Homework #5

Reading: Chapter 4

Textbook Problems: None

Special Problems:

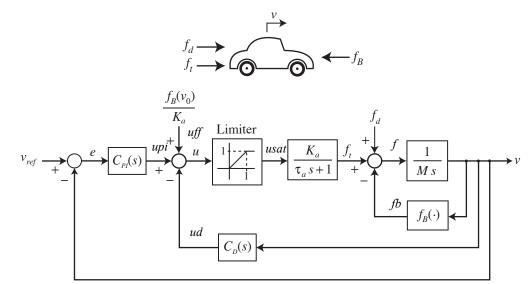


Figure 1. Analog speed control system.

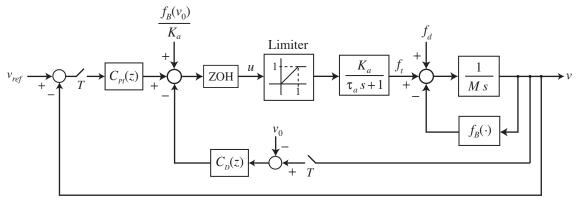


Figure 2. Digital speed control system.

1. For a previous homework assignment you determined digital controller transfer functions

$$C_{PI}(z) = \frac{(K_P + K_I T)z - K_P}{z - 1} \qquad \text{and} \qquad C_D(z) = \frac{K_D(z - 1)}{Tz}$$

that, for any given sampling period T, correspond to the analog controller transfer functions

$$C_{PI}(s) = \frac{K_P s + K_I}{s}$$
 and  $C_D(s) = K_D s$ 

You went on to analyze the performance of these analog and digital controller transfer functions in applications to a linearized small-perturbation model of the dynamics of the speed control system for a car for the operating point velocity  $v_0 = 25$  m/sec.

Here you will compare, via simulations, the performance of the same analog and digital controller transfer functions applied to the corresponding nonlinear model of the dynamics of the speed control system (the nonlinear model that you used in an earlier homework assignment to derive the linearized small-perturbation model) for the same operating point velocity  $v_0 = 25$  m/sec. The analog and digital control systems to be simulated are shown in Figures 1 and 2. Here

 $v_{ref}$  is the desired velocity (m/sec)

 $v_0$ =25 m/sec is the constant operating point velocity

*u* is the normalized throttle position (dimensionless)

 $K_a = 1599$  N is the gain constant for the engine and drive-train thrust response

 $\tau_a$ =0.5 sec is the time constant for the engine and drive-train thrust response

 $f_t$  is the forward thrust developed by the engine and drive-train (N)

M=1670 kg is the mass of the car

v is the actual velocity (m/sec)

 $f_d$  is a disturbance force that pushes the car forward (e.g., due to gravity when the car goes down a hill) (N)

 $f_R(\cdot) = B(\cdot)^2 \operatorname{sign}(\cdot) = 0.5559(\cdot)^2 \operatorname{sign}(\cdot)$  is the aerodynamic drag force (N)<sup>1</sup>

For the control law parameters, use

$$K_P = 0.6 \text{ 1/m/sec}$$

$$K_I = 0.01 \text{ 1/m}$$

$$K_P = 0.6 \text{ 1/m/sec}$$
  $K_I = 0.01 \text{ 1/m}$   $K_D = 0.08 \text{ 1/m/sec}$   $T = 0.5 \text{ sec}$ 

$$T = 0.5 \text{ sec}$$

For your comparison, generate the following plots:

- (a) A single plot on which the continuous-time v responses when the car is traveling on a level road, with the analog and digital controllers, to  $v_{ref} = 27$  m/sec are superimposed.
- (b) A single plot on which the continuous-time u responses and the throttle-limited continuous-time u responses when the car is on a level road, with the analog and digital controllers, to  $v_{ref} = 27$  m/sec are superimposed.
- (c) A single plot on which the continuous-time v responses when the car is traveling up a long hill that has a constant 5 percent grade, with the analog and digital controllers, to  $v_{ref} = 25$  m/sec are superimposed.
- (d) A single plot on which the continuous-time u responses and the throttle-limited continuous-time u responses when the car is traveling up a long hill that has a constant 5 percent grade, with the analog and digital controllers, to  $v_{ref} = 25$  m/sec are superimposed.

Assume, in all cases, that the car starts out at time t = 0 with  $v = v_0 = 25$  m/sec and dv/dt = 0.

## MATLAB Users:

A Simulink model that can be used to obtain the responses with the analog controller is shown below. The corresponding Simulink model file spdcntrl.slx and a Matlab m-file hw5spdemo.m that makes use of spdcntrl.slx are available on the course web site.

## **Python Users:**

A Jupyter notebook that can be used to obtain the responses with the analog controller is provided on the course web site. The signal names used by interconnect are shown above in Figure 1. Please see the corresponding Python tutorial video and Jupyter notebook, "Simulating sampled-data systems," which provides a function you will need to simulate the sampled-data controller.

<sup>1.</sup>  $\operatorname{sign}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$ 

