Problem 1.

a).

$$u_p(k) \longrightarrow U_p(Z) = K_p$$

$$u_I(k) \longrightarrow U_I(Z) = U_I(Z)\,Z^{-1} + K_i T$$

$$\Rightarrow U_I(Z) = \frac{Z}{Z-1} \cdot K_i T$$

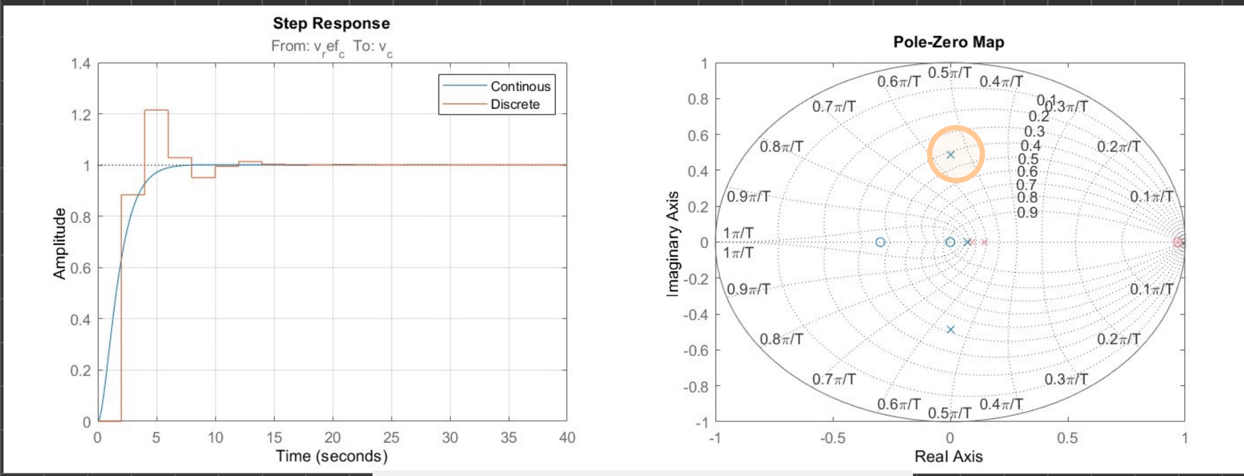
$$U_{PI} = \frac{K_p Z - K_p + K_i T Z}{Z - 1} = \frac{Z(K_p + K_i T) - K_p}{Z - 1}$$

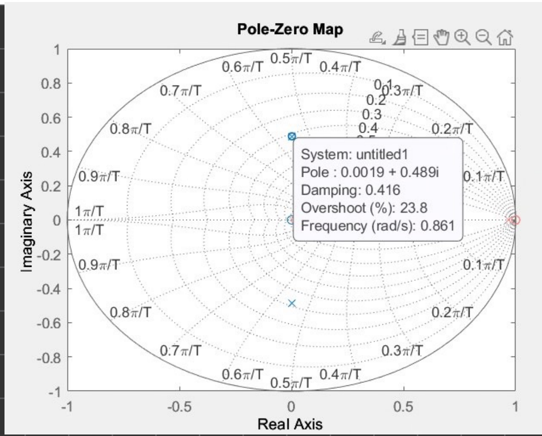
$$u_D(k) = \frac{K_D}{T}\left(e(k) - e(k-1)\right)$$

$$\hookrightarrow U_D(Z) = \frac{K_D}{T}\left(1 - Z^{-1}\right)$$

$$= \frac{K_D}{T} \cdot \left(\frac{Z-1}{Z}\right)$$

b, c, d, e, f).





Dominant Pole $\zeta = 0.416$ OS % = 23.8% $\omega_n = 0.861$

We look at left plot, it overshoot nearly 20% ↑.

Hence, fit ☺

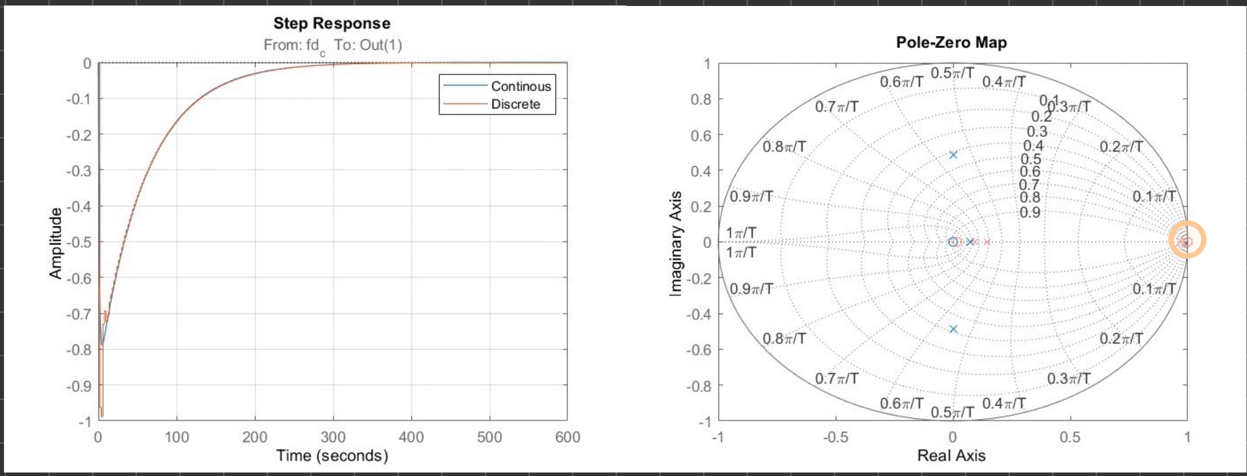
$$\tau = \frac{1}{0.416 \cdot 0.861} = 2.79 \text{ s}$$

However, sampling time is 2 sec, 2.79 s lay between 2 ~ 4 sec.

Therefore, we cannot distinguish by the plot.

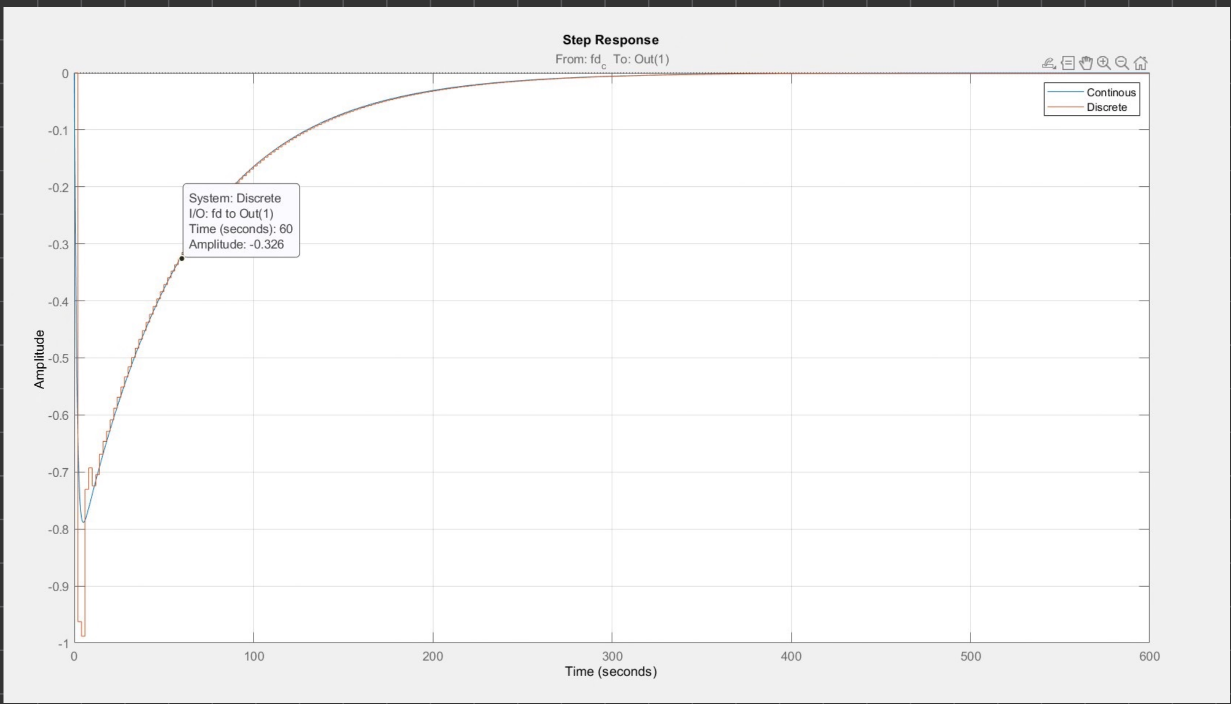
Continous system $z = e^{sT}$ all poles and zeros lie on

OS % = 0% line. Therefore, no overshoot.



Dominant Pole $\zeta = 1$ $OS\% = 0\%$ $\omega_n = 0.0164$

$$\tau = \frac{1}{0.0164} = 60.97 \text{ sec}$$



Problem 2.

a)

Using $z = e^{sT}$ $\quad s = -0.3 \pm 1.2j \quad T = \pi$

Nyquist frequency $\frac{\pi}{T} = 1 \text{ rad/s}$ (Img. Part

By observation, $s = -0.3 \pm 1.2j$ is out of primary strip.

Hence, convert it into $s = -0.3 \pm (1.2 - \frac{2\pi}{T})j$

Therefore, $s_p = -0.3 \pm 0.8j$

MATLAB Drive > ON I PAD > InClass.m

```
close all, clear all, clc

s = -0.3 - 0.8i;

T = pi;

z = exp(s*T)

w_n = abs(log(z)/T)

zeta = -cos(angle(log(z)))

tau = 1/(zeta*w_n)

PO = 100*exp(-pi*zeta/sqrt(1-zeta*zeta))
```

```
z =

  -0.3152 - 0.2290i

w_n =

    0.8544

zeta =

    0.3511

tau =

    3.3333

PO =

   30.7864

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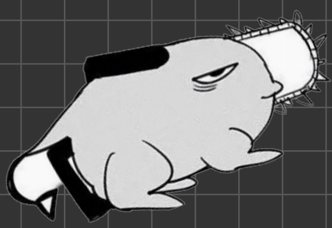
Problem 3.

$$G(Z) = \frac{1}{Z + 0.5} \qquad dt = 0.1 \qquad \omega = 20\ ^{rad}/_{s}$$

a). Poles $Z = -0.5$ Stable

b).

$$\Omega = 20 \cdot dt = 2\ ^{rad}/\ \text{sample}$$



Knowing $Z = e^{j\Omega} = e^{2j}$

Therefore, $G(z) = \dfrac{1}{e^{2j} + 0.5}$. Magnitude $= |G(Z)|$

$$e^{2j} + 0.5 = \cos 2 + j \sin 2 + 0.5$$

$$= \underbrace{(\cos 2 + 0.5)}_{\text{Real Part}} + \underbrace{j \sin 2}_{\text{Img Part}}$$

$$|G(z)| = \frac{1}{\sqrt{7.0314 \times 10^{-3} + 0.8268}} = 1.0951$$