Applying Unscented Kalman Filter for Estimating the Position of Marker With respect to Drone

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EMCH792 – State Estimation

Project Report

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Abstract

In most scenarios, unmanned aerial vehicles (UAVs), needs to have the capability of autonomous flying to carry out their mission successfully. To allow these autonomous flights, drones need to know their location constantly. Then, based on the current position and the destination, navigation commands will be generated, to guide drone to the desired destination. In indoor scenarios we use markers to localize its position. This work focuses on estimating the marker's position with respect to drone. To do this we use the position data that is be measured from a fixed marker and the pinhole camera that is attached to the drone, then we convert those measurements with respect to done and the marker. We use IMU data as inputs for linear and angular rate. We then measure the estimated states with the true state measurement and plot the error between the estimated state and the true state.

Nomenclature

H_{Base}	= Homogenous Matrix [R T; 0 0 1]
H_C^m	= marker in Camera's Frame
H_D^c	= camera in Drone's Frame
H_D^m	= marker in Drone's Frame
$Opti\_data$	= True data from Optitrack (reference data)
Cam_data	= Data from Stag marker
Vipose_data	= Externally converted H_D^m measurements for comparison
P	= Error covariance
Q	= Process Noise
R	= Measurement Noise
K	= Kalman Gain

I. State Space Representation

The State of the system is given by $x_{(123)}$ represent positions in X, Y, Z- axis, $x_{(456)}$ represent velocities and $x_{(789)}$ represent roll, pitch and yaw along X, Y, Z- axis respectively.

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \\ \dot{x_5} \\ \dot{x_6} \\ \dot{x_7} \\ \dot{x_8} \\ \dot{x_9} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \\ ttax \\ phiy \\ shiz \end{bmatrix} + \mathrm{W}$$

The measurement of where Marker is with respect to Drone is calculate by

$$H_D^m = H_D^m * H_D^c$$

II. Methodology

We have 3 data inputs, (a) Measurement form camera H_C^m Cam_data the pose data of where Marker is in Camera's frame, (b) The data form Optitrack the True of the reference data, which is in World frame, (c) Externally processed H_D^m pose data from Vipose_data.

While data in (b), (c) can be plotted directly we need to convert data in (a) Measurement form camera H_C^m; to obtain relation between marker with respect to Drone (H_D^m) by multiplication of the following the homogenous as shown in equation 1. We have the measurement matrix H_C^m that is recorded while camera is facing the marker this data is extracted as cam_data, we can obtain a static H_D^c matrix as the camera is rigidly attached to the Drone.

$$H_D^m = H_D^m * H_D^c$$

$$H_D^c = \begin{bmatrix} 0 & 0 & 1 & 0.1 \\ -1 & 0 & 0 & 0.025 \\ 0 & -1 & 0 & -0.085 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

<u>Steps in developing Unscented Kalman Filter (UKF):</u>

1. Generate Sigma points:

   Here as we consider 9 states, we have 2*(n) =18 Sigma points which are calculated by :

$$x_{k-1}^{(i)} = \hat{x}_{k-1}^{+} + \tilde{x}^{(i)} \qquad i=1,\dots,2n$$

$$\tilde{x}^{(i)} = \left(\sqrt{nP_{k-1}^{+}}\right)^T \qquad i=1,\dots,2n$$

$$\tilde{x}^{(n+i)} = -\left(\sqrt{nP_{k-1}^{+}}\right)^T \qquad i=1,\dots,2n$$

2. Pass the Sigma points through the discrete non-linear function of States $f(.)$:

$$\hat{x}_k^{(i)} = x_{k-1}^{(i)} + f(x_k, u_k)\,dt$$

3. Obtain the a priori State estimate and a priori error covariance at time k:

$$\hat{x}_k^- = \frac{1}{2n}\sum_{i=1}^{2n} \hat{x}_k^{(i)}$$

$$P_k^- = \frac{1}{2n}\sum_{i=1}^{2n} \left(\hat{x}_k^{(i)} - \hat{x}_k^-\right)\left(\hat{x}_k^{(i)} - \hat{x}_k^-\right)^T + Q_{k-1}$$

4. Now from time update equations we implement measurement update, Update the Sigma points with the current best guess of mean and covariance and the pass these new Sigma points through the measurement equation to get the $y_{estimate}$:

$$\hat{x}_k^{(i)} = \hat{x}_k^- + \tilde{x}^{(i)} \qquad i = 1 \ldots 2n$$

$$\tilde{x}^{(i)} = \left(\sqrt{nP_k^-}\right)^T \qquad i = 1 \ldots 2n$$

$$\tilde{x}^{(n+i)} = -\left(\sqrt{nP_k^-}\right)^T \qquad i = 1 \ldots n$$

$$\hat{y}_k^{(i)} = h\left(\hat{x}_k^{(i)}, v_k\right)$$

$$\boxed{H_D^m = H_D^c * H_C^m}$$

$$\hat{y}_k = \frac{1}{2n}\sum_{i=1}^{2n} \hat{y}_k^{(i)}$$

5. Estimate the covariance of the estimated measurement also add accounting for the measurement noise:

$$P_y = \frac{1}{2n}\sum_{i=1}^{2n} \left(\hat{y}_k^{(i)} - \hat{y}_k\right)\left(\hat{y}_k^{(i)} - \hat{y}_k\right)^T + R_k$$

6. Estimate the cross covariance:

$$P_{xy} = \frac{1}{2n}\sum_{i=1}^{2n} \left(\hat{x}_k^{(i)} - \hat{x}_k^-\right)\left(\hat{y}_k^{(i)} - \hat{y}_k\right)^T$$

7. Calculate the Kalman gain:

$$K_k = P_{xy} P_y^{-1}$$

8. Update the State estimate and the process covariance:

$$\hat{x}_k^+ = \hat{x}_k^- + K_k(y_k - \hat{y}_k)$$

$$P_k^+ = P_k^- - K_k P_y K_k^T$$

III. Results and Discussion

In the Figures (1- 4) shown below the estimates of States are plotted in blue, while the measurement from camera and the True data form the Optitrack are plotted with yellow and black lines.

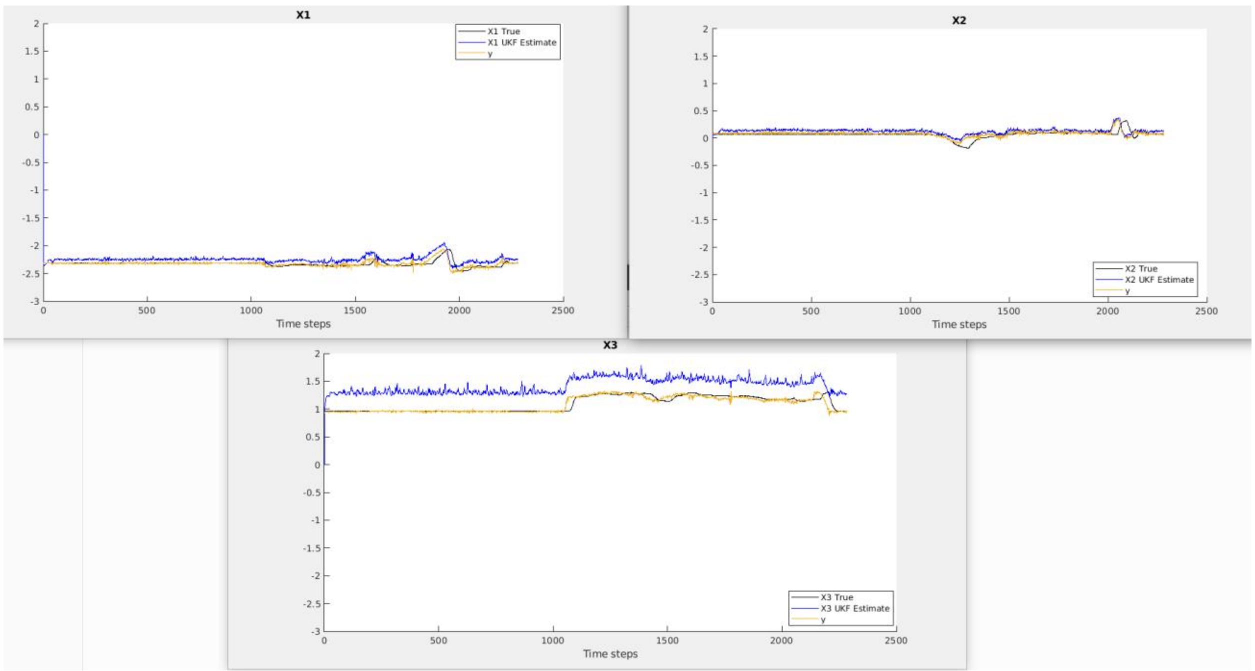

Figure 1: Position estimate Vs Measurement and True State

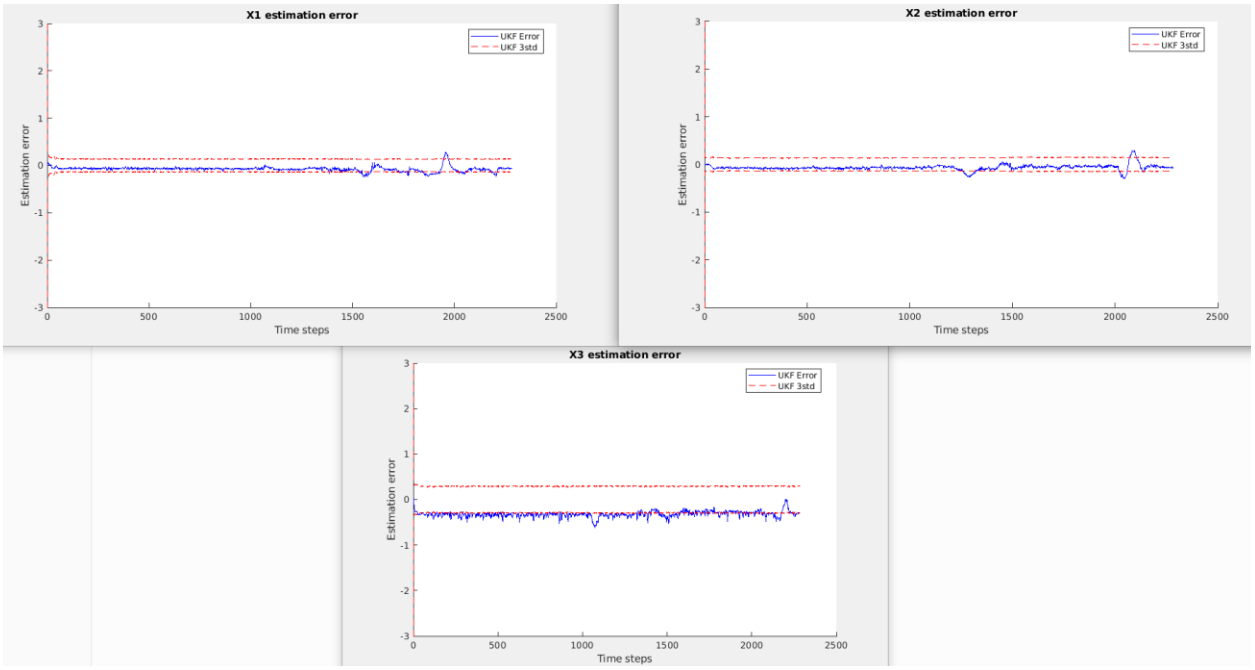

Figure 2: Error Estimate of Position

Figures: 1 represent the Position estimate of Marker with respect to Drone in X, Y, Z- axis. In X1 and X2 the estimate seems to closely follow the measurement and the True State. With a maximum of 19 ,14, 25cm variation from True value along X1, X2, X3 states. The error between the True and the estimated states lies between the $\pm 3\sigma$ as shown in Figure2.

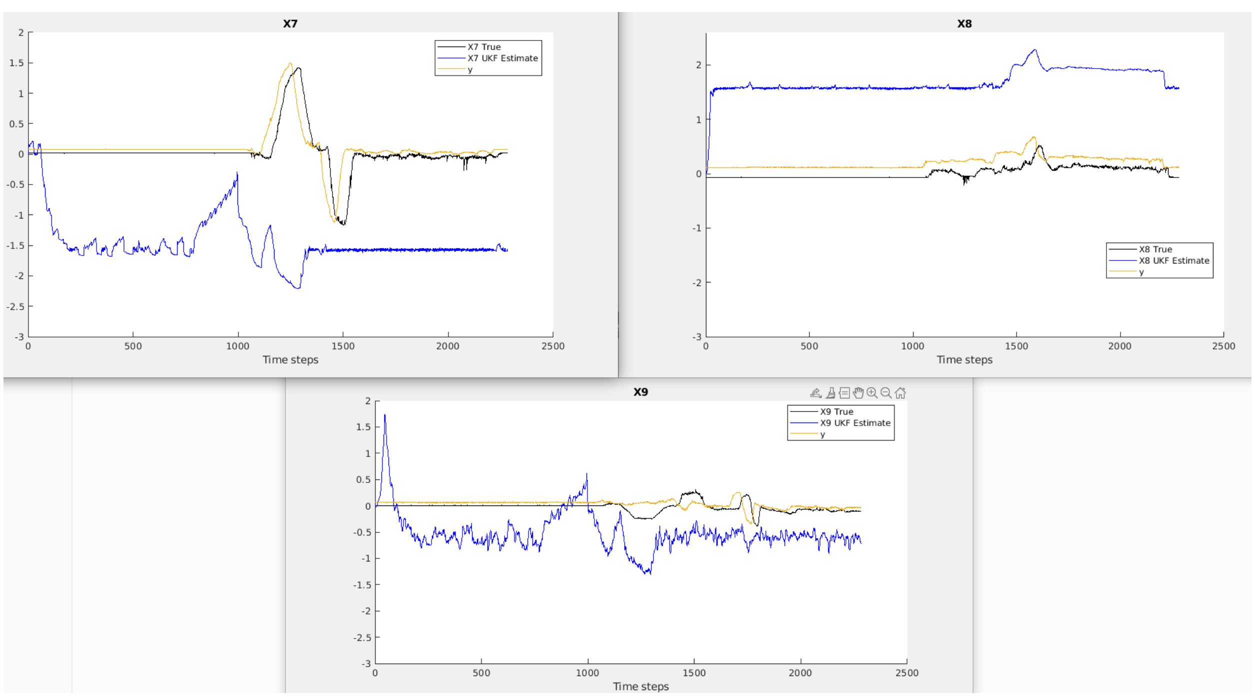

Figure 3: Roll, Pitch and Yaw estimate

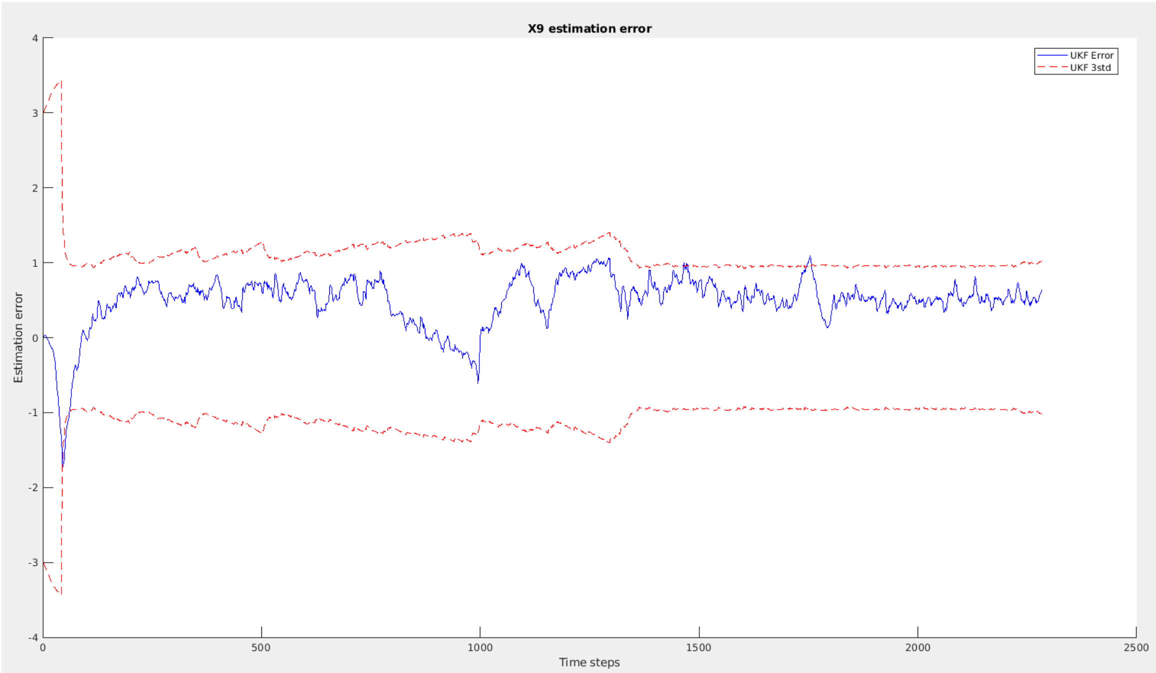

Figure 4: Error estimate of Yaw

Though the estimate in Figure 3, follows the similar trend it is not precise and as smooth as it was for the position estimates, and error bounds are roughly under $\pm 3\sigma$. These results further need to be analysed and possible reasons for offset in the estimates need to be detected. One of the approaches can be find fine tuning the process noise and covariance matrix.

IV. Conclusion

While the position estimates obtained seems to be within ± 3 times the standard deviation, the estimates of roll, pitch and yaw are not satisfactory. Possible factors for this error need to be further studied.

References

[1] Optimal State Estimation: Kalman, H∞, and Nonlinear Approaches. Author: Dan Simon