Assignment 3

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https://github.com/KBVijayVarma/EE3900/tree/main/Assignment_3

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Problem Q 2.30(d)

For each of the following systems, determine whether the system is (1) stable, (2) casual, (3) linear, (4) time invariant.
(d) $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$

Solution

Given $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$ there
(1) Stable

$$y(n) = \sum_{k=n-1}^{\infty} x[k] \tag{0.0.1}$$

$$y(n) = x[k-1] + x[n] + x[n+1] + x[n+2] + \cdots + \infty \tag{0.0.2}$$

If x[n] is finite also, the above sum y(n) extends to infinity.
$\therefore$ The system $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$ is **Unstable**.
(2) Casual

$$y(n) = \sum_{k=n-1}^{\infty} x[k] \tag{0.0.3}$$

$$y(n) = x[k-1] + x[n] + x[n+1] + x[n+2] + \cdots + \infty \tag{0.0.4}$$

Here the output y(n) depends on the future input (x[n+1], x[n+2], $\cdots$).
We know that if the output y(n) depends on future inputs then the system is a non casual system.
Hence, the given system $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$ is a **Non - Casual** System.

(3) Linear
Let us take

$$y_1(n) = \sum_{k=n-1}^{\infty} x_1[k]$$
$$y_2(n) = \sum_{k=n-1}^{\infty} x_2[k]$$
$$x[n] = ax_1[n] + bx_2[n]$$

Given $y(n) = \sum_{k=n-1}^{\infty} x[k]$

$$y[n] = \sum_{k=n-1}^{\infty} [ax_1[k] + bx_2[k]] \tag{0.0.5}$$

$$y[n] = \sum_{k=n-1}^{\infty} ax_1[k] + \sum_{k=n-1}^{\infty} bx_2[k] \tag{0.0.6}$$

$$y[n] = ay_1[n] + by_2[n] \tag{0.0.7}$$

$\therefore y[n] = ay_1[n] + by_2[n]$
The given system holds the Superposition and Homogenity property.
Hence, the given system $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$ is **Linear**.
(4) Time Invariant
Let

$$y_1(n) = \sum_{k=n-1}^{\infty} x_1[k]$$
$$y_2(n) = \sum_{k=n-1}^{\infty} x_2[k]$$
$$x_2[n] = x_1[n-t]$$

where t is a real number
Now,

$$y_2(n) = \sum_{k=n-1}^{\infty} x_2[k] \tag{0.0.8}$$

$$y_2(n) = \sum_{k=n-1}^{\infty} x_1[k-t] \tag{0.0.9}$$

Let us change the limits of the summation.

Let $k^{'} = k - t$
The lower limit $k = n - 1$ changes to $k^{'} = n - t - 1$.
The upper limit $k = \infty$ changes to $k^{'} = \infty$.
The above summation changes to

$$y_2(n) = \sum_{k^{'}=n-t-1}^{\infty} x_1[k^{'}] \tag{0.0.10}$$

Now changing $k^{'}$ with k we get,

$$y_2(n) = \sum_{k=n-t-1}^{\infty} x_1[k] \tag{0.0.11}$$

$$y_2(n) = y_1(n - t) \tag{0.0.12}$$

Time Invariance can be verified from above.
Hence, the given system $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$ is **Time Invariant**.