#### Honor statement:

"I have completed this work independently. The solutions given are entirely my own work"

## 1a.)

The following is the code used to perform Ridge Regression on the Piso2009 dataset:

View(Pisa2009) library(glmnet)

# Get all rows and columns except for the 1st and 25th columns.

x <- data.matrix(Pisa2009[,2:24])

# Get the response variable and save it as a double data-type.

y <- as.double(Pisa2009[,25])

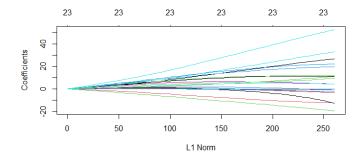
# Create the model w/ alpha = 0 for Ridge.

pisa\_model <- glmnet(x, y, alpha = 0)
summary(pisa\_model)</pre>

> summary(pisa_model)					
	Length	Class	Mode		
a0	100	-none-	numeric		
beta	2300	dgCMatrix S	S4		
df	100	-none-	numeric		
dim	2	-none-	numeric		
lambda	100	-none-	numeric		
dev.ratio	100	-none-	numeric		
nulldev	1	-none-	numeric		
npasses	1	-none-	numeric		
jerr	1	-none-	numeric		
offset	1	-none-	logical		
call	5	-none-	call		
nobs	1	-none-	numeric		

# # Displays Trace Plot

 $plot(pisa\_model)$ 



# # cv automatically does cross-validation to identify the lambda

pisa\_ridge\_cv <- cv.glmnet(x,y, family="gaussian", alpha=0) pisa\_ridge\_cv

> pisa\_ridge\_cv

Call: cv.glmnet(x = x, y = y, family = "gaussian", alpha = 0)

Measure: Mean-Squared Error

	Lambda	Index	Measu	re SE N	Vonzero
min	3.36	99	5705	128.1	23
1se	41.41	72	5819	136.7	23

#### # Get the lambda

pisa\_lambda <- pisa\_ridge\_cv\$lambda.min pisa\_lambda

```
> pisa_lambda <- pisa_ridge_cv$lambda.min
> pisa_lambda
[1] 3.359216
```

# # Find coefficients of the model

coef(pisa\_ridge\_cv, s=pisa\_lambda)

> coef(pisa_ridge_cv, s=pisa_lambda)				
24 x 1 sparse Matrix of class "dgCMatrix"				
	1			
(Intercept)	178.907609883			
grade	26.561537211			
male	-12.406794130			
raceeth	10.999647245			
preschool	-0.740149794			
expectBachelors	52.282541085			
motherHS	4.342749265			
motherBachelors	11.154201099			
motherWork	-3.198076587			
fatherHS	11.604885058			
fatherBachelors	19.515312833			
fatherWork	4.246623659			
selfBornUS	0.134092464			
motherBornUS	-12.584452833			
fatherBornUS	-2.535264505			
englishAtHome	9.588211699			
computerForSchoolwork	21.916035046			
read30MinsADay	32.661212423			
minutesPerWeekEnglish	0.014312649			
studentsInEnglish	-0.027115779			
schoolHasLibrary	-1.045897572			
publicSchool	-19.436026300			
urban	-2.768863426			
schoolSize	0.006535571			

The following is the single-order model that was created with the coefficients of the Ridge Regression technique:

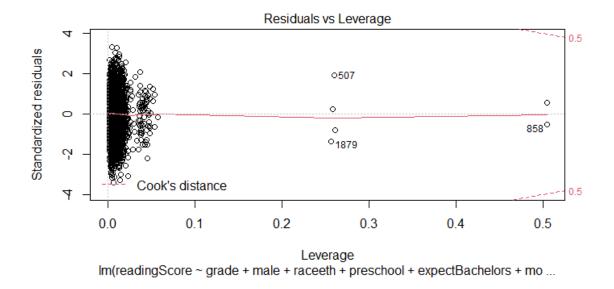
```
> pisa_model_1 <- lm(readingScore ~ grade + male + raceeth + preschool + expectBachelors + motherHS
            + motherBachelors + motherWork + fatherHS + fatherBachelors + fatherWork + selfBornUS
            + motherBornUS + fatherBornUS + englishAtHome + computerForSchoolwork + read30MinsADay
+
            + minutesPerWeekEnglish + studentsInEnglish + schoolHasLibrary + publicSchool
+
            + urban + schoolSize, data = Pisa2009)
> summary(pisa model 1)
Call:
lm(formula = readingScore ~ grade + male + raceeth + preschool +
  expectBachelors + motherHS + motherBachelors + motherWork +
  fatherHS + fatherBachelors + fatherWork + selfBornUS + motherBornUS +
  fatherBornUS + englishAtHome + computerForSchoolwork + read30MinsADay +
  minutesPerWeekEnglish + studentsInEnglish + schoolHasLibrary +
  publicSchool + urban + schoolSize, data = Pisa2009)
Residuals:
  Min
          1Q Median
                          30
                                Max
-252.698 -48.479 0.481 49.936 247.243
Coefficients:
                                         Estimate
                                                         Std. Error
                                                                         t value Pr(>|t|)
                                         299.488737
(Intercept)
                                                         55.703871
                                                                         5.376
                                                                                 8.11e-08 ***
grade9
                                                         52.722943
                                                                         0.764
                                                                                 0.444859
                                         40.285804
grade10
                                                                         1.719
                                         90.414303
                                                         52.582799
                                                                                 0.085621.
                                                                         1.994
grade11
                                         104.977136
                                                         52.650018
                                                                                 0.046247 *
grade12
                                         153.124134
                                                         64.477284
                                                                         2.375
                                                                                 0.017612 *
male1
                                                                                 1.87e-06 ***
                                         -12.629264
                                                         2.644476
                                                                         -4.776
raceethAsian
                                         59.289097
                                                         15.422137
                                                                                 0.000123 ***
                                                                         3.844
raceethBlack
                                         -3.245780
                                                         14.086553
                                                                         -0.230
                                                                                 0.817782
raceethHispanic
                                        28.478156
                                                         13.967874
                                                                         2.039
                                                                                 0.041545 *
raceethMore than one race
                                        42.834427
                                                         15.091587
                                                                         2.838
                                                                                 0.004563 **
raceethNative Hawaiian/Other Pacific Islander 52.643342
                                                         20.069600
                                                                         2.623
                                                                                 0.008754 **
                                                                                 3.66e-06 ***
raceethWhite
                                        62.865269
                                                         13.555355
                                                                         4.638
                                                                         -0.679
                                                                                 0.497026
preschool1
                                         -2.008505
                                                         2.956941
expectBachelors1
                                         53.227613
                                                         3.576399
                                                                         14.883
                                                                                 < 2e-16 ***
motherHS1
                                         4.375418
                                                                         0.864
                                                         5.063943
                                                                                 0.387631
                                                                                 0.000687 ***
motherBachelors1
                                         11.151077
                                                         3.281944
                                                                         3.398
motherWork1
                                         -2.268512
                                                         2.953436
                                                                         -0.768
                                                                                 0.442486
fatherHS1
                                         6.891077
                                                                         1.476
                                                                                 0.139905
                                                         4.667189
                                                                                 2.09e-07 ***
fatherBachelors1
                                         17.604801
                                                         3.384319
                                                                         5.202
fatherWork1
                                         3.033776
                                                         3.695971
                                                                         0.821
                                                                                 0.411799
                                                                         0.133
                                                                                 0.894016
selfBornUS1
                                        0.796308
                                                         5.976750
motherBornUS1
                                         -8.337533
                                                         5.669328
                                                                         -1.471
                                                                                 0.141482
fatherBornUS1
                                                         5.369816
                                                                         0.476
                                                                                 0.634005
                                         2.556788
englishAtHome1
                                         10.905428
                                                         5.835964
                                                                         1.869
                                                                                 0.061757.
                                                                                 4.63e-05 ***
computerForSchoolwork1
                                         19.807206
                                                         4.856144
                                                                         4.079
read30MinsADay1
                                         32.736380
                                                         2.862328
                                                                         11.437 < 2e-16 ***
minutesPerWeekEnglish
                                                                         1.324
                                                                                 0.185567
                                        0.011938
                                                         0.009016
studentsInEnglish
                                                                         -0.944
                                         -0.182103
                                                         0.192915
                                                                                 0.345260
schoolHasLibrary1
                                        -1.019002
                                                         7.570382
                                                                         -0.135 0.892933
publicSchool1
                                                                         -3.362 0.000782 ***
                                        -18.794210
                                                         5.590012
urban1
                                        -1.563926
                                                         3.320545
                                                                         -0.471
                                                                                 0.637682
schoolSize
                                        0.007573
                                                                         4.177
                                                                                 3.03e-05 ***
                                                         0.001813
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' '1
Residual standard error: 74.07 on 3372 degrees of freedom
                                Adjusted R-squared: 0.3099
Multiple R-squared: 0.3161,
F-statistic: 50.29 on 31 and 3372 DF, p-value: < 2.2e-16
```

The following were continued to check the Residual Standard Error and the Adjusted R-Square value that was calculated in the above model:

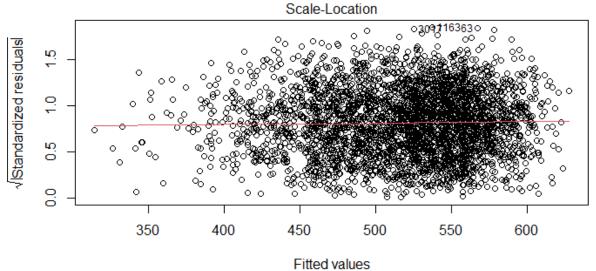
```
> y_predicted <- predict(pisa_ridge_cv, s = pisa_lambda, newx = x)
> sst <- sum((y - mean(y))^2)
> sse <- sum((y_predicted - y)^2)
> rsq <- 1 - sse/sst
> rsq
[1] 0.2925447
> RMSE = sqrt(sse/nrow(Pisa2009))
> RMSE
[1] 74.98547
```

The Ridge Trace Plot listed earlier demonstrates how Ridge Regression adds a degree of bias to the estimates and reduces the standard errors. The idea is for this technique to provide more reliable estimates.

The residual plots are as follows:

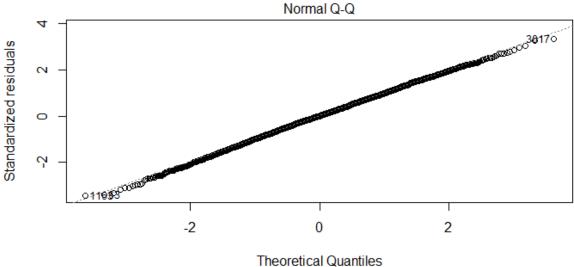


The "Residuals vs Leverage" plot will assist us with identifying outliers which may have influence on the change in the slope of the line.



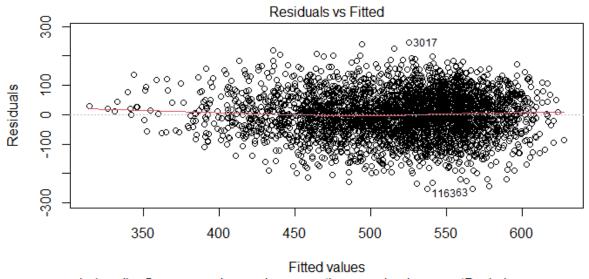
Im(readingScore ~ grade + male + raceeth + preschool + expectBachelors + mo ...

The Scale-Location Plot above verifies the homoskedasticity assumption.



Im(readingScore ~ grade + male + raceeth + preschool + expectBachelors + mo ...

The above QQ Plot displays that a majority of the data are on the line, which indicates that the residuals are normal.



 $Im(readingScore \sim grade + male + raceeth + preschool + expectBachelors + mo \dots \\$ 

The above "Residuals vs Fitted" Plot displays heteroscedacity as there is different variance between that data points at the left-end and right-end of the plot. I suppose that there could be some argument for homoscedacity as well. This is something that I'll need to investigate further.

The following commands were used to generate the LASSO coefficients:

```
set.seed(123) # random numbers
pisa_lasso <- cv.glmnet(x,y, family="gaussian", alpha=1)
coef(pisa_lasso, s=pisa_lambda)
```

The coefficients are as follows:

```
> set.seed(123) # random numbers
> pisa_lasso <- cv.glmnet(x,y, family="gaussian", alpha=1)
> coef(pisa_lasso, s=pisa_lambda)
24 x 1 sparse Matrix of class "dgCMatrix"
(Intercept)
                             196.693536580
grade
                             23.868517417
male
                             -6.181739917
                             8.823500814
raceeth
preschool
expectBachelors
                             51.560027629
motherHS
motherBachelors
                             7.896835697
motherWork
fatherHS
                             5.882282620
fatherBachelors
                             20.590565216
fatherWork
selfBornUS
motherBornUS
fatherBornUS
englishAtHome
computerForSchoolwork
                             17.527065546
read30MinsADay
                             28.089591247
minutesPerWeekEnglish
studentsInEnglish
schoolHasLibrary
publicSchool
                             -4.643123913
urban
schoolSize
                             0.001126722
```

The following is the model created that uses the features generated via LASSO:

```
Call:
lm(formula = readingScore ~ grade + male + raceeth + expectBachelors +
  motherBachelors + fatherHS + fatherBachelors + computerForSchoolwork +
  read30MinsADay + publicSchool + schoolSize, data = Pisa2009)
Residuals:
  Min
          1Q Median
                         30
                               Max
-256.107 -48.660 1.594 49.687 244.674
Coefficients:
                                            Estimate
                                                          Std. Error
                                                                         t value Pr(>|t|)
                                            304.822253
                                                                         5.598 2.34e-08 ***
(Intercept)
                                                          54.450077
grade9
                                            38.727433
                                                          52.641785
                                                                         0.736 0.461977
grade10
                                            88.635051
                                                          52.493357
                                                                         1.689
                                                                                0.091407.
grade11
                                            103.518161
                                                          52.557953
                                                                         1.970
                                                                                0.048966 *
grade12
                                                                         2.419 0.015608 *
                                            155.501148
                                                          64.278458
                                                                         -4.722 2.43e-06 ***
male1
                                            -12.442632
                                                          2.634792
raceethAsian
                                            58.293463
                                                          14.887589
                                                                         3.916 9.20e-05 ***
raceethBlack
                                                                         -0.262 0.793159
                                            -3.686041
                                                          14.056466
raceethHispanic
                                            26.763415
                                                          13.724340
                                                                         1.950 0.051250.
raceethMore than one race
                                            43.606069
                                                          15.051915
                                                                         2.897
                                                                                0.003791 **
raceethNative Hawaiian/Other Pacific Islander
                                           54.683162
                                                          19.837983
                                                                         2.756
                                                                                0.005874 **
raceethWhite
                                            62.922968
                                                          13.517890
                                                                         4.655
                                                                                3.37e-06 ***
expectBachelors1
                                            53.096680
                                                          3.559075
                                                                         14.919 < 2e-16 ***
motherBachelors1
                                            11.212567
                                                          3.233992
                                                                         3.467
                                                                                0.000533 ***
fatherHS1
                                            9.574209
                                                          4.179756
                                                                         2.291
                                                                                0.022047 *
fatherBachelors1
                                                          3.355885
                                                                         5.408
                                            18.147346
                                                                                6.83e-08 ***
computerForSchoolwork1
                                                          4.797978
                                                                         4.236 2.34e-05 ***
                                            20.322678
read30MinsADay1
                                                                         11.532 < 2e-16 ***
                                            32.886562
                                                          2.851702
publicSchool1
                                                                         -3.543 0.000401 ***
                                            -17.583400
                                                          4.962743
schoolSize
                                            0.006759
                                                          0.001623
                                                                         4.165 3.19e-05 ***
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 74.05 on 3384 degrees of freedom Multiple R-squared: 0.3141, Adjusted R-squared: 0.3102 F-statistic: 81.55 on 19 and 3384 DF, p-value: < 2.2e-16

## 1c.)

In this particular case, the models are not identical, but they are very close as the Adj. R-Squared value of the model based on Ridge is 30.99% and the Adj. R-Squared value for the model based on LASSO is 31%. Another difference is how the model variables are created. Ridge creates all of the variables and they are then added to model. For LASSO, the variables that are not initially needed are zeroed out. This demonstrates how useful LASSO regression is with feature selection.

#### 2a. & 2b.)

The following code was used to create the Logistic model:

# Convert to factor because the current variable contains two-levels

remission\$remiss <- as.factor(remission\$remiss) summary(remission)

# Using "family = binomial" to tell us that we're using Logistic Regression

rem\_model <- glm(remiss ~., family = "binomial", data=remission) summary(rem\_model)

Call:

glm(formula = remiss ~ ., family = "binomial", data = remission)

Deviance Residuals:

Min 1Q Median 3Q Max -1.95165 -0.66491 -0.04372 0.74304 1.67069

Coefficients:

	Estimate	Std. Error	z value $Pr(> z )$
(Intercept)	58.0385	71.2364	0.815 0.4152
cell	24.6615	47.8377	0.516 0.6062
smear	19.2936	57.9500	0.333 0.7392
infil	-19.6013	61.6815	-0.318 0.7507
li	3.8960	2.3371	1.667 0.0955.
blast	0.1511	2.2786	0.066 0.9471
temp	-87.4339	67.5735	-1.294 0.1957

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 34.372 on 26 degrees of freedom Residual deviance: 21.751 on 20 degrees of freedom

AIC: 35.751

Number of Fisher Scoring iterations: 8

# 2c.)

The glm() function means "General Linear Model". The lm() function fits models in the form of Y = Xb + e, where glm() fits models in the form of f(Y) = Xb + e and the "e" or distribution of the error term can be specified.

# 2d.)

Using the glm() function doesn't appear to create a model that fits the dataset "remission". The p-values for each of the variables aren't good.

The following will display the confidence interval and coefficients of the model, however, since this model appears to be a bad fit, I'm unsure if we should proceed with these steps:

```
# Confidence interval of the model at 95% confint(rem_model)

# Coefficients of the model coef(rem_model)

# This "delogs" the coefficient exp(coef(rem_model)) -1
```

```
> confint(rem_model) # Confidence interval of the model.. at 95%
                  2.5 %
                            97.5 %
(Intercept)
              -70.9683777
                            222.202990
cell
              -27.7332544 138.404531
              -60.4544868 152.174139
smear
              -159.7565104 67.536927
infil
li
              0.1944541
                            9.526820
              -4.5238625
blast
                            4.715064
              -244.7720744 24.913187
temp
There were 26 warnings (use warnings() to see them)
> coef(rem_model) # Coefficients of the model
(Intercept)
                                           infil
              cell
                            smear
                                                        li
                                                               blast
58.0384871
              24.6615439 19.2935746 -19.6012612 3.8959633 0.1510923 -87.4339024
>
>
> exp(coef(rem_model)) -1 # This "delogs" the coefficient..
 (Intercept)
                     cell
                                   smear
                                                 infil
                                                               li
                                                                              blast
1.606182e + 25 5.133014e + 10 2.393828e + 08 -1.000000e + 00 4.820343e + 01 1.631040e - 01
    temp
-1.000000e+00
```