$$D_x = D_{y_1} = D_{y_2} = D_{y_3} = D_z = \mathbb{Z}$$

$$\varphi(x) \equiv x \geq 0$$
$$\psi(x, z) \equiv z^3 \leq x < (z+1)^3$$

по индукции

$$SA : \varphi(x) \to P(x, 0, 1, 1)$$
$$ATH : \varphi(x) \wedge P(x, \bar{y}) \wedge (y_2 > x) \to \psi(x, P(x, \bar{y}))$$
$$AFA : \varphi(x) \wedge P(x, \bar{y}) \wedge (y_2 \leq x) \to P(x, y_1+1, y_2+(y_3+6(y_1+1)), y_3+6(y_1+1))$$

START:
$(y_1, y_2, y_3) \leftarrow (0, 0, 1)$

$y_2 \leftarrow y_2 + y_3$

—A

$y_2 > x$ F   T

$y_1 \leftarrow y_1 + 1$

HALT:
$z \leftarrow y_1$

$y_3 \leftarrow y_3 + 6 y_1$

Пусть $P(x, \bar{y}) \equiv x \geq y_1^3 \wedge y_2 = (y_1+1)^3 \wedge y_3 = (y_1+1)^3 - y_1^3$

$\Rightarrow SA : x \geq 0 \Rightarrow x \geq 0 \wedge 1 = 1 \wedge 1 - 1$

$ATH : x \geq 0 \wedge P(x, \bar{y}) \wedge (y_2 > x)$
$$\to \psi : y_1^3 \leq x < (y_1+1)^3$$

$AFA : x \geq 0 \wedge P(x, \bar{y}) \wedge (y_2 \leq x)$
$$\to x \geq (y_1+1)^3 \wedge y_2+(y_3+6(y_1+1)) = (y_1+1+1)^3$$

$$\wedge y_3 + 6(y_1+1) = (y_1+1+1)^3 - (y_1+1)^3$$

$$\Rightarrow \begin{cases} y_2 + y_3 + 6y_1 + 6 = (y_1+2)^3 \\ y_3 + 6(y_1+1) = (y_1+2)^3 - (y_1+1)^3 \end{cases} \Rightarrow y_2 = (y_1+1)^3$$

доказано