DISTRIBUTIONS

$$a\overline{Y}_{A\cdot} - b\overline{Y}_{B\cdot} \sim N(a(\mu+\tau_A), a^2\sigma^2) + N(-b(\mu+\tau_A), (-b)^2\sigma^2); \quad \sum_{i=1}^{n} Y_i^2 \sim \chi_n^2$$

$$W_1 \sim \chi_(^2 d_1); \quad W_2 \sim \chi_(^2 d_2); \quad \frac{W_1/d_1}{W_2/d_2} = Q \sim F_{d_1,d_2} \iff d_1 = 1, \sqrt{Q} \sim T_{d_2}$$

ESTIMABILITY

A function is estimable iff:

$$\sum_{i=1}^{v} b_i\left(\hat{\mu}+\hat{\tau}_i\right) = \sum_{i=1}^{v} b_i \overline{Y}_i \text{ (LSE)}; \quad \overline{Y}_{i\cdot} = \hat{\mu}+\hat{\tau}_i = \frac{1}{r_i}\sum_{t=1}^{n} Y_{it} \sim N\left(\mu+\tau_i, \frac{1}{r_i}\sigma^2\right)$$

ANOVA

Model

$$Y_{it} = \mu+\tau_i+\epsilon_{it}, \quad \epsilon_{it} \sim N(0,\sigma^2); \quad i = 1,\ldots,v; \quad t = 1,\ldots,r_i; \quad r_i = val$$

Sums of Squares

$$\text{SSE} = \sum_{i=1}^{v}\sum_{t=1}^{r_i}\left(\overline{Y}_{it} - \overline{Y}_{i\cdot}\right)^2; \quad \text{SST} = \sum_{i=1}^{v} r_i\left(\overline{Y}_{i\cdot} - \overline{Y}_{\cdot\cdot}\right)^2; \quad \text{SSTOT} = \sum_{i=1}^{v}\sum_{t=1}^{r_i}\left(\overline{Y}_{it} - \overline{Y}_{i\cdot}\right)^2$$

Variance

$$\widehat{\sigma^2} \approx \frac{\text{SSE}}{n-v}; \quad \frac{\text{SSE}}{\sigma^2} \sim \chi^2_{n-v}$$

Test Statistics

$$T^* = \frac{\text{SST}/(v-1)}{\text{SSE}/(n-v)} \sim F_{(v-1),(n-v)}$$

$$T^* = \frac{\left(\overline{Y}_{A\cdot} - \overline{Y}_{C\cdot}\right)^2 / (K(\sigma))^2}{\text{SSE}/\left[(n-v)\sigma^2\right]} \sim F_{1,(n-v)} \Longrightarrow \frac{\sqrt{\frac{r_i+r_j}{r_i r_j}}\left(\overline{Y}_i - \overline{Y}_j\right)}{\sqrt{SSE/(n-v)}} \sim F_{1,(n-v)}$$

Hypotheses

$$H_0 : \tau_i = \tau_j \quad H_A : \tau_i \neq \tau_{j \forall i \neq j}$$

ANOVA Table

	DF	Sum Sq	Mean Sq	F-Value
Treatment	v-1	SST	$\text{SST}/(v-1)$	$\frac{\text{SST}/(v-1)}{\text{SSE}/(n-v)}$
Error	n-v	SSE	$\text{SSE}/(n-v)$	NA
Total	n-1	SSTOT	NA	NA