STAT 461: Homework 4

Name: Kyle Salitrik | ***ID#:*** *997543474* | ***PSU ID:*** *kps168*

September 19, 2018

Problem 1

a)

This model is estimable where: $b_1 = b_2 = 1; b_3 = -2$

$$\sum_{n=1}^{3} b_i(\mu+\tau_i) = 1(\mu+\tau_1)+1(\mu+\tau_2)-2(\mu+\tau_3)$$
$$= 2\mu-2\mu+\tau_1+\tau_2-2\tau_3$$
$$= \tau_1+\tau_2-2\tau_3$$
$$\sum_{n=1}^{3} b_i\overline{Y}_i = \overline{Y}_1+\overline{Y}_2-2\overline{Y}_3$$

b)

This model is estimable where: $b_1 = b_2 = 0; b_3 = 1$

$$\sum_{n=1}^{3} b_i(\mu+\tau_i) = 0(\mu+\tau_1)+0(\mu+\tau_2)+1(\mu+\tau_3)$$
$$= \mu+\tau_3$$
$$\sum_{n=1}^{3} b_i\overline{Y}_i = 0\overline{Y}_1+0\overline{Y}_2+1\overline{Y}_3$$
$$= \overline{Y}_3$$

c)

This model is not estimable for any real values of b_i

d)

This model is estimable where: $b_1 = b_2 = b_3 = \frac{1}{3}$

$$\sum_{n=1}^{3} b_i(\mu+\tau_i) = \frac{1}{3}(\mu+\tau_1)+\frac{1}{3}(\mu+\tau_2)+\frac{1}{3}(\mu+\tau_3)$$
$$= 3*\frac{1}{3}\mu+\frac{1}{3}\tau_1+\frac{1}{3}\tau_2+\frac{1}{3}\tau_3$$
$$= \mu+\frac{1}{3}(\tau_1+\tau_2+\tau_3)$$
$$\sum_{n=1}^{3} b_i\overline{Y}_i = \frac{1}{3}\overline{Y}_1+\frac{1}{3}\overline{Y}_2+\frac{1}{3}\overline{Y}_3$$

Problem 2

a)

$$Y_{it} = \mu+\tau_i+\epsilon_{it}; \quad i = 1,2,3; \quad t = 1,2,3,4$$
$$\epsilon_{it} \overset{iid}{\sim} N\left(0,\sigma^2\right)$$

1 = Regular 2 = Deodorant 3 = Moisturizing

b)

For future calculations:

$$\overline{Y_i} = \overline{Y} + \hat{\tau}_i$$
$$\hat{\tau}_i = \overline{Y_i} - \overline{Y}$$
$$\overline{Y} = \frac{1}{12}(-.3 - .1 - .14 + .40 + 2.63 + 2.61 + 2.41 + 3.15 + 1.86 + 2.03 + 2.26 + 1.82) = 1.5525\bar{3}$$
$$\overline{Y_1} = \frac{1}{4}(-.3 - .1 - .14 + .40) = -0.035$$
$$\overline{Y_2} = \frac{1}{4}(2.63 + 2.61 + 2.41 + 3.15) = 2.7$$
$$\overline{Y_3} = \frac{1}{4}(1.86 + 2.03 + 2.26 + 1.82) = 1.9925$$
$$\hat{\tau}_1 = \overline{Y_1} - \overline{Y} = -0.035 - 1.5525\bar{3} \approx -1.5875$$
$$\hat{\tau}_2 = \overline{Y_2} - \overline{Y} = 2.7 - 1.5525\bar{3} \approx 1.475$$
$$\hat{\tau}_3 = \overline{Y_3} - \overline{Y} = 1.9925 - 1.5525\bar{3} \approx 0.44$$

$$\sum_{n=1}^{3} b_i \overline{Y}_i = b_1\overline{Y}_1 + b_2\overline{Y}_2 + b_3\overline{Y}_3$$

$$\sum_{n=1}^{3} b_i \overline{Y}_i = -0.035b_1 + 2.7b_2 + 1.9925b_3$$

The LSE for a bar of Deodorant Soap is where: $b_1 = b_3 = 0; \quad b_2 = 1$

$$\sum_{n=1}^{3} b_i \overline{Y}_i = -0.035 * 0 + 2.7 * 1 + 1.9925 * 0$$
$$= 2.7$$

c)

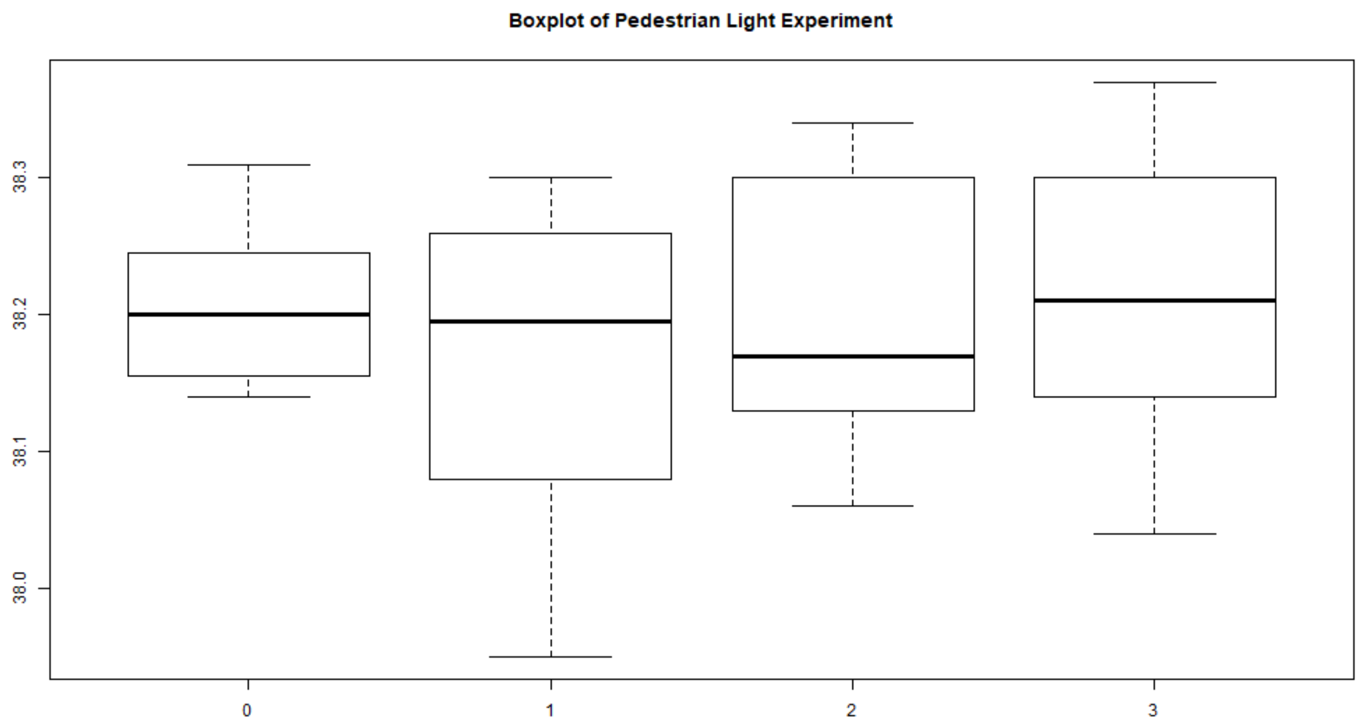
This model is estimable where: $b_1 = 1; b_2 = b_3 = -\frac{1}{2}$

$$\sum_{n=1}^{3} b_i(\mu + \tau_i) = 1(\mu + \tau_1) - \frac{1}{2}(\mu + \tau_2) - \frac{1}{2}(\mu + \tau_3)$$
$$= \mu - 2\left(\frac{1}{2}\mu\right) + \tau_1 - \frac{1}{2}\tau_2 - \frac{1}{2}\tau_3$$
$$= \tau_1 - \frac{1}{2}(\tau_2 + \tau_3)$$
$$\sum_{n=1}^{3} b_i \overline{Y}_i = \overline{Y}_1 - \frac{1}{2}\overline{Y}_2 - \frac{1}{2}\overline{Y}_3$$
$$= -0.035 - \frac{1}{2}(2.7 + 1.9925) = -2.38125$$

d)

```
> # Part B Re-calculation:
> contrast(lsmeans_losses, list("deodorant only"=c(0,1,0), "t1 - 1/2(t2+t3)"=c
    (1,-0.5,-0.5)))
 contrast          estimate        SE df t.ratio p.value
 deodorant only     1.99250 0.1389019  9  14.345  <.0001
 t1 - 1/2(t2+t3)    1.72125 0.1701194  9  10.118  <.0001
```

a)



b)

$Y_{it} = \mu + \tau_i + \epsilon_{it}; \quad i = 0,1,2,3; \quad t = 1,\ldots,r_i$

$r_1 = 7, r_1 = r_2 = 10, r_3 = 5$

$\epsilon_{it} \overset{iid}{\sim} N\left(0, \sigma^2\right)$

c)

```
> lsmeans_lights = lsmeans(aov_lights, "presses")
> lsmeans_lights
 presses    lsmean         SE df lower.CL upper.CL
 0        38.20714 0.03950929 28 38.12621 38.28807
 1        38.17100 0.03305584 28 38.10329 38.23871
 2        38.19400 0.03305584 28 38.12629 38.26171
 3        38.21200 0.04674802 28 38.11624 38.30776
```

d)

This model is estimable where: $b_0 = -1; b_1 = 1; b_2 = b_3 = 0$

$$\sum_{n=0}^{3} b_i(\mu + \tau_i) = b_0(\mu + \tau_0) + b_1(\mu + \tau_1) + b_2(\mu + \tau_2) + b_3(\mu + \tau_3)$$

$$\approx \sum_{n=0}^{3} b_i \overline{Y}_i$$

$$\sum_{n=0}^{3} b_i \overline{Y}_i = -\overline{Y}_0 + \overline{Y}_1 + 0\overline{Y}_2 + 0\overline{Y}_3$$

$$= 38.171 - 38.20714 = -0.03614286$$

R output is below

e)

This model is estimable where: $b_0 = -1; b_1 = b_2 = b_3 = \frac{1}{3}$

$$
\begin{aligned}
\sum_{n=0}^{3} b_i(\mu + \tau_i) &= b_0(\mu+\tau_0) + b_1(\mu+\tau_1) + b_2(\mu+\tau_2) + b_3(\mu+\tau_3) \\
&\approx \sum_{n=0}^{3} b_i \overline{Y}_i \\
\sum_{n=0}^{3} b_i \overline{Y}_i &= -\overline{Y}_0 + \frac{1}{3}\overline{Y}_1 + \frac{1}{3}\overline{Y}_2 + \frac{1}{3}\overline{Y}_3 \\
&= \frac{1}{3}(38.171 + 38.194 + 38.212) - 38.20714 = -0.01480952
\end{aligned}
$$

```
> contrast(lsmeans_lights, list("Part D"=c(-1, 1, 0, 0), "Part E"=c(-1, 1/3, 1/3, 1/
    3)))
contrast    estimate          SE  df  t.ratio  p.value
Part D    -0.03614286  0.05151381  28   -0.702   0.4887
Part E    -0.01480952  0.04523962  28   -0.327   0.7458
```

Code Appendix

```r
####################################################################
#### Setup
####################################################################
## Install and load libraries
# ipak function taken from: https://gist.github.com/stevenworthington/3178163
ipak <- function(pkg) {
  new.pkg <- pkg[!(pkg %in% installed.packages()[, "Package"])]
  if (length(new.pkg))
    install.packages(new.pkg, dependencies = TRUE)
  sapply(pkg, require, character.only = TRUE)
}
packages <- c("ggplot2", "reshape2", "gridExtra", "TSA", "astsa", "orcutt",
              "nlme", "fGarch", "vars", "lsmeans")
ipak(packages)

####################################################################
#### Problem 2
####################################################################
# From HW1
loss = c(-.3,-.1,-.14,.4,2.63,2.61,2.41,3.15,1.86,2.03,2.26,1.82)
type = c(rep('reg',4), rep('deo',4), rep('moi',4))
losses = data.frame(loss, type)

# Part D
aov_losses = aov(loss~type)
lsmeans_losses = lsmeans(aov_losses, "type")
lsmeans_losses

# Part B & C Re-calculation:
contrast(lsmeans_losses, list("deodorant only"=c(0,1,0), "t1 - 1/2(t2+t3)"=c
    (1,-0.5,-0.5)))

####################################################################
#### Problem 3
####################################################################
time = c(38.14, 38.20, 38.31, 38.14, 38.29, 38.17, 38.20,
         38.28, 38.17, 38.08, 38.25, 38.18, 38.03, 37.95, 38.26, 38.30, 38.21,
         38.17, 38.13, 38.16, 38.30, 38.34, 38.34, 38.17, 38.18, 38.09, 38.06,
         38.14, 38.30, 38.21, 38.04, 38.37)
presses = c(rep('0',7), rep('1',10), rep('2',10), rep('3',5))
lights = data.frame(time, presses)

# Part A
png("./figures/p3.png", width = 1024, height = 576)
  boxplot(time ~ presses, main="Boxplot of Pedestrian Light Experiment")
dev.off()

# Part C
aov_lights = aov(time ~ presses)
lsmeans_lights = lsmeans(aov_lights, "presses")
lsmeans_lights

# Part D & E
contrast(lsmeans_lights, list("Part D"=c(-1, 1, 0, 0), "Part E"=c(-1, 1/3, 1/3, 1/3))
    )
```