Assignment 4

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Download latex-tikz codes from

https://github.com/KBVijayVarma/EE3900/tree/main/Assignment_4

Download python codes from

https://github.com/KBVijayVarma/EE3900/tree/main/Assignment_4/code

Problem (Linear Forms Q 2.12)

The hypotenuse of a right angled triangle has its ends at the points $\begin{pmatrix}1\\3\end{pmatrix}$ and $\begin{pmatrix}-4\\1\end{pmatrix}$. Find an equation of the legs of the triangle.

Solution

Let Δ ABC be a right angle triangle, where AC is hypotenuse and $\angle$ B = 90°
Therefore hypotenuse end points are

$$\mathbf{A} = \begin{pmatrix}1\\3\end{pmatrix} \tag{0.0.1}$$

$$\mathbf{B} = \begin{pmatrix}-4\\1\end{pmatrix} \tag{0.0.2}$$

Now, to calculate the equations of legs of triangle, i.e., equation of **AB** and **BC**
Let slope of line **AB** be m.
Product of slopes of perpendicular lines is equal to -1.
Here, **AB** $\perp$ **BC**
$\therefore$ Slope of line **BC** = $\frac{-1}{\mathbf{m}}$
Equation of a line having slope m and passing through point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \tag{0.0.3}$$

Now, equation of line **AB** passing through $A(1, 3)$ and slope m is

$$(y - 3) = m(x - 1) \tag{0.0.4}$$

Equation of line **BC** passing through $C(-4, 1)$ and slope $\frac{-1}{m}$ is

$$(y - 1) = \frac{-1}{m}(x + 4) \tag{0.0.5}$$

The point **B** lies on the circle having end points of the diameter as **A** and **C** since angle in a Semi Circle is 90°.
$\therefore$ m can have Infinite values.
General Equations of the lines **AB** and **BC** are,

$$\text{Line AB is } mx - y + 3 - m = 0 \tag{0.0.6}$$

$$\text{Line BC is } x + my + 4 - m = 0 \tag{0.0.7}$$

We can take any value of m to get the equations of legs of the triangle.
The below figure is drawn using taking value of m to be infinity.

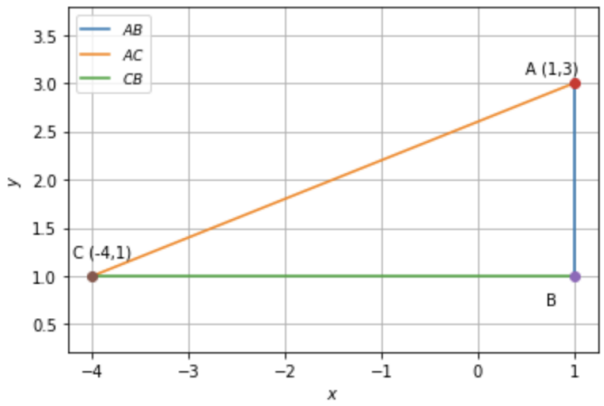

Fig. 0: Triangle ABC