

# Supplementary Materials for:

## Real-time 3D Ultrasonic Needle Tracking with a Photoacoustic Beacon

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## 1 Determination of Range Measurement Probability Density Functions

The maximum likelihood estimation multilateration method required knowledge of the probabilities of measuring all possible ranges, given a known true range and the signal amplitude. This section describes how the probability density functions describing this were determined.

### 1.1 Data Collection

The ultrasound tracking and imaging probe assembly was fixed at the surface of a water tank, with its face immersed and pointing vertically downwards. The needle was attached to a 3-axis motion stage and moved sequentially to 936 positions in a rectilinear grid aligned to the axial, lateral and elevational axes of the imaging system as described in Section 3.1 of the main paper. At each location, approximately 40 tracking frames were recorded, and each frame comprised 16 waveforms. In total, 577,520 waveforms were recorded.

### 1.2 Range Measurement

The index  $s$  of the peak sample in each waveform was measured and converted to a range  $r$  in millimetres:

$$r = c \cdot \delta t \cdot s \quad (1)$$

where  $c$  is the sound speed in water at room temperature ( $1480 \text{ m s}^{-1}$ ) and  $\delta t$  is the sampling interval of the digitiser (125 ns).

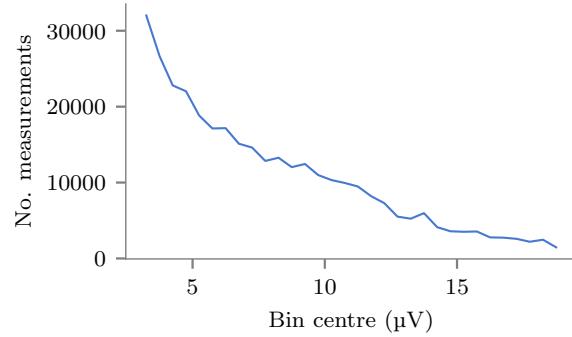
### 1.3 Range Measurement Error

For each of the 557,520 measured ranges, the true range from the location  $e$  of the tracking array element to the known needle tip location  $p$  was determined:

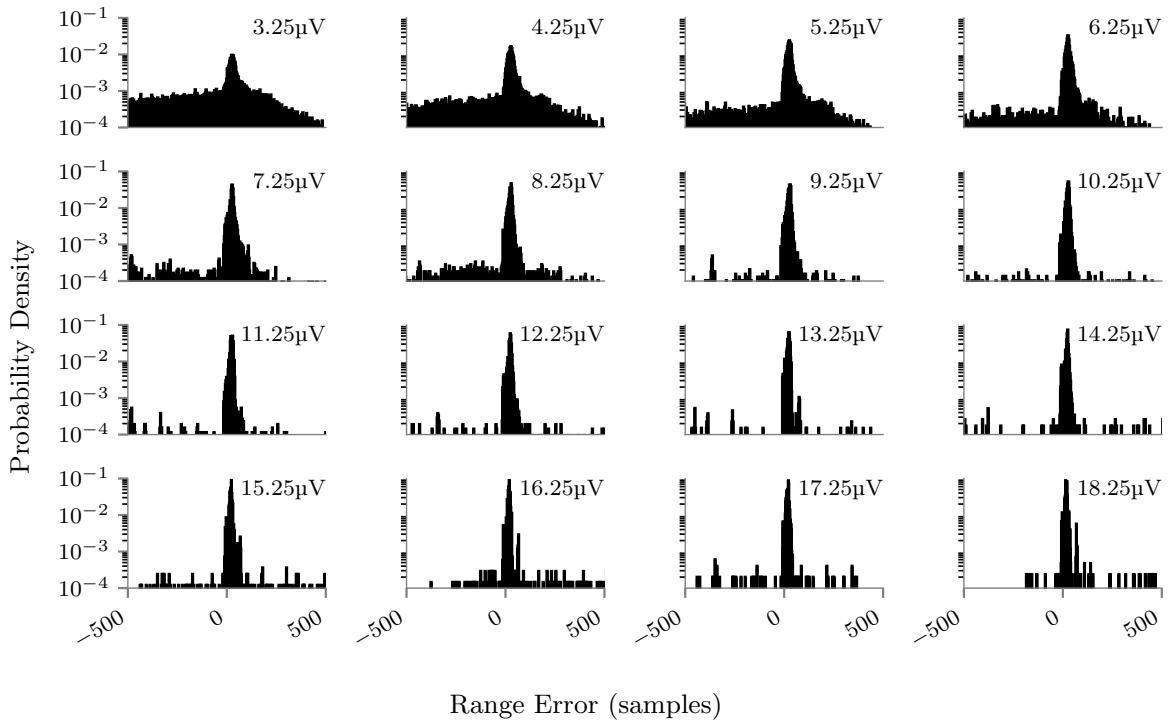
$$r_{\hat{p},i} = \|\hat{p} - e\| \quad (2)$$

where  $\|\cdot\|$  denotes the Euclidean norm. An associated error was calculated for each range measurement by subtracting from it the corresponding known true range. The 557,520 range errors were organised by the amplitudes of the waveforms from which they were derived. The analysis was performed on low-amplitude signals, for which noise has more influence on range measurement and therefore understanding of range measurement uncertainty is more critical; measurements from waveforms with amplitudes greater than  $19 \mu\text{V}$  were discarded. Measurements from waveforms with no pulse present (amplitude less than  $3 \mu\text{V}$ ) were also discarded. The remaining 338,925 range errors were collected into 32 amplitude bins between  $3 \mu\text{V}$  and  $19 \mu\text{V}$ , each bin being  $0.5 \mu\text{V}$  wide. The number of measurements

in each bin is plotted in [Figure S1](#). A histogram representing the distribution of error in each amplitude bin was then created; 16 of these are shown in [Figure S2](#).



[Figure S1](#): Number of measurements in each signal amplitude bin.



[Figure S2](#): Sixteen of the 32 range measurement histograms generated for determination of the range measurement probability density functions. Each histogram was generated from range measurements grouped into the signal amplitude bin shown above each histogram (bin mid-points are shown).

## 1.4 Gaussian Mixture Modelling

Each of the 32 histograms was fitted with a mixture of two Gaussians. The Gaussian Mixture Models were created using the SciKit-Learn Python library. For each histogram, the fitted Gaussian with the smallest variance was labelled as representing times when the ultrasound pulse was successfully detected by the range finding measurement, and the larger variance Gaussian as representing times when the ultrasound pulse was missed. An example of the fitted Gaussians is shown in [Figure S3](#).

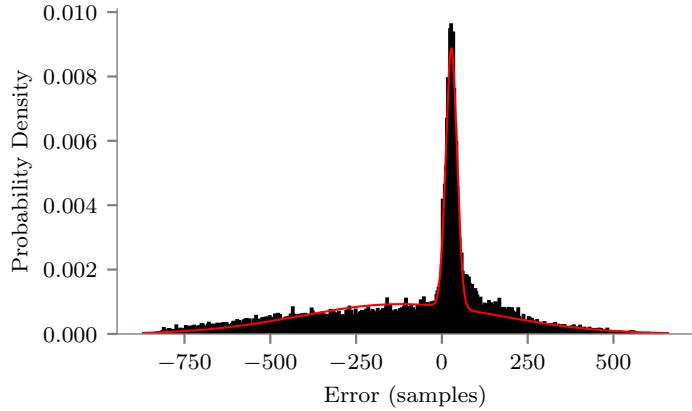


Figure S3: Range measurement histogram for the amplitude bin centred at  $3.25 \mu\text{V}$  fitted with a 2 component Gaussian mixture model.

## 1.5 Relationship Between Signal Amplitude and PDF Properties

The variance and relative weight of the “pulse detected” Gaussian components of the mixture model were plotted against signal amplitude. Variance was fitted with a exponential function of the form:

$$ae^{bx} + c. \quad (3)$$

Weight was fitted with a saturation growth function of the form:

$$a(1 - e^{-bx}) + c. \quad (4)$$

For both of the above equations,  $x$  represents signal amplitude and  $a$ ,  $b$  and  $c$  are constants. The fitted functions are plotted in [Figure S4](#). The variance of the “no pulse detected” Gaussians was independent of signal amplitude, as it related only to the measurement system; on average it was 10000 samples. These functions were used by the tracking application to dynamically generate probability density functions from measured signal amplitudes, which were provided to the Maximum Likelihood Estimation algorithm for multilateration.

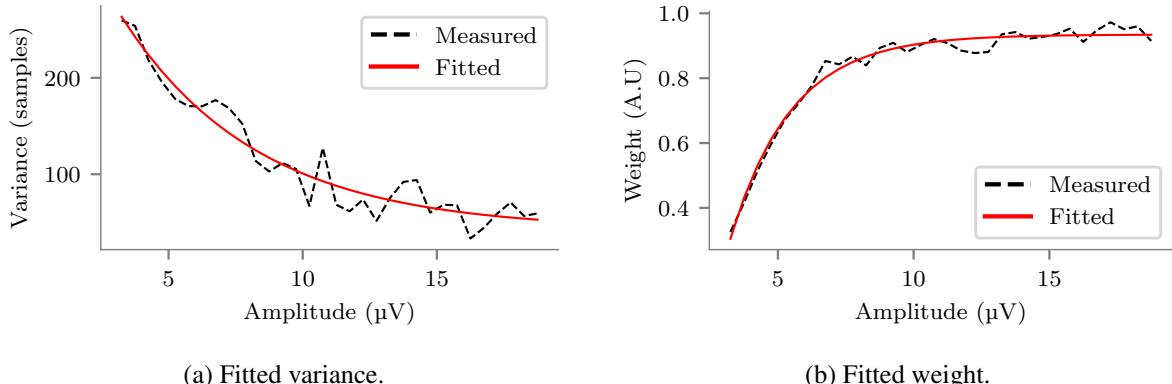


Figure S4: Fitted functions for success variance and success weight as a function of amplitude.

## 2 Tracking Accuracy in an *ex vivo* Tissue Phantom

A photograph of the *ex vivo* tissue phantom within the CT system gantry is shown in [Figure S6a](#). Volumes segmented from the CT data representing the probe assembly and multiple needle locations are

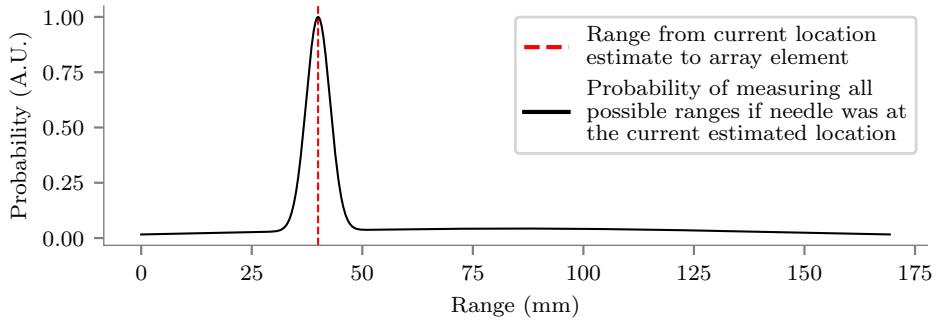


Figure S5: Example range measurement probability density function, describing the probability of measuring all possible ranges, given a true range of 40 mm and a signal amplitude of 5  $\mu$ V (the lowest accepted signal amplitude).

shown in [Figure S6b](#), annotated to show the CT-derived needle tip locations and corresponding tracked locations.

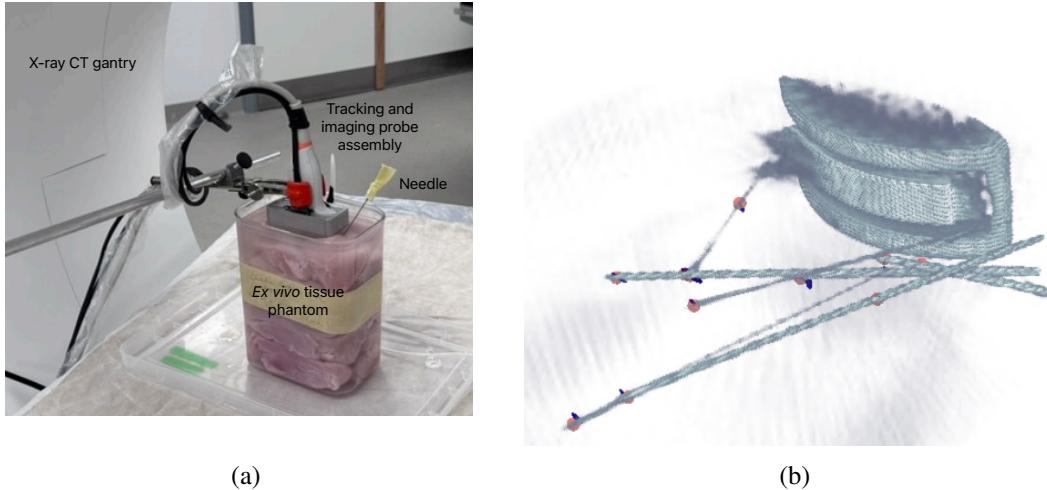


Figure S6: (a) Photograph of the *ex vivo* tissue phantom within the CT gantry during accuracy measurement. (b) Segmented tracking probe, imaging probe and needles annotated to show CT-derived tip locations (red circles) and tracked locations (blue ellipses).

### 3 Usability Tests Survey

The full list of statements in the Likert survey are shown in [Table 1](#) with their respective categories (used for the aggravated results) and topics (used in the individual results table).

Table 1: Full Likert survey statements with their respective categories and topics.

<b>Category</b>	<b>Topic</b>	<b>Statement</b>
Hardware	Stylet Fit in Needle	“The trackable stylet fit as well within the introducer needle as the solid stylet”
	Stylet Withdrawal Ease	“It was as easy to withdraw the trackable stylet from the introducer needle as the solid stylet”
	Cable Acceptability	“The fibre-optic cable between the trackable stylet and the tracking system did not affect my ability to carry out the procedure”
	Cable Length	“The fibre-optic cable between the trackable stylet and tracking system was the right length”
Imaging	Array Comfort	“The presence of the tracking array on the imaging probe did not affect my ability to carry out the procedure”
	Image Quality	“The ultrasound imaging feed was displayed on the tracking system with suitable image quality”
Visualisation	Satisfactory Latency	“Any noticeable latency on the imaging feed did not affect my ability to carry out the procedure”
	Tracking Clarity	“The tracking visualisation was clearly visible on top of the ultrasound imaging feed”
	Cursor Acceptability	“The tracking cursor did not obstruct the ultrasound image in a way that hindered my ability to perform the procedure”
Performance	Elevation Clarity	“I could easily interpret the distance of the needle tip from the imaging plane from the tracking visualisation”
	Satisfactory Latency	“Any noticeable latency of the displayed tracked location did not affect my ability to carry out the procedure”
	Satisfactory Rate	“The update rate of the tracking cursor was appropriate for the procedure”
	Depth Effect	“The depth of the needle tip within the phantom did not negatively affect the tracking accuracy”
	Angle Effect	“The insertion angle of the needle did not negatively affect the tracking accuracy”
Overall Impact	Satisfactory Accuracy	“The tracking was accurate enough to aid with performing the procedure”
	Tracking Benefit	“The presence of the tracking visualisation made it easier for me to carry out the procedure”

Table 2: Individual usability study survey results. Each statement used in the survey has been summarised here as a “topic”; see [Table 1](#) for the full statements. The categories used for the aggregated results in the main paper are shown. The ratings from “strongly disagree” to “strongly agree”, shown in the results figure in the main paper, are given integer values from -3 to 3 in this table, and the number of responses at each rating, for each statement, are shown.

Category	Topic	Rating					
		-3	-2	-1	0	1	2
Hardware	Stylet Fit in Needle				2	10	
	Stylet Withdrawal Ease			1	2	9	
	Cable Acceptability				2	10	
	Cable Length				3	9	
	Array Comfort		2		5	5	
Imaging	Image Quality					12	
	Satisfactory Latency			1	3	8	
Visualisation	Tracking Clarity				3	9	
	Cursor Acceptability			1	2	8	1
	Elevation Clarity			1	5	6	
Performance	Satisfactory Latency	1		1	4	6	
	Satisfactory Rate			1	4	7	
	Depth Effect	2	2		3	5	
	Angle Effect			2	2	8	
	Satisfactory Accuracy				5	7	
Overall Impact	Tracking Benefit			2	2	7	1