Modelling the Trojan Asteroids

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Trojan asteroids are v important. I'm going to make that paragraph a little longer to fill out the space in case this is forming some kind of error. i really hope this is long enough.

1 Introduction

The Jupiter trojans, commonly known as the Trojan asteroids, are two large groups of asteroids that share the planet Jupiter's orbit around the Sun. These two groups are called the Greeks and the Trojans, named after opposing sides in the mythological Trojan war, and lead/trail Jupiter respectively in its orbit. They correspond to Jupiter's two stable Lagrange points: L4, lying 60° ahead of the planet in its orbit, and L5, 60° behind, with asteroids distributed in two elongated, curved regions around these Lagrangian points. CITE

These Lagrange points are taken from Lagrange's initial analysis in 1772 [1].

The first Jupiter trojan discovered, 588 Achilles, was spotted in 1906 by German astronomer Max Wolf.[2] A total of 7,040 Jupiter trojans have been found as of October 2018.[3] By convention, they are each named from Greek mythology after a figure of the Trojan War, hence the name "Trojan". The total number of Jupiter trojans larger than 1 km in diameter is believed to be about 1 million, approximately equal to the number of asteroids larger than 1 km in the asteroid belt.[1] Like main-belt asteroids, Jupiter trojans form families.[4]

1.1 Trojan Asteroids

previous lit Alternatively we consider [2]

1.2 Orbit Geometry

Including mathematical assumptions in analysis

1.2.1 Assumptions

Circular orbit Constant Jupiter-Sun separation Planar orbit Negligible asteroid mass Newtonian gravity

2 Methodology

2.1 Integration Method

The default solver is RK45 (an explicit Runge-Kutta method of order 5(4) [3]) however this is non-stiff, giving a deviation in asteroid position (from the Lagrange point) in the order of 10^{-4} AU in the rotating frame over 50 years. This is larger than expected, suggesting the system of equations requires an unreasonable small step size for numerically stability with respect to this numerical method, even regions where the solution curve is smooth [4]. This suggests the system is stiff, and solvers designed for this typically do more work per step, allowing them to take much larger steps, and have improved numerical stability compared to the non-stiff solvers [5].

Instead the stiff "Radau" solver (an implicit Runge-Kutta method of the Radau IIA family of order 5 [6]) is used for increased stability [7], and achieves a deviation in asteroid position in the order of 10^{-13} AU instead. This also ensures stability in the rotating frame, with deviations of 0.76% in asteroid separation from Jupiter over

10^3 years, compared to 53% for the best non-stiff solvers.

3 Results

3.1 Orbit Stability

3.2 Wander Analysis

3.2.1 Perturbations in z-direction

4 Discussion

5 Conclusion

References

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