

# Assignment 1

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## PROBLEM (RAMSEY-1.1 POINTS-Q.8)

Prove that the points  $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ ,  $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$  and  $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$  are the vertices of a square.

### SOLUTION

Let us first prove that the given points form a **Parallelogram** (using parallel direction vectors of opposite sides). And then we will prove that **parallelogram** having equal and perpendicular diagonals form a **Square**.

Let the given points be

$$\mathbf{A} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad (0.0.1)$$

### Direction Vectors:

Two lines can be said parallel if their Direction Vectors are in the same ratio.

The Directional Vector of **AB** is:

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} -1 - 0 \\ 0 - 3 \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \quad (0.0.2)$$

The Directional Vector of **BC** is:

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 0 - 3 \\ 3 - 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \quad (0.0.3)$$

The Directional Vector of **CD** is:

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix} 3 - 2 \\ 2 - (-1) \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (0.0.4)$$

The Directional Vector of **DA** is:

$$\mathbf{D} - \mathbf{A} = \begin{pmatrix} 2 - (-1) \\ -1 - 0 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \quad (0.0.5)$$

The Directional Vector of **AC** is:

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} -1 - 3 \\ 0 - 2 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix} \quad (0.0.6)$$

The Directional Vector of **BD** is:

$$\mathbf{B} - \mathbf{D} = \begin{pmatrix} 0 - 2 \\ 3 - (-1) \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} \quad (0.0.7)$$

From (0.0.2) and (0.0.4), we can say that Side  $\overline{AB}$  and Side  $\overline{CD}$  are parallel to each other.

Also from (0.0.3) and (0.0.5), we can say that Side  $\overline{BC}$  and Side  $\overline{DA}$  are parallel to each other.

From above, since two pairs of opposites sides of Quadrilateral ABCD are parallel to each other, ABCD is a **Parallelogram**.

### Diagonal Length:

We know that the distance between the points  $\mathbf{P}(x_1, y_1)$  and  $\mathbf{Q}(x_2, y_2)$  is given by

$$\|\mathbf{Z}\| = \|\mathbf{P} - \mathbf{Q}\| \quad (0.0.8)$$

Now computing the length of the Diagonals  $\overline{AC}$ ,  $\overline{BD}$ ,

$$\overline{AC} = \|\mathbf{A} - \mathbf{C}\| = \sqrt{(-1 - 3)^2 + (0 - 2)^2} = \sqrt{20} \quad (0.0.9)$$

$$\overline{BD} = \|\mathbf{B} - \mathbf{D}\| = \sqrt{(0 - 2)^2 + (3 - (-1))^2} = \sqrt{20} \quad (0.0.10)$$

The Diagonals  $\overline{AC}$ ,  $\overline{BD}$  of the Parallelogram are of equal length  $\sqrt{20}$  units.

### Angle between Diagonals:

Now, let us check if the Diagonals  $\overline{AC}$  and  $\overline{BD}$  are perpendicular by using Orthogonality Condition,

$$(\mathbf{A} - \mathbf{C})^T (\mathbf{B} - \mathbf{D}) = 0 \quad (0.0.11)$$

We have,

$$\mathbf{A} - \mathbf{C} = (-1 - 3, 0 - 2) \quad (0.0.12)$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} -4 \\ -2 \end{pmatrix} \quad (0.0.13)$$

$$\mathbf{B} - \mathbf{D} = (0 - 2, 3 - (-1)) \quad (0.0.14)$$

$$\mathbf{B} - \mathbf{D} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} \quad (0.0.15)$$

For orthogonality, product of transpose of one and other must be 0. Here, checking for

$$(\mathbf{A} - \mathbf{C})^T (\mathbf{B} - \mathbf{D}) = \begin{pmatrix} -4 & -2 \end{pmatrix}^T \begin{pmatrix} -2 \\ 4 \end{pmatrix} \quad (0.0.16)$$

$$= 0 \quad (0.0.17)$$

Hence, using Orthogonality, Diagonal  $\overline{AC}$  is perpendicular to Diagonal  $\overline{BD}$ .

$\therefore \overline{AC} \perp \overline{BD}$

Since, the Parallelogram ABCD has equal diagonals perpendicular to each other, Hence it is a Square.

Therefore, ABCD is a **Square**.

Hence, the 4 points **A, B, C, D** are vertices of a **Square**.

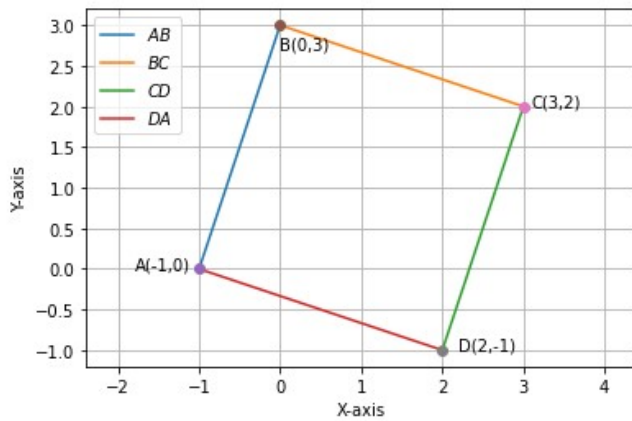


Fig. 0: Plot

This can be verified from the Figure 0.