CS559/659 HOMEWORK #1

Problem 1.
Problem 2.7. from Bishop.

Problem 2.
Problem 2.12. from Bishop.

Problem 3. Poisson distribution

The Poisson distribution is used to model the number of random arrivals to a system over a fixed period of time. Examples of systems in which events are determined by random arrivals are: arrivals of customers requesting the service, occurrence of natural disasters, such as floods, etc. The Poisson distribution is defined as:

$$p(x \mid \lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!} \text{ for } x = 0,1,2...$$

Answer the following questions:

- (a) Using the definition of the Poisson distribution show that the sum of probabilities of all events is 1. (Hint: use the definition of e^{λ} in terms of a sum).

- (b) Derive the mean of the Poisson distribution.

- (c) Assume we have n independent samples of x. What is the ML estimate of the parameter λ.

- (d) The conjugate prior for the Poisson distribution is Gamma distribution. It is defined as:

$$p(\lambda \mid a,b) = \frac{1}{b^{a}\Gamma(a)}\lambda^{a-1}e^{-\frac{\lambda}{b}}$$

Show that the posterior density of the parameter λ is again a Gamma distribution.

Now we are ready to do some Matlab experiments:

- (e) plot the probability function for Poisson distributions with parameters λ = 2 and λ = 6. Note that the Poisson model is defined over nonnegative integers only.

- (f) Assume the data in 'poisson.txt' that represent the number of incoming phone calls received over a fixed period of time. Compute and report the ML estimate of the parameter λ.

• (g) Assume the prior on λ is given by Gamma(a, b). Plot the Gamma distribution for the following set of parameters (a = 1, b = 2) and (a = 3, b = 5).

• (h) Plot the posterior density for λ after observing samples in 'poission.txt' and using priors in part (g). What changes in the distribution do you observe?

Problem 4. Exponential family.

Assume the Poisson distribution:

$$p(x \mid \lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!} \text{ for } x = 0,1,2...$$

Show that it belongs to the exponential family by defining: its natural parameters, the sufficient statistic and the partition function.

Note: it is not neccessary to submit any Matlab code for this assignment, just include the plots in your report.