Gate Assignment 2

Vijay Varma - AI20BTECH11012

Download latex-tikz codes from

https://github.com/KBVijayVarma/EE3900/tree/main/Gate_Assignment_2

Problem (GATE EC-2008 Q 78)

In the following network, the switch is closed at t = 0 and the sampling starts from t = 0. The sampling frequency is 10Hz.

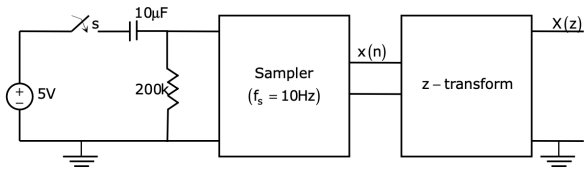

Fig. 0: question

The samples x(n) (n = 0, 1, 2, $\cdots$) are given by

1) $5(1-e^{-0.05n})$
2) $5e^{-0.05n}$
3) $5(1-e^{-5n})$
4) $5e^{-5n}$

Solution

The charge q, current i, voltage V in a circuit are,

$$q = cV \tag{0.0.1}$$

$$i = \frac{dq}{dt} = c\frac{dV}{dt} \tag{0.0.2}$$

$$i = \frac{V}{R} \tag{0.0.3}$$

$$\therefore c\frac{dV}{dt} = \frac{V}{R} \tag{0.0.4}$$

In the given circuit, let V be the voltage at a given time t. From above,

$$c\frac{d(5-V)}{dt} = \frac{V}{R} \tag{0.0.5}$$

$$-c\frac{dV}{dt} = \frac{V}{R} \tag{0.0.6}$$

$$\frac{dV}{V} = -\frac{dt}{Rc} \tag{0.0.7}$$

Integrating on both sides,

$$[\ln V]_5^V = -\frac{1}{Rc}[t]_0^t \tag{0.0.8}$$

$$\ln\frac{V}{5} = -\frac{t}{Rc} \tag{0.0.9}$$

$$\frac{V}{5} = e^{-\frac{t}{Rc}} \tag{0.0.10}$$

$$V = 5e^{-\frac{t}{Rc}} \tag{0.0.11}$$

Substituting the values of

$$R = 200K = 2\times10^5\Omega \tag{0.0.12}$$

$$c = 10\mu F = 10^{-5}F \tag{0.0.13}$$

$$Rc = 2 \tag{0.0.14}$$

We get

$$V = 5e^{-\frac{t}{2}} = 5e^{-0.5t} \tag{0.0.15}$$

Now, given Sampling frequency f = 10 Hz.
Sampling period T is given by

$$T = \frac{1}{f} = 0.1s \tag{0.0.16}$$

Now, samples x[n] are obtained by replacing t with nT in (0.0.15),

$$x[n] = 5e^{-0.5nT} \tag{0.0.17}$$

$$x[n] = 5e^{-0.05n} \tag{0.0.18}$$

The samples x(n) (n = 0, 1, 2, $\cdots$) are given by $x[n] = 5e^{-0.05n}$.
Hence, the correct answer is **Option 2**.