

Database Systems, CSCI 4380-01
 Homework # 1 Practice Problem for Question 3 Version 1.02

Question 3 [30 points]. For the following relations, find and list all the keys.

1. $R1(A, B, C, D, E, F, G)$, $\mathcal{F} = \{AB \rightarrow CD, DEF \rightarrow ACG, G \rightarrow BD, D \rightarrow E\}$

Solution.

1. Relation R has seven attributes. The total number of subsets of attributes would be $2^7 - 1 = 127$ (excluding the empty set). In principle, we have to consider each from these subsets but hopefully we can take some shortcuts, so that we don't have to test every single one of them. Let's start with the full list of subsets though:

A	AB	ABC	ABCD	ABCDE	ABCDEF	ABCDEFG
B	AC	ABD	ABCE	ABCDF	ABCDEG	
C	AD	ABE	ABCF	ABCDG	ABCDFG	
D	AE	ABF	ABCG	ABCEF	ABCEFG	
E	AF	ABG	ABDE	ABCEG	ABDEFG	
F	AG	ACD	ABDF	ABCFG	ACDEFG	
G	BC	ACE	ABDG	ABDEF	BCDEFG	
	BD	ACF	ABEF	ABDEG		
	BE	ACG	ABEG	ABDFG		
	BF	ADE	ABFG	ABEFG		
	BG	ADF	ACDE	ACDEF		
	CD	ADG	ACDF	ACDEG		
	CE	AEF	ACDG	ACDFG		
	CF	AEG	ACEF	ACEFG		
	CG	AFG	ACEG	ADEFG		
	DE	BCD	ACFG	BCDEF		
	DF	BCE	ADEF	BCDEG		
	DG	BCF	ADEG	BCDFG		
	EF	BCG	ADFG	BCEFG		
	EG	BDE	AEFG	BDEFG		
	FG	BDF	BCDE	CDEFG		
	BDG	BCDF				
	BEF	BCDG				
	BEG	BCEF				
	BFG	BCEG				
	CDE	BCFG				
	CDF	BDEF				
	CDG	BDEG				
	CEF	BDFG				
	CEG	BEFG				
	CFG	CDEF				
	DEF	CDEG				
	DEG	CDFG				
	DFG	CEFG				
	EFG	DEFG				

Now, looking at the functional dependencies, we see that none of them have attribute F on the right-hand side. It means that this attribute will be included in every key. Therefore, attribute sets without this attribute cannot be keys and can be crossed out:

A	AB	ABC	ABCD	ABCDE	ABCDEF	ABCDEFG
B	AC	ABD	ABCE	ABCDF	ABCDEG	
C	AD	ABE	ABCF	ABCDG	ABCDFG	
D	AE	ABF	ABC	ABCEF	ABCEFG	
E	AF	ABG	ABDE	ABCEG	ABDEFG	
F	AG	ACD	ABDF	ABCFG	ACDEFG	
G	BC	ACE	ABDG	ABDEF	BCDEFG	
	BD	ACF	ABEF	ABDEG		
	BE	ACG	ABEG	ABDFG		
	BF	ADE	ABFG	ABEFG		
	BG	ADF	ACDE	ACDEF		
	CD	ADG	ACDF	ACDEG		
	CE	AEF	ACD	ACDFG		
	CF	AEG	ACEF	ACEFG		
	CG	AFG	ACEG	ADEFG		
	DE	BCD	ACFG	BCDEF		
	DF	BCE	ADEF	BCDEG		
	DG	BCF	ADEG	BCDFG		
	EF	BCG	ADFG	BCEFG		
	EG	BDE	AEFG	BDEFG		
	FG	BDF	BCDE	CDEFG		
	BG	BCDF				
	BEF	BCDG				
	BEG	BCEF				
	BFG	BCEG				
	CDE	BCFG				
	CDF	BDEF				
	CDG	BDEG				
	CEF	BDFG				
	CEG	BEFG				
	CFG	CDEF				
	DEF	CDEG				
	DEG	CDFG				
	DFG	CEFG				
	EFG	DEFG				

Now, we only have 64 sets to test. Much easier than 127! Then let's pick a set and check if it is a key. For example, we consider $ACEF$: $\{A, C, E, F\}^+ = \{A, C, E, F\}$

It is not a key, so we can cross it out. More importantly, we can make an observation that any set of attributes that does not contain all the attributes from at least one of the following sets $\{A, B\}$, $\{D, E, F\}$, $\{G\}$, and $\{D\}$ would not be a key. It is due to the fact that we would not be able to apply any of the functional dependencies to add more attributes to the closure of such a set of attributes.

A	AB	ABC	ABCD	ABCDE	ABCDEF	ABCDEFG
B	AC	ABD	ABCE	ABCDF	ABCDEF	
C	AD	ABE	ABC F	ABCD G	ABCDF G	
D	AE	ABF	ABC G	ABCEF	ABCEFG	
E	AF	ABG	ABDE	ABCE G	ABDEFG	
F	AG	ACD	ABDF	ABC FG	ACDEF G	
G	BC	ACE	ABDG	ABDEF	BCDEF G	
	BD	ACF	ABEF	ABDE G		
	BE	ACG	ABEG	ABDF G		
	BF	ADE	ABFG	ABEFG		
	BG	ADF	ACDE	ACDEF		
	CD	ADG	ACDF	ACDE G		
	CE	AEF	ACDG	ACDF G		
	CF	AEG	ACE F	ACEFG		
	CG	AFG	ACE G	ADEFG		
	DE	BCD	ACFG	BCDEF		
	DF	BCE	ADEF	BCDE G		
	DG	BCF	ADE G	BCDF G		
	EF	BCG	ADFG	BCEFG		
	EG	BDE	AEFG	BDEFG		
	FG	BDF	BCDE	CDEFG		
		BDG	BCDF			
		BEF	BCDG			
		BEG	BCEF			
		BFG	BCEG			
		CDE	BCFG			
		CDF	BDEF			
		CDG	BDE G			
		CEF	BDFG			
		CEG	BEFG			
		CFG	CDEF			
		DEF	CDE G			
		DEG	CDFG			
		DFG	CEFG			
		EFG	DEFG			

We are down to just 52 sets to test. Let's pick a set $ABCDF$:

$$\{A, B, C, D, F\}^+ = \{A, B, C, D, F\} \cup \{E\} \text{ (after using } D \rightarrow E) \cup$$

$\{A, C, G\}$ (after using $DEF \rightarrow ACG$) = $\{A, B, C, D, E, F, G\}$. So, we found at least a superkey. But we don't know whether it is minimal or not. Let's list all of the subsets of $\{A, B, C, D, F\}$ that have not been crossed out:

- $\{D, F\}$
- $\{A, B, F\}$
- $\{A, D, F\}$
- $\{B, D, F\}$
- $\{C, D, F\}$
- $\{A, B, C, F\}$
- $\{A, B, D, F\}$
- $\{A, C, D, F\}$
- $\{B, C, D, F\}$

- $\{A, B, C, D, F\}$

Let's compute the closure of $\{A, B, F\}$:

$\{A, B, F\}^+ = \{A, B, F\} \cup \{C, D\}$ (after using $AB \rightarrow CD$) \cup
 $\{E\}$ (after using $D \rightarrow E$) $\cup \{A, C, G\}$ (after using $DEF \rightarrow ACG$) $= \{A, B, C, D, E, F, G\}$.
We conclude that $\{A, B, C, D, F\}$ was not a key since it was not minimal. $\{A, B, F\}$ is a superkey. Is it a key? Yes, because all proper subsets of $\{A, B, F\}$ would contain at most two elements and would also be subsets of $\{A, B, C, D, F\}$. However, there are no single-element subsets of $\{A, B, C, D, F\}$ listed above. The only two-element subset of $\{A, B, C, D, F\}$ listed above is $\{D, F\}$ which is not a subset of $\{A, B, F\}$. It means that $\{A, B, F\}$ is a key. Since we found a key, let's mark it and cross out all of its superkeys:

A	AB	ABC	ABCD	ABCDE	ABCDEF	ABCDEFG
B	AC	ABD	ABCE	ABCDF	ABCDEG	
C	AD	ABE	ABCF	ABCDG	ABCDFG	
D	AE	ABF	ACBG	ABCEF	ABCEFG	
E	AF	ABG	ABDE	ABCEG	ABDEFG	
F	AG	ACD	ABDF	ABCFG	ACDEFG	
G	BC	ACE	ABDG	ABDEF	BCDEFG	
	BD	ACF	ABEF	ABDEG		
	BE	ACG	ABEG	ABDFG		
	BF	ADE	ABFG	ABEFG		
	BG	ADF	ACDE	ACDEF		
	CD	ADG	ACDF	ACDEG		
	CE	AEF	ACDG	ACDFG		
	CF	AEG	ACEF	ACEFG		
	CG	AFG	ACEG	ADEFG		
	DE	BCD	ACFG	BCDEF		
	DF	BCE	ADEF	BCDEG		
	DG	BCF	ADEG	BCDFG		
	EF	BCG	ADFG	BCEFG		
	EG	BDE	AEFG	BDEFG		
	FG	BDF	BCDE	CDEFG		
	BDG	BCDF				
	BFF	BCDG				
	BEG	BCEF				
	BFG	BCEG				
	CDE	BCFG				
	CDF	BDEF				
	CDG	BDEG				
	CEF	BDFG				
	CEG	BEFG				
	CFG	CDEF				
	DEF	CDEG				
	DEG	CDFG				
	DFG	CEFG				
	EFG	DEFG				

We have 36 sets left to test. Let's pick a set AFG :

$\{A, F, G\}^+ = \{A, F, G\} \cup \{B, D\}$ (after using $G \rightarrow BD$) \cup
 $\{E\}$ (after using $D \rightarrow E$) $\cup \{A, C, G\}$ (after using $DEF \rightarrow ACG$) $= \{A, B, C, D, E, F, G\}$.
So, we found at least a superkey. But we don't know whether it is minimal or not. Let's list all of the subsets of $\{A, F, G\}$ that have not been crossed out:

- $\{F, G\}$
- $\{A, F, G\}$

Let's compute the closure of $\{F, G\}$:

$\{F, G\}^+ = \{F, G\} \cup \{B, D\}$ (after using $G \rightarrow BD$) \cup
 $\{E\}$ (after using $D \rightarrow E$) \cup $\{A, C, G\}$ (after using $DEF \rightarrow ACG$) $= \{A, B, C, D, E, F, G\}$.
We conclude that $\{A, F, G\}$ was not a key since it was not minimal. $\{F, G\}$ is a key since its closure contains all attributes, and none of the proper subsets of $\{F, G\}$ remains uncrossed in our list. Since we found a key, let's mark it and cross out all of its superkeys:

A	AB	ABC	ABCD	ABCDE	ABCDEF	ABCDEF	ABCDEF
B	AC	ABD	ABCE	ABCDF	ABCDEF	ABCDEF	ABCDEF
C	AD	ABE	ABCF	ABCDG	ABCDFG	ABCDFG	ABCDFG
D	AE	ABF	ABCG	ABCEF	ABCEFG	ABCEFG	ABCEFG
E	AF	ABG	ABDE	ABCEG	ABDEFG	ABDEFG	ABDEFG
F	AG	ACD	ABDF	ABCFG	ACDEFG	ACDEFG	ACDEFG
G	BC	ACE	ABDG	ABDEF	BCDEFG	BCDEFG	BCDEFG
	BD	ACF	ABEF	ABDEF	ABDEG	ABDEG	ABDEG
	BE	ACG	ABEG	ABEG	ABDFG	ABDFG	ABDFG
	BF	ADE	ABFG	ABFG	ABEFG	ABEFG	ABEFG
	BG	ADF	ACDE	ACDE	ACDEF	ACDEF	ACDEF
	CD	ADG	ACDF	ACDF	ACDEG	ACDEG	ACDEG
	CE	AEF	ACDG	ACDG	ACDFG	ACDFG	ACDFG
	CF	AEG	ACEF	ACEF	ACEFG	ACEFG	ACEFG
	CG	AFG	ACEG	ACEG	ADEFG	ADEFG	ADEFG
	DE	BCD	ACFG	ACFG	BCDEF	BCDEF	BCDEF
	DF	BCE	ADEF	ADEF	BCDEG	BCDEG	BCDEG
	DG	BCF	ADEG	ADEG	BCDFG	BCDFG	BCDFG
	EF	BCG	ADFG	ADFG	BCEFG	BCEFG	BCEFG
	EG	BDE	AEGF	AEGF	BDEFG	BDEFG	BDEFG
	FG	BDF	BCDE	BCDE	CDEFG	CDEFG	CDEFG
		BDG	BCDF	BCDF			
		BEF	BCDG	BCDG			
		BEG	BCEF	BCEF			
		BFG	BCEG	BCEG			
		CDE	BCFG	BCFG			
		CDF	BDEF	BDEF			
		CDG	BDEG	BDEG			
		CEF	BDFG	BDFG			
		CEG	BEFG	BEFG			
		CFG	CDEF	CDEF			
		DEF	CDEG	CDEG			
		DEG	CDFG	CDFG			
		DPG	CEFG	CEFG			
		EFG	DEFG	DEFG			

With that, we are down to only 12 sets to check. Note that all remaining sets are supersets of $\{D, F\}$. If we feel lucky, we can try to test $\{D, F\}$ and see if it is a key.

Let's compute the closure of $\{D, F\}$:

$\{D, F\}^+ = \{D, F\} \cup \{E\}$ (after using $D \rightarrow E$) \cup
 $\{A, C, G\}$ (after using $DEF \rightarrow ACG$) $\cup \{B, D\}$ (after using $G \rightarrow BD$) $= \{A, B, C, D, E, F, G\}$.
So, $\{D, F\}$ is a superkey. Since none of the proper subsets of $\{D, F\}$ remains uncrossed in our list, it is a key. Since we found a key, let's mark it and cross out all of its superkeys:

A	AB	ABC	ABCD	ABCDE	ABCDEF	ABCDEF	ABCDEFG
B	AC	ABD	ABCE	ABCDF	ABCDEF	ABCDEF	ABCDEFG
C	AD	ABE	ABCF	ABCDG	ABCDEF	ABCDEF	ABCDEFG
D	AE	ABF	ABCG	ABCEF	ABCDEF	ABCDEF	ABCDEFG
E	AF	ABG	ABDE	ABCEG	ABDEF	ABDEF	ABCDEFG
F	AG	ACD	ABDF	ABCFG	ACDEF	ACDEF	ACDEFG
G	BC	ACE	ABDG	ABDEF	BCDEF	BCDEF	BCDEFG
	BD	ACF	ABEF	ABDEG			
	BE	ACG	ABEG	ABDFG			
	BF	ADE	ABFG	ABEFG			
	BG	ADF	ACDE	ACDEF			
	CD	ADG	ACDF	ACDEG			
	CE	AEF	ACDG	ACDFG			
	CF	AEG	ACEF	ACEFG			
	CG	AFG	ACEG	ADEFG			
	DE	BCD	ACFG	BCDEF			
	DF	BCE	ADEF	BCDEG			
	DG	BCF	ADEG	BCDFG			
	EF	BCG	ADFG	BCEFG			
	EG	BDE	AEFG	BDEFG			
	FG	BDF	BCDE	CDEFG			
		BDG	BCDF				
		BEF	BCDG				
		BEG	BCEF				
		BFG	BCEG				
		CDE	BCFG				
		CDF	BDEF				
		CDG	BDEG				
		CEF	BDFG				
		CEG	BEEF				
		CFG	CDEF				
		DEF	CDEG				
		DEG	CDFG				
		DFG	CEFG				
		EFG	DEFG				

There are no subsets of arguments that are not keys and that remain uncrossed in our list. It means that we found all keys of $R1$.

Answer: Keys of $R1$ are DF , FG , and ABF .