Assignment 2

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Download latex-tikz codes from

https://github.com/KBVijayVarma/AI1103–Assignment–2

1 Problem

(GATE EC, Q. 12)
P and Q are considering to apply for a job. The probability that P applies for the job is $\frac{1}{4}$, the probability that P applies for the job given that Q applies for the job is $\frac{1}{2}$, and the probability that Q applies for the job given that P applies for the job is $\frac{1}{3}$. Then the probability that P does not apply for the job given that Q does not apply for the job is
(A) $\frac{4}{5}$ (B) $\frac{5}{6}$ (C) $\frac{7}{8}$ (D) $\frac{11}{12}$

2 Solution

Let A represent the event that P applies for the job. Let B represent the event that Q applies for the job. According to the given information in the question,

$$\Pr(A) = \frac{1}{4} \tag{2.0.1}$$

$$\Pr(A|B) = \frac{1}{2} \tag{2.0.2}$$

$$\Pr(B|A) = \frac{1}{3} \tag{2.0.3}$$

According to the definition of Conditional Probability,

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} \tag{2.0.4}$$

$$\Pr(B|A) = \frac{\Pr(AB)}{\Pr(A)} \tag{2.0.5}$$

On substituting the values of $\Pr(A)$, $\Pr(B|A)$ in (2.0.5), we get

$$\frac{1}{3} = \frac{\Pr(AB)}{\frac{1}{4}} \tag{2.0.6}$$

$$\Pr(AB) = \left(\frac{1}{3}\right)\left(\frac{1}{4}\right) \tag{2.0.7}$$

$$\therefore \Pr(AB) = \frac{1}{12} \tag{2.0.8}$$

Now substituting the values of $\Pr(A|B)$, $\Pr(AB)$ in (2.0.4), we get

$$\frac{1}{2} = \frac{\frac{1}{12}}{\Pr(B)} \tag{2.0.9}$$

$$\Pr(B) = \frac{\left(\frac{1}{12}\right)}{\left(\frac{1}{2}\right)} \tag{2.0.10}$$

$$\therefore \Pr(B) = \frac{1}{6} \tag{2.0.11}$$

The probability that P does not apply for the job given that Q does not apply for the job is given by $\Pr\left(A'|B'\right)$.
Now,

$$A'B' = (A+B)' \tag{2.0.12}$$

$$\Rightarrow \Pr\left(A'B'\right) = \Pr\left((A+B)'\right) \tag{2.0.13}$$

$$\therefore \Pr\left(A'B'\right) = 1 - \Pr(A+B) \tag{2.0.14}$$

As we know that,

$$\Pr(A+B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{2.0.15}$$

By substituting the values of $\Pr(A)$, $\Pr(B)$, $\Pr(AB)$ in (2.0.15), we get

$$\Pr(A+B) = \frac{1}{4} + \frac{1}{6} - \frac{1}{12} \tag{2.0.16}$$

$$= \frac{3+2-1}{12} \tag{2.0.17}$$

$$\therefore \Pr(A+B) = \frac{1}{3} \tag{2.0.18}$$

$\therefore$ Required Probability is

$$\Pr\left(A'|B'\right) = \frac{\Pr\left(A'B'\right)}{\Pr(B')} \tag{2.0.19}$$

By substituting the values of $\Pr(B)$, $\Pr\left(A'B'\right)$ [from

(2.0.14)] in (2.0.19), we get

$$\Pr\left(A^{'}|B^{'}\right) = \frac{1-\Pr\left(A+B\right)}{1-\Pr\left(B\right)} \tag{2.0.20}$$

$$\Pr\left(A^{'}|B^{'}\right) = \frac{1-\left(\frac{1}{3}\right)}{1-\left(\frac{1}{6}\right)} \tag{2.0.21}$$

$$\Rightarrow \Pr\left(A^{'}|B^{'}\right) = \frac{\left(\frac{2}{3}\right)}{\left(\frac{5}{6}\right)} = \frac{4}{5} \tag{2.0.22}$$

$$\tag{2.0.23}$$

$\therefore \Pr\left(A^{'}|B^{'}\right) = \frac{4}{5} = 0.8$

Hence, the probability that P does not apply for the job given that Q does not apply for the job is equal to $\frac{4}{5}$

$\therefore$ The correct option is (A) $\frac{4}{5}$.