Assignment 5

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Download latex-tikz codes from

https://github.com/KBVijayVarma/EE3900/tree/main/Assignment_5

Download python codes from

https://github.com/KBVijayVarma/EE3900/tree/main/Assignment_5/code

Problem (Quadratic Forms Q.2.9)

Find the smaller area enclosed by the circle $\mathbf{x}^\top\mathbf{x} = 4$ and the line $\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 2$.

Solution

General equation of circle is

$$\mathbf{x}^\top\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{0.0.1}$$

$$\|\mathbf{x}\|^2 + 2\mathbf{u}_1^\top\mathbf{x} + f_1 = 0 \tag{0.0.2}$$

$$\mathbf{x}^\top\mathbf{x} - 4 = 0 \tag{0.0.3}$$

$$\mathbf{u}_1 = \begin{pmatrix}0\\0\end{pmatrix} \tag{0.0.4}$$

$$f_1 = -4 \tag{0.0.5}$$

$$\mathbf{O}_1 = \begin{pmatrix}0\\0\end{pmatrix} \tag{0.0.6}$$

$$r = \sqrt{\mathbf{u}_1^\top\mathbf{u}_1 - f_1} = \sqrt{4} \tag{0.0.7}$$

$$\Rightarrow r = 2 \tag{0.0.8}$$

From (0.0.8), the points at which circle touches X-axis is $\begin{pmatrix}-2\\0\end{pmatrix}$ and $\begin{pmatrix}2\\0\end{pmatrix}$.

The direction vector of the given line $\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 2$ is $\begin{pmatrix}-1\\1\end{pmatrix}$.

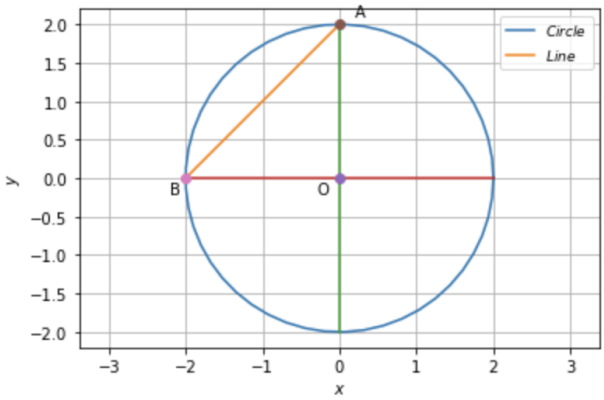

Fig. 0: Smaller area enclosed by line and circle

To find point **A** and **B**, the parametric form of line is,

$$\mathbf{A} = \mathbf{q} + \lambda\mathbf{m} \tag{0.0.9}$$

$$= \begin{pmatrix}1\\1\end{pmatrix} + \lambda\begin{pmatrix}-1\\1\end{pmatrix} \tag{0.0.10}$$

$$\lambda^2 = \frac{-f_1 - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} \tag{0.0.11}$$

$$= \frac{4-2}{2} = 1 \tag{0.0.12}$$

$$\Rightarrow \lambda = \pm 1 \tag{0.0.13}$$

$$\mathbf{A} = \begin{pmatrix}0\\2\end{pmatrix}\mathbf{B} = \begin{pmatrix}-2\\0\end{pmatrix} \tag{0.0.14}$$

$$\mathbf{O_1} = \begin{pmatrix}0\\0\end{pmatrix} \tag{0.0.15}$$

$$(\mathbf{A} - \mathbf{O_1}) = \begin{pmatrix}0\\2\end{pmatrix} \tag{0.0.16}$$

$$(\mathbf{B} - \mathbf{O_1}) = \begin{pmatrix}-2\\0\end{pmatrix} \tag{0.0.17}$$

Inner product of $(\mathbf{A} - \mathbf{O_1})$ and $(\mathbf{A} - \mathbf{O_1})$ is given as:

$$(\mathbf{A} - \mathbf{O_1})^\top(\mathbf{B} - \mathbf{O_1}) = 0 \tag{0.0.18}$$

Therefore, $(\mathbf{A} - \mathbf{O_1}) \perp (\mathbf{B} - \mathbf{O_1})$

Smaller area enclosed by circle and line **AB** is:
Area = (Area of circle in 2nd Quadrant) - (Area of right triangle formed by line AB, X and Y axis)

$$Area = \frac{\pi\theta_1}{360}r^2 - \frac{1}{2}\times 2\times 2 \tag{0.0.19}$$
$$= \frac{90}{360}\times 2^2 - 2 \tag{0.0.20}$$
$$= \pi - 2 \tag{0.0.21}$$

Hence, the smaller area enclosed by the circle $\mathbf{x}^\top\mathbf{x} = 4$ and the line $\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 2$ is $(\pi - 2)$ square units.