STAT 461: Homework 8

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November 6, 2018

a)

$$Y_{ijt} = \mu + \alpha_i + \beta_{j(i)} + \sigma$$
$$i = AcmeInd, GenWidget, WHSupply; j = 1,2,3,4; t = 1,2,3,4$$

b)

Batch is nested in supplier because each supplier is responsible for delivering 4 batches.

c)

```
1 > widgetNested = aov(WidgetSize ~ Supplier + Supplier:Batch)
2 > anova(widgetNested)
3 Analysis of Variance Table
4
5 Response: WidgetSize
6              Df Sum Sq Mean Sq F value    Pr(>F)
7 Supplier      2  6.792  3.3958  1.5234 0.2316974
8 Supplier:Batch 9 93.437 10.3819  4.6573 0.0003957 ***
9 Residuals    36 80.250  2.2292
10 ---
11 Signif. codes:  0 Š***Š 0.001 Š**Š 0.01 Š*Š 0.05 Š.Š 0.1 Š Š 1
```

Listing 1: Nested ANOVA Model for Widgets

d)

$$SS_{Supplier} = 6.792$$
$$SS_{Batch(Supplier)} = 93.437$$
$$SS_{Error} = 80.250$$

The nested interaction accounts for the highest variation and is the only significant factor.

e)

$$H_0^1 : \alpha_i + \overline{\beta}_{\cdot(i)} = \alpha_j + \overline{\beta}_{\cdot(j)} \quad \forall \{i, j\}$$
$$H_0^2 : \beta_{j(i)} = \beta_{k(i)} \quad \forall \{i, j\}$$

We reject H_0^2 because our anova results show that the nested interactions are different. Looking at the pairwise differences, we can see that the mean size between the suppliers themselves does not differ very much. However the batches have a large variation and can be put into 3 groups.

```
1 > widgetsLsmSupplier = lsmeans(widgetNested, ~ Supplier)
2 NOTE: Results may be misleading due to involvement in interactions
3 > cld(widgetsLsmSupplier, alpha=0.05)
4  Supplier  lsmean        SE df lower.CL upper.CL .group
5  AcmeInd   92.2500 0.3732599 36 91.49299 93.00701  1
6  GenWidget 92.9375 0.3732599 36 92.18049 93.69451  1
7  WHSupply  93.1250 0.3732599 36 92.36799 93.88201  1
8
9  Results are averaged over the levels of: Batch
10 Confidence level used: 0.95
11 P value adjustment: tukey method for comparing a family of 3 estimates
12 significance level used: alpha = 0.05
```

Listing 2: Pairwise Results for Supplier

```
1 > widgetsLsmInteract = lsmeans(widgetNested, ~ Supplier:Batch)
2 > cld(widgetsLsmInteract, alpha=0.05)
3  Batch Supplier  lsmean        SE df lower.CL upper.CL .group
4  2     AcmeInd   89.50 0.7465197 36 87.98599 91.01401  1
5  3     WHSupply  91.50 0.7465197 36 89.98599 93.01401  12
6  2     GenWidget 91.75 0.7465197 36 90.23599 93.26401  12
7  1     WHSupply  91.75 0.7465197 36 90.23599 93.26401  12
8  1     AcmeInd   92.50 0.7465197 36 90.98599 94.01401  12
9  3     AcmeInd   92.75 0.7465197 36 91.23599 94.26401  12
10 3     GenWidget 92.75 0.7465197 36 91.23599 94.26401  12
11 4     GenWidget 93.50 0.7465197 36 91.98599 95.01401   2
12 1     GenWidget 93.75 0.7465197 36 92.23599 95.26401   2
13 4     AcmeInd   94.25 0.7465197 36 92.73599 95.76401   2
14 4     WHSupply  94.50 0.7465197 36 92.98599 96.01401   2
15 2     WHSupply  94.75 0.7465197 36 93.23599 96.26401   2
16
17 Confidence level used: 0.95
18 P value adjustment: tukey method for comparing a family of 12 estimates
19 significance level used: alpha = 0.05
```

Listing 3: Pairwise Results for Nested Effects

a)

	m	f
str	○ ○ ○ ○ ○	△ △ △ △ △
end	○ ○ ○ ○ ○	△ △ △ △ △
cross	○ ○ ○ ○ ○	△ △ △ △ △

This is a crossed design

$$y_{ijt} = \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijt} \qquad \varepsilon \sim N(0, \sigma^2)$$
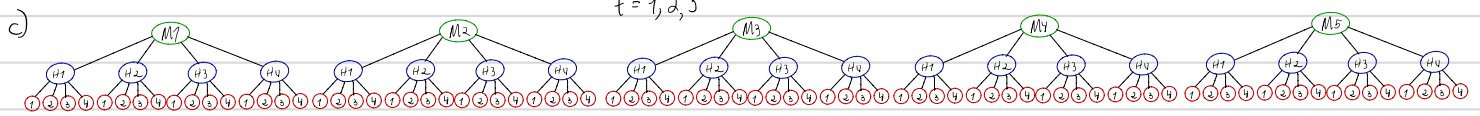
$i = m, f$

$j = str, end, cross$

$t = 1, \ldots, 5$

b)

This is a nested design (but actually a random effects)

$$y_{ijt} = \alpha_i + \beta_{j(i)} + \varepsilon_{ijt} \qquad \varepsilon \sim N(0, \sigma^2)$$

$i = stats, humanities$

$j = 1, 2, 3$

$t = 1, 2, 3$

c)

This is a nested design where head is nested in machine

$$y_{ijt} = \mu + \alpha_i + \beta_{j(i)} + \varepsilon_{ijt}$$

$i = M_1, \ldots, M_5$

$j = H_1, \ldots, H_4$

$t = 1, \ldots, 4$

$\varepsilon_{ijt} \sim N(0, \sigma^2)$

Code Appendix

```r
1  ##############################################################################
2  #### Setup
3  ##############################################################################
4  ## Install and load libraries
5  # ipak function taken from: https://gist.github.com/stevenworthington/3178163
6  ipak = function(pkg) {
7    new.pkg = pkg[!(pkg %in% installed.packages()[, "Package"])]
8    if (length(new.pkg))
9      install.packages(new.pkg, dependencies = TRUE)
10   sapply(pkg, require, character.only = TRUE)
11 }
12 packages = c("ggplot2","ggplotify", "reshape2", "gridExtra", "TSA", "astsa",
13               "orcutt", "nlme", "fGarch", "vars", "lsmeans", "multcompView",
14               "base2grob")
15 ipak(packages)
16
17 ##############################################################################
18 #### Problem 1
19 ##############################################################################
20 #####################################
21 #### Data Input
22 #####################################
23 widgets=read.table("widgets.txt",header=TRUE)
24 Batch=as.factor(widgets$Batch)
25 Supplier=widgets$Supplier
26 WidgetSize=widgets$WidgetSize
27
28 widgetTable=table(widgets[,c("Supplier","Batch")])
29
30 #####################################
31 ## Part C
32 #####################################
33 widgetNested = aov(WidgetSize ~ Supplier + Supplier:Batch)
34 anova(widgetNested)
35
36 #####################################
37 ## Part E
38 #####################################
39 widgetsLsmSupplier = lsmeans(widgetNested, ~ Supplier)
40 cld(widgetsLsmSupplier, alpha=0.05)
41
42 widgetsLsmInteract = lsmeans(widgetNested, ~ Supplier:Batch)
43 cld(widgetsLsmInteract, alpha=0.05)
```