Simple-as-possible stylized representation of the tradeoff between investment in income redistribution versus investment in emissions abatement

Abstract

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Model overview

In this work, we develop a highly styled model of an economy with income inequality, with a specified fraction of gross production allocated to social good, where that allocation choice is between emissions abatement and income redistribution.

The objective function of the model is to optimize the time-integral of aggregate utility, perhaps subject to a pure rate of time preference, by optimizing the allocation choice between emissions abatement, f. Thus the objective is to maximize

$$\frac{max}{f(t)} \int_0^{\infty} e^{-\rho t} \cdot U(t) \cdot L(t)\, dt \quad \text{, s.t. } 0 \le f(t) \le 1, \tag{1}$$

Where ϱ is the pure rate of time preference, U(t) is mean utility of the population at time t, and L(t) is the population at time t.

We make many simplifying assumptions to keep things analytically tractable, highlighting parameter dependencies. Later versions may relax some of these assumptions, making the model more realistic but necessitating numerical solution.

We extend a variant of the COIN model presented in [Caldeira et al. (2023)] to consider income inequality and diminishing marginal returns to utility of increased income at higher income levels.

The core economic model is a Solow-Swann growth model (ref) with a Cobb-Douglas production function (ref). Carbon dioxide emissions are assumed to be the product of a time-dependent carbon-intensity of GDP production time gross productivity. Global mean temperature change is assumed proportional to cumulative carbon dioxide emissions. Climate damage is assumed to be proportional to gross GDP times global mean temperature raised to some power. The cost of emissions abatement is assumed to be proportional to *a* time-dependent cost of emissions abatement times the amount of emissions abated.

The distribution of income is assumed to follow a Pareto distribution and can be described by a Lorenz curve (ref) corresponding to a Gini Index value. A shift in a reduction in the Gini Index implies an income transfer from high-income to low-income individuals.

If we assume a fixed fraction of GDP will be allocated to social good, one end-member is allocating it all to income redistribution. This would result in a decrease in the Gini index. The other member is if all of this income was taken from the same people but allocated instead to emissions abatement. Note that this tax on the wealthy would itself reduce the Gini index but would not increase anyone's present income so would be utility-neutral or utility-negative for everyone in the present. However, investment in emissions abatement would reduce current emissions and thus future temperatures and productivity losses to climate change.

We aim to find the optimal allocation of resources to income-redistribution versus emissions abatement, as a function fraction of GDP allocated to social good, and other model parameters.

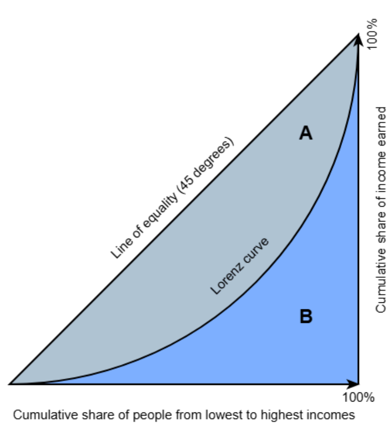

Figure M1. Illustration of Lorenz Curve showing cumulative share of total income as a function of fraction of population. The Gini Index is A/(A+B).

Income distribution, the Gini Index, utility, and aggregate utility

We assume a Pareto distribution of income that can be described by the following Lorenz curve:

$$\mathcal{L}(F) = 1 - (1 - F)^{1-1/a} , \tag{2}$$

where $\mathcal{L}(F)$ is the fraction of total income received by the fraction, F, of people with the least income. With this assumption, the Gini index, *G,* can be expressed as a function of *a*:

$$G = \frac{1}{2a-1} , \tag{3}$$

and therefore:

$$a = \frac{1+1/G}{2} . \tag{4}$$

If *y* is mean per-capita income, then income at rank *F* is:

$$c(F) = y\left(\frac{d\,\mathcal{L}}{d\,F}\right) = y\left(1 - \frac{1}{a}\right)(1 - F)^{-1/a} , \tag{5}$$

We further assume an isoelastic utility function, *u(c)*:

$$u(c) = \frac{c^{1-\eta}-1}{1-\eta} , \tag{6}$$

where η is the coefficient or relative risk aversion (which is a factor describing the diminishing returns of consumption marginal utility). Higher values of η imply more rapidly diminishing utility benefits of income increase at higher income levels; 0 means utility is proportional to income. If $\eta > 1$, this implies that maximum utility has an upper bound. If $\eta \leq 1$, this implies that maximum utility could potentially increase without bound. If $\eta = 1$, this means that each percentage increase in income produces the same increment in utility at all income levels and equation (6) becomes:

$$u(c) = ln(c) . \tag{7}$$

Note that under the assumption of a Pareto-Lorenz distribution of income and a CRRA utility function, the mean utility of a population, *U* (see equation 8), can be expressed as a function of mean income, y, the Gini index, G, and the coefficient of relative risk aversion, η:

$$U = \frac{y^{1-\eta}}{1-\eta}\left(\frac{(1+G)^{\eta}(1-G)^{1-\eta}}{1+G(2\eta-1)}\right) - \frac{1}{1-\eta} . \tag{16}$$

In the special case of $\eta = 1$, this reduces to:

$$U = ln\, y \ + \ ln\left(\frac{1-G}{1+G}\right) + \frac{2G}{1+G} \ . \tag{17}$$

Crossing rank, and amount of income transferred for a change in the Lorenz distribution parameter

If the income is redistributed from high-income to low-income people such that the Lorenz distribution parameter shifts from a_1 to a_2 , then $a_2 > a_1$ and the rank fraction, *F** that sees no change in income is:

$$F^* = 1 - \left(\frac{(1+G_1)(1-G_2)}{(1-G_1)(1+G_2)}\right)^{\frac{(1+G_1)(1-G_2)}{2(G_2-G_1)}} . \tag{10}$$

The average fraction, $\Delta L(F^*)$, of total income redistributed is then $L(F^*)$ evaluated with $G = G_2$ minus $L(F^*)$ evaluated with $G = G_1$;

$$\Delta\mathcal{L}(F^*) = \frac{2(G_1-G_2)}{(1-G_1)(1+G_2)}\left(\frac{(1+G_1)(1-G_2)}{(1-G_1)(1+G_2)}\right)^{\frac{(1+G_1)(1-G_2)}{2(G_2-G_1)}} . \tag{11}$$

While there is no closed form analytic expression solving equation (11) for G_2 as a function of the other variables,

The average amount, per capita, of income redistributed is $\Delta c(F^*)$, where:

$$\Delta c(F^*) = y\, \Delta\mathcal{L}(F^*) . \tag{12}$$

Income redistribution, abatement expenditures and utility

If resources that would have otherwise be redistributed from high-income to low-income earners would otherwise be allocated to emissions abatement, we need a way of representing tax policy in a simple way that is consistent with the complexity of this model. Differing assumptions, such as a constant tax amount or rate across income levels, would differing impacts on the Gini index. Nevertheless, our choice here is motivated by the question of the relative consequences for time-integrated aggregate utility of investing in income redistribution versus carbon emissions abatement.

In the redistribution case, the income transferred from high income earners would cause a change in the Gini index even if those resources were not transferred to low income earners.

If we assumed the tax system for people earning more than $c(F^*)$ does not cause anyone to have an income less than $c(F^*)$, if the Gini index prior to the income removal was G_1, then the Gini index after that removal would be G_{2r}:

$$G_{2r} = 1 - (1 - G_1)\left(\frac{1-\Delta\mathcal{L}(1-F^*)}{1-\Delta\mathcal{L}}\right) , \tag{13}$$

where *F** and *ΔL* are as defined in equation (6) and (7) respectively. Using equation (2b), one can calculate a value for a_{2r}, which would be the value of a if the resources that would be transferred in a shift from a_1 to a_2 was instead allocated to a purpose such as emissions abatement. Of course, with this removal of resources from people with an income greater than c(F*) we no longer have a Pareto-Lorenz function, and so equation (1) would not describe the income distribution. Nevertheless, we make this simplifying assumption so as not to increase the degrees of freedom in our model.

If some fraction, *f*, of *ΔL(F*)* is allocated to purposes such as emissions abatement and an amount proportional (1-*f*) is allocated to emissions abatement, then the corresponding new Gini coefficient would be:

$$G_2(f) = \frac{1-\Delta\mathcal{L}}{1-f\,\Delta\mathcal{L}}\left[1 - (1 - G_1)\left(\frac{1-\Delta\mathcal{L}(1-F^*)}{1-\Delta\mathcal{L}}\right)\right], \tag{14}$$

and the corresponding equivalent $a_2(f)$ can be calculated using equation (2b).

Allocation of resources to emissions abatement reduces current consumption. If y_1 is mean per-capita income in the absence of allocation to emissions abatement, then mean per-capita income after that allocation, $y_2(f)$, is:

$$y_2(f) = y_1 - f\,\Delta c\ . \tag{15}$$

Solow-Swann growth model with a simple representation of climate damage

We represent the economy using a Solow-Swann growth model (ref) with a Cobb-Douglas production function in their dimensional forms.

Gross production in the absence of climate damage, $Y_{gross}(t)$, is represented as:

$$Y_{gross}(t) = A(t)\cdot K(t)^{\alpha}\cdot L(t)^{1-\alpha}, \tag{18}$$

where K(t) is capital stock at time *t*, L(t) is the size of the labor force at time *t*, A(t) is total factor productivity at time *t*, and α is the output elasticity of capital (which in an efficient market is the capital share of income). A(t) and L(t) are prescribed exogenously. We make no distinction between general population and the size of the labor force.

A fraction, $\Omega(t)$, of gross production is assumed to be lost to climate damage. Production net of climate damage, $Y_{damaged}(t)$, is then:

$$Y_{damaged}(t) = (1 - \Omega(t))\cdot Y_{gross}(t)\ . \tag{19}$$

A fraction, $\Lambda(t)$, maybe allocated to emissions abatement. Production net of climate damage, $Y_{net}(t)$, is then:

$$Y_{net}(t) = (1 - \Lambda(t)) \cdot Y_{damaged}(t) \ . \quad (20)$$

The fraction, $\Omega(t)$, of $Y_{gross}(t)$ lost to climate damage is assumed to be an increasing function of the increase in global mean temperature at time t, $\Delta T(t)$:

$$\Omega(t) = k_{damage} \Delta T(t)^{\beta} \ . \quad (21)$$

In the absence of carbon dioxide emissions abatement, carbon dioxide emissions, $E_{base}(t)$ are assumed to be proportional to $\sigma(t)$ times gross productivity:

$$E_{base}(t) = \sigma(t) \ \cdot Y_{gross}(t) \ . \quad (22)$$

The carbon intensity of GDP in the absence of abatement, $\sigma(t)$, in units of tons of $CO2$ per dollar GDP, is prescribed exogenously.

Carbon dioxide emissions net of abatement, E(t), is represented:

$$E(t) = \sigma(t) \ \cdot (1 - \mu(t)) \cdot Y_{gross}(t) \ , \quad (23)$$

where $\mu(t)$ represents the share of emissions abated.

The fraction of gross production allocated to abatement, $\Lambda(t)$, is assumed to follow a power-law relationship with the fraction of emissions abated:

$$\Lambda(t) = \theta_1(t) \cdot \mu(t)^{\theta_2} , \quad (24)$$

where $\theta_1$and θ_2 are exogenously prescribed coefficients that reflect the cost of emissions abatement.

The global-mean temperature increase, $\Delta T(t)$, is assumed to be proportional to cumulative emissions:

$$\Delta T(t) = k_{climate} \int_0^t E\left(t_0\right) dt_0 \quad , \quad (25)$$

where k is a constant of proportionality.

Capital stock increases by investing a share, s, of production net climate damage and decreases due depreciation losses at rate δ:

$$\frac{dK}{dt}(t) = s\, Y_{net}(t) - \delta\, K(t) \quad . \tag{26}$$

Average per capita consumption is the part of net production that is not invested in capital divided by the population:

$$c(t) = (1 - s)\, Y_{net}(t)/L(t) \quad . \tag{27}$$

Computing fraction of emissions abated based on allocation of resources to either income redistribution of abatement

The central scenario of this optimization is the case where at each time step, some fraction, f_Y, of gross output is allocated to either redistribution of income or emissions abatement. The control variable in this optimization is *f*, the fraction of resources that could be allocated to income distribution that will instead be allocated to abatement. Thus, the fraction of emissions abated, μ, as a function of *f* can be described by:

$$\mu(t) = \left(\frac{f \cdot \Delta c \cdot L(t)}{\theta_1(t) \cdot L(t)}\right)^{1/\theta_2} \quad , \tag{28}$$

Table 1. Glossary of symbols used

Symbol	Meaning	Units	Source
ϱ	Pure rate of time preference	yr^{-1}	specified
t	Time	yr	calculated
F	Fraction of population with an income lower than some specified amount	–	calculated
$\mathcal{L}(F)$	Lorenz function describing share of income to population to the fraction of population described by F	–	calculated
a	Shape parameter of the Pareto distribution underlying the Lorenz function (lower values imply more inequality, a ≥ 0)	–	specified
G	Gini index (higher values imply more inequality; 0 ≤ G≤ 1)	–	calculated
c	Mean per-capita income	$\$ yr^{-1} person^{-1}$	calculated
c(F)	Per-capita income at rank F	$\$ yr^{-1} person^{-1}$	calculated
η	Coefficient or relative risk aversion	–	specified
u(c)	Utility at income level c	utils $person^{-1}$	calculated
U	Mean utility of a population	utils $person^{-1}$	specified
F*	Population rank that experiences no income change due to a change in a	–	calculated
ΔL	Mean fraction of income redistributed due to a change in a	–	calculated
Δc	Mean amount of income redistributed due to a change in a	$\$ yr^{-1} person^{-1}$	calculated
f	Fraction of ΔL that is allocated to emissions abatement rather than redistribution	–	control variable
G(f)	New Gini coefficient due after allocation to abatement of fraction f or resources that could have gone to redistribution of income, assuming that the income distribution remains a Pareto-Lorenz distribution	–	calculated
μ(f)	New mean per-capita income after allocation to abatement of fraction f or resources that could have gone to redistribution of income	$\$ yr^{-1} person^{-1}$	calculated
Y_{gross}	Gross production in the absence of climate change or abatement cost	$\$ yr^{-1}$	calculated
A(t)	Total factor productivity	$\$^{1-\alpha} yr^{-1} people^{\alpha-1}$	specified
K(t)	Capital stock	\$	calculated
L(t)	Labor pool, population	people	specified

α	Output elasticity of capital	–	specified
$\Lambda(t)$	Fraction of gross output allocated to emissions abatement	–	calculated
$\Omega(t)$	Fraction of gross output lost to climate damage	–	calculated
$Y_{net}(t)$	Output net of emissions abatement cost and climate damage	\$ yr^{-1}	calculated
k_{damage}	Coefficient relating temperature change to fractional loss of output	$°C^{-\beta}$	specified
β	Exponent relating temperature change to fractional loss of output	–	specified
$\sigma(t)$	Carbon intensity of GDP	tCO_2 \$$^{-1}$	specified
$E_{base}(t)$	Carbon dioxide missions in the absence of emissions abatement	tCO_2	calculated
$\mu(t)$	Fraction of potential emissions that are abated	–	calculated
E(t)	Carbon dioxide emissions net of carbon emissions abatement	tCO_2	calculated
$\Lambda(t)$	Fraction of gross output allocated to emissions abatement	\$ yr^{-1}	calculated
$\theta_1(t)$	Coefficient relating fractional loss of output to fraction of emissions abatement	–	specified
θ_2	Exponent relating loss of output to fraction of emissions abatement	–	specified
$k_{climate}$	Change in global mean temperature per cumulative ton CO_2 emitted	$°C\ tCO_2^{-1}$	specified
s	Savings rate	–	specified
δ	Depreciation rate	yr^{-1}	specified

f_Y	Fraction of gross output that is allocated to either income redistribution or abatement	–	calculated