Gate Assignment 2

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Download latex-tikz codes from

https://github.com/KBVijayVarma/EE3900/tree/main/Gate_Assignment_2

Problem (GATE EC-2008 Q 78)

In the following network, the switch is closed at t = 0 and the sampling starts from t = 0. The sampling frequency is 10Hz.

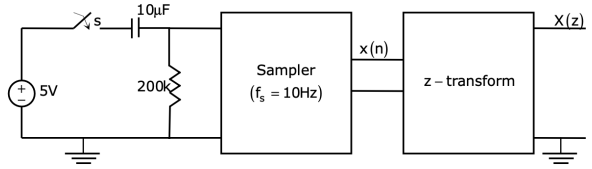

Fig. 0: question

The samples x(n) (n = 0, 1, 2, $\cdots$) are given by

1) $5(1-e^{-0.05n})$
2) $5e^{-0.05n}$
3) $5(1-e^{-5n})$
4) $5e^{-5n}$

Solution

The charge q, current i, voltage V in a circuit are,

$$q = cV \tag{0.0.1}$$

$$i = \frac{dq}{dt} = c\frac{dV}{dt} \tag{0.0.2}$$

$$i = \frac{V}{R} \tag{0.0.3}$$

$$\therefore c\frac{dV}{dt} = \frac{V}{R} \tag{0.0.4}$$

In the given circuit, let V(t) be the voltage at a given time t. Converting all variables into s domain, From above,

$$\frac{-V(s)+\frac{5}{s}}{\frac{1}{sc}} = \frac{V(s)}{R} \tag{0.0.5}$$

$$\frac{5}{sV(s)} - 1 = \frac{1}{sRc} \tag{0.0.6}$$

$$V(s) = \frac{5}{s(1+\frac{1}{sRc})} \tag{0.0.7}$$

$$V(s) = \frac{5}{s+\frac{1}{Rc}} \tag{0.0.8}$$

Now applying Inverse Laplace Transform we get,

$$V(t) = 5e^{-\frac{t}{Rc}} \tag{0.0.9}$$

Substituting the values of

$$R = 200K = 2\times10^{5}\Omega \tag{0.0.10}$$

$$c = 10\mu F = 10^{-5}F \tag{0.0.11}$$

$$Rc = 2 \tag{0.0.12}$$

We get

$$V(t) = 5e^{-\frac{t}{2}} = 5e^{-0.5t} \tag{0.0.13}$$

Now, given Sampling frequency f = 10 Hz.
Sampling period T is given by

$$T = \frac{1}{f} = 0.1s \tag{0.0.14}$$

Now, samples x[n] are obtained by replacing t with nT in (0.0.13),

$$x[n] = 5e^{-0.5nT} \tag{0.0.15}$$

$$x[n] = 5e^{-0.05n} \tag{0.0.16}$$

The samples x(n) (n = 0, 1, 2, $\cdots$) are given by $x[n] = 5e^{-0.05n}$.
Hence, the correct answer is **Option 2**.