

# Hieroglyphs

A team of researchers is studying the similarities between sequences of hieroglyphs. They represent each hieroglyph with a non-negative integer. To perform their study, they use the following concepts about sequences.

For a fixed sequence A, a sequence S is called a *subsequence* of A if and only if S can be obtained by removing some elements (possibly none) from A.

The table below shows some examples of subsequences of a sequence A = [3, 2, 1, 2].

Subsequence	How it can be obtained from $\boldsymbol{A}$	
[3, 2, 1, 2]	No elements are removed.	
[2, 1, 2]	[ <del>3</del> , 2, 1, 2]	
[3, 2, 2]	[3, 2, <del>1</del> , 2]	
[3, 2]	[3, <del>2</del> , <del>1</del> , 2] or [3, 2, <del>1</del> , <del>2</del> ]	
[3]	[3, <del>2</del> , <del>1</del> , <del>2</del> ]	
[]	[ <del>3</del> , <del>2</del> , <del>1</del> , <del>2</del> ]	

On the other hand, [3,3] or [1,3] are not subsequences of A.

Consider two sequences of hieroglyphs, A and B. A sequence S is called a *common subsequence* of A and B if and only if S is a subsequence of both A and B. Moreover, we say that a sequence U is a *universal common subsequence* of A and B if and only if the following two conditions are met:

- U is a common subsequence of A and B.
- ullet Every common subsequence of A and B is also a subsequence of U.

It can be shown that any two sequences A and B have at most one universal common subsequence.

The researchers have found two sequences of hieroglyphs A and B. Sequence A consists of N hieroglyphs and sequence B consists of M hieroglyphs. Help the researchers compute a universal common subsequence of sequences A and B, or determine that such a sequence does not exist.

# Implementation details

You should implement the following procedure.

```
std::vector<int> ucs(std::vector<int> A, std::vector<int> B)
```

- A: array of length N describing the first sequence.
- B: array of length M describing the second sequence.
- If there exists a universal common subsequence of A and B, the procedure should return an array containing this sequence. Otherwise, the procedure should return [-1] (an array of length 1, whose only element is -1).
- This procedure is called exactly once for each test case.

### **Constraints**

- $1 \le N \le 100\,000$
- $1 \le M \le 100\,000$
- $0 \leq A[i] \leq 200\,000$  for each i such that  $0 \leq i < N$
- $0 \leq B[j] \leq 200\,000$  for each j such that  $0 \leq j < M$

### **Subtasks**

Subtask	Score	Additional Constraints
1	3	N=M;A and $B$ both consist of $N$ distinct integers between $0$ and $N-1$ (inclusive)
2	15	For any integer $k$ , the number of elements of $A$ equal to $k$ plus the number of elements of $B$ equal to $k$ is at most $3$ .
3	10	$A[i] \leq 1$ for each $i$ such that $0 \leq i < N$ ; $B[j] \leq 1$ for each $j$ such that $0 \leq j < M$
4	16	There exists a universal common subsequence of $A$ and $B$ .
5	14	$N \leq$ 3000; $M \leq$ 3000
6	42	No additional constraints.

# **Examples**

### Example 1

Consider the following call.

```
ucs([0, 0, 1, 0, 1, 2], [2, 0, 1, 0, 2])
```

Here, the common subsequences of A and B are the following:  $[\ ]$ , [0], [1], [2], [0,0], [0,1], [0,2], [1,0], [1,2], [0,0,2], [0,1,0], [0,1,2], [1,0,2] and [0,1,0,2].

Since [0,1,0,2] is a common subsequence of A and B, and all common subsequences of A and B are subsequences of [0,1,0,2], the procedure should return [0,1,0,2].

#### Example 2

Consider the following call.

```
ucs([0, 0, 2], [1, 1])
```

Here, the only common subsequence of A and B is the empty sequence  $[\,]$ . It follows that the procedure should return an empty array  $[\,]$ .

### Example 3

Consider the following call.

```
ucs([0, 1, 0], [1, 0, 1])
```

Here, the common subsequences of A and B are  $[\,],[0],[1],[0,1]$  and [1,0]. It can be shown that a universal common subsequence does not exist. Therefore, the procedure should return [-1].

## Sample Grader

Input format:

```
N M
A[0] A[1] ... A[N-1]
B[0] B[1] ... B[M-1]
```

**Output format:** 

```
T
R[0] R[1] ... R[T-1]
```

Here, R is the array returned by ucs and T is its length.