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CSC 578 NN&DL Spring, 2022

HW7: Image Classification using a CNN

This code is slightly modified from the TensorFlow tutorial Convolutional Neural Network (CNN) for the purpose of our homework. The code first downloads the data, the CIFAR-10 dataset and partitions the training set into training and validation sets. Then the code builds a CNN network and trains the network with the training set. Finally the code evaluates the network performance using the validation set.

Note that there are **three places** in the code, indicated with **IMPORTANT**, where you have to choose the syntax that works for the version of TensorFlow (1 or 2) installed on your platform.

Import Tensorflow

IMPORTANT (1) Uncomment either import line(s) for the version of TensorFlow (TF1 or TF2) of your platform.

```
import matplotlib.pyplot as plt
import numpy as np
import tensorflow as tf
print(tf.__version__) # check the TF version!

2.8.0

In [215... # For TF version 2 (just one line)
from tensorflow.keras import datasets, layers, models
# For TF version 1 (need both lines)
# from tensorflow import keras
# from keras import datasets, layers, models
```

Download and prepare the CIFAR10 dataset¶

The CIFAR10 dataset contains 60,000 color images in 10 classes, with 6,000 images in each class. The dataset is (pre-)divided into 50,000 training images and 10,000 testing images.

```
In [216...
# Download the data from the repository site.
    (train_all_images, train_all_labels), (test_images, test_labels) = datasets.cifar10.loa
```

KeyboardInterrupt

Traceback (most recent call last)

```
~\AppData\Local\Temp/ipykernel 17924/907951196.py in <module>
              1 # Download the data from the repository site.
        ----> 2 (train_all_images, train_all_labels), (test_images, test_labels) = datasets.cif
        ar10.load_data()
        ~\anaconda3\lib\site-packages\keras\datasets\cifar10.py in load data()
                  dirname = 'cifar-10-batches-py'
             78
                  origin = 'https://www.cs.toronto.edu/~kriz/cifar-10-python.tar.gz'
        ---> 79
                 path = get_file(
                      dirname,
             80
             81
                      origin=origin,
        ~\anaconda3\lib\site-packages\keras\utils\data_utils.py in get_file(fname, origin, unta
        r, md5_hash, file_hash, cache_subdir, hash_algorithm, extract, archive_format, cache_di
        r)
                    # File found; verify integrity if a hash was provided.
            246
                    if file hash is not None:
            247
                      if not validate_file(fpath, file_hash, algorithm=hash_algorithm):
        --> 248
            249
                        io utils.print msg(
                             'A local file was found, but it seems to be '
            250
        ~\anaconda3\lib\site-packages\keras\utils\data_utils.py in validate_file(fpath, file_has
        h, algorithm, chunk size)
                  hasher = resolve hasher(algorithm, file hash)
            360
            361
                  if str( hash file(fpath, hasher, chunk size)) == str(file hash):
        --> 362
            363
                    return True
            364
                  else:
        ~\anaconda3\lib\site-packages\keras\utils\data_utils.py in hash file(fpath, algorithm,
         chunk_size)
            339
                 with open(fpath, 'rb') as fpath_file:
            340
                    for chunk in iter(lambda: fpath file.read(chunk size), b''):
                      hasher.update(chunk)
        --> 341
            342
                  return hasher.hexdigest()
            343
        KeyboardInterrupt:
In [ ]:
         # !! DO NOT REMOVE THIS LINE !!
         # Delete test labels (by making it an empty list) so that we don't accidentally
         # use it in the code.
         #test labels = []
         # Then split the training set ('train_all') into two subsets: train and
         # validation. After that, we have 3 subsets: train, validation and test.
         from sklearn.model selection import train test split
         # 80% train, 20% validation, and by using stratefied sampling.
         train_images, valid_images, train_labels, valid_labels \
           = train test split(train all images, train all labels,
                              stratify=train all labels, test size=0.2)
In [ ]:
         # Normalize pixel values of images to be between 0 and 1
         train images, valid images, test images \
           = train images / 255.0, valid images / 255.0, test images / 255.0
```

```
In [ ]: test_labels
In [ ]: train_labels
In [ ]: valid_labels
```

Verify the data

To verify that the dataset looks correct, plot the first 10 images from the training set and display the class name below each image.

Create a convolutional network

As input, a CNN takes tensors of shape (image_height, image_width, color_channels), ignoring the batch size, where color_channels refers to (R,G,B). The format of CIFAR images is 32 * 32 pixels, so the input shape is (32, 32, 3). The output layer has 10 nodes, corresponding to the number of categories of the images.

In this code, the activation function of the output layer is specified to be softmax for the purpose of aligning the two versions of TensorFlow (TF1 and TF2; in particular to make TF2 compatible with TF1's 'sparse_categorical_crossentropy' loss function).

```
In [ ]:
    model = models.Sequential()
    model.add(layers.Conv2D(32, (3, 3), activation='relu', input_shape=(32, 32, 3)))
    model.add(layers.MaxPooling2D((2, 2)))
    model.add(layers.Conv2D(64, (3, 3), activation='relu'))
    model.add(layers.Conv2D(64, (3, 3), activation='relu'))
    model.add(layers.Flatten())
    model.add(layers.Dense(64, activation='relu'))
    model.add(layers.Dense(10, activation='softmax')) # As noted above
```

Verify the model

```
In [ ]:
```

```
model.summary()
```

Compile the model

IMPORTANT (2) Uncomment either loss function for the version of TensorFlow (TF1 or TF2) of your platform.

Train the model

Evaluate the model

IMPORTANT (3) Uncomment either syntax for the version of TensorFlow (TF1 or TF2) of your platform.

```
plt.plot(history.history['accuracy'], label='training accuracy') # For TF2
#plt.plot(history.history['acc'], label='training accuracy') # For TF1
plt.plot(history.history['val_accuracy'], label = 'valid. accuracy') # For TF2
#plt.plot(history.history['val_acc'], label = 'valid. accuracy') # For TF1
plt.xlabel('Epoch')
plt.ylabel('Accuracy')
plt.ylim([0.5, 1])
plt.legend(loc='lower right')

# Evaluate the Learned model with validation set
valid_loss, valid_acc = model.evaluate(valid_images, valid_labels, verbose=2)
print ("valid_accuracy=%s, valid_loss=%s" % (valid_acc, valid_loss))
```

TO DO (by you): Make Predictions

Apply the learned network to 'test_images' and generate predictions.

Look at the code from HW#4 or other tutorial code for the syntax. You should generate predictions and create/write a KAGGLE submission file.

First Model

This model was built as a baseline to become familiar with the tools and project. Extra layers were added to this model to see the effect on its accuracy.

```
model 1.add(layers.MaxPooling2D((2, 2)))
         model_1.add(layers.Conv2D(64, (3, 3), activation='relu'))
         model_1.add(layers.Conv2D(64, (3, 3), activation='relu'))
         model 1.add(layers.MaxPooling2D((2, 2)))
         model 1.add(layers.Conv2D(64, (3, 3), activation='relu'))
         model_1.add(layers.Conv2D(64, (3, 3), activation='relu'))
         model 1.add(layers.Flatten())
         model 1.add(layers.Dense(64, activation='relu'))
         model 1.add(layers.Dense(10, activation='softmax')) # As noted above
In [ ]:
         model_1.summary()
In [ ]:
         model 1.compile(optimizer='adam',
                       loss=tf.keras.losses.SparseCategoricalCrossentropy(from logits=False), #
                       #loss='sparse categorical crossentropy', # For TF1
                       metrics=['accuracy'])
In [ ]:
         history_1 = model_1.fit(train_images, train_labels, epochs=10,
                             validation data=(valid images, valid labels))
In [ ]:
         history 1.history.keys()
In [ ]:
         plt.plot(history_1.history['accuracy'], label='training accuracy') # For TF2
         #plt.plot(history.history['acc'], label='training accuracy') # For TF1
         plt.plot(history_1.history['val_accuracy'], label = 'valid. accuracy') # For TF2
         #plt.plot(history.history['val acc'], label = 'valid. accuracy') # For TF1
         plt.xlabel('Epoch')
         plt.ylabel('Accuracy')
         plt.ylim([0.1, 1])
         plt.legend(loc='lower right')
         # Evaluate the learned model with validation set
         valid loss, valid acc = model 1.evaluate(valid images, valid labels, verbose=2)
         print ("valid_accuracy=%s, valid_loss=%s" % (valid_acc, valid_loss))
In [ ]:
         print(valid acc)
In [ ]:
         predictions = model 1.predict(valid images)
In [ ]:
         predictions[0]
In [ ]:
         np.argmax(predictions[0])
In [ ]:
```

```
# len(test labels)
         # len(valid_labels)
In [ ]:
         valid labels
         #indices = np.random.choice(range(len(valid labels)), replace=False)
         #valid labels=np.array(valid labels)[indices.astype(int)]
         valid labels
In [ ]:
         len(test_images)
In [ ]:
         #test_labels[0]
         valid labels[0]
In [ ]:
         valid_labels = np.concatenate(valid_labels)
In [ ]:
         def plot_image(i, predictions_array, true_label, img):
           true_label, img = true_label[i], img[i]
           plt.grid(False)
           plt.xticks([])
           plt.yticks([])
           plt.imshow(img, cmap=plt.cm.binary)
           predicted_label = np.argmax(predictions_array)
           if predicted label == true label:
             color = 'blue'
           else:
             color = 'red'
           plt.xlabel("{} {:2.0f}% ({})".format(class_names[predicted_label],
                                          100*np.max(predictions array),
                                          class names[true label]),
                                          color=color)
         def plot_value_array(i, predictions_array, true_label):
           true_label = true_label[i]
           plt.grid(False)
           plt.xticks(range(10))
           plt.yticks([])
           thisplot = plt.bar(range(10), predictions array, color="#777777")
           plt.ylim([0, 1])
           predicted_label = np.argmax(predictions_array)
           thisplot[predicted_label].set_color('red')
           thisplot[true label].set color('blue')
In [ ]:
         i = 9
```

plt.figure(figsize=(6,3))

```
plt.subplot(1,2,1)
#plot_image(i, predictions[i], test_labels, test_images)
plot_image(i, predictions[i], valid_labels, valid_images)
plt.subplot(1,2,2)
#plot_value_array(i, predictions[i], test_labels)
plot_value_array(i, predictions[i], valid_labels)
plt.show()
In []:
# Plot the first X test images, their predicted labels, and the true labels.
# Color correct predictions in blue and incorrect predictions in red.
num_rows = 5
num_cols = 3
num images = num rows*num cols
```

plt.figure(figsize=(2*2*num_cols, 2*num_rows))

plt.subplot(num rows, 2*num cols, 2*i+1)

plt.subplot(num rows, 2*num cols, 2*i+2)

plot_image(i, predictions[i], valid_labels, valid_images)
plot_image(i, predictions[i], test_labels, test_images)

plot_value_array(i, predictions[i], test_labels)
plot value array(i, predictions[i], valid labels)

for i in range(num images):

```
plt.show()
In []:
```

Model 2 </h2>

plt.tight layout()

A duplicate of the first model. Trying to understand why there was an increase in accuracy with no changes.

```
model_2 = models.Sequential()
model_2.add(layers.Conv2D(32, (3, 3), activation='relu', input_shape=(32, 32, 3)))
model_2.add(layers.Conv2D(64, (3, 3), activation='relu'))
model_2.add(layers.MaxPooling2D((2, 2)))
model_2.add(layers.Conv2D(64, (3, 3), activation='relu'))
model_2.add(layers.Conv2D(64, (3, 3), activation='relu'))
model_2.add(layers.MaxPooling2D((2, 2)))
model_2.add(layers.Conv2D(64, (3, 3), activation='relu'))
model_2.add(layers.Conv2D(64, (3, 3), activation='relu'))
model_2.add(layers.Flatten())
model_2.add(layers.Dense(64, activation='relu'))
model_2.add(layers.Dense(10, activation='relu'))
# As noted above
In []:
model_2.summary()
```

```
#loss='sparse categorical crossentropy', # For TF1
                       metrics=['accuracy'])
In [ ]:
         history_2 = model_2.fit(train_images, train_labels, epochs=10,
                             validation data=(valid images, valid labels))
In [ ]:
         history 2.history.keys()
In [ ]:
         plt.plot(history_2.history['accuracy'], label='training accuracy') # For TF2
         #plt.plot(history.history['acc'], label='training accuracy') # For TF1
         plt.plot(history_2.history['val_accuracy'], label = 'valid. accuracy') # For TF2
         #plt.plot(history.history['val_acc'], label = 'valid. accuracy') # For TF1
         plt.xlabel('Epoch')
         plt.ylabel('Accuracy')
         plt.ylim([0.1, 1])
         plt.legend(loc='lower right')
         # Evaluate the learned model with validation set
         valid loss, valid acc = model 2.evaluate(valid images, valid labels, verbose=2)
         print ("valid accuracy=%s, valid loss=%s" % (valid acc, valid loss))
In [ ]:
         print(valid acc)
In [ ]:
         predictions 2 = model 2.predict(valid images)
In [ ]:
         valid labels = np.concatenate(valid labels)
In [ ]:
         i = 17
         plt.figure(figsize=(6,3))
         plt.subplot(1,2,1)
         #plot image(i, predictions[i], test labels, test images)
         plot_image(i, predictions_2[i], valid_labels, valid_images)
         plt.subplot(1,2,2)
         #plot value array(i, predictions[i], test labels)
         plot_value_array(i, predictions_2[i], valid_labels)
         plt.show()
In [ ]:
         # Plot the first X test images, their predicted labels, and the true labels.
         # Color correct predictions in blue and incorrect predictions in red.
         num rows = 6
         num cols = 4
         num_images = num_rows*num_cols
         plt.figure(figsize=(2*2*num cols, 2*num rows))
         for i in range(num images):
           plt.subplot(num rows, 2*num cols, 2*i+1)
           plot_image(i, predictions_2[i], valid_labels, valid_images)
           # plot_image(i, predictions[i], test_labels, test_images)
           plt.subplot(num rows, 2*num cols, 2*i+2)
           # plot_value_array(i, predictions[i], test_labels)
```

```
plot_value_array(i, predictions_2[i], valid_labels)
plt.tight_layout()
plt.show()
In []:
```

Model 3 - Removing layers

Several layers from the initial model were removed. Unfortunately, this resulted in overfitting.

```
In [79]:
    model_3 = models.Sequential()
    model_3.add(layers.Conv2D(32, (3, 3), padding='same', activation='relu', input_shape=(3
    model_3.add(layers.MaxPooling2D((2, 2)))

    model_3.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
    model_3.add(layers.MaxPooling2D((2, 2)))

    model_3.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
    model_3.add(layers.Flatten())

    model_3.add(layers.Dense(64, activation='relu'))
    model_3.add(layers.Dense(10, activation='softmax')) # As noted above
```

In [80]: model_

```
model_3.summary()
```

Model: "sequential_4"

```
Layer (type)
                         Output Shape
                                                 Param #
______
conv2d 15 (Conv2D)
                         (None, 32, 32, 32)
                                                 896
max pooling2d 8 (MaxPooling (None, 16, 16, 32)
2D)
conv2d 16 (Conv2D)
                         (None, 16, 16, 64)
                                                18496
max pooling2d 9 (MaxPooling (None, 8, 8, 64)
2D)
conv2d_17 (Conv2D)
                         (None, 8, 8, 64)
                                                36928
flatten 4 (Flatten)
                         (None, 4096)
dense 8 (Dense)
                         (None, 64)
                                                 262208
dense 9 (Dense)
                         (None, 10)
                                                 650
Total params: 319,178
```

Total params: 319,178
Trainable params: 319,178
Non-trainable params: 0

```
In [81]: model_3.compile(optimizer='adam',
```

```
loss=tf.keras.losses.SparseCategoricalCrossentropy(from logits=False), #
         #loss='sparse categorical crossentropy', # For TF1
         metrics=['accuracy'])
history 3 = model 3.fit(train images, train labels, epochs=10,
             validation data=(valid images, valid labels))
Epoch 1/10
4691 - val_loss: 1.1707 - val_accuracy: 0.5891
Epoch 2/10
6279 - val loss: 1.0067 - val accuracy: 0.6491
Epoch 3/10
6914 - val_loss: 0.9186 - val_accuracy: 0.6787
7330 - val_loss: 0.9376 - val_accuracy: 0.6788
Epoch 5/10
7604 - val loss: 0.8591 - val accuracy: 0.7057
Epoch 6/10
7858 - val_loss: 0.8327 - val_accuracy: 0.7172
Epoch 7/10
8109 - val loss: 0.8657 - val accuracy: 0.7196
Epoch 8/10
8307 - val loss: 0.8865 - val accuracy: 0.7164
Epoch 9/10
8522 - val_loss: 0.9970 - val_accuracy: 0.7157
Epoch 10/10
8709 - val loss: 1.0529 - val accuracy: 0.7085
history_3.history.keys()
dict_keys(['loss', 'accuracy', 'val_loss', 'val_accuracy'])
plt.plot(history_3.history['accuracy'], label='training accuracy') # For TF2
#plt.plot(history.history['acc'], label='training accuracy') # For TF1
plt.plot(history_3.history['val_accuracy'], label = 'valid. accuracy') # For TF2
#plt.plot(history.history['val acc'], label = 'valid. accuracy') # For TF1
plt.xlabel('Epoch')
plt.ylabel('Accuracy')
plt.ylim([0.1, 1])
plt.legend(loc='lower right')
# Evaluate the learned model with validation set
valid loss, valid acc = model 3.evaluate(valid images, valid labels, verbose=2)
```

In [82]:

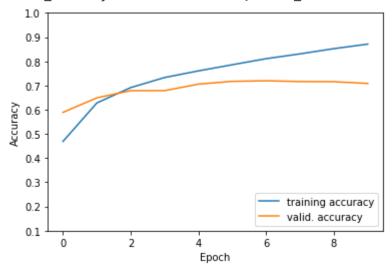
In [83]:

Out[83]:

In [84]:

print ("valid accuracy=%s, valid loss=%s" % (valid acc, valid loss))

valid accuracy=0.7085000276565552, valid loss=1.0528604984283447



In []:

Model 4 - Trial and error with layers and padding

Unfortunately, the final result for this model was overfitting.

```
model_4 = models.Sequential()
model_4.add(layers.Conv2D(32, (3, 3), padding='same', activation='relu', input_shape=(3
model_4.add(layers.Conv2D(32, (3, 3), padding='same', activation='relu'))
model_4.add(layers.MaxPooling2D((2, 2), strides=(2,2)))

model_4.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_4.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_4.add(layers.MaxPooling2D((2, 2), strides=(2,2)))

model_4.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_4.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_4.add(layers.Platten())
model_4.add(layers.Dense(64, activation='relu'))
model_4.add(layers.Dense(64, activation='relu'))
model_4.add(layers.Dense(10, activation='softmax')) # As noted above
```

In [87]: model_4.summary()

Model: "sequential 5"

Layer (type)	Output Shape	Param #
conv2d_18 (Conv2D)	(None, 32, 32, 32)	896
conv2d_19 (Conv2D)	(None, 32, 32, 32)	9248
<pre>max_pooling2d_10 (MaxPoolin g2D)</pre>	(None, 16, 16, 32)	0
conv2d_20 (Conv2D)	(None, 16, 16, 64)	18496

```
conv2d 21 (Conv2D)
                 (None, 16, 16, 64)
                                36928
max_pooling2d_11 (MaxPoolin (None, 8, 8, 64)
g2D)
conv2d_22 (Conv2D)
                 (None, 8, 8, 64)
                                36928
conv2d 23 (Conv2D)
                 (None, 8, 8, 64)
                                36928
flatten 5 (Flatten)
                 (None, 4096)
dense 10 (Dense)
                 (None, 64)
                                262208
dense 11 (Dense)
                 (None, 10)
                                650
______
Total params: 402,282
Trainable params: 402,282
Non-trainable params: 0
model_4.compile(optimizer='adam',
        loss=tf.keras.losses.SparseCategoricalCrossentropy(from logits=False), #
        #loss='sparse categorical crossentropy', # For TF1
        metrics=['accuracy'])
history_4 = model_4.fit(train_images, train_labels, epochs=10,
            validation data=(valid images, valid labels))
Epoch 1/10
0.4390 - val loss: 1.2418 - val accuracy: 0.5564
Epoch 2/10
0.6253 - val_loss: 0.9948 - val_accuracy: 0.6435
Epoch 3/10
0.6945 - val loss: 0.8799 - val accuracy: 0.6968
0.7426 - val loss: 0.8082 - val accuracy: 0.7153
Epoch 5/10
0.7797 - val loss: 0.7710 - val accuracy: 0.7291
Epoch 6/10
0.8120 - val loss: 0.7993 - val accuracy: 0.7350
Epoch 7/10
0.8411 - val loss: 0.8470 - val accuracy: 0.7326
Epoch 8/10
0.8597 - val_loss: 0.8631 - val_accuracy: 0.7295
Epoch 9/10
0.8859 - val_loss: 0.9778 - val_accuracy: 0.7322
Epoch 10/10
```

In [88]:

In [89]:

```
0.9034 - val_loss: 1.0310 - val_accuracy: 0.7243
In [91]:
         history_4.history.keys()
        dict_keys(['loss', 'accuracy', 'val_loss', 'val_accuracy'])
Out[91]:
In [92]:
         plt.plot(history_4.history['accuracy'], label='training accuracy') # For TF2
         #plt.plot(history.history['acc'], label='training accuracy') # For TF1
         plt.plot(history_4.history['val_accuracy'], label = 'valid. accuracy') # For TF2
         #plt.plot(history.history['val_acc'], label = 'valid. accuracy') # For TF1
         plt.xlabel('Epoch')
         plt.ylabel('Accuracy')
         plt.ylim([0.1, 1])
         plt.legend(loc='lower right')
         # Evaluate the learned model with validation set
         valid loss, valid acc = model 4.evaluate(valid images, valid labels, verbose=2)
         print ("valid accuracy=%s, valid loss=%s" % (valid acc, valid loss))
        313/313 - 8s - loss: 1.0310 - accuracy: 0.7243 - 8s/epoch - 25ms/step
        valid accuracy=0.7243000268936157, valid loss=1.0310277938842773
          1.0
           0.9
           0.8
           0.7
          0.6
```

```
In [95]: print(valid_acc)
0.7243000268936157
In []:
```

0.5 0.4 0.3

0.2

0.1

Model 5 - Adjusting layers and experimenting with Spatial Dropout and Strides

training accuracy

valid. accuracy

The final result appearing to be a well fitting model, according to the plot. Unfortunately the accuracy of this model was below the accuracy of the initial model.

```
In [107...
    model_5 = models.Sequential()
    model_5.add(layers.Conv2D(32, (3, 3), padding='same', activation='relu', input_shape=(3)
```

```
model_5.add(layers.Conv2D(32, (3, 3), padding ='same', activation='relu'))
model_5.add(layers.SpatialDropout2D(0.25))
model_5.add(layers.MaxPooling2D((2, 2), strides=(2,2)))

model_5.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_5.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_5.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_5.add(layers.SpatialDropout2D(0.25))
model_5.add(layers.MaxPooling2D((2, 2), strides=(2,2)))

model_5.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_5.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_5.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_5.add(layers.MaxPooling2D((2, 2), strides=(2,2)))

model_5.add(layers.Flatten())
model_5.add(layers.Dense(64, activation='relu'))
model_5.add(layers.Dense(10, activation='softmax')) # As noted above
```

In [108...

model_5.summary()

Model: "sequential_14"

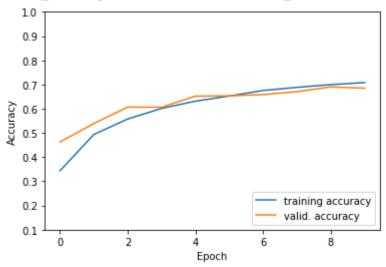
Layer (type)	Output Shape	Param #
conv2d_43 (Conv2D)	(None, 32, 32, 32)	896
conv2d_44 (Conv2D)	(None, 32, 32, 32)	9248
<pre>spatial_dropout2d_8 (Spatia lDropout2D)</pre>	(None, 32, 32, 32)	0
<pre>max_pooling2d_13 (MaxPoolin g2D)</pre>	(None, 16, 16, 32)	0
conv2d_45 (Conv2D)	(None, 16, 16, 64)	18496
conv2d_46 (Conv2D)	(None, 16, 16, 64)	36928
conv2d_47 (Conv2D)	(None, 16, 16, 64)	36928
<pre>spatial_dropout2d_9 (Spatia lDropout2D)</pre>	(None, 16, 16, 64)	0
<pre>max_pooling2d_14 (MaxPoolin g2D)</pre>	(None, 8, 8, 64)	0
conv2d_48 (Conv2D)	(None, 8, 8, 64)	36928
conv2d_49 (Conv2D)	(None, 8, 8, 64)	36928
conv2d_50 (Conv2D)	(None, 8, 8, 64)	36928
<pre>max_pooling2d_15 (MaxPoolin g2D)</pre>	(None, 4, 4, 64)	0

```
flatten 6 (Flatten)
                     (None, 1024)
      dense_12 (Dense)
                      (None, 64)
                                     65600
      dense_13 (Dense)
                      (None, 10)
                                      650
     _____
     Total params: 279,530
     Trainable params: 279,530
     Non-trainable params: 0
In [109...
      model 5.compile(optimizer='adam',
              loss=tf.keras.losses.SparseCategoricalCrossentropy(from logits=False), #
              #loss='sparse categorical crossentropy', # For TF1
              metrics=['accuracy'])
In [110...
      history_5 = model_5.fit(train_images, train_labels, epochs=10,
                 validation data=(valid images, valid labels))
     Epoch 1/10
     0.3436 - val_loss: 1.4596 - val_accuracy: 0.4627
     Epoch 2/10
     0.4936 - val loss: 1.2639 - val accuracy: 0.5388
     Epoch 3/10
     0.5576 - val_loss: 1.0964 - val_accuracy: 0.6068
     Epoch 4/10
     0.6012 - val loss: 1.1101 - val accuracy: 0.6061
     Epoch 5/10
     0.6312 - val_loss: 0.9832 - val_accuracy: 0.6519
     Epoch 6/10
     0.6524 - val_loss: 0.9679 - val_accuracy: 0.6530
     Epoch 7/10
     0.6754 - val loss: 0.9713 - val accuracy: 0.6585
     Epoch 8/10
     0.6883 - val_loss: 0.9228 - val_accuracy: 0.6705
     Epoch 9/10
     0.6995 - val loss: 0.8886 - val accuracy: 0.6902
     Epoch 10/10
     0.7079 - val loss: 0.8878 - val accuracy: 0.6845
In [111...
      history_5.history.keys()
     dict_keys(['loss', 'accuracy', 'val_loss', 'val_accuracy'])
Out[111...
In [112...
      plt.plot(history_5.history['accuracy'], label='training accuracy') # For TF2
```

```
#plt.plot(history.history['acc'], label='training accuracy') # For TF1
plt.plot(history_5.history['val_accuracy'], label = 'valid. accuracy') # For TF2
#plt.plot(history.history['val_acc'], label = 'valid. accuracy') # For TF1
plt.xlabel('Epoch')
plt.ylabel('Accuracy')
plt.ylim([0.1, 1])
plt.legend(loc='lower right')

# Evaluate the learned model with validation set
valid_loss, valid_acc = model_5.evaluate(valid_images, valid_labels, verbose=2)
print ("valid_accuracy=%s, valid_loss=%s" % (valid_acc, valid_loss))
```

313/313 - 8s - loss: 0.8878 - accuracy: 0.6845 - 8s/epoch - 26ms/step valid_accuracy=0.684499979019165, valid_loss=0.8878217935562134



Model 6 - Experimenting with layers, Spatial Dropout and Strides

More layers were added to this model in addition to additional Epochs. Unfoturnately, the final result was a poor model. This particular model fails.

```
model 6 = models.Sequential()
model_6.add(layers.Conv2D(32, (3, 3), padding='same', activation='relu', input_shape=(3)
model_6.add(layers.Conv2D(32, (3, 3), padding ='same', activation='relu'))
model_6.add(layers.SpatialDropout2D(0.2))
model 6.add(layers.MaxPooling2D((2, 2), strides=(2,2)))
model_6.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_6.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_6.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model 6.add(layers.SpatialDropout2D(0.2))
model 6.add(layers.MaxPooling2D((2, 2), strides=(2,2)))
model_6.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_6.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_6.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model 6.add(layers.SpatialDropout2D(0.2))
model_6.add(layers.MaxPooling2D((2, 2), strides=(2,2)))
model_6.add(layers.Conv2D(128, (3, 3), padding='same', activation='relu'))
model_6.add(layers.Conv2D(128, (3, 3), padding='same', activation='relu'))
model 6.add(layers.Conv2D(128, (3, 3), padding='same', activation='relu'))
model 6.add(layers.SpatialDropout2D(0.2))
model 6.add(layers.MaxPooling2D((2, 2), strides=(2,2)))
model_6.add(layers.Flatten())
model_6.add(layers.Dense(64, activation='relu'))
model 6.add(layers.Dense(10, activation='softmax')) # As noted above
```

In [116...

```
model_6.summary()
```

Model: "sequential_15"

Layer (type)	Output Shape	Param #
conv2d_51 (Conv2D)	(None, 32, 32, 32)	896
conv2d_52 (Conv2D)	(None, 32, 32, 32)	9248
<pre>spatial_dropout2d_10 (Spati alDropout2D)</pre>	(None, 32, 32, 32)	0
<pre>max_pooling2d_16 (MaxPoolin g2D)</pre>	(None, 16, 16, 32)	0
conv2d_53 (Conv2D)	(None, 16, 16, 64)	18496
conv2d_54 (Conv2D)	(None, 16, 16, 64)	36928
conv2d_55 (Conv2D)	(None, 16, 16, 64)	36928
<pre>spatial_dropout2d_11 (Spati alDropout2D)</pre>	(None, 16, 16, 64)	0
<pre>max_pooling2d_17 (MaxPoolin g2D)</pre>	(None, 8, 8, 64)	0

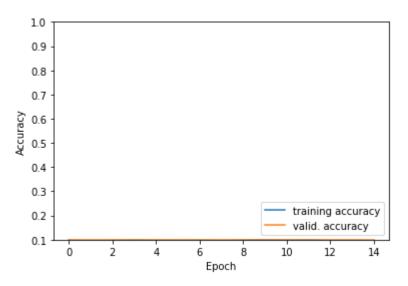
```
conv2d 56 (Conv2D)
                    (None, 8, 8, 64)
                                       36928
                    (None, 8, 8, 64)
conv2d_57 (Conv2D)
                                       36928
                    (None, 8, 8, 64)
conv2d 58 (Conv2D)
                                       36928
spatial_dropout2d_12 (Spati (None, 8, 8, 64)
alDropout2D)
max pooling2d 18 (MaxPoolin (None, 4, 4, 64)
                                       0
g2D)
conv2d_59 (Conv2D)
                    (None, 4, 4, 128)
                                       73856
conv2d 60 (Conv2D)
                    (None, 4, 4, 128)
                                       147584
conv2d 61 (Conv2D)
                    (None, 4, 4, 128)
                                       147584
spatial dropout2d 13 (Spati (None, 4, 4, 128)
alDropout2D)
max_pooling2d_19 (MaxPoolin (None, 2, 2, 128)
g2D)
flatten_7 (Flatten)
                    (None, 512)
dense_14 (Dense)
                    (None, 64)
                                       32832
dense 15 (Dense)
                    (None, 10)
                                       650
______
Total params: 615,786
Trainable params: 615,786
Non-trainable params: 0
model 6.compile(optimizer='adam',
          loss=tf.keras.losses.SparseCategoricalCrossentropy(from logits=False), #
          #loss='sparse categorical crossentropy', # For TF1
          metrics=['accuracy'])
history 6 = model 6.fit(train images, train labels, epochs=15,
              validation data=(valid images, valid labels))
Epoch 1/15
0.0975 - val loss: 2.3026 - val accuracy: 0.1000
Epoch 2/15
0.0954 - val loss: 2.3026 - val accuracy: 0.1000
Epoch 3/15
0.0967 - val_loss: 2.3026 - val_accuracy: 0.1000
Epoch 4/15
0.0984 - val_loss: 2.3026 - val_accuracy: 0.1000
Epoch 5/15
```

In [117...

In [118...

```
0.0970 - val loss: 2.3027 - val accuracy: 0.1000
     Epoch 6/15
     0.0971 - val_loss: 2.3026 - val_accuracy: 0.1000
     Epoch 7/15
     0.0978 - val loss: 2.3026 - val accuracy: 0.1000
     Epoch 8/15
     0.0962 - val loss: 2.3026 - val accuracy: 0.1000
     Epoch 9/15
     0.0983 - val_loss: 2.3026 - val_accuracy: 0.1000
     Epoch 10/15
     0.0994 - val loss: 2.3026 - val accuracy: 0.1000
     Epoch 11/15
     0.0979 - val loss: 2.3027 - val accuracy: 0.1000
     Epoch 12/15
     0.0992 - val_loss: 2.3026 - val_accuracy: 0.1000
     Epoch 13/15
     0.0965 - val_loss: 2.3026 - val_accuracy: 0.1000
     Epoch 14/15
     0.0970 - val_loss: 2.3026 - val_accuracy: 0.1000
     Epoch 15/15
     0.0956 - val_loss: 2.3026 - val_accuracy: 0.1000
In [119...
      plt.plot(history 6.history['accuracy'], label='training accuracy') # For TF2
      #plt.plot(history.history['acc'], label='training accuracy') # For TF1
      plt.plot(history_6.history['val_accuracy'], label = 'valid. accuracy') # For TF2
      #plt.plot(history.history['val acc'], label = 'valid. accuracy') # For TF1
      plt.xlabel('Epoch')
      plt.ylabel('Accuracy')
      plt.ylim([0.1, 1])
      plt.legend(loc='lower right')
      # Evaluate the learned model with validation set
      valid loss, valid acc = model 6.evaluate(valid images, valid labels, verbose=2)
      print ("valid_accuracy=%s, valid_loss=%s" % (valid_acc, valid_loss))
     313/313 - 9s - loss: 2.3026 - accuracy: 0.1000 - 9s/epoch - 30ms/step
```

valid accuracy=0.10000000149011612, valid loss=2.302598714828491



Model 7 - Experiment with Batch Normalization

In this model, Batch Normalization was added in addition to reducing the number of layers and adjusting the Spatial Dropout. The final result appears to be a decent model with an accuracy of 74%.

```
model_7.add(layers.SpatialDropout2D(0.25))
model_7.add(BatchNormalization())
model_7.add(layers.MaxPooling2D((2, 2), strides=(2,2)))

model_7.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_7.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_7.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
model_7.add(layers.SpatialDropout2D(0.25))
model_7.add(BatchNormalization())
model_7.add(layers.MaxPooling2D((2, 2), strides=(2,2)))

model_7.add(layers.Flatten())
model_7.add(layers.Dense(64, activation='relu'))
model_7.add(layers.Dense(10, activation='relu'))
model_7.add(layers.Dense(10, activation='softmax')) # As noted above
```

In [124...

```
model_7.summary()
```

Model: "sequential_17"

Layer (type)	Output Shape	Param #
conv2d_64 (Conv2D)	(None, 32, 32, 32)	896
conv2d_65 (Conv2D)	(None, 32, 32, 32)	9248
<pre>spatial_dropout2d_15 (Spati alDropout2D)</pre>	(None, 32, 32, 32)	0
<pre>batch_normalization (BatchN ormalization)</pre>	(None, 32, 32, 32)	128
<pre>max_pooling2d_20 (MaxPoolin g2D)</pre>	(None, 16, 16, 32)	0
conv2d_66 (Conv2D)	(None, 16, 16, 64)	18496
conv2d_67 (Conv2D)	(None, 16, 16, 64)	36928
conv2d_68 (Conv2D)	(None, 16, 16, 64)	36928
<pre>spatial_dropout2d_16 (Spati alDropout2D)</pre>	(None, 16, 16, 64)	0
<pre>batch_normalization_1 (Batc hNormalization)</pre>	(None, 16, 16, 64)	256
<pre>max_pooling2d_21 (MaxPoolin g2D)</pre>	(None, 8, 8, 64)	0
conv2d_69 (Conv2D)	(None, 8, 8, 64)	36928
conv2d_70 (Conv2D)	(None, 8, 8, 64)	36928
conv2d_71 (Conv2D)	(None, 8, 8, 64)	36928
<pre>spatial_dropout2d_17 (Spati alDropout2D)</pre>	(None, 8, 8, 64)	0

```
hNormalization)
     max pooling2d 22 (MaxPoolin (None, 4, 4, 64)
     g2D)
     flatten_8 (Flatten)
                     (None, 1024)
                     (None, 64)
     dense 16 (Dense)
                                   65600
     dense 17 (Dense)
                     (None, 10)
                                   650
     ______
     Total params: 280,170
     Trainable params: 279,850
     Non-trainable params: 320
In [125...
     model 7.compile(optimizer='adam',
             loss=tf.keras.losses.SparseCategoricalCrossentropy(from logits=False), #
             #loss='sparse_categorical_crossentropy', # For TF1
             metrics=['accuracy'])
In [126...
     history_7 = model_7.fit(train_images, train_labels, epochs=10,
                validation data=(valid images, valid labels))
     Epoch 1/10
     0.3345 - val_loss: 1.5204 - val_accuracy: 0.4427
     Epoch 2/10
     0.4876 - val_loss: 1.2131 - val_accuracy: 0.5540
     Epoch 3/10
     0.5633 - val loss: 1.1156 - val accuracy: 0.6081
     Epoch 4/10
     0.6144 - val loss: 0.9936 - val accuracy: 0.6460
     Epoch 5/10
     0.6481 - val loss: 0.8790 - val accuracy: 0.6910
     Epoch 6/10
     0.6803 - val loss: 0.8026 - val accuracy: 0.7149
     Epoch 7/10
     0.7005 - val_loss: 0.8234 - val_accuracy: 0.7104
     Epoch 8/10
     0.7222 - val loss: 0.7482 - val accuracy: 0.7342
     Epoch 9/10
     0.7376 - val loss: 0.7219 - val accuracy: 0.7470
     Epoch 10/10
```

256

batch normalization 2 (Batc (None, 8, 8, 64)

0.7468 - val loss: 0.7079 - val accuracy: 0.7486

```
plt.plot(history 7.history['accuracy'], label='training accuracy') # For TF2
In [127...
         #plt.plot(history.history['acc'], label='training accuracy') # For TF1
         plt.plot(history_7.history['val_accuracy'], label = 'valid. accuracy') # For TF2
         #plt.plot(history.history['val_acc'], label = 'valid. accuracy') # For TF1
          plt.xlabel('Epoch')
          plt.ylabel('Accuracy')
         plt.ylim([0.1, 1])
         plt.legend(loc='lower right')
         # Evaluate the learned model with validation set
         valid loss, valid acc = model 7.evaluate(valid images, valid labels, verbose=2)
         print ("valid_accuracy=%s, valid_loss=%s" % (valid_acc, valid_loss))
         313/313 - 9s - loss: 0.7079 - accuracy: 0.7486 - 9s/epoch - 29ms/step
         valid accuracy=0.7486000061035156, valid loss=0.7078680396080017
           1.0
           0.9
           0.8
           0.7
           0.6
           0.5
           0.4
           0.3
                                              training accuracy
           0.2
                                              valid. accuracy
           0.1
                         2
                                                    8
                                  Epoch
In [128...
         test_loss, test_acc = model_7.evaluate(valid_images, valid_labels)
          print('\nTest accuracy:', test acc)
         print ('\n')
         train loss, train acc = model 7.evaluate(train images, train labels, verbose = 2)
         print('Training accuracy:', train_acc)
         86
         Test accuracy: 0.7486000061035156
         1250/1250 - 40s - loss: 0.5335 - accuracy: 0.8130 - 40s/epoch - 32ms/step
```

Model 8 - Experimenting with Optimizers

Training accuracy: 0.8130499720573425

In []:

This model is built on top of model 7 and will experiment with several optimizers. The Adagrad optimizer fails as it causes the model performance to suffer. The Adadelta model also fails. The

introduction of Adamax results in a very close fit for the data after the 8th Epoch, and its accuracy is stead at 74%.

```
In [129...
          model_8 = models.Sequential()
          model_8.add(layers.Conv2D(32, (3, 3), padding='same', activation='relu', input_shape=(3
          model_8.add(layers.Conv2D(32, (3, 3), padding ='same', activation='relu'))
          model_8.add(layers.SpatialDropout2D(0.25))
          model 8.add(BatchNormalization())
          model_8.add(layers.MaxPooling2D((2, 2), strides=(2,2)))
          model_8.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
          model_8.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
          model_8.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
          model_8.add(layers.SpatialDropout2D(0.25))
          model 8.add(BatchNormalization())
          model 8.add(layers.MaxPooling2D((2, 2), strides=(2,2)))
          model 8.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
          model_8.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
          model_8.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
          model_8.add(layers.SpatialDropout2D(0.25))
          model 8.add(BatchNormalization())
          model 8.add(layers.MaxPooling2D((2, 2), strides=(2,2)))
          model 8.add(layers.Flatten())
          model_8.add(layers.Dense(64, activation='relu'))
          model 8.add(layers.Dense(10, activation='softmax')) # As noted above
```

In [130...

```
model_8.summary()
```

Model: "sequential_18"

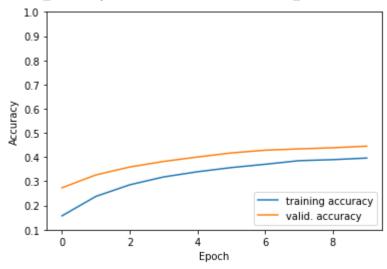
Layer (type)	Output Shape	Param #
conv2d_72 (Conv2D)		896
conv2d_73 (Conv2D)	(None, 32, 32, 32)	9248
<pre>spatial_dropout2d_18 (Spati alDropout2D)</pre>	(None, 32, 32, 32)	0
<pre>batch_normalization_3 (Batc hNormalization)</pre>	(None, 32, 32, 32)	128
<pre>max_pooling2d_23 (MaxPoolin g2D)</pre>	(None, 16, 16, 32)	0
conv2d_74 (Conv2D)	(None, 16, 16, 64)	18496
conv2d_75 (Conv2D)	(None, 16, 16, 64)	36928
conv2d_76 (Conv2D)	(None, 16, 16, 64)	36928
<pre>spatial_dropout2d_19 (Spati alDropout2D)</pre>	(None, 16, 16, 64)	0

```
batch_normalization_4 (Batc (None, 16, 16, 64)
                                                  256
hNormalization)
max_pooling2d_24 (MaxPoolin (None, 8, 8, 64)
g2D)
conv2d 77 (Conv2D)
                         (None, 8, 8, 64)
                                                  36928
conv2d_78 (Conv2D)
                          (None, 8, 8, 64)
                                                  36928
conv2d 79 (Conv2D)
                          (None, 8, 8, 64)
                                                  36928
spatial_dropout2d_20 (Spati (None, 8, 8, 64)
alDropout2D)
batch_normalization_5 (Batc (None, 8, 8, 64)
                                                  256
hNormalization)
max pooling2d 25 (MaxPoolin (None, 4, 4, 64)
g2D)
flatten_9 (Flatten)
                          (None, 1024)
dense 18 (Dense)
                          (None, 64)
                                                  65600
dense 19 (Dense)
                          (None, 10)
                                                  650
______
Total params: 280,170
Trainable params: 279,850
Non-trainable params: 320
```

Adagrad Optimizer

```
0.3180 - val loss: 1.6866 - val accuracy: 0.3821
      Epoch 5/10
      0.3395 - val_loss: 1.6340 - val_accuracy: 0.4005
      Epoch 6/10
      0.3568 - val loss: 1.5913 - val accuracy: 0.4173
      Epoch 7/10
      0.3704 - val loss: 1.5596 - val accuracy: 0.4287
      Epoch 8/10
      0.3853 - val_loss: 1.5354 - val_accuracy: 0.4340
      Epoch 9/10
      0.3896 - val loss: 1.5175 - val accuracy: 0.4387
      Epoch 10/10
      0.3961 - val loss: 1.4925 - val accuracy: 0.4454
In [133...
      plt.plot(history_8.history['accuracy'], label='training accuracy') # For TF2
      #plt.plot(history.history['acc'], label='training accuracy') # For TF1
      plt.plot(history_8.history['val_accuracy'], label = 'valid. accuracy') # For TF2
      #plt.plot(history.history['val acc'], label = 'valid. accuracy') # For TF1
      plt.xlabel('Epoch')
      plt.ylabel('Accuracy')
      plt.ylim([0.1, 1])
      plt.legend(loc='lower right')
      # Evaluate the learned model with validation set
      valid loss, valid acc = model 8.evaluate(valid images, valid labels, verbose=2)
      print ("valid accuracy=%s, valid loss=%s" % (valid acc, valid loss))
```

313/313 - 9s - loss: 1.4925 - accuracy: 0.4454 - 9s/epoch - 28ms/step valid_accuracy=0.4453999996185303, valid_loss=1.492461919784546

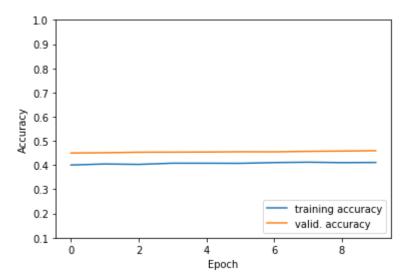


Adadelta Optimizer

In [137...

```
Epoch 1/10
     0.4005 - val_loss: 1.4882 - val_accuracy: 0.4499
     Epoch 2/10
     0.4047 - val loss: 1.4866 - val accuracy: 0.4512
     Epoch 3/10
     0.4030 - val_loss: 1.4815 - val_accuracy: 0.4533
     Epoch 4/10
     0.4080 - val loss: 1.4805 - val accuracy: 0.4537
     0.4078 - val loss: 1.4788 - val accuracy: 0.4541
     Epoch 6/10
     0.4074 - val_loss: 1.4767 - val_accuracy: 0.4552
     Epoch 7/10
     0.4106 - val loss: 1.4736 - val accuracy: 0.4549
     Epoch 8/10
     0.4121 - val loss: 1.4686 - val accuracy: 0.4569
     Epoch 9/10
     0.4103 - val_loss: 1.4663 - val_accuracy: 0.4581
     Epoch 10/10
     0.4110 - val_loss: 1.4630 - val_accuracy: 0.4597
In [138...
      plt.plot(history 8.history['accuracy'], label='training accuracy') # For TF2
      #plt.plot(history.history['acc'], label='training accuracy') # For TF1
      plt.plot(history 8.history['val accuracy'], label = 'valid. accuracy') # For TF2
      #plt.plot(history.history['val_acc'], label = 'valid. accuracy') # For TF1
      plt.xlabel('Epoch')
      plt.ylabel('Accuracy')
      plt.ylim([0.1, 1])
      plt.legend(loc='lower right')
      # Evaluate the learned model with validation set
      valid loss, valid acc = model 8.evaluate(valid images, valid labels, verbose=2)
      print ("valid_accuracy=%s, valid_loss=%s" % (valid_acc, valid_loss))
```

313/313 - 9s - loss: 1.4630 - accuracy: 0.4597 - 9s/epoch - 29ms/step valid accuracy=0.45969998836517334, valid loss=1.4630039930343628



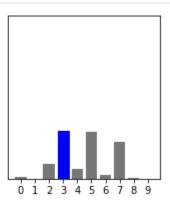
Adamax Optimizer

```
In [141...
     model_8.compile(optimizer='adamax',
            loss=tf.keras.losses.SparseCategoricalCrossentropy(from logits=False), #
            #loss='sparse categorical crossentropy', # For TF1
            metrics=['accuracy'])
In [142...
     history_8 = model_8.fit(train_images, train_labels, epochs=10,
               validation data=(valid images, valid labels))
    Epoch 1/10
    0.4182 - val loss: 1.3284 - val accuracy: 0.5175
    0.5052 - val_loss: 1.2312 - val_accuracy: 0.5565
    Epoch 3/10
    0.5676 - val loss: 1.0539 - val accuracy: 0.6180
    Epoch 4/10
    0.6096 - val loss: 0.9752 - val accuracy: 0.6478
    Epoch 5/10
    0.6446 - val_loss: 0.8990 - val_accuracy: 0.6801
    Epoch 6/10
    0.6691 - val_loss: 0.8351 - val_accuracy: 0.7040
    Epoch 7/10
    0.6914 - val loss: 0.7992 - val accuracy: 0.7163
    Epoch 8/10
    0.7090 - val_loss: 0.7812 - val_accuracy: 0.7270
    Epoch 9/10
    0.7218 - val_loss: 0.7428 - val_accuracy: 0.7438
    Epoch 10/10
    0.7318 - val loss: 0.7362 - val accuracy: 0.7396
```

```
In [143...
          plt.plot(history_8.history['accuracy'], label='training accuracy') # For TF2
          #plt.plot(history.history['acc'], label='training accuracy') # For TF1
          plt.plot(history_8.history['val_accuracy'], label = 'valid. accuracy') # For TF2
          #plt.plot(history.history['val_acc'], label = 'valid. accuracy') # For TF1
          plt.xlabel('Epoch')
          plt.ylabel('Accuracy')
          plt.ylim([0.1, 1])
          plt.legend(loc='lower right')
          # Evaluate the learned model with validation set
          valid loss, valid acc = model 8.evaluate(valid images, valid labels, verbose=2)
          print ("valid accuracy=%s, valid loss=%s" % (valid acc, valid loss))
         313/313 - 9s - loss: 0.7362 - accuracy: 0.7396 - 9s/epoch - 29ms/step
         valid accuracy=0.7396000027656555, valid loss=0.7362483143806458
           1.0
           0.9
           0.8
           0.7
         Accuracy
           0.6
           0.5
           0.4
           0.3
                                               training accuracy
           0.2
                                               valid. accuracy
           0.1
                         ż
                                  4
                                   Epoch
In [144...
          test_loss, test_acc = model_2.evaluate(test_images, test_labels)
          print('\nTest accuracy:', valid acc)
          print ('\n')
          train_loss, train_acc = model.evaluate(train_images, train_labels, verbose = 2)
          print('Training accuracy:', train_acc)
         0000e+00
         Test accuracy: 0.7396000027656555
         1250/1250 - 8s - loss: 0.5887 - accuracy: 0.7922 - 8s/epoch - 7ms/step
         Training accuracy: 0.7921749949455261
In [146...
          predictions_8 = model_8.predict(valid_images)
In [149...
          #valid labels = np.concatenate(valid labels)
          #valid_labels
```

```
i = 21
plt.figure(figsize=(6,3))
plt.subplot(1,2,1)
#plot_image(i, predictions[i], test_labels, test_images)
plot_image(i, predictions_2[i], valid_labels, valid_images)
plt.subplot(1,2,2)
#plot_value_array(i, predictions[i], test_labels)
plot_value_array(i, predictions_2[i], valid_labels)
plt.show()
```

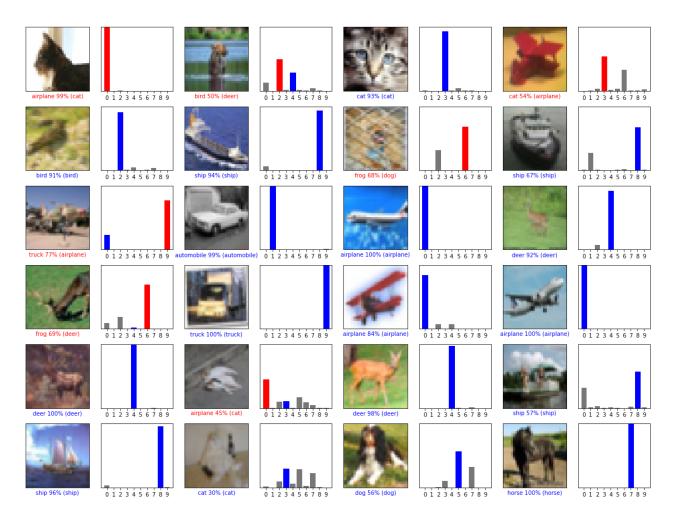




```
# Plot the first X test images, their predicted labels, and the true labels.

# Color correct predictions in blue and incorrect predictions in red.
```

```
# Color correct predictions in blue and incorrect predictions in red.
num_rows = 6
num_cols = 4
num_images = num_rows*num_cols
plt.figure(figsize=(2*2*num_cols, 2*num_rows))
for i in range(num_images):
   plt.subplot(num_rows, 2*num_cols, 2*i+1)
   plot_image(i, predictions_2[i], valid_labels, valid_images)
# plot_image(i, predictions[i], test_labels, test_images)
   plt.subplot(num_rows, 2*num_cols, 2*i+2)
# plot_value_array(i, predictions[i], test_labels)
   plot_value_array(i, predictions_2[i], valid_labels)
plt.tight_layout()
plt.show()
```



BEST FITTING MODEL

Model 10

For this model, the "Adam" optimizer was used, in addition to adjustments made by adding more layers, Batch Normalization, and Dropout. This model was tested on 10 and 15 Epochs.

The final result is a model with an accuracy of 80%. Unfortunately, there appears to be some overfitting in the early Epochs, but with additional Epochs, the area between the training and validation data may decrease.

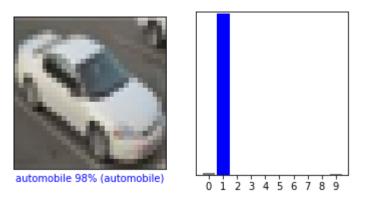
```
In [183...
    model_10 = models.Sequential()
    model_10.add(layers.Conv2D(32, (3, 3), padding='same', activation='relu', input_shape=(
    model_10.add(layers.Conv2D(32, (3, 3), padding='same', activation='relu'))
    model_10.add(BatchNormalization())
    model_10.add(layers.Conv2D(32, (3, 3), padding='same', activation='relu'))
    model_10.add(layers.Dropout(0.1))
    model_10.add(BatchNormalization())
    model_10.add(layers.MaxPooling2D((2, 2), strides=(2,2)))

model_10.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
    model_10.add(BatchNormalization())
    model_10.add(BatchNormalization())
    model_10.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu'))
    model_10.add(BatchNormalization())
    model_10.add(BatchNormalization())
```

```
model 10.add(layers.Dropout(0.1))
       model 10.add(layers.MaxPooling2D((2, 2), strides=(2,2)))
       model 10.add(layers.Conv2D(128, (3, 3), padding='same', activation='relu'))
       model 10.add(BatchNormalization())
       model_10.add(layers.Conv2D(128, (3, 3), padding='same', activation='relu'))
       model 10.add(BatchNormalization())
       model_10.add(layers.Conv2D(128, (3, 3), padding='same', activation='relu'))
       model 10.add(BatchNormalization())
       model_10.add(layers.Conv2D(128, (3, 3), padding='same', activation='relu'))
       model 10.add(BatchNormalization())
       model 10.add(layers.Dropout(0.1))
       model_10.add(layers.MaxPooling2D((2, 2), strides=(2,2)))
       model 10.add(layers.Flatten())
       model_10.add(layers.Dense(128, activation='relu'))
       model_10.add(BatchNormalization())
       model 10.add(layers.Dropout(0.1))
       model 10.add(layers.Dense(10, activation='softmax')) # As noted above
In [184...
       model 10.compile(optimizer='adam',
                loss=tf.keras.losses.SparseCategoricalCrossentropy(from logits=False), #
                #loss='sparse categorical crossentropy', # For TF1
                metrics=['accuracy'])
In [173...
       history_10 = model_10.fit(train_images, train_labels, epochs=15,
                     validation_data=(valid_images, valid_labels))
      Epoch 1/15
      0.4789 - val loss: 1.3672 - val accuracy: 0.5144
      0.6648 - val_loss: 1.6568 - val_accuracy: 0.4793
      Epoch 3/15
      0.7312 - val_loss: 1.1284 - val_accuracy: 0.6595
      Epoch 4/15
      0.7694 - val loss: 0.8895 - val accuracy: 0.6960
      Epoch 5/15
      0.7984 - val loss: 0.9126 - val accuracy: 0.6957
      Epoch 6/15
      0.8224 - val loss: 0.8137 - val accuracy: 0.7240
      Epoch 7/15
      0.8422 - val loss: 0.9060 - val accuracy: 0.6970
      Epoch 8/15
      0.8594 - val_loss: 0.8081 - val_accuracy: 0.7381
      Epoch 9/15
      0.8750 - val_loss: 0.6932 - val_accuracy: 0.7837
      Epoch 10/15
```

```
0.8915 - val loss: 0.6889 - val accuracy: 0.7835
    Epoch 11/15
    0.9040 - val loss: 0.7120 - val accuracy: 0.7770
    Epoch 12/15
    0.9121 - val_loss: 0.6087 - val_accuracy: 0.8050
    Epoch 13/15
    0.9227 - val_loss: 0.7350 - val_accuracy: 0.7693
    Epoch 14/15
    0.9292 - val_loss: 0.7470 - val_accuracy: 0.7781
    Epoch 15/15
    0.9361 - val_loss: 0.6612 - val_accuracy: 0.8042
In [185...
     history 10 = model 10.fit(train images, train labels, epochs=10,
              validation_data=(valid_images, valid_labels))
    Epoch 1/10
    0.5167 - val_loss: 1.2339 - val_accuracy: 0.5835
    Epoch 2/10
    0.6955 - val_loss: 0.8144 - val_accuracy: 0.7130
    Epoch 3/10
    0.7515 - val_loss: 0.8029 - val_accuracy: 0.7269
    Epoch 4/10
    0.7888 - val_loss: 0.6781 - val_accuracy: 0.7725
    Epoch 5/10
    0.8196 - val loss: 0.5918 - val accuracy: 0.8000
    Epoch 6/10
    0.8474 - val_loss: 0.6852 - val_accuracy: 0.7716
    Epoch 7/10
    0.8675 - val loss: 0.6669 - val accuracy: 0.7859
    Epoch 8/10
    0.8866 - val loss: 0.5784 - val accuracy: 0.8107
    Epoch 9/10
    0.9032 - val_loss: 0.7755 - val_accuracy: 0.7685
    Epoch 10/10
    0.9137 - val loss: 0.6264 - val accuracy: 0.8040
In [186...
     test_loss, test_acc = model_10.evaluate(test_images, test_labels)
     print('\nTest accuracy:', valid acc)
     print ('\n')
     train loss, train acc = model 10.evaluate(train images, train labels, verbose = 2)
     print('Training accuracy:', train_acc)
```

```
0000e+00
         Test accuracy: 0.8041999936103821
         1250/1250 - 67s - loss: 0.2064 - accuracy: 0.9313 - 67s/epoch - 53ms/step
         Training accuracy: 0.9313499927520752
In [187...
          plt.plot(history_10.history['accuracy'], label='training accuracy') # For TF2
          #plt.plot(history.history['acc'], label='training accuracy') # For TF1
          plt.plot(history 10.history['val accuracy'], label = 'valid. accuracy') # For TF2
          #plt.plot(history.history['val acc'], label = 'valid. accuracy') # For TF1
          plt.xlabel('Epoch')
          plt.ylabel('Accuracy')
          plt.ylim([0.1, 1])
          plt.legend(loc='lower right')
          # Evaluate the learned model with validation set
          valid loss, valid acc = model 10.evaluate(valid images, valid labels, verbose=2)
          print ("valid_accuracy=%s, valid_loss=%s" % (valid_acc, valid_loss))
         313/313 - 16s - loss: 0.6264 - accuracy: 0.8040 - 16s/epoch - 50ms/step
         valid accuracy=0.8040000200271606, valid loss=0.626409649848938
           1.0
           0.9
           0.8
           0.7
           0.6
           0.5
           0.4
           0.3
                                               training accuracy
           0.2
                                               valid. accuracy
           0.1
                         ż
                                                     8
                                  4
                                   Epoch
In [188...
          predictions 10 = model 10.predict(valid images)
In [190...
          i = 35
          plt.figure(figsize=(6,3))
          plt.subplot(1,2,1)
          #plot image(i, predictions[i], test labels, test images)
          plot_image(i, predictions_2[i], valid_labels, valid_images)
          plt.subplot(1,2,2)
          #plot_value_array(i, predictions[i], test_labels)
          plot value array(i, predictions 2[i], valid labels)
          plt.show()
```



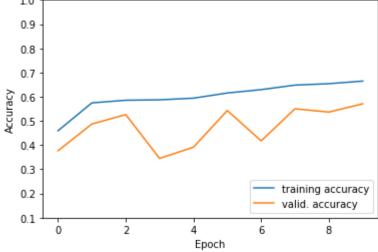
```
In [193...
          # Plot the first X test images, their predicted labels, and the true labels.
          # Color correct predictions in blue and incorrect predictions in red.
          num\_rows = 5
          num cols = 5
          num images = num rows*num cols
          plt.figure(figsize=(2*2*num_cols, 2*num_rows))
          for i in range(num_images):
            plt.subplot(num rows, 2*num cols, 2*i+1)
            plot_image(i, predictions_2[i], valid_labels, valid_images)
            # plot_image(i, predictions[i], test_labels, test_images)
            plt.subplot(num_rows, 2*num_cols, 2*i+2)
            # plot_value_array(i, predictions[i], test_labels)
            plot_value_array(i, predictions_2[i], valid_labels)
          plt.tight_layout()
          plt.show()
 In [ ]:
```

Model 11 - Experiment with Kernel Regularization

Regularization was added as an attempt to improve on Model 10. Unfortunately, the addition of regularization on decreased the model's performance.

```
model 11 = models.Sequential()
In [203...
         model_11.add(layers.Conv2D(32, (3, 3), padding='same', kernel_regularizer=tf.keras.regu
         model_11.add(layers.Conv2D(32, (3, 3), padding = 'same', kernel_regularizer=tf.keras.reg
         model 11.add(BatchNormalization())
         model 11.add(layers.Conv2D(32, (3, 3), padding = 'same', kernel regularizer=tf.keras.reg
         model 11.add(BatchNormalization())
         model 11.add(layers.Dropout(0.1))
         model_11.add(layers.MaxPooling2D((2, 2), strides=(2,2)))
         model 11.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu', kernel regula
         model 11.add(BatchNormalization())
         model_11.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu', kernel_regula
         model 11.add(BatchNormalization())
         model_11.add(layers.Conv2D(64, (3, 3), padding='same', activation='relu', kernel_regula
         model 11.add(BatchNormalization())
         model 11.add(layers.Dropout(0.1))
         model_11.add(layers.MaxPooling2D((2, 2), strides=(2,2)))
         model_11.add(layers.Conv2D(128, (3, 3), padding='same', activation='relu', kernel_regul
         model 11.add(BatchNormalization())
         model_11.add(layers.Conv2D(128, (3, 3), padding='same', activation='relu', kernel_regul
         model 11.add(BatchNormalization())
         model 11.add(layers.Conv2D(128, (3, 3), padding='same', activation='relu', kernel regul
         model 11.add(BatchNormalization())
         model_11.add(layers.Conv2D(128, (3, 3), padding='same', activation='relu', kernel_regul
         model 11.add(BatchNormalization())
         model 11.add(layers.Dropout(0.1))
         model_11.add(layers.MaxPooling2D((2, 2), strides=(2,2)))
         model 11.add(layers.Flatten())
         model_11.add(layers.Dense(128, activation='relu', kernel_regularizer=tf.keras.regulariz
         model 11.add(BatchNormalization())
         model 11.add(layers.Dropout(0.1))
         model_11.add(layers.Dense(10, activation='softmax')) # As noted above
In [204...
         model_11.compile(optimizer='adam',
                     loss=tf.keras.losses.SparseCategoricalCrossentropy(from logits=False), #
                     #loss='sparse categorical crossentropy', # For TF1
                     metrics=['accuracy'])
In [205...
         history 11 = model 11.fit(train images, train labels, epochs=10,
                          validation data=(valid images, valid labels))
        Epoch 1/10
        0.4594 - val loss: 2.8641 - val accuracy: 0.3766
        Epoch 2/10
        0.5745 - val loss: 2.1093 - val accuracy: 0.4871
        Epoch 3/10
        0.5854 - val_loss: 1.8941 - val_accuracy: 0.5261
        Epoch 4/10
        0.5872 - val_loss: 2.7312 - val_accuracy: 0.3452
        Epoch 5/10
```

```
0.5940 - val loss: 2.1880 - val accuracy: 0.3913
      Epoch 6/10
      0.6154 - val loss: 1.9090 - val accuracy: 0.5430
      Epoch 7/10
      0.6293 - val_loss: 2.0737 - val_accuracy: 0.4176
      Epoch 8/10
      0.6479 - val loss: 1.7119 - val accuracy: 0.5503
      Epoch 9/10
      0.6538 - val loss: 1.8745 - val accuracy: 0.5364
      0.6646 - val_loss: 1.6443 - val_accuracy: 0.5708
In [206...
       plt.plot(history 11.history['accuracy'], label='training accuracy') # For TF2
       #plt.plot(history.history['acc'], label='training accuracy') # For TF1
       plt.plot(history_11.history['val_accuracy'], label = 'valid. accuracy') # For TF2
       #plt.plot(history.history['val_acc'], label = 'valid. accuracy') # For TF1
       plt.xlabel('Epoch')
       plt.ylabel('Accuracy')
       plt.ylim([0.1, 1])
       plt.legend(loc='lower right')
       # Evaluate the learned model with validation set
       valid loss, valid acc = model 11.evaluate(valid images, valid labels, verbose=2)
       print ("valid_accuracy=%s, valid_loss=%s" % (valid_acc, valid_loss))
      313/313 - 16s - loss: 1.6443 - accuracy: 0.5708 - 16s/epoch - 51ms/step
      valid accuracy=0.5708000063896179, valid loss=1.644324779510498
        1.0
        0.9
```



```
In [ ]: test_loss, test_acc = model_11.evaluate(valid_images, valid_labels)
    print('\nTest accuracy:', test_acc)
    print ('\n')
    train_loss, train_acc = model_11.evaluate(train_images, train_labels, verbose = 2)
    print('Training accuracy:', train_acc)
```

```
In [ ]:
In [200...
          predictions = model 10.predict(test images)
          print(predictions)
          [[9.8308057e-01 1.8212999e-05 2.5517668e-03 ... 8.5479184e-04
           7.4558267e-03 1.0308787e-03]
          [1.9771393e-02 4.8074129e-04 5.3020746e-01 ... 3.2225266e-02
            2.6721442e-02 1.1811379e-04]
          [1.1857492e-04 1.8295294e-03 1.2232403e-03 ... 6.4079337e-05
           4.3495267e-04 5.2719499e-04]
          [7.2197413e-06 1.5024254e-06 6.4147240e-04 ... 4.3253703e-03
           2.1635567e-06 7.2113835e-07]
          [9.4684534e-04 4.8566353e-06 5.7106060e-03 ... 4.6219058e-02
           8.2565202e-05 8.7258177e-06]
           [1.8457818e-03 1.5198733e-06 9.9797016e-01 ... 2.4859485e-06
           3.8984173e-05 7.5319730e-07]]
 In [ ]:
          import pandas as pd
In [209...
          predict DF = pd.DataFrame(predictions)
          predict DF = predict DF.reset index()
          predict_DF.columns = ('id','cat0','cat1','cat2','cat3','cat4','cat5','cat6','cat7','cat
          predict_DF['id'] = predict_DF['id'] + 1
          predict DF.to csv('predictions.csv', index=False)
In [210...
          test_images
         array([[[[9.52876345e-06, 6.75456649e-06, 2.95512284e-06],
Out[210...
                   [9.58907208e-06, 6.69425786e-06, 2.83450558e-06],
                   [9.95092385e-06, 6.99580101e-06, 3.07574010e-06],
                   [8.26228223e-06, 5.72931979e-06, 2.17111066e-06],
                   [7.59888731e-06, 5.48808528e-06, 2.17111066e-06],
                   [6.99580101e-06, 5.12623350e-06, 1.99018477e-06]],
                  [[9.16691167e-06, 6.75456649e-06, 3.07574010e-06],
                   [9.10660304e-06, 6.63394924e-06, 2.41234518e-06],
                   [9.58907208e-06, 6.87518375e-06, 2.71388832e-06],
                   [8.20197360e-06, 5.72931979e-06, 1.86956751e-06],
                   [7.53857868e-06, 5.48808528e-06, 1.92987614e-06],
                   [7.17672690e-06, 5.30715939e-06, 2.05049340e-06]],
                  [9.10660304e-06, 6.63394924e-06, 2.83450558e-06],
                   [9.10660304e-06, 6.57364061e-06, 1.99018477e-06],
                   [9.52876345e-06, 6.69425786e-06, 2.17111066e-06],
                   [8.38289949e-06, 5.91024568e-06, 2.05049340e-06],
                   [7.84012182e-06, 5.72931979e-06, 2.05049340e-06],
                   [7.23703553e-06, 5.36746802e-06, 1.99018477e-06]],
                  . . . ,
```

```
[[4.10098680e-06, 7.47827005e-06, 1.06746274e-05],
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 [2.29172792e-06, 5.84993705e-06, 8.80505989e-06],
 [7.84012182e-07, 3.85975228e-06, 6.51333198e-06],
 [2.41234518e-06, 5.12623350e-06, 7.65919594e-06]],
 [[3.67882639e-06, 6.99580101e-06, 1.01318497e-05],
 [2.95512284e-06, 6.15148020e-06, 8.92567715e-06],
 [2.11080203e-06, 5.12623350e-06, 7.96073908e-06],
 [1.56802436e-06, 4.94530761e-06, 7.84012182e-06],
 [1.74895025e-06, 4.94530761e-06, 7.59888731e-06],
 [1.20617259e-06, 3.85975228e-06, 6.45302335e-06]],
 [[3.25666599e-06, 6.45302335e-06, 9.64938071e-06],
 [3.37728325e-06, 6.33240609e-06, 8.98598578e-06],
 [2.71388832e-06, 5.36746802e-06, 7.96073908e-06],
 [1.44740711e-06, 4.64376446e-06, 7.47827005e-06],
 [2.05049340e-06, 5.06592487e-06, 7.77981319e-06],
 [1.26648122e-06, 4.04067817e-06, 6.63394924e-06]]],
[[[1.41725279e-05, 1.41725279e-05, 1.41725279e-05],
 [1.39312934e-05, 1.39312934e-05, 1.39312934e-05],
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 [1.40519107e-05, 1.40519107e-05, 1.40519107e-05],
 [1.39916020e-05, 1.39916020e-05, 1.39916020e-05]],
 [[1.43534538e-05, 1.43534538e-05, 1.43534538e-05],
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 [1.41725279e-05, 1.41725279e-05, 1.41725279e-05],
 [1.42328365e-05, 1.42328365e-05, 1.42328365e-05],
 [1.42328365e-05, 1.42328365e-05, 1.42328365e-05],
 [1.41725279e-05, 1.41725279e-05, 1.41725279e-05]],
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