## **FairRankTune**

# RankTune Data Generation Technical Report

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#### 1 RankTune Data Generator

A primary component of FairRankTune is the fairness-tunable ranked data generator RANKTUNE.

#### 1.1 Notation

We use the symbol  $X=x_1,x_2,...,x_n$  to represent items to be ordered in a ranked list  $\tau$ .  $\tau(x_i)$  denotes the ordinal position of item  $x_i$  in the ranking  $\tau$ . Items, sometimes called documents, candidates, or providers, belong to a group defined by a shared protected attribute value, such as, gender = "woman". We represent the groups associated with the items as  $G=g_1,g_2,...,g_m$ . We use  $D_X=(p_{X:g_1},...,p_{X:g_m})$  to represent the distribution of groups in the item set X, where the proportion of each group is  $p_{X:g_m}=|g_m|/|X|$ . For example,  $D_X=(0.2,0.3,0.5)$  indicates that  $g_1$  corresponds to 20% of items in X, and  $g_2$  and  $g_3$  to 30% and 50% of X, respectively.

#### 1.2 Underlying Core Idea

RANKTUNE is a fairness-tunable ranked data generation method. It constructs a ranking  $\tau$  by placing items into  $\tau$  from top to bottom. The idea behind RANKTUNE is that to construct a "fair" ranking, each time we place an item in the generated ranking, the likelihood of placing an item  $\in g_i$  should be equal to  $g_i$ 's proportion of the items (i.e., if  $p_{x:g_i}=0.2$ , meaning  $g_i$  is 20% of X then  $g_i$  should have a 20% chance of being placed). Then, on the other side of the spectrum, if we want a completely "unfair" ranking, we should place items into  $\tau$  such that groups are ordered by increasing size from small to large. In this way, smaller groups would get bigger proportions of favorable positions, which maximally violates statistical parity fairness [1]. These are the two ends of the statistical parity spectrum.

To generate rankings along this spectrum, RANKTUNE samples a random number r in the [0,1] interval each time it places an item. We design this interval to have "regions" that map to groups. In this way, the unfairness tuning parameter  $\phi$  controls representativeness, i.e., how fairly each group is represented in the ranking. Specifically, when  $\phi=0$ , then each group is fairly represented. Thus each group's region is equal to the group's proportion of X (fair). As  $\phi$  increases, the fair representation of each group degrades because regions are distorted in such a way that smaller groups have larger regions compared to their proportion of X (unfair). The fairness tuning parameter  $\phi$  is used to create the regions prior to placing any items into ranking  $\tau$ .

#### 1.3 Description of RankTune Data Generation Algorithm

Algorithm 1 displays the pseudocode for the core algorithmic strategy of the RANKTUNE tool. This algorithm operates in three stages: (1) *creating group regions stage*, the assignment of groups to regions in the [0,1] interval using  $\phi$ ; (2) *pseduo-random item placement stage*, the repeated sampling of the [0,1] interval to place items into the to-be-constructed ranking and (3) *filling-out stage*, once a group has no items remaining, the rest of the items are placed.

**Creating Group Regions:** This begins by identifying the smallest group  $g_{min}$ , or in the case of multiple such groups, it chooses a random group among those of the smallest size. Next,  $\phi$  is scaled to a new value  $\phi_{scaled}$  which represents  $g_{min}$ 's artificially adjusted new proportion of the item set (line 4). This new proportion can be the same (case of  $\phi = 0$ , i.e., fair) or larger all the way

to  $\phi=0$  resulting in  $\phi_{scaled}=1$  meaning the smallest group is certainly at the top of the ranking. Then the original distribution of groups  $D_x$  is adjusted to a new group distribution  $D_{target}$ . That is, all other groups are proportionally scaled down to accommodate  $g_{min}$ 's new proportion of the item set  $\phi_{scaled}$ . Then we take the target group distribution and map each group's proportion of X into representing a corresponding region of the [0,1] space. Each group's proportion of the [0,1] interval is represented via a lower bound (lowBound) and upper bound (upperBound). This stage sets the fairness of the placement procedure.

**Pseudo-random Item Placement:** Subsequently the ranking  $\tau$  is constructed from top to bottom (i.e. appending), by repeatedly sampling a random number r in the uniform [0,1] interval. In the prior stage, groups were mapped to regions of the interval. Thus, when r is sampled, we know what group to place  $(g_{2p})$ . Here,  $g_{2p}$  is the group for which  $lowBound[g_{2p}] < r < upperBound[g_{2p}]$ .

Filling Out Stage: Once RANKTUNE has placed an entire group the remaining positions in  $\tau$  are filled by placing groups according to increasing group size. RANKTUNE then returns the ranking  $\tau$ .

### 1.4 Assumptions underpinning RankTune

The underlying assumption of RANKTUNE is that in order to generate rankings satisfying statistical parity fairness, the likelihood of a group receiving a positive outcome should be equal to that group's proportion of the candidate pool. Then unfairness can be added by distorting this proportional relationship between the likelihood of the group receiving the positive outcome and its proportion of the of candidate pool. We view the positive outcome to be the placement of a candidate into the generated ranking and each placement is made by sampling a random number in the uniform [0,1] interval. In the case of perfect statistical parity, the likelihood of placing a candidate from each group is equal to that group's proportion of all candidates. RankTune is driven by a stochastic process (using a random number generator to sample the [0,1] interval), thus we are unable to formally prove that the approach always generates a ranking of a desired degree of fairness.

### Algorithm 1: RANKTUNE Data Generation Algorithm

```
Data: Item set X and groups G, representativeness parameter \phi
Result: Ranking \tau
\tau \leftarrow [], D_X \leftarrow (p_{x:g_1}, ..., p_{x:g_m})
g_{min} \leftarrow \text{group id of smallest group};
\phi_{scaled} \leftarrow (1-\phi) * (1-p_{x:g_{min}}) + p_{x:g_{min}};
D_{target} \leftarrow (D_X/(1-p_{x:q_{min}})*(1-\phi_{scaled});
                                                                                     /* so \sum D_{target} = 1 */
D_{target}(g_{min}) \leftarrow \phi_{scaled};
lowCount \leftarrow 0
lowBound \& upperBound \leftarrow empty arrays of size |G|
forall q_i \in G do
    lowBound[g_i] \leftarrow lowCount;
    upperBound[g_i] \leftarrow lowCount + D_{target}(g_i);
    lowCount \leftarrow lowCount + D_{target}(g_i);
while each group has unplaced items do
    r \leftarrow random([0,1])
    g_{2p} \leftarrow \text{s.t. } lowBound[g_{2p}] < r < upperBound[g_{2p}]
  \tau.append(next item from group g_{2p})
G_{rem} \leftarrow \text{sorted (smallest to largest) groups with unplaced items}
                                                   /* place items by increasing group size */
forall g_i \in G_{rem};
 do
    forall x_i \in g_i do
     | \tau.append(x_j)
Return \tau
```

### 1.5 Demonstration of RankTune Algorithm

#### 1.5.1 Fairness Metrics

In this section we employ measures of statistical parity with specific formulations. For brevity, they are referenced by their abbreviation. Table 1 describes the specific formulation we use here.

Abbreviation	Formulation	Range	More Fair
EXP	Min-max ratio of avg. group exposures as in [2]	(0,1]	1
EE-D	$L_2$ norm of avg. group exposure as in [3]	$[0,\infty)$	0
AWRF	Min-max ratio avg. group attention as in [4]	(0, 1]	1
NDKL	Proportional group representation across rank prefixes as in [5]	$[0,\infty)$	0
ARP	Max-min difference avg. mixed pairs as in [6]	[0,1]	0

Table 1: Overview of fair ranking metrics used in this work.

#### 1.5.2 Empirical Demonstration

We now demonstrate the capabilities of FairRankTune's RANKTUNE using a variety of group memberships. Table 2 depicts the group distributions we utilize in our demonstration.

Name	<b>Distribution</b> - $D_X$
Dist A	(0.2, 0.3, 0.5)
Dist B	(0.1, 0.3, 0.6)
Dist C	(0.2, 0.3, 0.1, 0.05, 0.03)
Dist D	(0.2, 0.2, 0.2, 0.2, 0.2)
Dist E	(0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1,
Dist F	(0.6, 0.08, 0.02, 0.15, 0.1, 0.05)

Table 2: Group distributions employed in our empirical analysis and case study. We use X=1,000 items. For example, *Dist A* has three groups: the first has 200 members, the second has 300 members, and the third has 500 members.

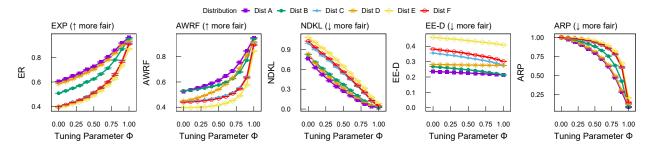


Figure 1: Average metric values (with 95% confidence intervals) from 200 RANKTUNE constructed rankings with the distributions in Table 2 for 1, 000 items. As  $\phi$  increases, RANKTUNE outputs increasingly fairer rankings. EXP and AWRF are more fair at 1 (thus upward slopes), and NDKL, EE-D, and ARP are more fair at 0 (thus downward slopes).

Figure 1, displays the results of generating 200 rankings for different  $\phi$  representativeness values for the multiple groups distributions from Table 2(*Dist A - F.* Specifically, we generate 200 rankings for  $\phi$  ranging from 0 to 1 in increments of 0.1.

We observe that across all fairness metrics and all group distributions (colored lines), RANK-TUNE outputs progressively fairer rankings as  $\phi$  is adjusted from 0 to 1. The metrics follow the same overall trends, but vary in their ranges and sensitivity to different group distributions as seen by how different metrics relatively order  $Dist\ A$  - F when  $\phi=1$  or other  $\phi$  values. Critically, for each distribution and metric, we consistently see fairness increase as  $\phi$  increases. Robustness is seen through the small 95% confidence intervals over 200 t rials. Thus, we illustrate that while RANKTUNE is agnostic to particular formulations of group fairness, it is generally applicable to settings in which statistical parity is studied.

#### References

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