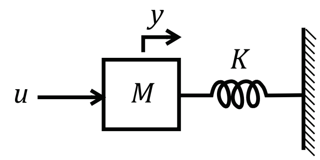
Homework #7

Reading: Chapter 6

Textbook problems: None

Special problems:

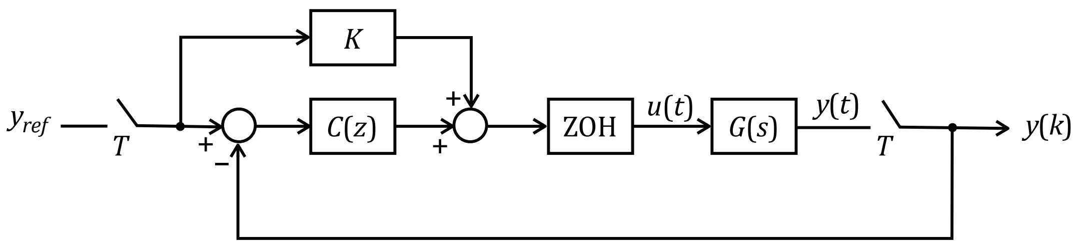
$u =$ force (N)




$y =$ position (m)

$M = 0.01$ kg $\qquad K = 44.567$ N/m

$$u - K\,y = M\,\frac{d^2y}{dt^2}$$




1. Consider the above closed-loop system, with $T = 0.1$ sec, and with $G(s)$ representing the $u$-to-$y$ dynamics of the mass-spring system above it.

(a) Use the direct $z$-plane root locus design method to determine a lead compensator, $C(z)$, that yields $z$-plane closed-loop poles that have, according to the MATLAB/python-control `damp` command, damping ratio precisely $\zeta = 1/5$ and time constant precisely $1/(\zeta\,\omega_n) = 1$ sec.
**Note**: you are may find it helpful to use the `rlocus` command to visualize the path of the closed-loop poles, but you are to use the magnitude condition to calculate the precise gain needed.

(b) Plot (using symbols, rather than a continuous curve) the first 4 seconds of the discrete-time $y$ response of the resulting closed-loop system to a unit step $y_{ref}$ input. Superimpose on this plot a dashed horizontal line at the maximum percent overshoot predicted by the Simple Oscillator Model i.e., the percent overshoot predicted by

$$\text{PO} = 100\ e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

Also superimpose on this plot horizontal dashed lines at 98% and 102% of the steady-state $y$ response to a unit step $y_{ref}$ input.

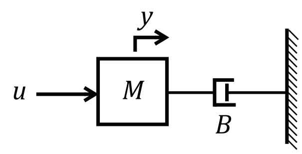
(c) Explain the difference, if any, between the maximum percent overshoot in the $y$ response in your plot for part (b) and that predicted by the Simple Oscillator Model.

(d) Explain the difference, if any, between the 2% Settling Time in the $y$ response in your plot for part (b) and that predicted by the Simple Oscillator Model.

(e) Plot (using a continuous curve, using Simulink or the provided example tools in Python in "simulating sampled data systems" ) the first 1 second of the continuous-time $y$ response of the resulting closed-loop system to a unit step $y_{ref}$ input. Superimpose on this plot (using dots rather than a continuous curve) the first 1 second of the discrete-time $y$ response to the same $y_{ref}$ input.

(f) Explain the differences, if any, between the maximum percent overshoots of the continuous-time and discrete-time responses in your plot for part (e).

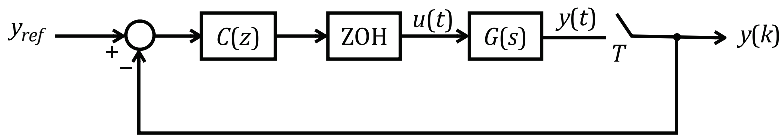
$u = \text{force (N)}$




$y = \text{position (m)}$

$M = 0.01$ kg $\quad B = 0.062832$ N/m/sec

$$u - B\frac{dy}{dt} = M\frac{d^2y}{dt^2}$$




2. Consider the above closed-loop system, with $T = 0.1$ sec, and with $G(s)$ representing the $u$-to-$y$ dynamics of the mass-damper system above it.

(a) Use the direct $z$-plane root locus design method to determine a proportional controller $C(z) = K$ that yields $z$-plane closed-loop poles that have, according to the MATLAB/python `damp` command, damping ratio $\zeta = 1/5$ and time constant $1/(\zeta\,\omega_n) = 2/3$ sec.

(b) Is the above closed-loop system Type 0, Type 1, or Type 2? Determine the value of its corresponding finite tracking error constant ($K_p$, $K_v$ or $K_a$).

(c) Use the direct $z$-plane root locus design method to add lag compensation to your $C(z)$ from part (a) to: (1) increase the tracking error constant you reported for part (b) by a factor of 10; (2) keep two closed-loop poles very near those you achieved with just your proportional controller for part (a); and (3) yield a time constant, as reported by the `damp` command, within 20% of 10 sec for the resulting third closed-loop pole.

(d) Superimpose (using symbols, rather than a continuous curve), on one plot: (1) the first 4 seconds of the discrete-time $y$ response, to a unit step $y_{ref}$ input, of the closed-loop system that utilizes the proportional controller you designed for part (a); and (2) the first 4 seconds of the discrete-time $y$ response, to a unit step $y_{ref}$ input, of the closed-loop system that utilizes the proportional-plus-lag compensator you designed for part (c).

(e) Superimpose (using symbols, rather than a continuous curve) on one plot: (1) the first 40 seconds of the discrete-time $y_{ref} - y$ response, to a unit ramp $y_{ref}$ input, of the closed-loop system that utilizes the proportional controller you designed for part (a); and (2) the first 40 seconds of the discrete-time $y_{ref} - y$ response, to a unit ramp $y_{ref}$ input, of the closed-loop system that utilizes the proportional-plus-lag compensator you designed for part (c). Also superimpose on this plot horizontal dotted lines at 1/[the tracking error constant you determined for part (b)] and at 1/[10 × (the tracking error constant you determined for part (b))].
**Note:** one way to simulate the response of a system to a ramp input is to pre-multiply it by an integrator and apply a step input to it. For a discrete-time system, you can use an integrator of the form $\frac{T}{z-1}$.

(f) In your plot for part (e) you should see an exponential decay having a comparatively long time constant. Precisely what determined the length of this time constant?