Homework 3

1

Show that if P and Q are stochastic matrices of the same size, then PQ is also stochastic. 2.4.29

2

The digraph H that results from reversing all the arrows in a digraph G is called reverse digraph of G. Show that if A is the adjacency matrix for G then A^T is the adjacency matrix for the reverse digraph H. 2.4.36

3

Solve the PageRank problem with P as in Example 2.46, teleportation vector $\mathbf{v} = \frac{1}{6}\mathbf{e}$ and teleportation parameter $\alpha = 0.8$. 2.5.21

4

Modify the surfing matrix P of Example 2.46 by using the correction vector $\frac{1}{5}(1, 1, 1, 0, 1, 1)$ and solve the resulting PageRank problem with teleportation vector $\mathbf{v} = \frac{1}{6}\mathbf{e}$ and teleportation parameter $\alpha = 0.8$. 2.5.22

5

Show that there is more than one stationary state for the Markov chain of Example 2.46. 2.5.29

6

Repair the dangling node problem of the graph of Figure 2.7 by using transition to all nodes as equally likely and find all stationary states for the resulting Markov chain. 2.5.30

7

Solve the nonlinear system of equations of Example 2.48 by using nine iterations of the vector Newton formula (2.5), starting with the initial guess $\mathbf{x}^{(0)} = (0, 1)$. Evaluate $F\left(\mathbf{x}^{(9)}\right)$. 2.5.25

8

Find the minimum value of the function $F(x, y) = \left(x^2 + y + 1\right)^2 + x^4 + y^4$ by using the Newton method to find critical points of the function $F(x, y)$, i.e., points where $f(x, y) = F_x(x, y) = 0$ and $g(x, y) = F_y(x, y) = 0$. 2.5.26

9

Apply the following digital filter to the noisy data of Example 2.71 and graph the results. Does it appear to be a low pass filter? 2.8.9

$$y_k = \frac{1}{2}x_k + \frac{1}{2}x_{k-1}, \quad k = 1, 2, \ldots, 33$$

10

Apply the following digital filter to the noisy data of Example 2.71 and graph the results. Does it appear to be a high pass filter? 2.8.10

$$y_k = \frac{1}{2}x_k - \frac{1}{2}x_{k-1}, \quad k = 1, 2, \ldots, 33$$

11

(I'll talk about LU factorization in class on the Wednesday that this homework is due; you may want to hold off on the next few problems until then.) 2.8.1

Show that $L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 4 & -3 \\ 0 & 0 & -1 \end{bmatrix}$ is an LU factorization of $A = \begin{bmatrix} 2 & -1 & 1 \\ 2 & 3 & -2 \\ 4 & 2 & -2 \end{bmatrix}$.

12

By hand:

Find an LU factorization of the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ -4 & -1 & -1 \\ 2 & 3 & -3 \end{bmatrix}$. 2.8.5

13

By hand:

Find a PLU factorization of the matrix $A = \begin{bmatrix} 2 & 1 & 3 \\ -4 & -2 & -1 \\ 2 & 3 & -3 \end{bmatrix}$. 2.8.6