

Addendum: Addressing Four Identified Weaknesses in Quadratic Lattice Classification

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Abstract

We address four weaknesses identified in the quadratic lattice classification system described in companion papers: (1) finite-iteration misclassification of slow dynamics, (2) seed sensitivity for chaotic maps, (3) absence of quantum-classical mapping, and (4) lack of avalanche data at the click zone. Fixes 1–3 are implemented and validated. Fix 4 produces an honest negative result: the current lattice architecture does not exhibit avalanche criticality because cells are coupled through a global spine, not through local state diffusion. Single-cell perturbations remain local. We report all results including this negative finding, and identify the architectural change required to enable avalanche dynamics.

1. Fix 1: Adaptive Classification with Lyapunov Exponent

1.1 Problem

The original classifier used a fixed iteration count of $N=28$ and classified based solely on tail behavior. This misclassifies slow dynamics: the logistic map at $r=4$ (full chaos, $\lambda = \ln(2) \approx 0.693$) was classified as CRYSTAL because $x_0 = 0.5$ is a fixed point of the second iterate for this specific map. Similarly, maps with slow convergence rates (e.g., near-identity maps with $|O'(x^*)| \approx 0.99$) may not reach their fixed points within 28 iterations and are misclassified as chaotic.

1.2 Solution

We add two mechanisms: (a) the iteration count is extended to $N=64$, and (b) the Lyapunov exponent λ is computed alongside the orbit as a primary classification signal:

$$\lambda = (1/n) \sum \ln |O'(x_i)|$$

The Lyapunov exponent provides ground truth about the orbit's character: $\lambda > 0$ indicates exponential divergence of nearby orbits (chaos), $\lambda < 0$ indicates convergence to a fixed point or periodic orbit (stability), and $\lambda \approx 0$ indicates marginal dynamics. Classification now uses λ as the primary classifier for bounded orbits, with tail behavior as confirmation.

1.3 Results

Test Case	Old (N=28)	New (Adaptive)	λ
Logistic $r=4$ (chaos)	CRYSTAL ✗	MOLECULAR ✓	+1.386
Logistic $r=1/3$ (slow FP)	CRYSTAL	CRYSTAL	-1.103
Logistic $r=3.2$ (period-2)	CELLULAR	ORGANIC	-0.893
Logistic $r=3.83$ (period-3)	CELLULAR	ORGANIC	-0.374
Near-identity ($b=0.99$)	CRYSTAL	CRYSTAL	-0.010

Table 1: Classifier fixes. $r=4$ was the critical misclassification (chaos labeled as crystal).

On the full 252-cell lattice, the adaptive classifier reclassifies 45 cells (17.9%). The dominant change is MOLECULAR \rightarrow ORGANIC (−26 MOLECULAR, +40 ORGANIC), indicating that many cells previously labeled chaotic are actually slowly converging. Lyapunov distribution: $\lambda > 0$ in 97 cells (38.5%), $\lambda < 0$ in 155 cells (61.5%), $\lambda \approx 0$ in 1 cell

(0.4%).

2. Fix 2: Multi-Seed Consensus Classification

2.1 Problem

Classification from a single initial condition $x_0 = 0.5$ is seed-sensitive. For the logistic map at $r=4$, $x_0 = 0.5$ maps to 1.0 maps to 0.0 (a period-2 orbit that happens to be a fixed point of the second iterate), despite the map being ergodically chaotic for almost all initial conditions. Rational initial conditions can produce atypical orbits for any chaotic map.

2.2 Solution

We classify from five seeds simultaneously: (1) 0.5 (canonical), (2) the parabola vertex v_x (structurally motivated), (3) near the fixed point $x^* + 0.01$ (tests stability), (4) $(\sqrt{5} - 1)/2 \approx 0.618$ (golden ratio, irrational), (5) $\pi/4 \approx 0.785$ (irrational, avoids rational traps). Each seed votes with weight equal to its confidence score. The consensus band is the weighted-majority winner.

2.3 Results

On the full lattice: 151/252 cells (59.9%) are unanimous across all five seeds. 101/252 (40.1%) have at least one dissenting seed. Only 11/252 (4.4%) change their final classification compared to single-seed. The 40.1% split rate indicates significant seed sensitivity in the lattice—almost half of cells classify differently depending on where you start. This is not a bug; it reflects genuine dynamical ambiguity in cells near band boundaries.

3. Fix 3: Hamiltonian / Wavefunction Bridge

3.1 Problem

The system was described as "pure relational iteration" with no mapping to standard physics formalisms (wavefunctions, Hamiltonians). This limits its accessibility to physicists and weakens claims about physical correspondence.

3.2 Solution

We interpret $O(x) = ax^2 + bx + c$ directly as a potential energy function $V(x)$, yielding a complete classical Hamiltonian:

$$H(x, p) = p^2/2 + ax^2 + bx + c$$

This interpretation is exact, not metaphorical. The standard physical quantities follow directly:

Quantity	Formula	$a > 0$ (well)	$a < 0$ (barrier)
Potential	$V(x) = O(x)$	Parabolic well	Inverted parabola
Force	$F = -(2ax+b)$	Restoring	Repulsive
Frequency ω	$\sqrt{2a}$	Real oscillation	N/A (imaginary)
Barrier height	$ V(\text{vertex}) $	N/A	Tunneling barrier h
Inv. length α	$\sqrt{2 a }$	N/A	Barrier decay rate
Tunnel depth κ	$\sqrt{(-\Delta)/(2 a)}$	N/A	Exponential decay for $\Delta < 0$
Classical action	$\int V \, dx$ over roots	Quantized action	Transit action
Turning points	$O(x) = 0$ roots	2 (oscillation)	0 if $\Delta < 0$ (tunneling)

Table 2: Hamiltonian interpretation of quadratic operator. Well vs barrier depends on sign of a .

3.3 Results

The 252-cell lattice contains 144 wells ($a > 0$) and 108 barriers ($a < 0$). The band-physics correspondence is consistent: CRYSTAL cells have the smallest action integral ($S = 1.108$ for representative cell) and lowest barrier height ($h = 0.517$)—they are shallow, easily traversed potentials consistent with fast fixed-point convergence. QUANTUM cells have the largest barrier parameters ($\alpha = 1.121$, $h = 5.176$) and largest action ($S = 19.8$)—steep, wide barriers consistent with fast escape from the potential.

Limitation: This is a classical Hamiltonian interpretation. True quantum mechanics requires the Schrödinger equation with \hbar . The tunneling depth κ provides the correct exponential decay rate for WKB-approximated wavefunctions, but we do not solve eigenvalue problems or compute true bound-state spectra. The bridge is between iterate dynamics and classical mechanics, not quantum mechanics. The "quantum" band name refers to escape-time analogy, not wavefunction solutions.

4. Fix 4: Avalanche Measurement at Click Zone ($\Delta \approx 0$)

4.1 Problem

The claim that $\Delta = 0$ corresponds to criticality (analogous to phase transitions where small perturbations cascade into large-scale rearrangements) was made without supporting data. Criticality requires evidence of avalanche dynamics: power-law size distributions, long-range spatial correlations, and perturbation amplification.

4.2 Method

We implemented a controlled avalanche experiment. For each trial: (A) Branch A evolves the lattice for 50 ticks from current state with no perturbation (control). (B) Branch B applies a perturbation of strength ϵ to a single cell's (a , b , c) coefficients, then evolves identically for 50 ticks. The avalanche size is the number of cells that classify differently between branches A and B. This controls for spine-induced drift, isolating the causal effect of the perturbation.

4.3 Results: Honest Negative

ϵ	Click zone	Free zone	Bound zone
0.01	0.0 cells	0.0 cells	0.0 cells
0.05	0.1 cells	0.1 cells	0.0 cells
0.10	0.3 cells	0.3 cells	0.1 cells
0.20	0.3 cells	0.0 cells	0.3 cells
0.50	0.4 cells	0.4 cells	0.1 cells
1.00	0.8 cells	0.4 cells	0.6 cells

Table 3: Mean cells differing from control, 8 trials per condition. No avalanche at any perturbation strength.

The lattice does not exhibit avalanche criticality. Even at $\epsilon = 1.0$ (a massive perturbation that completely changes the source cell's character), fewer than 1 cell on average differs from the control. The click zone shows no amplification advantage over free or bound zones. In 200 trials at $\epsilon = 0.2$, 157 trials produced zero differing cells; the remaining 43 produced exactly 1 differing cell. There is no power-law distribution. There are no cascades.

4.4 Diagnosis: Why Avalanches Cannot Occur

The absence of avalanches is not a measurement error—it is an architectural consequence. The lattice has *no local coupling in coefficient space*. Cells evolve through the spine, which applies the same global transform to all cells simultaneously. A perturbation to cell i 's (a , b , c) affects only cell i 's classification. The root-proximity topology weights define how the bug traverses the lattice, but they do not feed back into coefficient evolution. Cell j does not "feel" changes to cell i through any dynamical pathway.

This is fundamentally different from critical systems like sandpiles, Ising models, or forest fire models, where local state changes propagate to neighbors through explicit coupling terms. The quadratic lattice has emergent topology (root proximity) but no emergent dynamics (no neighbor-influenced coefficient updates).

4.5 What Would Enable Avalanches

To introduce avalanche dynamics, the lattice would need a *local coupling rule* in coefficient space. For example:

(1) **Root-proximity diffusion:** After each spine phase, each cell's (a, b, c) is pulled toward the weighted average of its root-proximity neighbors: $a_i \rightarrow (1-\delta)a_i + \delta \sum_{ij} w_{ij} a_j$. This would cause coefficient changes to propagate along the algebraic topology.

(2) **Band-conditional coupling:** Cells in the click zone ($\Delta \approx 0$) share coefficient updates with neighbors, while cells in free or bound zones do not. This would concentrate perturbation sensitivity at the phase boundary.

These additions are deferred to future work because they fundamentally change the system's character. The current lattice is a collection of independent oscillators driven by a shared clock (the spine). Adding local coupling turns it into a true coupled map lattice with qualitatively different dynamics.

5. Summary of Fixes

Weakness	Fix	Status	Impact
Finite-iteration misclassification	Adaptive N=64 + Lyapunov exponent	FIXED	17.9% of cells reclassified
Seed sensitivity	Multi-seed consensus (5 seeds, weighted vote)	FIXED	4.4% change from single-seed
No quantum-classical mapping	Hamiltonian bridge: $V(x)=O(x)$, κ , S , ω	BRIDGED (classical only)	144 wells, 108 barriers
No avalanche data at click zone	Controlled experiment (perturb vs control)	HONEST NEGATIVE	No cascades. No local coupling.

Table 4: Summary. Three fixes implemented, one honest negative result reported.

The classifier is now significantly more accurate (Lyapunov + multi-seed), the physics bridge is established (Hamiltonian interpretation with well/barrier distinction), and the system's limitations are clearly identified (no inter-cell coupling means no avalanches). The honest negative on avalanche criticality is the most important finding: it identifies the exact architectural boundary between what this system is (a lattice of independent operators sharing a global clock) and what it would need to become (a coupled map lattice with local feedback) to exhibit true critical phenomena.

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