STAT 461: Homework 4

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Problem 1

a)

This model is estimable where: $b_1 = b_2 = 1; b_3 = -2$

$$\begin{aligned}\sum_{n=1}^{3} b_i(\mu+\tau_i) &= 1(\mu+\tau_1)+1(\mu+\tau_2)-2(\mu+\tau_3)\\ &= 2\mu-2\mu+\tau_1+\tau_2-2\tau_3\\ &= \tau_1+\tau_2-2\tau_3\\ \sum_{n=1}^{3} b_i\overline{Y}_i &= \overline{Y}_1+\overline{Y}_2-2\overline{Y}_3\end{aligned}$$

b)

This model is estimable where: $b_1 = b_2 = 0; b_3 = 1$

$$\begin{aligned}\sum_{n=1}^{3} b_i(\mu+\tau_i) &= 0(\mu+\tau_1)+0(\mu+\tau_2)+1(\mu+\tau_3)\\ &= \mu+\tau_3\\ \sum_{n=1}^{3} b_i\overline{Y}_i &= 0\overline{Y}_1+0\overline{Y}_2+1\overline{Y}_3\\ &= \overline{Y}_3\end{aligned}$$

c)

This model is not estimable for any real values of b_i

d)

This model is estimable where: $b_1 = b_2 = b_3 = \frac{1}{3}$

$$\begin{aligned}\sum_{n=1}^{3} b_i(\mu+\tau_i) &= \frac{1}{3}(\mu+\tau_1)+\frac{1}{3}(\mu+\tau_2)+\frac{1}{3}(\mu+\tau_3)\\ &= 3*\frac{1}{3}\mu+\frac{1}{3}\tau_1+\frac{1}{3}\tau_2+\frac{1}{3}\tau_3\\ &= \mu+\frac{1}{3}(\tau_1+\tau_2+\tau_3)\\ \sum_{n=1}^{3} b_i\overline{Y}_i &= \frac{1}{3}\overline{Y}_1+\frac{1}{3}\overline{Y}_2+\frac{1}{3}\overline{Y}_3\end{aligned}$$

Problem 2

a)

$$Y_{it} = \mu+\tau_i+\epsilon_{it}; \quad i = 1,2,3; \quad t = 1,2,3,4$$

$$\epsilon_{it} \overset{iid}{\sim} N\left(0,\sigma^2\right)$$

1 = Regular 2 = Deodorant 3 = Moisturizing

b)

For future calculations:

$$\overline{Y_i} = \overline{Y} + \hat{\tau}_i$$
$$\hat{\tau}_i = \overline{Y_i} - \overline{Y}$$
$$\overline{Y} = \frac{1}{12}(-.3 - .1 - .14 + .40 + 2.63 + 2.61 + 2.41 + 3.15 + 1.86 + 2.03 + 2.26 + 1.82) = 1.5525\bar{3}$$
$$\overline{Y_1} = \frac{1}{4}(-.3 - .1 - .14 + .40) = -0.035$$
$$\overline{Y_2} = \frac{1}{4}(2.63 + 2.61 + 2.41 + 3.15) = 2.7$$
$$\overline{Y_3} = \frac{1}{4}(1.86 + 2.03 + 2.26 + 1.82) = 1.9925$$
$$\hat{\tau}_1 = \overline{Y_1} - \overline{Y} = -0.035 - 1.5525\bar{3} \approx -1.5875$$
$$\hat{\tau}_2 = \overline{Y_2} - \overline{Y} = 2.7 - 1.5525\bar{3} \approx 1.475$$
$$\hat{\tau}_3 = \overline{Y_3} - \overline{Y} = 1.9925 - 1.5525\bar{3} \approx 0.44$$

$$\sum_{n=1}^{3} b_i \overline{Y}_i = b_1 \overline{Y}_1 + b_2 \overline{Y}_2 + b_3 \overline{Y}_3$$
$$\sum_{n=1}^{3} b_i \overline{Y}_i = -0.035 b_1 + 2.7 b_2 + 1.9925 b_3$$

The LSE for a bar of Deodorant Soap is where: $b_1 = b_3 = 0; \quad b_2 = 1$

$$\sum_{n=1}^{3} b_i \overline{Y}_i = -0.035 * 0 + 2.7 * 1 + 1.9925 * 0$$
$$= 2.7$$

c)

This model is estimable where: $b_1 = 1; b_2 = b_3 = -\frac{1}{2}$

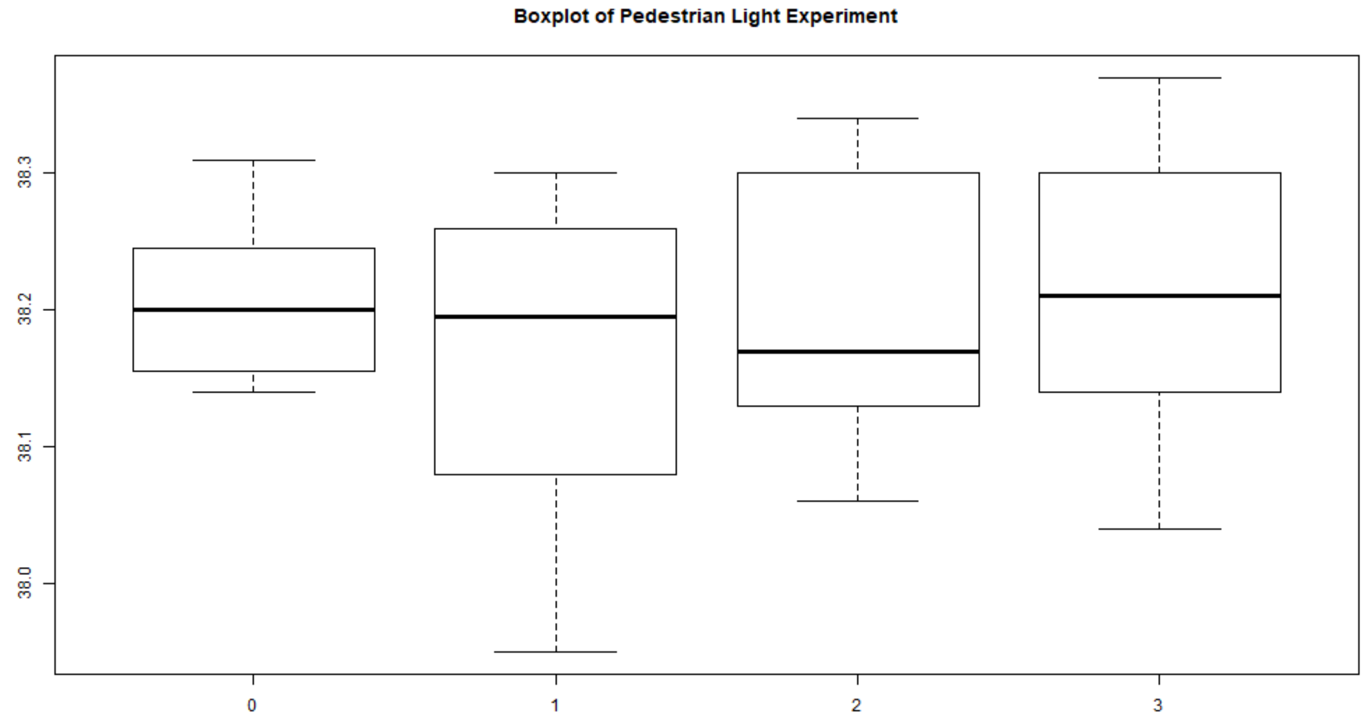
$$\sum_{n=1}^{3} b_i(\mu + \tau_i) = 1(\mu + \tau_1) - \frac{1}{2}(\mu + \tau_2) - \frac{1}{2}(\mu + \tau_3)$$
$$= \mu - 2\left(\frac{1}{2}\mu\right) + \tau_1 - \frac{1}{2}\tau_2 - \frac{1}{2}\tau_3$$
$$= \tau_1 - \frac{1}{2}(\tau_2 + \tau_3)$$
$$\sum_{n=1}^{3} b_i \overline{Y}_i = \overline{Y}_1 - \frac{1}{2}\overline{Y}_2 - \frac{1}{2}\overline{Y}_3$$
$$= -0.035 - \frac{1}{2}(2.7 + 1.9925) = -2.38125$$

d)

```
> #Part D
> reg_mean = mean(loss[type=='reg'])
> deo_mean = mean(loss[type=='deo'])
> moi_mean = mean(loss[type=='moi'])
>
> #Part B Re-calculation:
> deo_mean
[1] 2.7
>
> #Part C Re-calculation:
> reg_mean -(deo_mean + moi_mean)/2
[1] -2.38125
```

Problem 3

a)



b)

$$Y_{it} = \mu + \tau_i + \epsilon_{it}; \quad i = 0,1,2,3; \quad t = 1,\dots,r_i$$

$$r_1 = 7, r_1 = r_2 = 10, r_3 = 5$$

$$\epsilon_{it} \overset{iid}{\sim} N(0,\sigma^2)$$

c)

```
> mean_0 = mean(time[presses == '0'])
> mean_1 = mean(time[presses == '1'])
> mean_2 = mean(time[presses == '2'])
> mean_3 = mean(time[presses == '3'])
> mean_0
[1] 38.20714
> mean_1
[1] 38.171
> mean_2
[1] 38.194
> mean_3
[1] 38.212
```

d)

This model is estimable where: $b_0 = -1; b_1 = 1; b_2 = b_3 = 0$

$$\begin{aligned}
\sum_{n=0}^{3} b_i(\mu+\tau_i) &= b_0(\mu+\tau_0) + b_1(\mu+\tau_1) + b_2(\mu+\tau_2) + b_3(\mu+\tau_3) \\
&\approx \sum_{n=0}^{3} b_i \overline{Y}_i \\
\sum_{n=0}^{3} b_i \overline{Y}_i &= -\overline{Y}_0 + \overline{Y}_1 + 0\overline{Y}_2 + 0\overline{Y}_3 \\
&= 38.171 - 38.20714 = -0.03614286
\end{aligned}$$

```
> # Part D
> mean_1 - mean_0
[1] -0.03614286
```

e)

This model is estimable where: $b_0 = -1; b_1 = b_2 = b_3 = \frac{1}{3}$

$$\begin{aligned}
\sum_{n=0}^{3} b_i(\mu+\tau_i) &= b_0(\mu+\tau_0) + b_1(\mu+\tau_1) + b_2(\mu+\tau_2) + b_3(\mu+\tau_3) \\
&\approx \sum_{n=0}^{3} b_i \overline{Y}_i \\
\sum_{n=0}^{3} b_i \overline{Y}_i &= -\overline{Y}_0 + \frac{1}{3}\overline{Y}_1 + \frac{1}{3}\overline{Y}_2 + \frac{1}{3}\overline{Y}_3 \\
&= \frac{1}{3}(38.171 + 38.194 + 38.212) - 38.20714 = -0.01480952
\end{aligned}$$

```
> # Part E
> (1/3)*(mean_1 + mean_2 + mean_3) - mean_0
[1] -0.01480952
```

Code Appendix

```r
###################################################################
#### Setup
###################################################################
## Install and load libraries
# ipak function taken from: https://gist.github.com/stevenworthington/3178163
ipak <- function(pkg) {
  new.pkg <- pkg[!(pkg %in% installed.packages()[, "Package"])]
  if (length(new.pkg))
    install.packages(new.pkg, dependencies = TRUE)
  sapply(pkg, require, character.only = TRUE)
}
packages <- c("ggplot2", "reshape2", "gridExtra", "TSA", "astsa", "orcutt",
              "nlme", "fGarch", "vars", "lsmeans")
ipak(packages)

###################################################################
#### Problem 2
###################################################################
# From HW1
loss = c(-.3,-.1,-.14,.4,2.63,2.61,2.41,3.15,1.86,2.03,2.26,1.82)
type = c(rep('reg',4), rep('deo',4), rep('moi',4))
losses = data.frame(loss, type)

# Part D
mean_reg = mean(loss[type=='reg'])
mean_deo = mean(loss[type=='deo'])
mean_moi = mean(loss[type=='moi'])
ybar = mean(loss)
tau1 = mean_reg - ybar
tau2 = mean_deo - ybar
tau3 = mean_moi - ybar

# Part B Re-calculation:
mean_deo

# Part C Re-calculation:
mean_reg -(mean_deo + mean_moi)/2

###################################################################
#### Problem 3
###################################################################
time = c(38.14, 38.20, 38.31, 38.14, 38.29, 38.17, 38.20,
         38.28, 38.17, 38.08, 38.25, 38.18, 38.03, 37.95, 38.26, 38.30, 38.21,
         38.17, 38.13, 38.16, 38.30, 38.34, 38.34, 38.17, 38.18, 38.09, 38.06,
         38.14, 38.30, 38.21, 38.04, 38.37)
presses = c(rep('0',7), rep('1',10), rep('2',10), rep('3',5))
lights = data.frame(time, presses)

# Part A
png("./figures/p3.png", width = 1024, height = 576)
  boxplot(time ~ presses, main="Boxplot of Pedestrian Light Experiment")
dev.off()

# Part C
mean_0 = mean(time[presses == '0'])
mean_1 = mean(time[presses == '1'])
mean_2 = mean(time[presses == '2'])
mean_3 = mean(time[presses == '3'])

# Part D
mean_1 - mean_0

# Part E
(1/3)*(mean_1 + mean_2 + mean_3) - mean_0
```