vec{E} = -\vec{grad}(V)$

That expression permits the calculus of the components of $\vec{E}$ by applying the vectorial operator « *gradient* » noted « $\vec{grad}$ ». Applying this operator to the scalar potential V(M) means that you do the derivation of this function wrt the 3 space-variables.

$$\vec{E}_i = 5 + \vec{AB} - \vec{grad}\{\frac{\partial V_i}{\partial t}\}$$
$$\nabla(q\vec{v} \wedge \vec{B})$$

a) Components of the $\vec{grad}$ operator in different systems of coordinates.

Coordinates	Cartesian	Polar	Cylindrical	Spherical
Variables	(x, y, z)	(r, θ)	(r, θ, z)	(r, θ, φ)
$\vec{grad}$	$\frac{\partial}{\partial x}$ $\frac{\partial}{\partial y}$ $\frac{\partial}{\partial z}$	$\frac{\partial}{\partial r}$ $\frac{1}{r}\frac{\partial}{\partial \theta}$	$\frac{\partial}{\partial r}$ $\frac{1}{r}\frac{\partial}{\partial \theta}$ $\frac{\partial}{\partial z}$	$\frac{\partial}{\partial r}$ $\frac{1}{r}\frac{\partial}{\partial \theta}$ $\frac{1}{r\sin(\theta)}\frac{\partial}{\partial \varphi}$

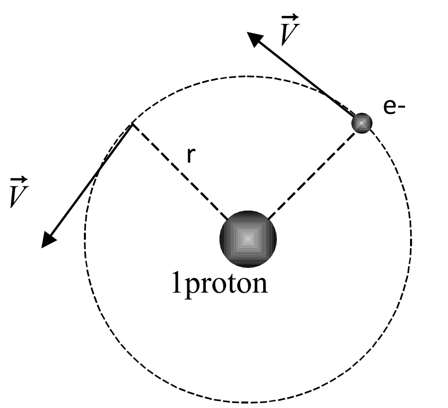
V – *Potential electrostatic energy*

A point charge q positionned at point M where the electric potential is V(M) has a potential electric energy given by: $E_{pe} = q.V(M)$

Proof_

Application to atomic physics (Atome of hydrogen).

Potential electric energy of the electron in circular motion around the nucleus.



IV – Continuous Distribution

1- Calculus of the charge Q of a continuous distribution

Nature of the distribution	Lineic	Surfacic	Volumic
Density	λ (in C/m)	σ (in C/m²)	ρ (in C/m³)
dQ	$\lambda.dl$	$\sigma.dS$	$\rho.d\tau$
Q	$\int_L \lambda dl$	$\iint_S \sigma\, dS$	$\iiint_V \rho\, d\tau$

1- Calculus of the $\vec{E}$ field created by continuous distributions of charges

	Discrete Distribution	Continuous Distribution
Elementary Charge	q_i	dQ
Elementary Field	$E_i = k\frac{q_i}{r_i^2}$	$dE = \frac{kdQ}{r^2}$
Resulting field (or total)	$\vec{E} = \sum_i \vec{E}_i$	Lineic charge distribution $\vec{E} = \int_L d\vec{E}$ Surfacic charge distribution $\vec{E} = \iint_S d\vec{E}$ Volumic charge distribution $\vec{E} = \iiint_\tau d\vec{E}$

Application

<u>Infinite wire</u>

Let's us consider an infinite charged wire with a constant and positive density λ . Retrieve E(M) :