

Quadratic equations

1. Quadratic polynomial:

$$ax^2 + bx + c = 0$$

2. . . . equation:

$$a \neq 0$$

Solve quadratic equations.

Method 1: factorise. e.g. $x^2 + 5x - 6 = 0$.

$$(x+6)(x-1)=0 \Rightarrow x=-6. x=1.$$

Method 2: quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\text{Sum of roots} = -\frac{b}{a}$$

Method 3: Complete the square.

$$ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x\right) + c$$

$$= a \left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 \right] + c$$

$$\left(\frac{\text{Coefficient}}{2} \right)^2$$

$$= a \left(x + \frac{b}{2a} \right)^2 - \frac{b^2}{4a} + c$$

$$\left(-\frac{b}{2a}, -\frac{b^2}{4a} + c \right)$$

vertex

Method 4: Solving without factorising

$$(x-1)^2=5 \Rightarrow x-1=\pm\sqrt{5} \therefore x=1\pm\sqrt{5}$$

Method 5

: $x - 6\sqrt{x} + 8 = 0$

$$(\sqrt{x})^2 - 6 \cdot \underbrace{\sqrt{x}}_u + 8 = 0$$

$$u^2 - 6u + 8 = 0$$

$$(u-2)(u-4)=0$$

$$1^\circ \therefore u=2=\sqrt{x} \Rightarrow x=4$$

$$2^\circ u=4=\sqrt{x} \Rightarrow x=16$$

Note: $x^2 - 5x = 0.$

$$x(x-5)=0.$$

Properties of roots : discriminant $= b^2 - 4ac$

"has real roots"

$$b^2 - 4ac \geq 0$$

$b^2 - 4ac > 0$. 2 real distinct roots

$b^2 - 4ac = 0$. 1 real root

$b^2 - 4ac < 0$. no real roots

Sketch quadratic

