Assignment 1

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Download latex-tikz codes from

https://github.com/KBVijayVarma/EE3900/tree/main/Assignment_1

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Problem (Ramsey-1.1 Points-Q.8)

Prove that the points $\begin{pmatrix}-1\\0\end{pmatrix}, \begin{pmatrix}0\\3\end{pmatrix}, \begin{pmatrix}3\\2\end{pmatrix}$ and $\begin{pmatrix}2\\-1\end{pmatrix}$ are the vertices of a square.

Solution

Let the given points be

$$\mathbf{A} = \begin{pmatrix}-1\\0\end{pmatrix}, \mathbf{B} = \begin{pmatrix}0\\3\end{pmatrix}, \mathbf{C} = \begin{pmatrix}3\\2\end{pmatrix}, \mathbf{D} = \begin{pmatrix}2\\-1\end{pmatrix} \tag{0.0.1}$$

Two lines can be said parallel if their Direction Vectors are in the same ratio.
The Directional Vector of **AB** is:

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix}-1-0\\0-3\end{pmatrix} = \begin{pmatrix}-1\\-3\end{pmatrix} \tag{0.0.2}$$

The Directional Vector of **BC** is:

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix}0-3\\3-2\end{pmatrix} = \begin{pmatrix}-3\\1\end{pmatrix} \tag{0.0.3}$$

The Directional Vector of **CD** is:

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix}3-2\\2-(-1)\end{pmatrix} = \begin{pmatrix}1\\3\end{pmatrix} \tag{0.0.4}$$

The Directional Vector of **DA** is:

$$\mathbf{D} - \mathbf{A} = \begin{pmatrix}2-(-1)\\-1-0\end{pmatrix} = \begin{pmatrix}3\\-1\end{pmatrix} \tag{0.0.5}$$

The Directional Vector of **AC** is:

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix}-1-3\\0-2\end{pmatrix} = \begin{pmatrix}-4\\-2\end{pmatrix} \tag{0.0.6}$$

The Directional Vector of **BD** is:

$$\mathbf{B} - \mathbf{D} = \begin{pmatrix}0-2\\3-(-1)\end{pmatrix} = \begin{pmatrix}-2\\4\end{pmatrix} \tag{0.0.7}$$

From (0.0.2) and (0.0.4), we can say that Side $\overline{AB}$ and Side $\overline{CD}$ are parallel to each other.
Also from (0.0.3) and (0.0.5), we can say that Side $\overline{BC}$ and Side $\overline{DA}$ are parallel to each other.
From above, since two pairs of opposites sides of Quadrilateral ABCD are parallel to each other, ABCD is a **Parallelogram**.
We know that the distance between the points $\mathbf{P}(x_1,y_1)$ and $\mathbf{Q}(x_2,y_2)$ is given by

$$\|\mathbf{Z}\| = \|\mathbf{P} - \mathbf{Q}\| \tag{0.0.8}$$

Now computing the length of the Diagonals $\overline{AC}$, $\overline{BD}$,

$$\overline{AC} = \|\mathbf{A} - \mathbf{C}\| = \sqrt{(-1-3)^2 + (0-2)^2} = \sqrt{20} \tag{0.0.9}$$

$$\overline{BD} = \|\mathbf{B} - \mathbf{D}\| = \sqrt{(0-2)^2 + (3-(-1))^2} = \sqrt{20} \tag{0.0.10}$$

The Diagonals $\overline{AC}$, $\overline{BD}$ of the Parallelogram are of equal length $\sqrt{20}$ units.

Now, let us check if the Diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular by using Orthogonality Condition,

$$(\mathbf{A} - \mathbf{C})^T(\mathbf{B} - \mathbf{D}) = 0 \tag{0.0.11}$$

We have,

$$\mathbf{A} - \mathbf{C} = (-1-3, 0-2) \tag{0.0.12}$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix}-4\\-2\end{pmatrix} \tag{0.0.13}$$

$$\mathbf{B} - \mathbf{D} = (0-2, 3-(-1)) \tag{0.0.14}$$

$$\mathbf{B} - \mathbf{D} = \begin{pmatrix}-2\\4\end{pmatrix} \tag{0.0.15}$$

For orthogonality, product of transpose of one and other must be 0. Here, checking for

$$(\mathbf{A} - \mathbf{C})^T(\mathbf{B} - \mathbf{D}) = \begin{pmatrix}-4\\-2\end{pmatrix}^T \begin{pmatrix}-2\\4\end{pmatrix} \tag{0.0.16}$$

$$= 0 \tag{0.0.17}$$

Hence, using Orthogonality, Diagonal $\overline{AC}$ is perpendicular to Diagonal $\overline{BD}$.
$\therefore \overline{AC} \perp \overline{BD}$
Since, the Parallelogram ABCD has equal diagonals perpendicular to each other, Hence it is a Square.
Therefore, ABCD is a **Square**.
Hence, the 4 points $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are vertices of a **Square**.

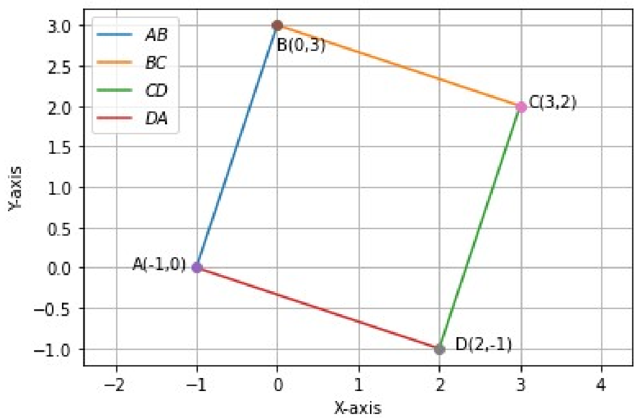

Fig. 0: Plot

This can be verified from the Figure 0.