Assignment 1

Vijay Varma - AI20BTECH11012

Download latex-tikz codes from

https://github.com/KBVijayVarma/EE3900/tree/main/Assignment_1

Download python code from

https://github.com/KBVijayVarma/EE3900/tree/main/Assignment_1

Problem (Ramsey-1.1 Points-Q.8)

Prove that the points $\begin{pmatrix}-1\\0\end{pmatrix}, \begin{pmatrix}0\\3\end{pmatrix}, \begin{pmatrix}3\\2\end{pmatrix}$ and $\begin{pmatrix}2\\-1\end{pmatrix}$ are the vertices of a square.

Solution

Let us first prove that the given points form a **Rectangle** (using all sides are perpendicular). And then we will prove that **Rectangle** having perpendicular diagonals is a **Square**.
Let the given points be

$$\mathbf{A} = \begin{pmatrix}-1\\0\end{pmatrix}, \mathbf{B} = \begin{pmatrix}0\\3\end{pmatrix}, \mathbf{C} = \begin{pmatrix}3\\2\end{pmatrix}, \mathbf{D} = \begin{pmatrix}2\\-1\end{pmatrix} \tag{0.0.1}$$

Direction Vectors:
Two lines can be said parallel if their Direction Vectors are in the same ratio.
The Directional Vector of **AB** is:

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix}-1-0\\0-3\end{pmatrix} = \begin{pmatrix}-1\\-3\end{pmatrix} \tag{0.0.2}$$

The Directional Vector of **BC** is:

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix}0-3\\3-2\end{pmatrix} = \begin{pmatrix}-3\\1\end{pmatrix} \tag{0.0.3}$$

The Directional Vector of **CD** is:

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix}3-2\\2-(-1)\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix} \tag{0.0.4}$$

The Directional Vector of **DA** is:

$$\mathbf{D} - \mathbf{A} = \begin{pmatrix}2-(-1)\\-1-0\end{pmatrix} = \begin{pmatrix}3\\-1\end{pmatrix} \tag{0.0.5}$$

The Directional Vector of **AC** is:

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix}-1-3\\0-2\end{pmatrix} = \begin{pmatrix}-4\\-2\end{pmatrix} \tag{0.0.6}$$

The Directional Vector of **BD** is:

$$\mathbf{B} - \mathbf{D} = \begin{pmatrix}0-2\\3-(-1)\end{pmatrix} = \begin{pmatrix}-2\\4\end{pmatrix} \tag{0.0.7}$$

Angle between Sides:

Let us check the angles between sides AB, BC, CD, DA.

$$(\mathbf{A} - \mathbf{B})^\top(\mathbf{B} - \mathbf{C}) = \begin{pmatrix}-1\\-3\end{pmatrix}^\top \begin{pmatrix}-3\\1\end{pmatrix} \tag{0.0.8}$$

$$(\mathbf{A} - \mathbf{B})^\top(\mathbf{B} - \mathbf{C}) = 0 \tag{0.0.9}$$

$$\therefore \mathbf{AB} \perp \mathbf{BC} \tag{0.0.10}$$

$$(\mathbf{B} - \mathbf{C})^\top(\mathbf{C} - \mathbf{D}) = \begin{pmatrix}-3\\1\end{pmatrix}^\top \begin{pmatrix}1\\3\end{pmatrix} \tag{0.0.11}$$

$$(\mathbf{B} - \mathbf{C})^\top(\mathbf{C} - \mathbf{D}) = 0 \tag{0.0.12}$$

$$\therefore \mathbf{BC} \perp \mathbf{CD} \tag{0.0.13}$$

$$(\mathbf{C} - \mathbf{D})^\top(\mathbf{D} - \mathbf{A}) = \begin{pmatrix}1\\3\end{pmatrix}^\top \begin{pmatrix}3\\-1\end{pmatrix} \tag{0.0.14}$$

$$(\mathbf{C} - \mathbf{D})^\top(\mathbf{D} - \mathbf{A}) = 0 \tag{0.0.15}$$

$$\therefore \mathbf{CD} \perp \mathbf{DA} \tag{0.0.16}$$

$$(\mathbf{D} - \mathbf{A})^\top(\mathbf{A} - \mathbf{B}) = \begin{pmatrix}3\\-1\end{pmatrix}^\top \begin{pmatrix}-1\\-3\end{pmatrix} \tag{0.0.17}$$

$$(\mathbf{D} - \mathbf{A})^\top(\mathbf{A} - \mathbf{B}) = 0 \tag{0.0.18}$$

$$\therefore \mathbf{DA} \perp \mathbf{AB} \tag{0.0.19}$$

All the sides of Quadrilateral ABCD are perpendicular to one another.

Therefore, ABCD is a **Rectangle**.

Angle between Diagonals:
Now, let us check if the Diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular by using Orthogonality Condition,

$$(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{D}) = 0 \tag{0.0.20}$$

We have,

$$\mathbf{A}-\mathbf{C} = (-1-3, 0-2) \tag{0.0.21}$$
$$\mathbf{A}-\mathbf{C} = \begin{pmatrix}-4\\-2\end{pmatrix} \tag{0.0.22}$$
$$\mathbf{B}-\mathbf{D} = (0-2, 3-(-1)) \tag{0.0.23}$$
$$\mathbf{B}-\mathbf{D} = \begin{pmatrix}-2\\4\end{pmatrix} \tag{0.0.24}$$

For orthogonality, product of transpose of one and other must be 0. Here, checking for

$$(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{D}) = \begin{pmatrix}-4\\-2\end{pmatrix}^T\begin{pmatrix}-2\\4\end{pmatrix} \tag{0.0.25}$$
$$= 0 \tag{0.0.26}$$

Hence, using Orthogonality, Diagonal $\overline{AC}$ is perpendicular to Diagonal $\overline{BD}$.
$\therefore \overline{AC} \perp \overline{BD}$
Since, the Parallelogram ABCD has equal diagonals perpendicular to each other, Hence it is a Square.
Therefore, ABCD is a **Square**.
Hence, the 4 points $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are vertices of a **Square**.

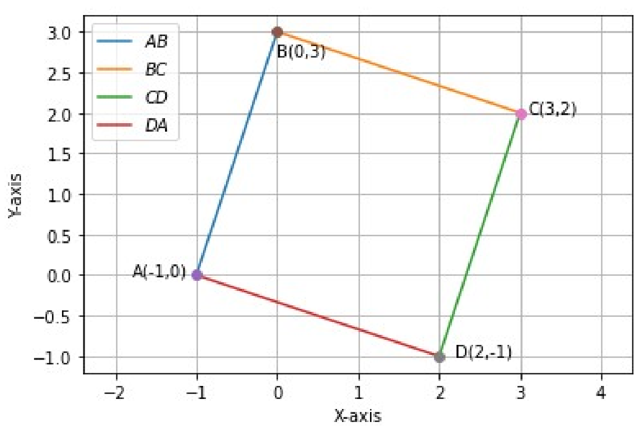

Fig. 0: Plot

This can be verified from the Figure 0.