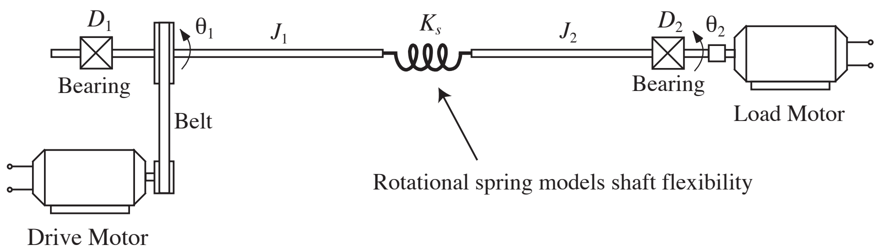
Homework #6

Reading: Chapter 6

1:



The apparatus for a DC motors experiment is represented schematically above. The voltage applied to the Drive Motor's amplifier, e_1, is the control input. The voltage applied to the Load Motor's amplifier, e_2, is the disturbance input. A controller is to be designed to make the angle, θ_2, of the load motor end of the flexible shaft follow a reference angle, θ_{2ref}, in spite of the disturbance e_2. Only θ_{2ref} and θ_2 are measured. The dynamics of the system are represented by:

$$L_1 \frac{di_1}{dt} = -\left(R_{A1} + R_1\right) i_1 - n\, K_1 \frac{d\theta_1}{dt} + K_{A1}\, e_1$$

$$L_2 \frac{di_2}{dt} = -\left(R_{A2} + R_2\right) i_2 - K_2 \frac{d\theta_2}{dt} + K_{A2}\, e_2$$

$$\left(J_1 + n^2 J_{m1}\right) \frac{d^2\theta_1}{dt^2} = n\, K_1\, i_1 - \left(D_1 + n^2\, D_{m1}\right) \frac{d\theta_1}{dt} - K_s \left(\theta_1 - \theta_2\right)$$

$$\left(J_2 + J_{m2}\right) \frac{d^2\theta_2}{dt^2} = K_2\, i_2 - \left(D_2 + D_{m2}\right) \frac{d\theta_2}{dt} + K_s \left(\theta_1 - \theta_2\right)$$

where

- L_1, L_2 are the drive and load motor armature inductances (H)
- i_1, i_2 are the drive and load motor currents (A)
- R_{A1}, R_{A2} are the drive and load motor amplifier resistances (Ω)
- R_1, R_2 are the drive and load motor armature resistances (Ω)
- K_1, K_2 are the drive and load motor back EMF constants (V/rad/sec)
- K_{A1}, K_{A2} are the drive and load motor amplifier gains (dimensionless)
- J_{m1}, J_{m2} are the drive and load motor rotational mass moment of inertias (kg m^2)
- n is the gear ratio of the pulley and belt assembly (dimensionless)
- D_{m1}, D_{m2} are the drive and load motor viscous damping constants (N m/rad/sec)
- D_1, D_2 are the drive and load motor bearing viscous damping constants (N m/rad/sec)
- J_1, J_2 are the drive and load motor shaft section rotational mass moments of inertia (kg m^2)

or, in state equation form,

$$\frac{dx}{dt} = A\, x(t) + B\, u(t)$$

with

$$x = \begin{bmatrix} i_1 & i_2 & \theta_1 & \frac{d\theta_1}{dt} & \theta_2 & \frac{d\theta_2}{dt} \end{bmatrix}^{\mathrm{T}} \qquad u = \begin{bmatrix} e_1 & e_2 \end{bmatrix}^{\mathrm{T}}$$

$$
A = \begin{bmatrix}
-\frac{R_{A1}+R_1}{L_1} & 0 & 0 & -\frac{nK_1}{L_1} & 0 & 0 \\
0 & -\frac{R_{A2}+R_2}{L_2} & 0 & 0 & 0 & -\frac{K_2}{L_2} \\
0 & 0 & 0 & 1 & 0 & 0 \\
\frac{nK_1}{J_1+n^2J_{m1}} & 0 & -\frac{K_s}{J_1+n^2J_{m1}} & -\frac{D_1+n^2D_{m1}}{J_1+n^2J_{m1}} & \frac{K_s}{J_1+n^2J_{m1}} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & \frac{K_2}{J_2+J_{m2}} & \frac{K_s}{J_2+J_{m2}} & 0 & -\frac{K_s}{J_2+J_{m2}} & -\frac{D_2+D_{m2}}{J_2+J_{m2}}
\end{bmatrix}
\qquad
B = \begin{bmatrix}
\frac{K_{A1}}{L_1} & 0 \\
0 & \frac{K_{A2}}{L_2} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
$$

Matlab users: the script `dcmtrs.m` gives these matrices for a particular set of parameters: `[A,B] = dcmtrs;`
Python users: use `dcmtrs.ipynb`

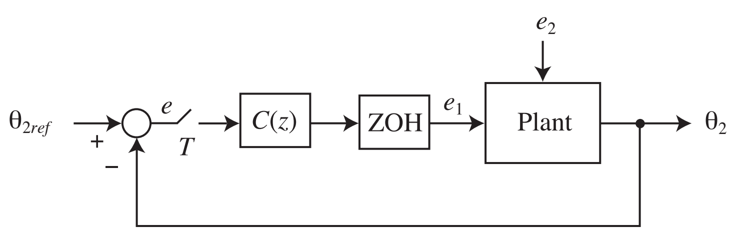


Figure 1. Closed-loop control system

1. In Figure 1, the Plant block represents the dynamics of the DC motors apparatus defined by the `dcmtrs` file. The controller transfer function is

$$C(z) = \frac{55}{100}\,\frac{z-0.46415}{z-0.2}\,\frac{z-0.991}{z-1}$$

and the sampling period is $T = 0.02$ sec.

(a) Determine the Figure 1 system's gain and phase margins.

(b) Determine the smallest positive gain for the sensor (larger than the nominal gain for the sensor, which is one) that generates the measurement of θ_2 for the Figure 1 system's feedback path such that the system will be unstable.

(c) Use Matlab and Simulink or Python-Control to generate a plot that has superimposed on it the first 1 sec of the $\theta_1(t)$ and $\theta_2(t)$ responses of the Figure 1 system to a unit step θ_{2ref} input when the sensor that generates the measurement of θ_2 for the feedback path has the destabilizing gain you determined for part (b).

(d) Suppose that the sensor that generates the measurement of θ_2 for the Figure 1 system's feedback path generates its measurement with a time delay. Determine the smallest such time delay (larger than the nominal time delay for the sensor, which is zero) such that Figure 1 system will be unstable.[1]

(e) Use Matlab and Simulink or Python-Control to generate a plot that has superimposed on it the first 1 sec of the $\theta_1(t)$ and $\theta_2(t)$ responses of the Figure 1 system to a unit step θ_{2ref} input when the sensor that generates the measurement of θ_2 for the system's feedback path generates its measurement with the time delay you determined for part (d).

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[1] Such a time delay could also model, equally well, the computation time for a digital computer to update the control signal following each sampling instant.