

Quiz - 1

Vijay Varma - AI20BTECH11012

Download latex-tikz codes from

https://github.com/KBVijayVarma/EE3900/tree/main/Quiz_1

PROBLEM Q 2.30(D)

For each of the following systems, determine whether the system is (1) stable, (2) casual, (3) linear, (4) time invariant.

(d) $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$

SOLUTION

Given $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$ there

(1) Stable

$$y(n) = \sum_{k=n-1}^{\infty} x[k] \quad (0.0.1)$$

$$y(n) = x[k-1] + x[n] + x[n+1] + x[n+2] + \dots + \infty \quad (0.0.2)$$

If $x[n]$ is finite also, the above sum $y(n)$ extends to infinity.

\therefore The system $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$ is **Unstable**.

(2) Casual

$$y(n) = \sum_{k=n-1}^{\infty} x[k] \quad (0.0.3)$$

$$y(n) = x[k-1] + x[n] + x[n+1] + x[n+2] + \dots + \infty \quad (0.0.4)$$

Here the output $y(n)$ depends on the future input ($x[n+1]$, $x[n+2]$, \dots).

We know that if the output $y(n)$ depends on future inputs then the system is a non casual system.

Hence, the given system $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$ is a **Non - Casual System**.

(3) Linear

Let us take

$$y_1(n) = \sum_{k=n-1}^{\infty} x_1[k]$$

$$y_2(n) = \sum_{k=n-1}^{\infty} x_2[k]$$

$$x[n] = ax_1[n] + bx_2[n]$$

Given $y(n) = \sum_{k=n-1}^{\infty} x[k]$

$$y[n] = \sum_{k=n-1}^{\infty} [ax_1[k] + bx_2[k]] \quad (0.0.5)$$

$$y[n] = \sum_{k=n-1}^{\infty} ax_1[k] + \sum_{k=n-1}^{\infty} bx_2[k] \quad (0.0.6)$$

$$y[n] = ay_1[n] + by_2[n] \quad (0.0.7)$$

$$\therefore y[n] = ay_1[n] + by_2[n]$$

The given system holds the Superposition and Homogeneity property.

Hence, the given system $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$ is **Linear**.

(4) Time Invariant

Let

$$y_1(n) = \sum_{k=n-1}^{\infty} x_1[k]$$

$$y_2(n) = \sum_{k=n-1}^{\infty} x_2[k]$$

$$x_2[n] = x_1[n-t]$$

where t is a real number

Now,

$$y_2(n) = \sum_{k=n-1}^{\infty} x_2[k] \quad (0.0.8)$$

$$y_2(n) = \sum_{k=n-1}^{\infty} x_1[k-t] \quad (0.0.9)$$

Let us change the limits of the summation.

Let $k' = k - t$

The lower limit $k = n - 1$ changes to $k' = n - t - 1$.

The upper limit $k = \infty$ changes to $k' = \infty$.

The above summation changes to

$$y_2(n) = \sum_{k'=n-t-1}^{\infty} x_1[k'] \quad (0.0.10)$$

Now changing k' with k we get,

$$y_2(n) = \sum_{k=n-t-1}^{\infty} x_1[k] \quad (0.0.11)$$

$$y_2(n) = y_1(n - t) \quad (0.0.12)$$

Time Invariance can be verified from above.

Hence, the given system $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$ is

Time Invariant.

\therefore The given system $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$ is **Unsta-**

ble, Non - Casual, Linear, Time Invariant.