Assignment 4

Vijay Varma - AI20BTECH11012

Download latex-tikz codes from

https://github.com/KBVijayVarma/AI1103-Assignment-3

Problem GATE 2017 (CS-SET 2), Q.60

There are 3 red socks, 4 green socks and 3 blue socks. You choose 2 socks. The probability that they are of the same colour is

1) $\frac{1}{5}$ 2) $\frac{7}{30}$ 3) $\frac{1}{4}$ 4) $\frac{4}{15}$

Solution

Let $X_1 \in \{0, 1, 2\}$ and $X_2 \in \{0, 1, 2\}$ be two Random Variables representing the colour of socks taken in 1^{st} draw and in 2^{nd} draw respectively.
$X_1 = 0$, $X_1 = 1$, $X_1 = 2$ represent choosing Red, Green, Blue socks in the first draw respectively.
Similarly, $X_2 = 0$, $X_2 = 1$, $X_2 = 2$ represent choosing Red, Green, Blue socks in the second draw respectively.
Now, the probability that the socks drawn in 1^{st} draw and 2^{nd} draw are of the same colour is given by

$$\Pr(X_1 = X_2)$$

Now,

$$\Pr(X_1 = X_2) = \sum_{k=0}^{k=2} \Pr(X_1 = X_2 = k) \tag{0.0.1}$$

$$= \sum_{k=0}^{k=2} \Pr(X_1 = k, X_2 = k) \tag{0.0.2}$$

$$= \sum_{k=0}^{k=2} \Pr(X_1 = k) \Pr((X_2 = k)|(X_1 = k)) \tag{0.0.3}$$

$$\begin{aligned} &= \Pr(X_1 = 0) \Pr((X_2 = 0)|(X_1 = 0)) \\ &\quad + \Pr(X_1 = 1) \Pr((X_2 = 1)|(X_1 = 1)) \\ &\quad + \Pr(X_1 = 2) \Pr((X_2 = 2)|(X_1 = 2)) \end{aligned} \tag{0.0.4}$$

From the given information in the question,

$$\Pr(X_1 = X_2) = \left(\frac{3}{10}\right)\left(\frac{2}{9}\right) + \left(\frac{4}{10}\right)\left(\frac{3}{9}\right) + \left(\frac{3}{10}\right)\left(\frac{2}{9}\right) \tag{0.0.5}$$

$$= \left(\frac{6}{90}\right) + \left(\frac{12}{90}\right) + \left(\frac{6}{90}\right) \tag{0.0.6}$$

$$= \frac{24}{90} = \frac{4}{15} \tag{0.0.7}$$

Therefore, the probability that the two socks are of same colour is $\frac{4}{15}$.
Hence, the correct option is 4) $\frac{4}{15}$.