



2016

Modeling Extrasolar Trojan Asteroids in Gravitational Potentials of Migrating Jovian-like Planets to Inform Future Observations

Austin Hinkel

University of Kentucky, austin.hinkel7@gmail.com

Right click to open a feedback form in a new tab to let us know how this document benefits you.

Follow this and additional works at: <https://uknowledge.uky.edu/honprog>

 Part of the [External Galaxies Commons](#), [Other Astrophysics and Astronomy Commons](#), [Physical Processes Commons](#), and the [Stars, Interstellar Medium and the Galaxy Commons](#)

Recommended Citation

Hinkel, Austin, "Modeling Extrasolar Trojan Asteroids in Gravitational Potentials of Migrating Jovian-like Planets to Inform Future Observations" (2016). *Lewis Honors College Capstone Collection*. 22.
<https://uknowledge.uky.edu/honprog/22>

This Article is brought to you for free and open access by the Lewis Honors College at UKnowledge. It has been accepted for inclusion in Lewis Honors College Capstone Collection by an authorized administrator of UKnowledge. For more information, please contact UKnowledge@lsv.uky.edu.

Modeling Extrasolar Trojan Asteroids in Gravitational Potentials of Migrating Jovian-like Planets to Inform Future Observations

Austin Hinkel
Department of Physics & Astronomy
University of Kentucky
Honors Capstone: Senior Thesis
5 May 2016

Abstract:

In this paper, I construct a program to map the evolution of the potential in a planet-star system where a planet with a few Jupiter masses migrates inward. Given a trojan asteroid librating around the fourth or fifth Lagrange point, the asteroid follows the evolving equipotential lines of the slowly changing potential map. As the planet and its trojan asteroids migrate inward towards the host star, the trojan asteroid librations become tighter, providing a denser “cloud” of trojan asteroids. Such a change in the density of trojan asteroids is examined with the intent to deduce the likelihood of detection via the transit method. As the librations become tighter due to an inward planetary migration, a “deeper” transit depth is predicted for the transit method of extrasolar object detection due to a higher fraction of the asteroids transiting simultaneously.

I. Introduction & Motivation:

First discovered in 1801, asteroids have fascinated humanity for centuries. Indeed, our civilization is intimately tied to these celestial objects. It is theorized that asteroids and their comet relatives were responsible for depositing water and organic material onto the early Earth some billions of years ago.

Though possibly responsible for delivering the building blocks of life, asteroids and comets have also been theorized to inhibit life. The end of the Late Heavy Bombardment (LHB), which occurred approximately 700 million years after the Earth formed ^[1], seems to have coincided with the rise of life on Earth. One could reasonably conclude, as many have, that the constant barrage of asteroids made life impossible.

The LHB has also been shown to coincide with the migration of the giant planets in our solar system ^[1]. Such a finding provides an opportunity to learn more about solar system formation in general by examining the “leftovers” and how they interacted with the planets and planetesimals. As such, I look specifically at “Trojan Asteroids” (asteroids, which orbit in the fourth and fifth Lagrange Points, L4 and L5) of Jovian-like planets as the planets migrate. The comparison of behavior of

simulated asteroids and those of our own solar system can then inform us of the way the planets behaved in the early solar system.

Clearly, asteroids and other small bodies pose an intriguing topic to study in order to determine our solar system’s past, but they have also been quite a hot topic recently. As a new paper by Dr. Renu Malhotra et. al. explains, the orbital resonances of long period Kuiper Belt Objects (KBO) are consistent with the existence of a “Planet Nine,” as proposed (to much public fanfare) by Batygin and Brown in early 2016 ^[2,3]. Such a huge discovery on the forefront of science involved inferences (not so different from the ones I propose) from these small asteroids and comets. Not bad for so called leftover pieces of planetary formation.

Another instance of asteroids in the recent science spotlight is the formation of NASA’s Planetary Defense Coordination Office, which aims to precisely track all asteroids with even the remotest chance of impacting the Earth. Perhaps by learning about the stability or instability of Trojan asteroids in our own solar system, we will be able to determine if any danger exists from potentially ejected Trojan asteroids. Conceivable application of my results in this field would be the definition of practical.

The Trojan-Group of Asteroids:

Let us digress for a moment and delve deeper into the topic of trojan asteroids. For those with knowledge of these objects, feel free to skip this short section.

Trojan asteroids were actually predicted by Joseph-Louis Lagrange over a century before the first instance of observation in 1906. The first trojan asteroid discovered, 588 Achilles, was found to be on the same orbital trajectory as Jupiter around the sun at 60 degrees of arc from Jupiter, exactly as Lagrange predicted ^[4]. It is no surprise, then, that the Lagrange points (places where the potential is at an extrema) are named for the man who predicted such a complex phenomenon.

The trojan asteroids are named after famous Greeks and Trojans from the Trojan War, and include well known names like Odysseus, Hector, Achilles, Agamemnon, and the like ^[4]. Each Lagrange point is home to the different camps of “Greeks” and “Trojans.” Collectively, though, the asteroids are simply referred to as the trojan asteroids, or trojan-group asteroids.

Research Motivation and Outline:

Simulations have shown how our own solar system dynamics may have affected the Trojan asteroids of Jupiter ^[5], but what about applying this to other stellar systems and using those results to inform future observations? With the coming of next generation space telescopes like the 6.5-meter James Webb Space Telescope (JWST) and the 11.7-meter High-Definition Space Telescope (HDST), the potential for discovering swarms of smaller bodies in other solar systems increases dramatically ^[6]. It is therefore of utmost importance that the scientific community is able to predict what objects and phenomena can be observed with these new tools. This is the quintessential reason for this report.

Eventually, with improving observational abilities, astronomers will be able to find an ensemble of planetary systems at various stages of evolution. This fact affords us the ability to see the evolution of stellar systems in snapshots,

and seems to be the only way around the not-so-trivial problem of waiting hundreds of millions of years to watch a given system evolve. If large clouds of Trojan asteroids are indeed observable by then, as I hope to predict in this paper, then stellar system evolution theory will likely experience an astronomical leap forward.

Perhaps studying present day trojan asteroids will prove to be a way to deduce the past evolutionary activity of a stellar system. If this turns out to be true, astronomers might ultimately be able to learn about the origins of stellar systems, the paths towards a stable planetary system, the proto-planetary disk, and other very interesting topics in planetary science. While this is likely decades away, developing predictions and bounds ahead of time may be beneficial.

As one can see, there are observational, practical, and theoretical motivations for learning more about trojan asteroids in migrating planetary systems. The astronomical community stands to learn a great deal from the dynamics of these asteroids and other small bodies.

In this paper, I create a program to model the changing gravitational potential of a star-planet system to determine whether or not the trojan asteroids are retained upon slow migration of the planet. With the results from this program, I attempt to draw conclusions about the possibility of observation of large swarms of exo-trojan asteroids, to wit: Trojan asteroids around other stars. Should we expect to find Trojan asteroids accompanying a Hot-Jupiter? Do orbits of Trojan asteroids become unstable during fast planetary migration? What parameters determine the stability of a Trojan asteroid during planetary migration? What factors determine the population of the Lagrange points? Can early evolutionary activity be extrapolated from present day trojan asteroid conditions? These are a few of the questions that can be conceivably answered from the program created for this report or from future modifications of this type of program.

II. Theory:

For a large planet orbiting its parent star, there exists a sizeable region of space where

asteroids can accumulate into stable orbits. These points are called the fourth and fifth Lagrange Points, or L4 and L5 for short. These specific Lagrange points are considered “stable” but are actually maxima in the potential map. Stable orbits (also called librations) may seem counter intuitive, but one must also consider the Coriolis effect which is not able to be included in the potential, as it cannot be written as a gradient of a potential. It is this Coriolis force that “bends” the orbits of asteroids into elliptical orbits about the L4 and L5 points.

First, let’s look at the gravitational potential of the star-planet system. Note that for this paper, I use the potential, which is the potential energy per unit mass, such that asteroids of various masses can be input later. Taking into account the potential from the presence of both the star (primary) and the planet (secondary), we arrive at a gravitation potential, V_{grav} , such that ^[7]:

$$V_{grav} = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2} \quad (1)$$

The gravitational potential, however, is not the whole story. Now we must also include the “fictitious” centripetal force introduced by going to the rotating frame of reference, so that we as the observers are rotating at the same angular velocity as the planet around its host star. This “effective potential” is then added to the gravitational potential to arrive at the overall potential, V_{tot} ^[7]:

$$V_{tot} = V_{grav} - \frac{1}{2} * \omega^2 r'^2 \quad (2)$$

Where r' is the vector pointing from the center of mass of the system to the point of interest (namely the location of the tertiary, the trojan asteroid) and the Greek letter Omega is the angular speed.

As mentioned previously, the potential does not fully describe the situation, as the Coriolis force cannot be written as a gradient of the potential. The Coriolis force acts on the asteroids based on their velocity and their trajectory so that the asteroids curve into stable, orbits about the L4 and L5 points. Even though

the potential is a maximum near these points, a particle placed at the maximum will want to slide “down” the potential, which in turn causes the Coriolis force to bend the particle’s orbit into a *clockwise* orbit by the right hand rule (mind the minus sign). The total force acting on the asteroids is thus ^[7]:

$$\mathbf{F} = -2m\boldsymbol{\omega} \times \mathbf{v} - m * \nabla V_{tot} \quad (3)$$

Where F is the force, m is the mass of the asteroid, Greek letter Omega is the angular velocity vector, v is the velocity vector of the asteroid, and V is the effective potential (potential energy per unit mass) as detailed above.

Please note that in the regions where the potential is nearly flat the Coriolis force dominates. This condition can be satisfied in the region of L4 and L5, so the force on a trojan asteroid is entirely governed by the Coriolis force. Because the Coriolis force acts *perpendicular* to the velocity, the kinetic energy of the trojan asteroids remains constant, as ^[7]:

$$\mathbf{F} \cdot \mathbf{v} = 0 \quad (4)$$

The above condition occurs for asteroids with low velocities with respect to the rotating frame of reference (Note that such velocities for the Sun-Jupiter system can be calculated from Szebehely’s *Theory of Orbits* ^[8]). Because the kinetic and potential energies do not change appreciably (as a consequence of the perpendicular force and velocity vectors, see equation 4 above), the trojan asteroids can be expected to orbit along the contour, or equivalently the equipotential, lines of the potential map, as is outlined later in the report. This is the physical reason for the existence of the trojan group of asteroids. Indeed, as the Jupiter-specific program by Fowles and Cassiday shows, trojan asteroids do indeed orbit along the contour lines of the potential map, with librational periods on the order of hundreds of years ^[7].

III. Method:

In order to better understand what happens to trojan asteroids as their host planet migrates inward towards its host star, I have constructed a MATLAB program, based on a simple, open source, preexisting program [8]. First, I built a potential map and updated it for a slowly migrating gas giant, with the assumption that the planet remained in a roughly circular orbit. Additionally, I used these data to create contour and 3-dimensional plots to better illustrate the potential in the regions of L4 and L5. I have included a contour map below to qualitatively illustrate the potential:

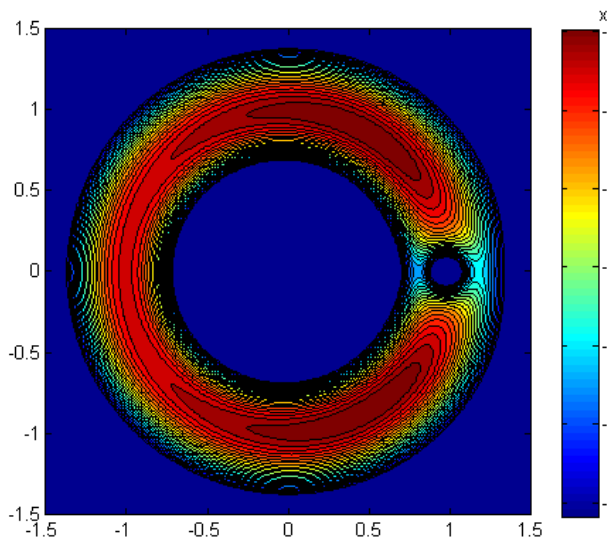


Figure 1: A 2D contour map of the potential.

Here is the 3-dimensional view of the same potential:

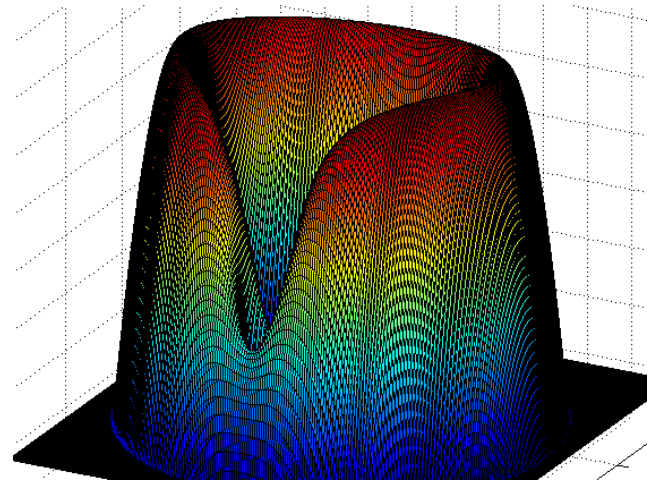


Figure 2: A 3D rendering of the potential map.

With the above qualitative potential maps in hand, I then added in a more physical representation complete with a system very much like the Jupiter-Sun system. The program input for the mass of a planet is in Jupiter masses and is easily changed for various planetary and stellar masses. Additionally, the unit of distance in my program is the Jupiter-Sun distance, and can be adjusted to simulate planetary migration.

By looping the program to run for successively smaller values of planet-star separation, the evolution of the potential map can be visualized and quantified. Because the trojan asteroids follow the equipotential lines, a simulated orbit is plotted on the initial potential map, and is then evolved with the potential maps for each successive value of planet-star distance. These lines, plotted in green in the example below, show the trojan asteroid's orbital change over time.

In this example, the first figure shows the asteroid's orbit whilst at one Jupiter distance, while the second figure shows the orbit after the planet migrates one-tenth of one percent of its original distance from the sun. Clearly, there appears to be a significant change in the libration of the trojan asteroid:

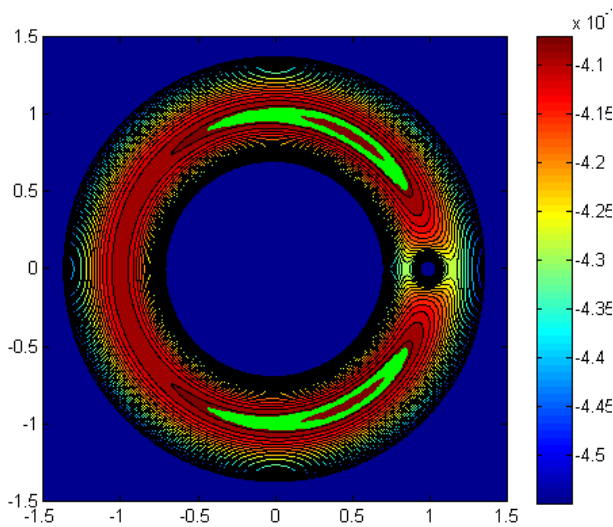


Figure 3: Trojan Asteroid orbits before migration.

Which after migration becomes:

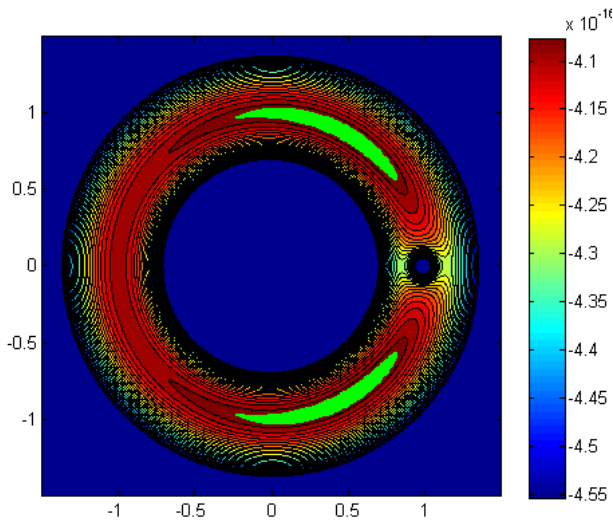


Figure 4: Trojan asteroid orbits after a slow planetary migration to 99.9% the original planetary orbital distance.

The next programmatic consideration I would like to point out is that the entire simulation is fit onto a grid. Obviously, MATLAB's interpolations can only go so far, so the number of grid points, hereafter called the resolution, is directly related to the precision to which the simulation can describe the system. For the qualitative examples above the grid was 400 by 400 points.

Finally, as with any simulation, there are a few assumptions that must be acknowledged. First, the Lagrange point derivations follow from the restricted 3-body problem, so it is implicitly assumed that the three bodies are constrained to a plane and that the primary and secondary masses are much more massive than the tertiary mass. Circular orbits are also assumed [7].

Because circular orbits are assumed, it follows that the planetary migration must be extremely slow to preserve such orbits. Without an approximately adiabatic change in orbit, circular orbits would not be likely after a migration. Thus it is doubtful that trojan asteroids would survive a catastrophic change in orbit, which would most certainly be non-circular. Because of this fact, it only makes sense to study the interesting part of the problem: the case of adiabatic migration.

IV. Results:

Upon the execution of my program, many relevant results were readily apparent. Firstly, the libration of the trojan asteroids around their Lagrange points actually compressed so that they became more densely distributed over a slow and short planetary migration. This can easily be seen in a qualitative sense in Figures 3 and 4 above.

For a more quantitative analysis, let us first consider a few things. First, just by virtue of the new configuration being closer to the host star, the arc that the trojan asteroids occupy must shrink. In other words, when an angle subtends two concentric circles, the arc in the inner circle must be smaller than that of the outermost arc. Because of this, one should expect the arc length to have a linear dependence in R .

This simple linear dependence, however, is not all: the librational compression shown qualitatively above also clearly depends on R . Originally, I had assumed a power law relation between the radius and "librational arc length," which I define as the arc length in the planet's orbit that the trojan asteroids populate. However, after examining this arc length at numerous stages in the migratory process, it became apparent that the librational arc length was much more complex than that. Indeed, as evident in the

figure below, the relationship between migratory distance and librational arc length looks almost like a trigonometric function of sorts:

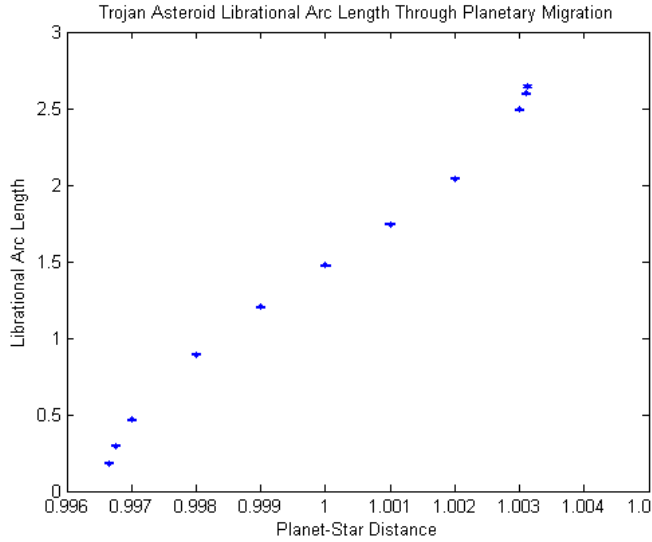


Figure 5: Librational Arc Length vs. Planet-Sun distance for a planet on the order of 5 times the mass of Jupiter

After further evaluation, I think that the relationship resembles a translation of an inverse sinusoid or inverse cosine function. The trigonometric relation makes sense from a mathematical point of view, as the librational arc length, "S," is actually just the angle from the sun that the trojan swarm subtends (in radians) multiplied by the planet-sun distance in the migrating system:

$$S = R\theta \quad (5)$$

Thus it is apparent that the librational arc length is not just an inverse trigonometric function that depends on R in some capacity, but also has an extra factor of R in the relation. This extra factor of R is simply from the smaller orbit as mentioned above. Once a fit can be determined for this rather exotic function, it makes sense to define an asteroid "arc density," where the number of asteroids, N, per length of arc is given as:

$$\rho = \frac{N}{S} \quad (6)$$

Clearly, as the librational arc length decreases upon inward planetary migration as

shown above, the arc density increases accordingly. Now, with a more densely packed extrasolar trojan asteroid population, more of these asteroids will transit the host star at the same time, as seen from the Earth. For example, the Jupiter L4 swarm of Trojan asteroids is estimated to have a total cross sectional area of about $5 \times 10^{12} m^2$ according to a calculation done by Jewitt et al ^[9].

Let's do a quick estimation calculation of transit observation potential. In an initial orbit, before planetary migration, perhaps only 2% of asteroids may transit the host star at a time. However, if the asteroid orbits are compressed, as my program seems to suggest, maybe 20% of the asteroids could transit simultaneously. In this example scenario, the cross sectional area is larger for the post short migration planet by a factor of ten. In this estimate, the cross sectional area would be about $10^{12} m^2$, or roughly a tenth of that of Mercury. Now, while I don't expect Mercury sized objects to be routinely discovered in the present time, it is possible that such findings will occur in the near future, as motivated in the introduction section of this report. Additionally, if planets larger than Jupiter can hold on to more asteroids, there stands a chance that the transiting cross sectional area will be even greater still, corresponding to a deeper transit depth.

Finally, please note, that my program does not address what happens to the asteroid libration when the contour line the asteroid follows ceases to exist due to a long enough migrational distance. Do the asteroid orbits simply become unstable? Is there some sort of mechanism that I haven't considered that becomes more crucial as the orbits tighten? If the asteroids are indeed ejected at that point, that will certainly imply that the cross sectional area would *decrease* as some orbits become unstable while *increasing* as the still stable orbits compress. Perhaps an N-body simulation with many different librational paths will be able to answer this question.

V. Conclusions:

Ultimately, this report is but a starting point for future research. It should be clear by

now that there are many interesting questions surrounding trojan asteroids in migrating planetary systems. Moreover, the program created for this report can feasibly be altered in a multitude of ways for other, novel simulations. One such simulation I hope to try in the future is the actual modeling of the trojan asteroids themselves to not only confirm the adiabatic approximation offered in the theory section, but to determine the time scales needed for a “slow enough” migration to retain trojan asteroids. Additionally, migrating a Jupiter-like planet through a simulated asteroid belt would also be a worthwhile simulation that could provide insights into the conditions for stable orbits around the fourth and fifth Lagrange points and the mechanisms for their capture.

For this experiment, though, there is still more to be said. From my results, I have learned that trojan-group asteroids are very fickle objects. Indeed, under even short distance, adiabatic planetary migration, the Trojans seem, surprisingly, to be quite unstable. This result seems to agree qualitatively with Gomes’s analysis for primordial Saturn specific trojan asteroids ^[5].

If asteroids seem to be ejected under short migratory distances even during long adiabatic migration processes, one must conclude that a gas giant with trojan asteroids inside the frost line would have had to capture new asteroids during the migration process. In other words, finding trojan asteroids with a Jovian-type planet that exists considerably closer to its star than where it could have possibly formed could actually imply the existence of an asteroid belt in that exoplanetary system.

Now, the above paragraph only makes sense if the migration happened well after the initial formation of the stellar system. Otherwise, the planetesimals would have conceivably been (almost) ubiquitously present for capture in the stable Lagrange points. This scenario is also of interest, though. If the migration happened very early in the exoplanet’s lifetime, the planetesimals captured in the Lagrange points during the early migration would be a time capsule of sorts for the building blocks of that particular system. If spectral-transit methods could be developed to detect the chemical

makeups or isotopic ratios of these bodies, astronomers would gain a huge insight into the diversity (or lack thereof) of exoplanetary system chemistry. Such a finding could herald a new approach to finding life-capable systems around other suns.

While such abilities mentioned above do not seem to be available today, my predictions will hopefully be testable in the near future. I contend that Trojan-group asteroids will be a vital-sign of nascent and old exoplanetary systems alike: a way to tell the past of the system or even its chemical makeup. Such a tool would be invaluable, and I propose that other such possible indicators of planetary migration history be researched.

In light of the above speculation, it is beneficial to predict the observational minutiae of exo-trojan asteroids. Clearly, if the arc that the trojan asteroids can populate decreases in size upon inward planetary migration as my simulation suggests, the density of the asteroids thus increases. With this more densely compacted configuration, it is apparent that the transit depth of such a swarm of trojan asteroids increases. For example, when the asteroids are in their original orbits before migration, their sparse distribution ensures that only a small fraction of the asteroids will transit their star at once. However, should a slow planetary migration occur like the one modeled in this paper, the higher “arc-density” of asteroids implies that a higher fraction of the trojan asteroids will transit the star simultaneously. To wit: the transit depth of the asteroids will indeed increase as calculated above.

Next Steps:

Perhaps the greatest feature of my simulation is that any parameter can be varied to learn more about the orbits of Trojan asteroids. Throughout the course of my study of this topic, I’ve come upon a myriad of interesting questions. For example, if an N-body simulation of orbits were run in my migrating potential, would the compression of orbits lead to a higher collision rate? It seems to me that a picture of the situation wouldn’t be too far from the ideal gas model. It would seem that the compression of

orbits would act like a pressure and the collision of the asteroids would be like a temperature. Obviously, the comparison isn't sterling, but it provides a starting point for thinking about the physical situation. Such a model could be useful for modeling ejections of the Trojans.

Another test I would like to run is migrating an empty Lagrange point through an asteroid belt to determine the capture parameters and to find a sort of "dispersion" curve that could inform our understanding of events in our own early solar system like the Late Heavy Bombardment. As was mentioned earlier, the migration of Jupiter seems to have coincided with the LHB. It is of understandable interest, then, for this topic to be studied. Indeed, it could even lead to a better knowledge of potential present day impact events from gravitationally perturbed Trojan asteroids. Such information would conceivably be of great use for NASA's Planetary Defense Coordination Office.

The questions are endless. Do more massive planets more fiercely "hang on" to asteroids during the migratory process? Are they able to capture asteroids more easily? One could even envision a simulation of this type to determine if the fourth or fifth Lagrange points are more likely to capture asteroids. Does the leading Lagrange point capture more asteroids, or does the trailing one?

Additionally, how does mass affect the librational arc length as defined earlier in this report, and thus transit depth? Are the timescales needed for a migration to be adiabatic, or "slow enough" altered for different masses of planets? Can the program be used to model the Lagrange points' effects on planetesimals in the early solar system and could the migrating planet be a mechanism for populating exo-Kuiper belts or exo-Oort clouds?

There exist many more such interesting questions just waiting to be uncovered. Because of this, I believe that the study of Trojan asteroids and migrating planets will be a fruitful field of discovery in the future. It is only a matter of time before we are able to detect these swarms of trojan asteroids.

Appendix:

To provide a concrete example of Trojan Asteroids in our own solar system, I referenced Szebehely's *Theory of Orbits*, in which a few trojan asteroids are tabulated along with their orbital parameters. As an example calculation, Szebehely shows that a trojan asteroid has an orbital period around L4 or L5 that is on the order of 150 years long. For more information regarding trojan asteroids specific to our own solar system, the interested reader is referred to the aforementioned source as well as the following trojan asteroids as case studies: 1647 Menelaus, 588 Achilles, & 1143 Odysseus ^[8].

Please note that all of my programs and figures are available upon request. Please contact me at aphi224@g.uky.edu for these requests.

Finally, I would like to extend my sincerest thanks to Dr. Ron Wilhelm at the University of Kentucky, for all of his help and without whom the idea for this report would likely never have been imagined.

References:

- [1] R. Gomes, H. F. Levison, K. Tsiganis, A. Morbidelli. "Origin of the Late Heavy Bombardment period of the terrestrial planets." *Nature*. Vol. 435. 26 May 2005. (doi:10.1038/nature03676)
- [2] R. Malhotra, K. Volk, X. Wang. "Corralling a distant planet with extreme resonant Kuiper belt objects." *ArXiv*. (arXiv:1603.02196v2)
- [3] K. Batygin, M. E. Brown. "Evidence for a Distant Giant Planet in the Solar System." *The Astronomical Journal*. Vol. 151 No. 2.
- [4] "Theory of Orbits" V. Szebehely. 1967.
- [5] R. S. Gomes. "Dynamical Effects of Planetary Migration on Primordial Trojan-type Asteroids." *The Astronomical Journal*. Vol. 116, pp. 2590-2597. 1998.

- [6] “From Cosmic Birth to Living Earths: The Future of UVOIR Astronomy.”
http://static1.squarespace.com/static/558adc44e4b002a448a04c1a/t/55b8c264e4b041851ef43883/1438171748257/hdst_report_final_072715.pdf.
- [7] “Analytical Mechanics.” G. R. Fowles & G. L. Cassiday. Seventh Edition. Section 7.4. 2005.
- [8] *Orbiter Forum*. Post by martins on February 5, 2009. <http://orbiter-forum.com/showthread.php?p=279860>.
- [9] D. C. Jewitt, C. A. Trujillo, J. X. Luu. “Population and Size Distribution of Small Jovian Trojan Asteroids.” *The Astronomical Journal*. 120: 1140-1147. August 2000.