INFO spe S3
Physics

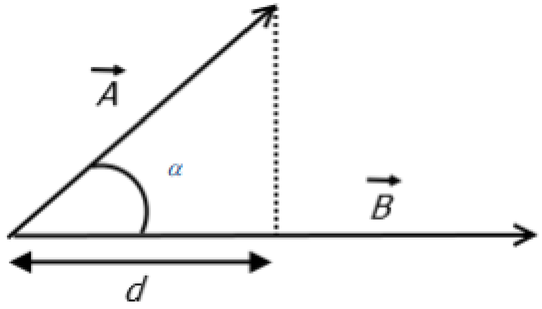
EPITA ENG

Teacher: Inès Mtir

Mathematical and vectors properties reminder

$$\vec{A}.\vec{B} = \|\vec{A}\|.\|\vec{B}\|.\cos(\alpha)$$

$$\vec{A}.\vec{B} = \|\vec{A}\|.\|\vec{B}\|.\cos(\alpha) = d.\|\vec{B}\|$$



$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

$$(\vec{A}+\vec{B}).\vec{C} = \vec{A}.\vec{C} + \vec{B}.\vec{C}$$

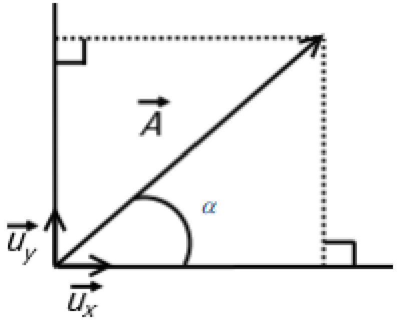
$$\vec{A}.\vec{A} = \|\vec{A}\|.\|\vec{A}\| = \|\vec{A}\|^2 \qquad \sqrt{\vec{A}.\vec{A}} = \|\vec{A}\|$$

$$(\vec{A}+\vec{B}).(\vec{A}+\vec{B}) = \|\vec{A}+\vec{B}\|^2 = \vec{A}.\vec{A} + \vec{B}.\vec{B} + 2\vec{A}.\vec{B}$$

$$\|\vec{A}+\vec{B}\| = \sqrt{\|\vec{A}\|^2 + \|\vec{B}\|^2 + 2\|\vec{A}\|\|\vec{B}\|.\cos(\vec{A},\vec{B})}$$

$$\vec{A} = A_x\vec{u_x} + A_y\vec{u_y}$$

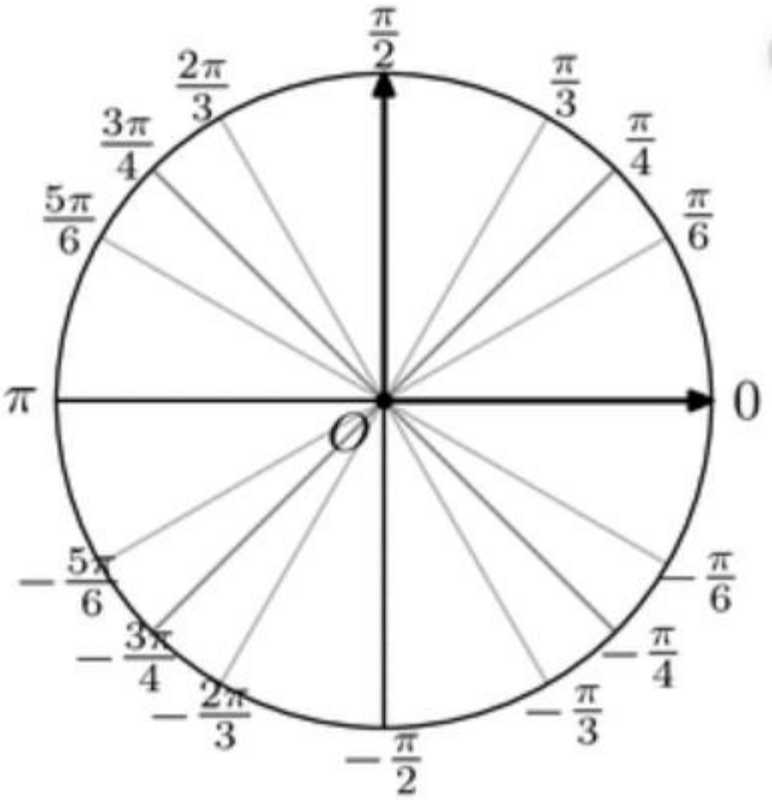
with $A_x = \vec{A}.\vec{u_x} = \|\vec{A}\|\cos(\alpha)$ et $A_y = \vec{A}.\vec{u_y} = \|\vec{A}\|\sin(\alpha)$



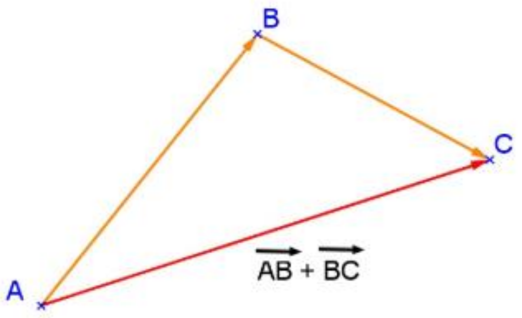
Scalar product: $\vec{A}.\vec{B} = x_A x_B + y_A y_B + z_A z_B$

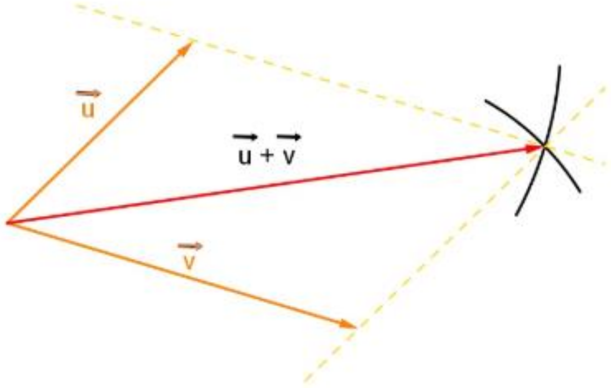
Magnitude of a vector: $\|\vec{A}\| = \sqrt{x_A{}^2 + y_A{}^2 + z_A{}^2}$ with $\vec{A},\vec{B}$ (x_A, y_A, z_A) (x_B, y_B, z_B)

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	2π
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	1
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	0
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Non définie	0	0

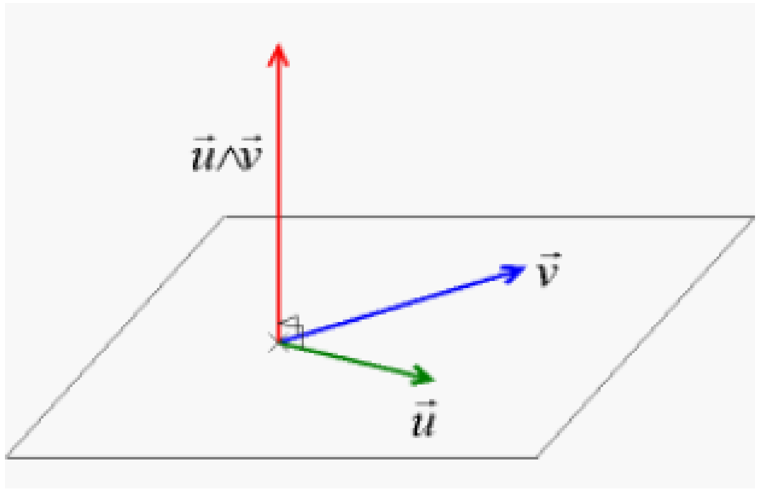


Vectorial addition





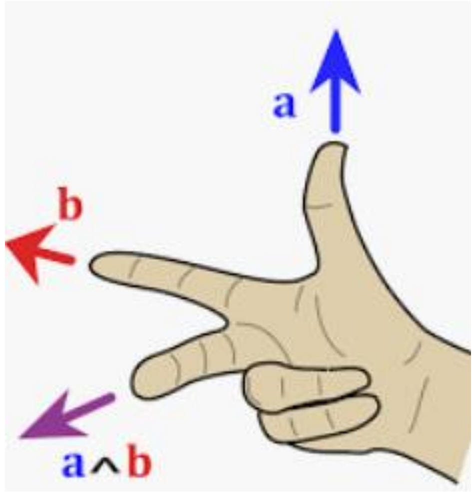
Vectorial product $\vec{W} = \vec{U} \wedge \vec{V}$



$$\vec{U} \wedge \vec{V} = \begin{bmatrix} U_x \\ U_y \\ U_z \end{bmatrix} \wedge \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} U_y V_z - U_z V_y \\ U_z V_x - U_x V_z \\ U_x V_y - U_y V_x \end{bmatrix}$$

$$\left\|\vec{W}\right\| = \left\|\vec{U}\right\| \left\|\vec{V}\right\| \left|\sin\left(\vec{U}, \vec{V}\right)\right|$$

$$\vec{W} \perp \vec{U} \quad \vec{W} \perp \vec{V}$$



Right-handed trihedron for the vectorial product direction

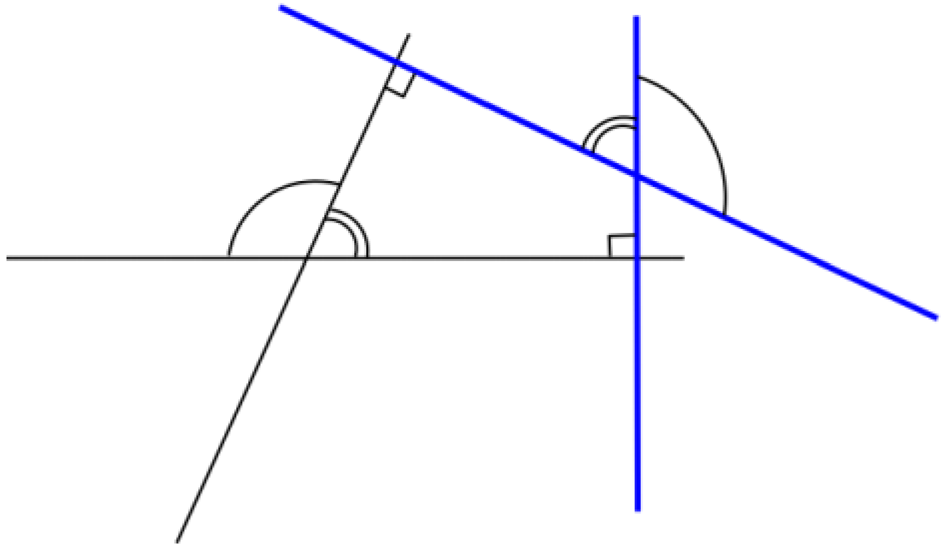
$$\vec{v_0} = v_{0x}\,\vec{i} + v_{0y}\,\vec{j}$$

$$v_0 \begin{pmatrix} v_{0x} \\ v_{0y} \end{pmatrix}$$

+ $v_{0x} = \|\vec{V_0}\| \,.\, \cos\alpha$

$v_{0y} = \|\vec{v_0}\| \,.\, \sin\alpha$

Useful …



Differential: scalar

$$f'(x) = \frac{df}{dx}$$

$$df = f'(x).dx$$

With 3 variables x,y and z:

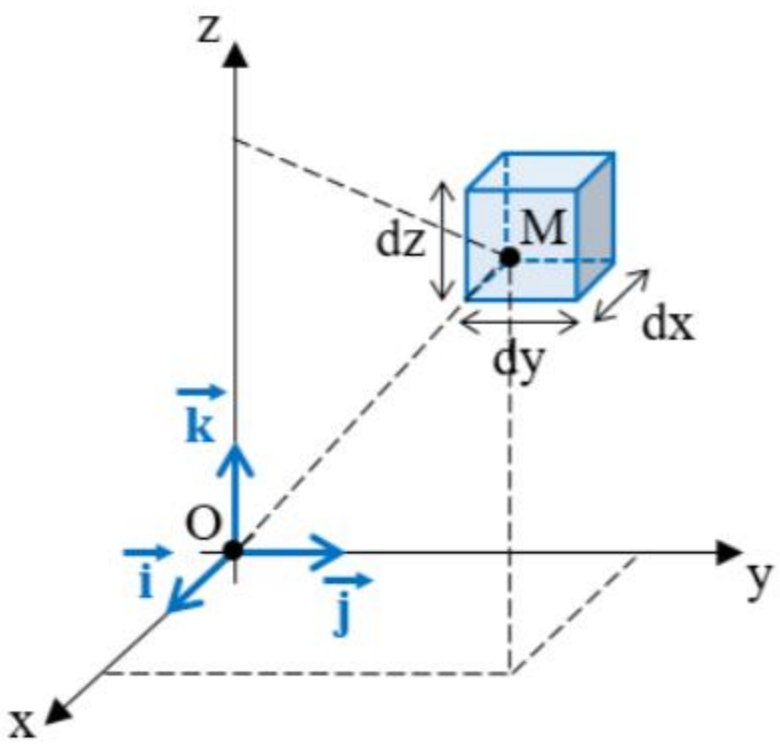
$$df(x,y,z) = \frac{\partial f}{\partial x}.dx + \frac{\partial f}{\partial y}.dy + \frac{\partial f}{\partial z}.dz$$

Gradiant: vector

Partial derivative of f with respect to x (y and z are supposed constant here in the partial derivative calculation).

In 3D cartesian frame: $$\overrightarrow{grad}(f) = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix} = \frac{\partial f}{\partial x}.\vec{u}_x + \frac{\partial f}{\partial y}.\vec{u}_y + \frac{\partial f}{\partial z}.\vec{u}_z$$

Cartesian coordinates



$\left(O, \vec{i}, \vec{j}, \vec{k}\right)$ $\left(O, \vec{u}_x, \vec{u}_y, \vec{u}_z\right)$

$\overrightarrow{OM} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$

$\vec{dl} = dx\,\vec{i} + dy\,\vec{j} + dz\,\vec{k}$

$dA = dx\,dy$ or $dA = dy\,dz$ or $dA = dx\,dz$

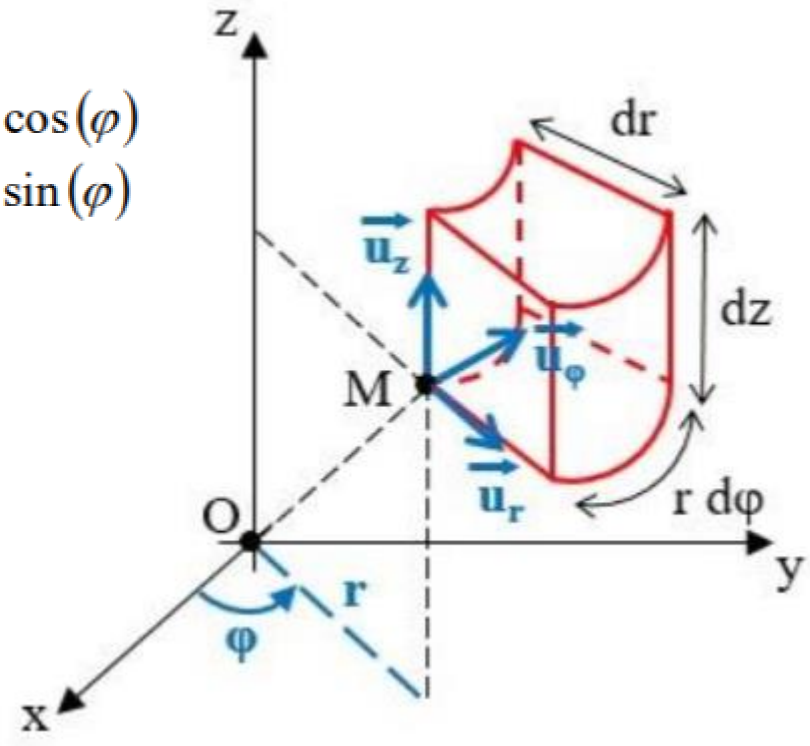
$d\tau = dx\,dy\,dz$

V(x,y,z) Gradient:

$$\vec{\nabla}V = \frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}$$

Cylindrical coordinates

$$x = r\cos(\varphi)$$
$$y = r\sin(\varphi)$$
$$z = z$$



$$\left(O, \vec{u}_r, \vec{u}_\varphi, \vec{u}_z\right)$$

$$\overrightarrow{OM} = r\,\vec{u}_r + z\,\vec{u}_z$$

$$\vec{dl} = dr\,\vec{u}_r + r\,d\varphi\,\vec{u}_\varphi + dz\,\vec{u}_z$$

$dA = r\,d\varphi\,dz$ or $dA = r\,dr\,d\varphi$

or $dA = dr\,dz$

$$d\tau = r\,dr\,d\varphi\,dz$$

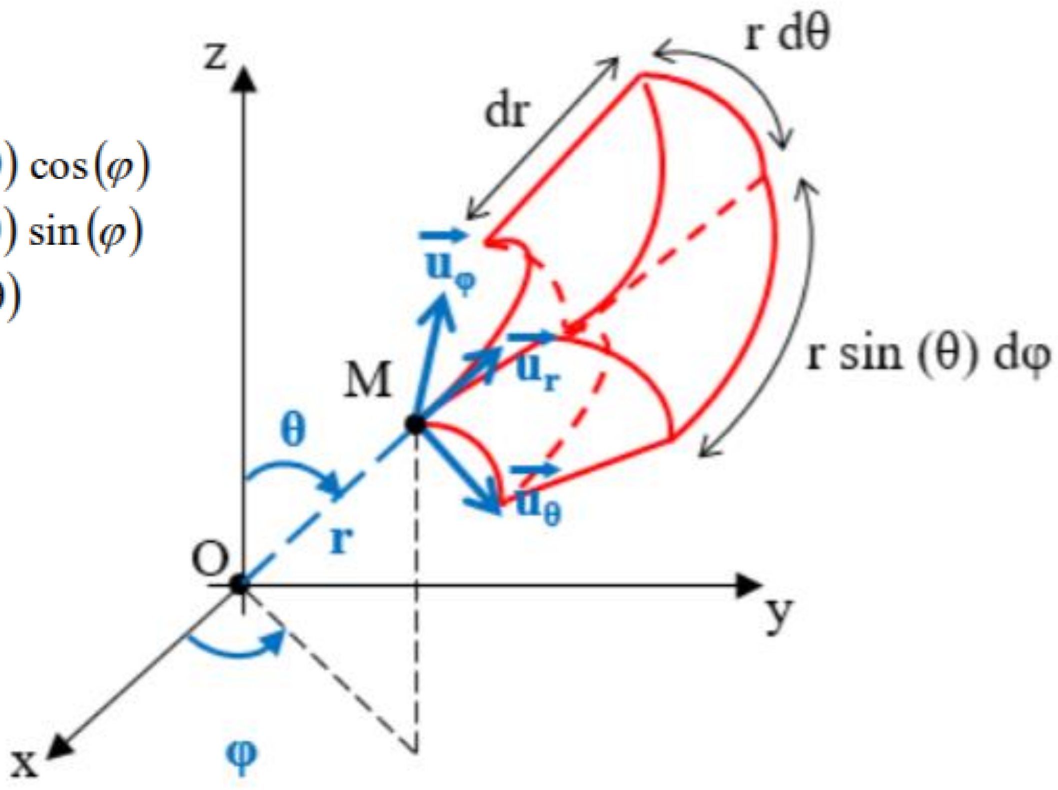
V(r,φ,z) Gradient in cylindrical frame:

$$\vec{\nabla} V = \frac{\partial V}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial V}{\partial \varphi}\vec{u}_\varphi + \frac{\partial V}{\partial z}\vec{u}_z$$

$$x = r\cos(\varphi)$$
$$y = r\sin(\varphi)$$
$$z = z$$

Spherical coordinates

$$x = r\sin(\theta)\cos(\varphi)$$
$$y = r\sin(\theta)\sin(\varphi)$$
$$z = r\cos(\theta)$$



$$(O, \vec{u}_r, \vec{u}_\theta, \vec{u}_\varphi)$$

$$\overrightarrow{OM} = r\,\vec{u}_r$$

$$\vec{dl} = dr\,\vec{u}_r + r\,d\theta\,\vec{u}_\theta + r\sin(\theta)d\varphi\,\vec{u}_\varphi$$

$dA = r^2\sin(\theta)d\theta\,d\varphi$ or $dA = \sin(\theta)r\,dr\,d\varphi$

or $dA = r\,dr\,d\theta$

$$d\tau = r^2\,dr\sin(\theta)d\theta\,d\varphi$$

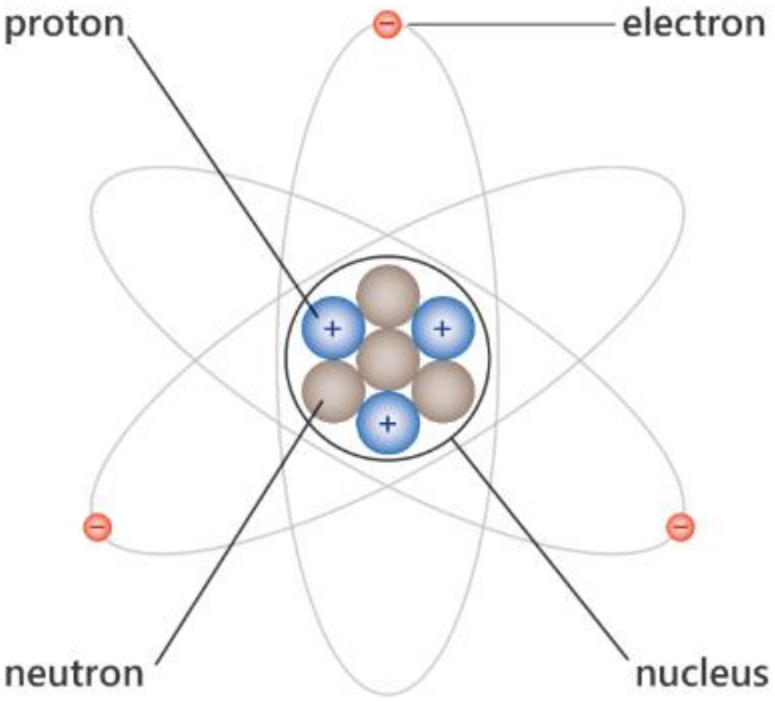
V(r,θ,φ) Gradient in spherical frame:

$$\vec{\nabla} V = \frac{\partial V}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{u}_\theta + \frac{1}{r\sin(\theta)}\frac{\partial V}{\partial \varphi}\vec{u}_\varphi$$

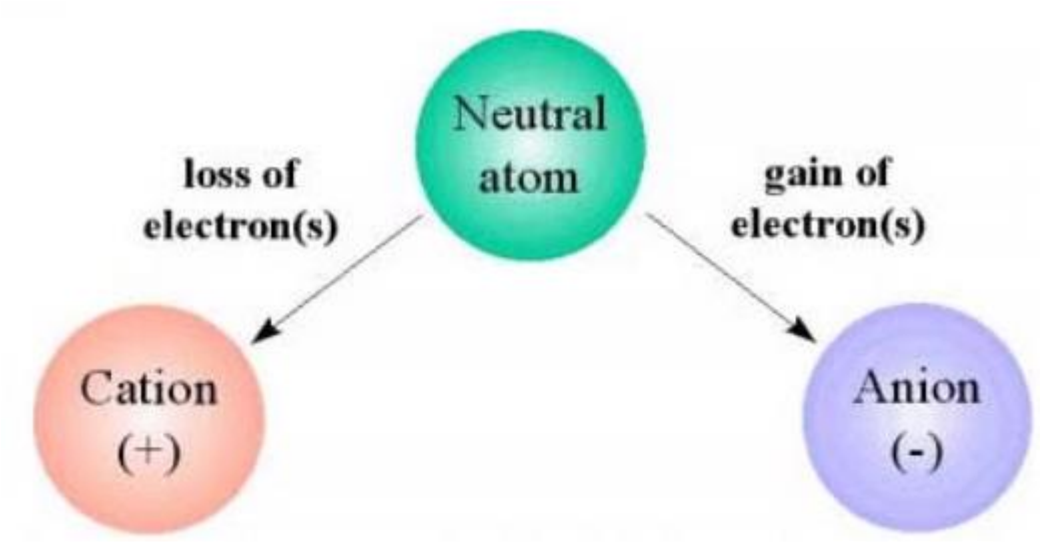
$$x = r\sin(\theta)\cos(\varphi)$$
$$y = r\sin(\theta)\sin(\varphi)$$
$$z = r\cos(\theta)$$

Coordinates	Cartesian	Polar	Cylindrical	Spherical
Variables	(x, y, z)	(r, θ)	(r, θ, z)	(r, θ, φ)
$\overrightarrow{grad}$	$\frac{\partial}{\partial x}$ $\frac{\partial}{\partial y}$ $\frac{\partial}{\partial z}$	$\frac{\partial}{\partial r}$ $\frac{1}{r}\frac{\partial}{\partial \theta}$	$\frac{\partial}{\partial r}$ $\frac{1}{r}\frac{\partial}{\partial \theta}$ $\frac{\partial}{\partial z}$	$\frac{\partial}{\partial r}$ $\frac{1}{r}\frac{\partial}{\partial \theta}$ $\frac{1}{r\sin(\theta)}\frac{\partial}{\partial \varphi}$

CHAPTER I: ELECTRODTATICS



Structure of an atom.



Positive charge

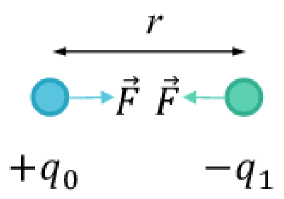
q+

Negative charge

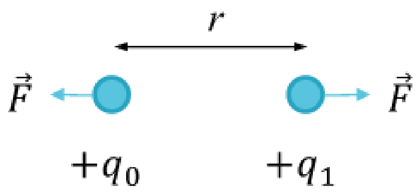
q-

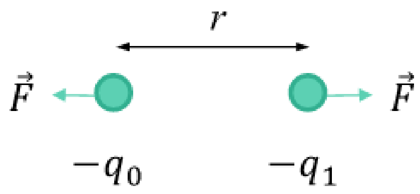
Electric force

Unlike charges attract,



Like charges repel,





Coulomb’s law

$$\vec{F} = K \frac{q_0 \, q_1}{r^2} \, \hat{r}$$

Where

- $\vec{F}$ is the electric force, directed on a line between the two charged bodies.
- K is a constant of proportionality that relates the left side of the equation (newtons) to the right side (coulombs and meters). It is needed to make the answer come out right when we do a real experiment.
- q_0 and q_1 represent the amount of charge on each body, in units of *coulombs* (the SI unit for charge).
- r is the distance between the charged bodies.
- $\hat{r}$ is a variable unit vector that reminds us the force points along the line between the two charges. If the charges are alike, the force is repulsive; if the charges are unlike, the force is attractive.

K, the constant of proportionality

$$K = \frac{1}{4\pi\epsilon_0}$$

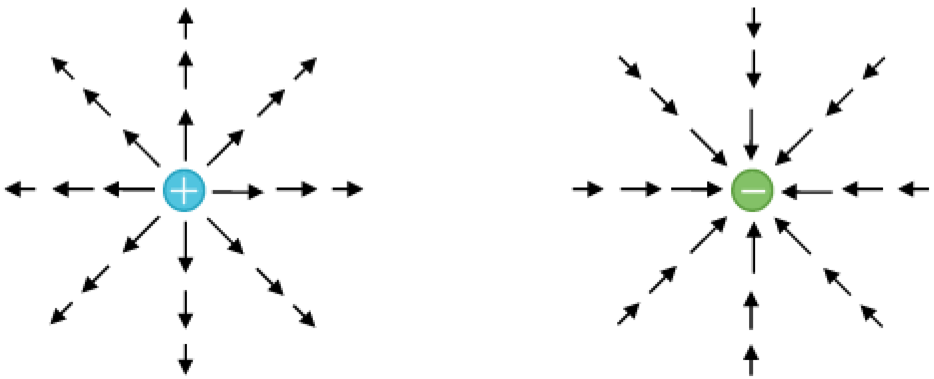
$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_0\, q_1}{r^2}\, \hat{r}$$

Permittivity of free space $\epsilon_0 = 8.854187817 \times 10^{-12}$ coulomb^2/ newton-meter^2

Electric field

Coulomb's Law: $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q\, q_i}{r^2}\, \hat{r}_i$ newtons

Electric field: $\vec{E} = \frac{\vec{F}}{q} = \frac{1}{4\pi\epsilon_0} \frac{q_i}{r^2}\, \hat{r}_i$ newtons/coulomb



$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_i \frac{\mathrm{q}_i}{r^2}\, \hat{r}_i$$

Electric potential Energy

$$U_r = \frac{q\,Q}{4\pi\epsilon_0}\frac{1}{r}$$

U_r represent the *electric potential energy* stored in charge q when it is distance r away from Q. The change in energy going from A to B can be written as,

$$\text{electric potential energy difference}_{AB} = U_B - U_A$$

electric potential

$$\text{electric potential } = \frac{U_r}{q}$$

$$\mathrm{V}_r = \frac{Q}{4\pi\epsilon_0}\frac{1}{r}$$

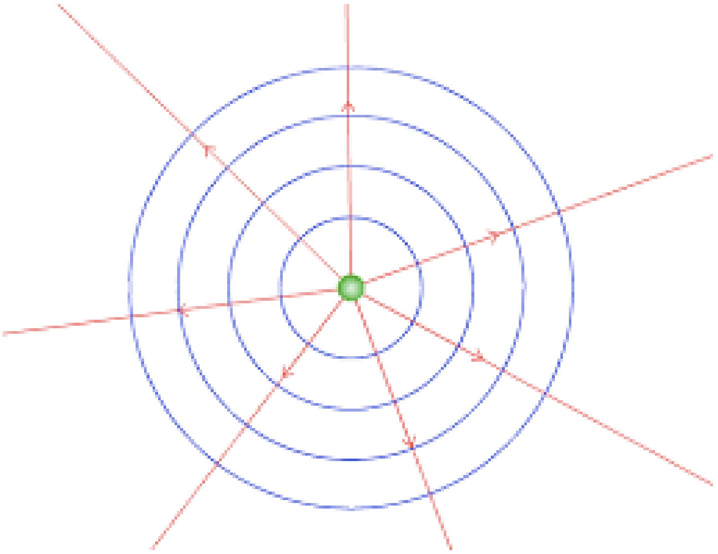
Electrical potential

$$V = k\frac{q}{r}$$

$$V(M) = k\sum_i \frac{q_i}{r_i}$$

For countinuous distribution:

$$V(M) = k\int \frac{dq}{r}$$



Blue: equipotential lines
Red: electric field lines

Electrical potential enregy

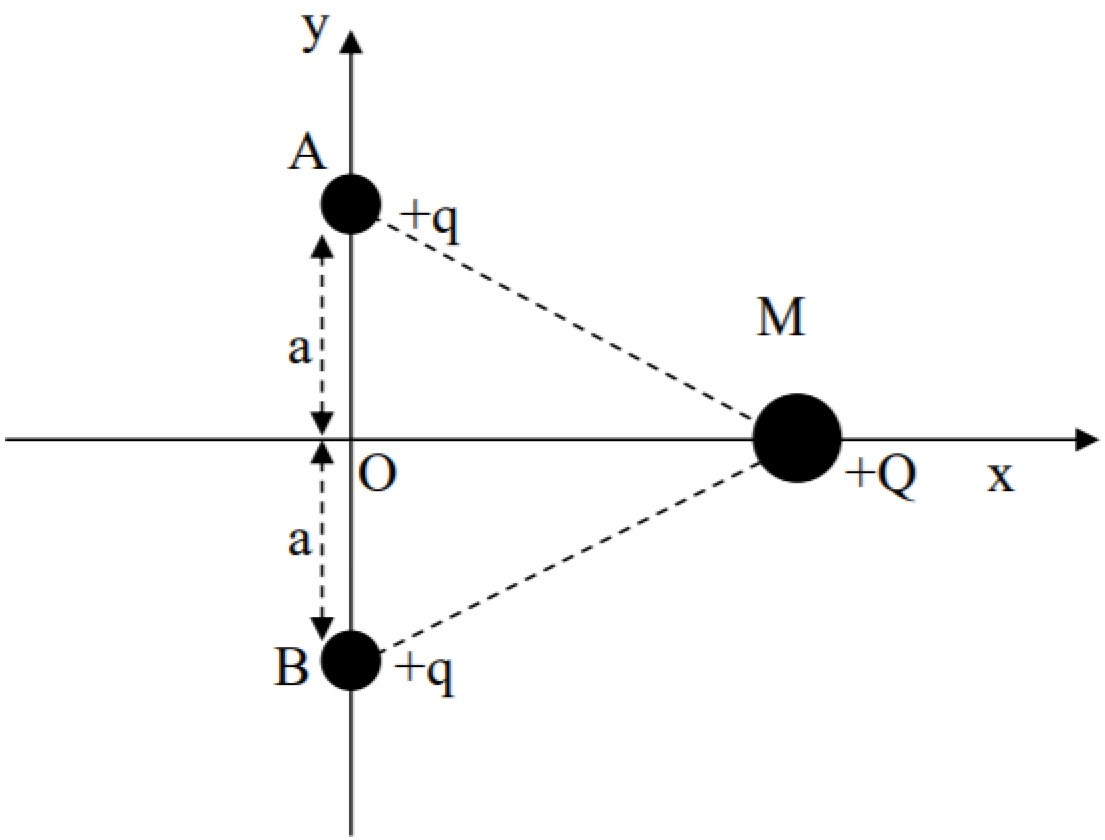
$$U_E = QV \qquad [J] = [C] \times [V]$$

Q the charge at the point that recive the potential V from the other sources qi

Tutorial 1

Exercise 2

Consider two pointlike charges, both with charge $q = 2\ \mu C$. They are located at points A and B along the y-axis separated from point O by a distance $a = 3$ cm.
A charge $+Q = 4\ \mu C$ is at point M on x-axis such that $OM = x$.

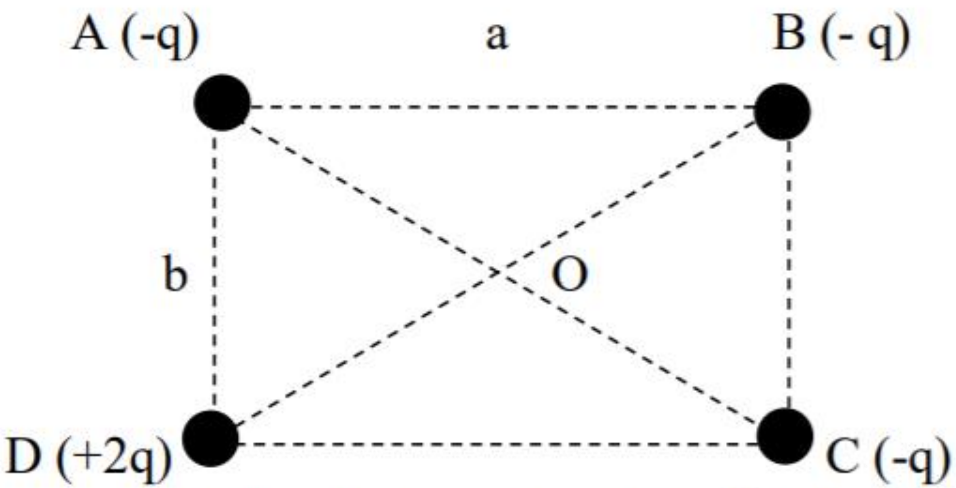


1- Sketch the net force $\vec{F}_e$ of all the electrostatic forces acting on the charge Q at point M.

2- Determine as a function of x the intensity of $\vec{F}_e$.

Exercise 3

Four point-like charges (with $q > 0$) are located at points A, B, C and D. These points are corners of a rectangle of length a, width b and center O such that angle (ABO) = 30°.



1- Express the norms of the electric vector fields $\vec{E}_A(B)$, $\vec{E}_D(B)$ and $\vec{E}_C(B)$ which are generated at B respectively by charges q_A, q_D et q_C. Write their expression in terms of k, q and a. Draw them.

2- Write the norm of the total electrostatic field $\vec{E}(B)$ which is created at B as function of k, q and a. Sketch $\vec{E}(B)$.

3- Write the norm of the total electrostatic field $\vec{E}(O)$ which is created at O as function of k, q and a. Sketch $\vec{E}(O)$.

4- Compute the electrostatic potential V(O) created at O in terms of k, q and a.